Closed-loop flow control using the resolvent operator about the mean flow

Colin Leclercq\(^1\), Charles Poussot-Vassal\(^2\), Denis Sipp\(^1\), Eric Garnier\(^1\)

\(^1\)DAAA - Aérodynamique, aéroélasticité, acoustique, Meudon
\(^2\)DTIS - Traitement de l'Information et Systèmes, Toulouse

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1. Introduction: receptivity to forcing in nonlinear flows

2. Feedback control of 2D cavity flow at Re=7500

3. Iterative procedure for nonlinear flows

4. Linear identification versus resolvent-based transfer function

5. Conclusions and outlook
Introduction: receptivity to "internal" forcing in nonlinear flows

Input-output reformulation of the Navier-Stokes equations (McKeon & Sharma, 2010):

1) Reynolds decomposition: \( u = \overline{u} + u' \)
2) Fourier transform

\[
\Rightarrow \hat{u}' = (i\omega \mathbb{I} - \mathcal{L}_\overline{u})^{-1} \hat{f}
\]

with "internal" forcing term (nonlinearities): \( f = \overline{u} \cdot \nabla u - u \cdot \nabla \overline{u} \)

The receptivity to "internal" forcing is well captured by a linear time-invariant (LTI) operator: the resolvent \( \mathcal{R}_\overline{u}(i\omega) \).

(Beneddine et al., 2016; Gómez et al., 2016; etc.)
Introduction: receptivity to external forcing in nonlinear flows

LES of D-shaped bluff body at $Re=10^4$:

The receptivity to external forcing may also be captured by a LTI input-output model (Dahan et al., 2012; Dalla Longa et al., 2017)

$\Rightarrow$ physics-based model using the resolvent operator?
Introduction: receptivity to forcing in nonlinear flows

"Internal" forcing: \( f = \mathbf{u} \cdot \nabla \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u} \)

External forcing: \( r(t) \mathcal{B}(\mathbf{x}) \)

Hypothesis for \( r \neq 0 \): frozen mean flow, frozen nonlinearity

A good assumption for flow control?
Feedback control using the resolvent operator

LTI-model:

\[ r \xrightarrow{G} y \]

resolvent-based
open-loop transfer function

Nonlinearity = output noise

\[ y = CR_u B r + CR_u f \]

Hypothesis for \( r \neq 0 \): frozen mean flow, frozen nonlinearity
Feedback control using the resolvent operator

Uncontrolled flow

\[ r = 0 \]

Output PSD:

\[ \Phi_{yy} = \Phi_{ww} \]

Define power spectral density (PSD)

\[
\Phi_{ab} = \lim_{T \to \infty} \frac{1}{2T} E[\hat{a}_T \hat{b}_T^*] \quad \text{with} \quad [\hat{a}_T, \hat{b}_T](\omega) = \int_{-T}^{T} [a(t), b(t)] e^{-i\omega t} dt
\]
Feedback control using the resolvent operator

Controlled flow

\[ r = 0 \]

\[ w \]

\[ y = S w \]

\[ G \]

\[ K \]

Sensitivity function:

\[ S \equiv (1 - GK)^{-1} \]

Output PSD:

\[ \Phi_{yy} = |S|^2 \Phi_{ww} \]

Nonlinearity rejection:

\[ |S(\omega)| < 1 \text{ at dominant frequencies in PSD spectrum of } w \]
Numerical model

Square 2D incompressible cavity at Re=7500
(Barbagallo, Sipp, Schmid, 2009)

FreeFem++
(P1b,P1b,P1)
193916 elements
681121 d.o.f.

initial condition: base flow + most unstable mode
Dynamics of unforced open cavity flow

Quasiperiodic dynamics: 2-torus

phase portrait

Poincaré section

output PSD

\begin{align*}
\Phi_{yy}(\omega) &= 10^{-7} \\
2\omega_1, \omega_0 - 2\omega_1, 2\omega_1, \omega_0 - \omega_1, \omega_0 + \omega_1, \omega_0 + \omega_1
\end{align*}
Open-loop transfer function

Mean flow
(streamwise velocity)

\[ G = C R \bar{u} B \]

Greatest receptivity to external forcing at \( \omega_0 \), then \( \omega_0 \pm \omega_1 \)
Robust controller design: $H_\infty$ structured synthesis

Stable controller $K_{\text{order 3 targeting } \omega_0, \omega_0 \pm \omega_1}$

Output

$\Phi_{yy} = |S|^2 \Phi_{ww}$ nonlinearity

$S = (1 - GK)^{-1}$ sensitivity function
Moderation of the Kelvin-Helmholtz instability
Results: dynamics of controlled flow

Closed-loop flow control using the resolvent operator about the mean flow

2-torus => limit cycle
Results: local and global measures

sensor
\[ y = C(u - u_B) \]

perturb. KE
\[ E = \|u - u_B\|^2 \]

| \( \|u - u_B\|^2 \) | \( \overline{y} \) | \( y_{\text{rms}} \) | \( |y|_{\text{max}} \) |
|----------------|--------|----------|----------|
| 0 \rightarrow 1 | -72\%  | -76\%    | -25\%    | -53\%    |

Can we do better?
Iterative procedure

Closed-loop flow control using the resolvent operator about the mean flow

\[ r = 0 \]

\[ G_0 \]

\[ K_1 \]

\[ w_0 \]

\[ y \]

**LTI model**

Frozen mean flow \( \bar{u} := \bar{u}_0 \), frozen nonlinearity \( f := f_0 \)

\[ \Rightarrow G_0 := \mathcal{C} \mathcal{R}_{\bar{u}_0} \mathcal{B} \quad \text{and} \quad w_0 := \mathcal{C} \mathcal{R}_{\bar{u}_0} f \]
Iterative procedure

New mean flow $\bar{u} \rightarrow \bar{u}_1$, new nonlinearity $f \rightarrow f_1$

$\Rightarrow G \rightarrow G_1 = CR\bar{u}_1 B$ and $w \rightarrow w_1 = CR\bar{u}_1 f_1$
Iterative procedure

Controller correction $K_2$ based on new frozen mean flow $\bar{u} := \bar{u}_1$, frozen nonlinearity $f := f_1$

$\Rightarrow \quad G_1 := C R_{\bar{u}_1} B \quad \text{and} \quad w_1 := C R_{\bar{u}_1} f$
Iterative procedure

Closed-loop flow control using the resolvent operator about the mean flow

\[ r = 0 \]

\[ w_2 \]

\[ y \]

\[ G_2 \]

\[ K_1 \]

\[ K_2 \]

LTI model

statistical equilibrium

New mean flow \( \bar{u} \rightarrow \bar{u}_2 \) , new nonlinearity \( f \rightarrow f_2 \)

\[ G \rightarrow G_2 = CR\bar{u}_2 B \quad \text{and} \quad w \rightarrow w_2 = CR\bar{u}_2 f \]

etc., …

Linear analog to Trust-Region POD ROM (Fahl, 2000)
Results: dynamics of controlled flow

Output PSD:
uncontrolled

\[ K_1 \]

\[ K_1 & K_2 \]

2-torus => limit cycle
Results: local and global measures

sensor
\[ y = C(u - u_B) \]

perturb. KE
\[ E = \|u - u_B\|^2 \]

| \( \|u - u_B\|^2 \) | \( \bar{y} \) | \( y_{\text{rms}} \) | \( |y|_{\text{max}} \) |
|---|---|---|---|
| 0 → 1 | -72% | -76% | -25% | -53% |
| 0 → 2 | -85% | -89% | -31% | -63% |

Yes we can do better!
Frozen mean flow/frozen nonlinearity model:

\[ r \neq 0 \]

\[
\begin{array}{c}
G \\
\downarrow \quad w \\
\downarrow \\
y = Gr + w
\end{array}
\]

with

\[ w = CR_uB \]

2 possible estimations of input/output relation with PSDs:

\[
H_1 \equiv \frac{\Phi_{yy}}{\Phi_{ry}} \quad \text{and} \quad H_2 \equiv \frac{\Phi_{yr}}{\Phi_{rr}}
\]

corresponding estimation error w.r.t. \( G \):

\[
H_1 - G = \Phi_{wy}/\Phi_{ry} \quad \text{and} \quad H_2 - G = \Phi_{wr}/\Phi_{rr}
\]
Linear identification vs resolvent-based transfer function

Harmonic forcing \( u = A \cos(\omega_f t) \)

\[
\begin{align*}
\omega_f &= 11 \\
\omega_f &= \omega_0 \\
\omega_f &= 13
\end{align*}
\]

Estimation error

nonlinearity \( w \) correlated with output \( y \)

\( \Rightarrow \) estimation error \( \Phi_{wy}/\Phi_{ry} \)

never negligible even if \( \omega_f \notin 2\text{-torus}. \)
Linear identification vs resolvent-based transfer function

Harmonic forcing  \[ u = A \cos(\omega_f t) \]

Estimation error

nonlinearity \( w \) correlated with input \( r \)

\[ H_2 - \left| \frac{G}{|G|} \right| \]

Frozen nonlinearity assumption

\[ \omega_f = 11 \]
\[ \omega_f = \omega_0 \]
\[ \omega_f = 13 \]
Conclusions

- Use of the **resolvent operator about the mean flow** to obtain LTI input/output model for feedback control + robust control methods, alternative to ROM/LQG, linear identification, etc.

- **Iterative method** for nonlinear control based on a sequence of controller corrections from **successive mean flows**

- Resolvent-based transfer function cannot be identified directly from I/O data because of **frozen nonlinearity assumption**
Efficiency at high Re?

Feedfoward strategy?

Ongoing ZDES computation of flow over a deep cavity at $Re=O(10^5)$

Feedforward control of the amplifier flow past a backward-facing step