Low-frequency dynamics of separated boundary-layer

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Motivations & Objectives

The dynamics of a separated boundary-layer is crucial in many engineering applications, such as airplane wings or turbomachinery blades.

Objectives: To analyse the space-time dynamics in non parallel separated flows with a global linear stability framework,

1. flat plate boundary-layer flow,
2. incompressible regime at low Reynolds number,
3. Separated zone is generated by normal velocity suction profile on upper side of the domain.

Movie from IAG/Stuttgart University, with courtesy of U. Rist.
Outline

1. Direct Numerical Simulation,
2. Global stability framework,
3. Base flow: configuration & computation,
4. Subcritical dynamics,
   a. Asymptotic dynamics,
   b. Transient dynamics,
   c. Nonlinear dynamics,
5. Supercritical dynamics,
   a. Asymptotic dynamics,
   b. Transient dynamics,
   c. Vortex-shedding phenomenon,
6. Conclusions
2D Incompressible Navier-Stokes equations

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u},
\]

\[\nabla \cdot \mathbf{u} = 0,\]

where \( \mathbf{u} = (u, v)^T \) is the velocity vector, \( p \) the pressure and \( Re = \frac{U_\infty \delta^*}{\nu} \)

- Fractional step on a staggered grid,
- Spatial discretization: centered second order for the linear terms, compact sixth order for the non-linear terms (Chu & Fan 1999),

Boundary conditions:
- inflow: Blasius profile,
- wall: viscous conditions,
- upper side: suction velocity profile,
- outflow: convective conditions + sponge zone.
General aspects:

- Instantaneous flow: $Q(x, y, z, t) = \overline{Q}(x, y) + \varepsilon q(x, y, z, t)$ with $\varepsilon \ll 1$
- Base flow: $\overline{Q}(x, y) = (U, 0, P)^t$, steady and two-dimensional.
- Small perturbation: $q(x, y, z, t) = (u, p)^t (x, y, t) e^{i\beta z}$.

Linearized perturbated evolution operator:

\[
\begin{aligned}
B \frac{\partial q}{\partial t} &= Aq + f e^{i\sigma t}, \\
q_0 \text{ the initial condition}
\end{aligned}
\]

- The operator $A$ and $B$ are deducted from the linearized Navier-Stokes equations,
- $f$ is a spatially localized forcing term,
- $\sigma$ is the forcing frequency.
Theoretical background: global stability approach (2)

\( f \equiv 0 \), homogeneous solution of:

\[
B \frac{\partial q}{\partial t} = Aq, \quad \text{and } q_0 \text{ the initial condition},
\]

The solution of the evolution equation is decomposed into a global modes basis where the normal and streamwise directions are taken as eigendirections:

**Decomposition of the perturbations in temporal modes basis**

\[
q(x, y, z, t) = \sum_{n=1}^{N} K_n \hat{q}_n(x, y) e^{i(\beta z - \Omega_n t)}
\]

where \( N \) is the number of modes, \( \hat{q}_n \) are the eigenvectors, \( \Omega_n \) are the complex eigenmodes and \( K_n \) is the initial amplitude of each mode.

Substituting in the N-S equations and a linearizing lead to the following large eigenvalues problem

\[
(A - \Omega B) \hat{q} = 0
\]

Which is discretized by Chebyshev spectral method, employing \( N = 850 \) modes on a \( 270 \times 50 \) grid, and it is solved with a shift and invert Arnoldi algorithm using the ARPACK library.
In supercritical regime

Separated boundary-layer flow is studied for $Re_{\delta^*} > Re_{\delta^*c} \simeq 220$

$\Downarrow$

the flow is globally unstable for two-dimensional perturbations

what are the physical mechanisms in this supercritical regime?
Base flows

General characteristics

a - The profile suction is.

b - The Reynolds number is \( \text{Re}_{\delta^*} = 225 \),

c - The base flow is obtained by Newton procedure,

d - When \( \text{Re}_{\delta^*} \) increase some topological changes occur: the presence of a secondary separation within the primary one.

Streamline velocity contours,
solid-line: separation streamline, dotted-line: \( U = 0 \).
Asymptotic global stability analysis, $\beta = 0$ - (1)

Global spectrum ($N_x = 250, N_y = 48$). Krylov subspace: 850, $L_x = 450$

- Eigenspectrum for flows BF1 (♦) at $Re_\delta = 225$,
- This separated flow is asymptotically globally unstable.

Seven slightly unstable modes,
- These modes are convective modes,
- Reminiscent of the KH–TS waves.
Asymptotic global stability analysis, $\beta \neq 0$

For BF-1: spectrum. $\beta = 0.08$

Isovorticity contours: $M_{GS}$ and $M_{Gort}$

Evolution of $M_{GS}$.

Strongly dependent of the bubble shape.
Theoretical background: transient growth analysis

- Transient growth is related to the non normality of the global operator. [1, 2].

- We compute the initial perturbation which maximize the quantity $G(t)$ for time $T$:

\[
G(t) = \max_{u_0} \frac{\|u(t)\|^2_E}{\|u_0\|^2_E} = sv_1 \left( \|F \exp(tD) F^{-1}\|_2 \right)^2
\]

with this inner product

\[
\langle u_i, u_j \rangle_E = \int_0^{Ly} \int_0^{Lx} (u_i^* u_j + v_i^* v_j) \, dx \, dy
\]

and with $D$, $D_{l,k} = -\delta_{lk} i\Omega_k$ and $M = F^* F$, $M_{i,j} = \langle u_i, u_j \rangle_E$

Precursor studies:

- C. Cossu and J.-M. Chomaz.
  Global measures of local convective instabilities.

- P.J. Schmid
  Nonmodal stability theory.
Transient global stability analysis, $\beta = 0$ - (1)

Optimal energy gain $G(t) = \max_{E_0} E(t)/E_0$ versus $\text{Re}_{\delta^*}$

- First peak and associated time: linear dependency with $\text{Re}_{\delta^*}$,
- Critical Reynolds number: $\text{Re}_c \approx 220$,
- Such a linear increase could be due to the linear increase of the size of the bubble with $\text{Re}_{\delta^*}$,
- At large times, modulations are recovered in the energy gain curves at all Reynolds numbers.
Transient global stability analysis, $\beta = 0$ - (2)

a - The initial energy is concentrated at the upstream part of the bubble,

b - The disturbance is convected downstream by the mean flow as a localized wave packet,

c - A second wave packet is generated due to the amplification of the disturbance carried back by the recirculation bubble,

d - A wave packet cycle is established asymptotically.

This result is very similar to Ehrenstein & Gallaire results (JFM, 2008).
Transient global stability analysis, $\beta = 0$ - (3)

Physical interpretation

a - The flow is not absolutely unstable (from local analysis) but it is nonparallel/global characteristics of the flow,

b - Self-sustained low-frequency: linear interaction between of most unstable modes of the spectrum.

$G(t)$

$\text{Re} = 225$

$\omega_{r_3} - \omega_{r_1} = 0.0075 \rightarrow T_I = 850$

$\omega_{r_2} - \omega_{r_3} = 0.02 \rightarrow T_{II} = 300$
Supercritical regime: nonlinear dynamics

Unstable modes

\[ \omega_1 \cdots \omega_n \]

\[ \Delta \omega_{ij} = |\text{Re}(\omega_i) - \text{Re}(\omega_j)| \]

Global Stability analysis:

- \( \text{a} - \text{Re}_{\delta^*} = 225 \),
- \( \text{b} - \) Two-dimensional perturbation: \( \beta = 0 \),
- \( \text{c} - \) Seven unstable modes.

Direct Numerical Simulation:

- \( \text{a} - \text{Re}_{\delta^*} = 230 \), \( A_0 = 10^{-6} \),
- \( \text{b} - \) Two dynamics are observed,
  - High frequency: related to global unstable modes \( \omega_1 \cdots \omega_7 \),
  - Low frequency: related to global unstable modes interaction.

Good accordance between stability analysis and numerical simulation.
Is there any characteristic scale for the flapping frequency?

Let us consider a Reynolds number based on a fixed length $L$, $\text{Re}_L = U_\infty L / \nu$, and the corresponding dimensionless frequency $F = L / \delta^* f$.

By the eigenvalue analysis, we find $F \approx 1$ for any $\text{Re}_L < 35000$ ($\text{Re}_{\delta^*} < 213$), that is the threshold for the onset of the secondary flapping frequency.

→ The values of the flapping frequencies are well converged with respect to grid resolution and domain length.
Scaling law

Is there any physical explication for the flapping phenomenon?

Hypothesis: the separation, carrying back the perturbation in the upstream part of the bubble, could induce an interaction of modes producing the flapping.

- A characteristic scale could be the time needed by the mean flow to carry back a wave packet from the reattachment to the separation point:

  \[ F \propto \frac{1}{t_L} \propto \frac{U_b}{L_b} \]

  \( L_b \) being the bubble size and \( U_b \) the base flow velocity within the bubble.

- \( L_b \) and \( U_b \) vary linearly with respect to \( Re_L \) for \( Re_L < 35000 \) (\( Re_{\delta^*} < 213 \))

- The flapping frequency \( F \) is constant with respect to \( Re_{\delta^*} < 213 \).
Role of topological changes in the flapping phenomenon (1)

What happens at $Re_L = 35000$?

- The secondary flapping frequency appears.
- The frequency $F$ increases with $Re_{\delta^*}$.
- $U_b$ and $L_b$ do not vary linearly with $Re_{\delta^*}$.
- Topological changes appear in the bubble.

Could be these events linked?

The inflection of the streamlines could lead the bubble to split in two smaller ones, A and B, which could carry back the perturbations at two different rates generating two distinct modulations.
Role of topological changes in the flapping phenomenon

(2)

How to validate the hypothesis of bubble splitting?

- The ratio of the size of bubble A with respect to the size of the bubble B ($L_B/L_A \approx 2.5$) is close to the ratio of the two flapping frequencies ($\omega_{II}/\omega_I \approx 2.7$)

- For $Re_L > 35000$ the primary beating is generated by the part B of the bubble, which is smaller than the entire bubble, and is able to carry back disturbances in a smaller time, originating a higher primary beating frequency.

- More validations need to be carried out, involving bubbles with different aspect ratio or geometry-induced-separations (generated by a bump or a backward-facing step).
Conclusions

- Possibility to analyse the transient and asymptotic behaviour of the separated flow with a reduced model based on global modes.

- At this Reynolds number, $Re_\delta^* = 200$, two different mechanisms: high 2D transient mechanism ($G_{\text{max}} \simeq 10^8$, $t_{\text{max}} \simeq 350$) and a 3D centrifugal mechanism, what is the dominant mechanism?

- Sensitivity of the complete spectrum: high sensitivity for the convective modes. This high sensitivity can triggered a vortex shedding phenomenon based on the Kelvin-Helmholtz instability.

- The considered separated flow become unstable when a secondary bubble originate within the primary one, supporting the hypothesis of Dallmann et al (1995).

- For $Re_\delta^* < 213$, a low-frequency beating is found within the flow, whose value is constant with respect to $Re$.

- For $Re_\delta^* \geq 213$, when topological changes are recovered on the base flow, a secondary flapping frequency appears, while the primary one increases.

- A scaling law has been developed, based on the assumption that the oscillations are due to the interaction of the main wave packet with the perturbations carried upstream by the backflow, explaining the previous findings.
Prospects

- Control strategy (for the short time / for the long time),
- Three dimensional transient growth, lift-up-effect,
- Direct-adjoint strategy, comparison with the reduced model,
- Comparison between direct numerical simulations and experiments.