Sensitivity of vortex shedding in 2D turbulent flow past a blunt trailing edge: a linearized approach

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Control of separated flows in ground vehicle applications

Turbulent flows, $\text{Re} \sim 10^6$: complex dynamics

Low frequency unsteadiness / large-scale structures
Detrimental noise, maneuverability, structural excitation, fatigue...
Dynamics of large scale structures

Laminar flows

2D cylinder (Williamson 1996)

2D shear layer (Konrad 1977)

axisymmetric base (Siegel et al. 2008)

sphere (Johnson & Patel 1999)

Hydrodynamic stability explains how structures of a specific frequency and scale are selected and emerge → control, optimization...
Dynamics of large scale structures

Turbulent flows

2D cylinder (Williamson 1996)

2D shear layer (Konrad 1977)

sphere (Taneda 1978)

Large-scale structures + small-scale turbulence
Present objective

Applying tools of linear algebra to turbulent flows dominated by large-scale structures

Toy problem: 2D D-shaped cylinder
Flow past a D-shaped cylinder

Experiments, Re=13000
Parezanović 2012, Parezanović & Cadot 2011-2012

Separated mean flow

Power spectra

Instantaneous flow

Periodic flow dominated by vortex shedding

Numerical approach using stability: model? validity?
Numerical approach

Governing equations: unsteady RANS equations

- Time integration for large scales
- Turbulence (Spalart-Allmaras) model for small scales

\[
\nabla \cdot u = 0 \\
\frac{\partial u}{\partial t} + \nabla u \cdot u - \nabla p + \nabla \cdot \left( \nu + \nu_T \right) \nabla u \\
\frac{\partial \tilde{v}}{\partial t} + \nabla \tilde{v} \cdot u = C_{b1} (1 - f_t) \tilde{S} \tilde{v} - \left( C_{w1} f_w - \frac{C_{b1}}{K^2} f_t \right) \frac{\tilde{v}^2}{d^2} + \frac{1}{\sigma} \left( \nabla \cdot \left( \nu + \tilde{v} \right) \nabla \tilde{v} \right) + C_{b2} \nabla \tilde{v} \cdot \nabla \tilde{v}
\]

Compact form: \[ B \frac{\partial q}{\partial t} + M q + N(q, q) = 0 \]

with \[ q = (u, p, \tilde{v})^T \]

Linear and bilinear

Finite elements FreeFem++ solver \( (F. \text{Hecht}, \text{UPMC Paris, France}) \)
Flow past a D-shaped cylinder

Unsteady RANS + Spalart-Allmaras
\(~200000\) triangles / \(~1000000\) DoF

Vorticity snapshots

Vortex shedding frequency: \(St \sim 0.23\) (\(St_{exp} \sim 0.22\))

*Meliga, Pujals & Serre 2012*
Flow past a D-shaped cylinder

Unsteady RANS + Spalart-Allmaras
~200000 triangles / ~1000000 DoF

Mean vorticity distribution

Fixed point equations for the mean flow:

\[
B \frac{\partial q}{\partial t} + M q + N(q, q) = 0 \quad \rightarrow \quad Mq + N(q, q) = -\overline{N(q', q')}
\]

Mean flow averaged Reynolds stress owing to large scale unsteadiness

*Meliga, Pujals & Serre 2012*
Mean flow global stability

Fate of disturbances superimposed on time-averaged mean flow?

\[ q = \bar{q} + \hat{q}(x, y)e^{\lambda t} \]

global mode (+ complex pulsation)

Assuming perturbation does not alter the Reynolds stress → the global mode satisfies the linearized unsteady RANS equations (Barkley 2006)

\[ \lambda B \hat{q} + L(\bar{q}) \hat{q} = 0 \]

Evolution operator linearized about the mean flow
Mean flow global stability

Shedding mode
Shift & Invert Arnoldi method / ARPACK

Streamwise velocity distribution

Eigenfrequency = 0.25 / Spatial wavelength = 4 ≈ Nonlinear vortex shedding

Application to flow control?

Meliga, Pujals & Serre 2012
Control by a small secondary cylinder

Experiments: sensitive regions for the shedding frequency
*Parezanović & Cadot 2012*

Problem: bottleneck approach, need to test all parameters
Control by a small secondary cylinder

How to identify the sensitive regions beforehand?

*Nonlinear* shedding frequency approximated by the eigenfrequency of the *linear* shedding mode (depends on the mean flow features)

\[
\dot{\lambda} \dot{B} + L(q) \dot{q} = 0 \quad \longrightarrow \quad \dot{\lambda} = \lambda(q)
\]

Effect of the control cylinder modeled by a steady source of momentum in the mean flow problem

\[
M \dot{q} + N(q, q) = -N(q', q') + f \quad \longrightarrow \quad q = \bar{q}(f)
\]

Sensitivity analysis to a steady forcing

\[
\lambda = \lambda(q) + \bar{q} = \bar{q}(f) \quad \longrightarrow \quad \dot{\lambda} = \dot{\lambda}(f)
\]
Sensitivity to a steady forcing

\[
\lambda = \lambda(\bar{f}) \quad \Rightarrow \quad \lambda = \lambda(\bar{f} + \delta \bar{f})
\]

forcing modification

\[
\delta \psi = \frac{\partial \psi}{\partial l} \delta l \quad \text{(1 DoF)}
\]

eigenvalue modification

\[
\|\delta \bar{f}\| \ll 1 \quad \Rightarrow \quad \delta \lambda = \langle \nabla_{\bar{f}} \lambda, \delta \bar{f} \rangle \quad \text{(n DoF)}
\]

gradient

\[
\langle a, b \rangle = \iint a^T \cdot b \, \delta x \, \delta y
\]
Sensitivity to a steady forcing

\[ \lambda = \lambda(\bar{f}) \]

\[ \lambda = \lambda(\bar{f} + \delta \bar{f}) \] \hspace{1cm} \text{forcing modification}

\[ \delta \psi = \frac{\partial \psi}{\partial l} \delta l \hspace{1cm} \text{(1 DoF)} \]

\[ \| \delta \bar{f} \| \ll 1 \rightarrow \delta \lambda = \langle \nabla \bar{f} \lambda, \delta \bar{f} \rangle \hspace{1cm} \text{(n DoF)} \]

\[ \langle a, b \rangle = \iint a^T \cdot b \delta x \delta y \]

Adjoint method: analytical expression of the gradient

\[ \nabla \bar{f} \lambda = \Pi(\overline{q}^+, \overline{q}, \hat{q}^+, \hat{q}) \]

Lagrange multiplier for the mean flow

\[ \overline{q}^+ = \Pi(\overline{q}, \hat{q}^+, \hat{q}) \]

Lagrange multiplier for the shedding mode

\[ \hat{q}^+ = \Pi(\overline{q}) \]
Sensitivity to a steady forcing

Force model

$\delta \lambda = \langle \nabla \lambda, \delta f \rangle$

Flow over cylinder $f_{\text{drag}} = \frac{1}{2} C_d \| \vec{u} \| \vec{u}$   Cylinder over flow $\delta f = -f_{\text{drag}}$

Capability to retrieve experimental sensitive regions in laminar flows

Hill 1992, Marquet et al. 2008, Meliga et al. 2010, Pralits et al. 2010...

Iso-lines of zero growth rate
Experimental measurements
(Strykowski & Sreenivasan 1990)

Theoretical predictions
(Marquet et al. 2008)
Sensitivity of the shedding mode

Theoretical predictions for shedding frequency...

Meliga, Pujals & Serre 2012
Sensitivity of the shedding mode

Theoretical predictions for shedding frequency...

...vs. Experiments

Parezanović & Cadot 2012

Agreement confirms the effect of the control at the mean flow level

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Summary & Perspectives

Possibility to apply generic tools from linear algebra to turbulent flows dominated by an instability mechanism

Spatio-temporal analyses based on instability modes

Adjoint-based sensitivity analyses

Paves the way for open-loop control: predictive/systematic approach

Next step: improvement of turbulence modelisation in view of the application
Thank you for your attention