Global stability of an isolated cylindrical rugosity

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Motivations

• **Problem** :
  - 45% of the drag on airplanes is due to skin friction,
  - turbulent boundary layers induce more skin friction than laminar ones.

• **Motivations** :
  - What are the mechanisms responsible for transition?
  - Can we predict threshold for it?
  - How to control and delay transition in boundary layer flows in order to reduce the drag?
Background

- Multiple transition scenarios have been found:
  - Tollmien-Schlichting waves (modal instability)
  - Streamwise streaks (non-modal instability)
  - Localized perturbation of specific shape (non-linear optimal perturbations)
  - ...

(a) Tollmien-Schlichting waves
(b) Streamwise streaks
Stability to TS waves can be improved introducing a spanwise modulation of the boundary layer:
- Periodic array of roughness elements creates horseshoe vortices inducing streamwise streaks further downstream,
- the resulting boundary layer is less sensitive to 2D TS waves.

Figures taken from Fransson et al.
Background

- **Problem**: If the amplitude of the streaks is too large, transition occurs right downstream the roughness element!
  - What is the mechanism responsible for this?
- Experimental work has been conducted in the 60’s by von Doenhoff & Braslow.

- Empirical criterion for transition:
  \[
  Re_c = 600 \left( \frac{k}{d} \right)^{\frac{2}{5}}
  \]
Geometry & Notations

Sketch of the computational arrangement and various scales used for DNS and stability analysis.

- \( \left( L_x, L_y, L_z \right) / h = (17, 4, 40) \),
- \( d/h = 1 \),
- Inflow : Blasius boundary layer
- Reynolds number : \( Re = U_e h / \nu \)
Global approach

- Linearized Navier-Stokes equations about a baseflow
  \[
  \frac{\partial u}{\partial t} + (U \cdot \nabla) u + (u \cdot \nabla) U = -\nabla p + \frac{1}{Re} \nabla^2 u
  \]
  \[
  \nabla \cdot u = 0
  \]
  \[
  u(x, 0) = u_0
  \]

- Initial value problem
  \[
  \frac{\partial u}{\partial t} = Au
  \]
  \[
  u(0) = u_0
  \]

- Formal Solution
  \[
  u(t) = e^{At} u_0
  \]

- Investigation of matrix exponential properties
Numerical Difficulties

- **Numerics**: Develop tools for fully 3D stability problems
  - Three inhomogeneous directions
  - Impossibility to store the Jacobian matrix and hence to perform any direct eigenvalue calculation

- **Dimension of discretized system**:

<table>
<thead>
<tr>
<th>Base Flow</th>
<th>Inhomogeneous direction(s)</th>
<th>Dimension of $u(t)$</th>
<th>Storage of $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poiseuille</td>
<td>$U(y)$</td>
<td>1D</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Blasius</td>
<td>$U(x, y)$</td>
<td>2D</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Roughness</td>
<td>$U(x, y, z)$</td>
<td>3D</td>
<td>$10^7$</td>
</tr>
</tbody>
</table>

- Use Navier-Stokes solver or any CFD code to approximate the action of exponential matrix:

  $$u(\Delta t) = e^{A(\Delta t)} u_0$$
Iterative eigenvalue methods

- **Eigenvalue problem**:
  \[ \mathcal{F}(\Delta t)u_j = \sigma_j u_j, \quad (n \times n), \quad n > 10^6 \]

- **Construct a very small subspace** (compared to the size of the initial problem) from snapshots:
  \[ \mathcal{K} = \text{span} [u_0, \mathcal{F}(\Delta t)u_0, \mathcal{F}(2\Delta t)u_0, \cdots, \mathcal{F}((m-1)\Delta t)u_0] \]

- **Solve small eigenvalue problem**
  - Orthonormalize (e.g. Arnoldi) \( V = [V_1, \cdots, V_m] \)
  - Project operator \( \mathcal{F}(\Delta t) \approx VHVT \)
  - Solve small eigenvalue problem \( HS = S\Sigma \quad (m \times m) \quad m < 1000 \)
Modal Stability

- Asymptotic behavior:

\[ e^{At} u_j = \sigma_j u_j \]

\[ | \sigma_1 | > 1 \quad \text{globally unstable} \]
\[ | \sigma_1 | \leq 1 \quad \text{globally stable} \]

- \( u_j \) global eigenmodes,
- Determine growth/decay as \( t \to \infty \).
Equations and numerics

- **Time-stepper technique**: Never store matrices and use only velocity fields seen as matrix-vector product.

  \[ u_0 \xrightarrow{\text{DNS}} u(\Delta t) = e^{A\Delta t}u_0 \]

- **Numerics**:
  - Nek 5000: Massively parallel Spectral elements code (DNS and LDNS)
  - Selective Frequency Damping: Base Flow computation (implemented into Nek 5000)
  - Timestepper Arnoldi: Matrix-free eigenvalue computation (coupled with Nek 5000 LDNS)
Three-dimensional Base Flow

- **SFD**:
  - Selective Frequency Damping (Akervik et al. 2006)
  - Solution of the steady Navier-Stokes equations.

- **Characteristic features**:
  - Central plan (xz-plane) is a symmetry plane,
  - Horse-shoe vortex,
  - Recirculation bubble.

**Figure**: Base Flow - $h/d = 1 - Re = 1000 - iso-U = 0$ and Q-criterion.
Global Spectrum

- 20 first global eigenmodes
  - Slightly supercritical case
  - Anti-symmetric vortex street modes
  - Symmetric modes

First anti-symmetric Eigenfunction real part of $u'(x, y, z)$.

Cylindrical roughness - EigenSpectrum - $h/d = 1$ - Re=1000.
Global Spectrum

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Cylindrical roughness - EigenSpectrum - h/d = 1 - Re=1000.

First symmetric Eigenfunction real part of $u'(x, y, z)$. 
Comparison with von Doenhoff-Braslow diagram

- Two cases have been considered so far: \( d/h = 1 \) and \( d/h = 2 \);
- For \( d/h = 1 \): \( Re^c_h(AS) = 950 \) and \( Re^c_h(S) = 1064 \);
- For \( d/h = 2 \): \( Re^c_h(AS) = 757 \) and \( Re^c_h(S) = 804 \);

Comparison between the von Doenhoff-Braslow diagram and stability results

- Stability predictions are consistent with experimental results.
- Potentially various dynamics following the Reynolds number.
Limits of the modal approach

- Global stability analysis only gives the upper threshold $Re_C$ for transition,
- Lower threshold $Re_G$ depends on the nature of the bifurcation:
  - if supercritical: $Re_G = Re_C$,
  - if subcritical: $Re_G < Re_C$.
- For $Re \in [Re_G, Re_C]$, the flow might undergo transition because of transient growth only.
Limits of the modal approach

- Linearized DNS initialized with random white noise,
- Energy gain: \( G \approx 10^3 \) for transients
- How large would be \( G_{\text{max}} \) if the LDNS is initialized with the optimal linear perturbation?
- What about the optimal perturbation as initial condition for DNS? Would it be sufficient to trigger transition?
Discussion

- **Previous study**: Mechanism responsible for transition is non-modal transient growth (Arnal, AIAA, 2011)

- **Present study**:
  - 3D linear stability: threshold for transition in reasonable agreements with experimental observations;
  - $Re_h < Re_h^c(AS)$: flow globally stable;
  - $Re_h^c(AS) < Re_h < Re_h^c(S)$: flow globally unstable for anti-symmetric perturbation;
  - $Re_h > Re_h^c(S)$: flow globally unstable for all perturbation.
  - But ... DNS shows a symmetrical development of the perturbation (Hairpin coherent structure)
  - The mechanism (modal vs. non-modal) is not yet clearly identified though nor understood,
Outlooks

- **Future work:**
  - Investigation of the dynamics for other aspect ratios,
  - Transient growth analysis and DNS for several transitional Reynolds numbers,
  - Influence of the rugosity shape (cylindrical, hemispherical, ...) on the dynamics,
  - Investigation of the non-modal/non-linear coupling of modes,
  - Design of Reduced Order Models for closed-loop control.