Real time implementation of delay control on chaotic systems

T. Duriez, F. Lusseyran, L. Pastur
Outline

• Context :
  - Cavity flow
  - Dynamical model : Rossler
  - Delay control : Pyragas
  - Objectives
• Real – time simulation design
  - Hard and software
  - What will be needed
  - Embedding
  - Delay detection
• The model
• Real – time demonstration
• Toward Experimental implementation
  - Sensor
  - Actuation
Flow over an open cavity

LIMSI

From Brès & Colonius (Youtube, 2008)
Flow over an open cavity: properties.

- Different regimes: when control parameter varies, instabilities are kicked in: 3D flow, KH, mode competition.

- Low dimensional dynamics: using Grassberger–Proccacia algorithm: $5 < \text{embedding space} < 10$

  *F. Lusseyran, L. Pastur, and C. Letellier, PoF 20, 114101 2008*
Control strategies

- Goal ?
  - reduce noise, fluctuations energy, circulation...
- Which strategy ?
  - passive, open-loop, closed-loop.
- Where to act ?
  - boundary layer, shear layer, recirculation, impigement...

We choose to stabilize periodic orbits of the system. To this end we will use close-loop control based on chaos control.
Periodic orbit? What for?

From Jcbandlab11 (Youtube, 2010)
Control strategy.

- Algorithms used to control low dimension chaos:

  - O.G.Y.: deform the system in order to put the current state on an UPO.
    
    *E. Ott, C. Grebog, and J. A. Yorke, Phys. Rev. Lett. 64, 1196 (1990)*

  - Pyragas (citation): displace the system to stay on an UPO.
    
    *Pyragas, Phys. Lett. A 170, 421 (1992).*
Context : delay control

• Principle: try to stabilize unstable periodic orbit embedded into the attractor.

• Formulation:

\[ \dot{y}_i = \dot{y}_{i0} + F(t) \\
F(t) = K(y(t-\tau) - y(t)) \]

\( \tau \) is the delay used and corresponds to the period of the stabilized UPO.

• Known things:
  - robustness ?:
  - successful experimental implementation

What is needed to use a Pyragas control on an experimental system.

- Sensor -> get y(t): pressure sensor
- System state evaluation -> evaluate τ and control effectiveness: embedding
- Control calculation -> get y(t-τ) – y(t): DSP
- Actuation ???: speaker, plasma actuator, jets.

First step: evaluate control strategy with real-time simulation on simple dynamical system.
Test case: the Rossler System

- Evolution equations:

\[
\begin{align*}
\frac{dx}{dt} &= -y - z \\
\frac{dy}{dt} &= x + ay \\
\frac{dz}{dt} &= b + z(x - c)
\end{align*}
\]

- Présence of Unstable Periodic Orbits.
- Previous attempts of control successful.
Real-time : DSpace

- Largely used in automatics
- Design of model using Simulink.
- Model is compiled on computing boards.
- Enjoy.
Embedding

• Get multidimensionnal dynamics from under-dimensionned measurement (1D for instance).
Real – time dynamics

\[
A_m(t_0, t_n, \delta t) = \begin{bmatrix}
    y(t_0) & \cdots & y(t_0 + j\delta t) & \cdots & y(t_0 + m\delta t) \\
    \vdots & \ddots & \vdots & \cdots & \vdots \\
    y(t_i) & \cdots & y(t_i + j\delta t) & \cdots & y(t_i + m\delta t) \\
    \vdots & \ddots & \vdots & \cdots & \vdots \\
    y(t_n - (m - 1)\delta t) & \cdots & y(t_n - (m - j)\delta t) & \cdots & y(t_n)
\end{bmatrix}
\]

Construction of delay matrix

\[
A_m(t_0, t_n, \delta t) = U(t_0, t_n) \times \Sigma \times V(\delta t)^*
\]

U contains principal components used to observe the dynamics.

\[
U(t_0, t_n) = A_m(t_0, t_n) \times V(\delta t) \times \Sigma^{-1}
\]
Real-time dynamics: example

\[ U(t_0, t_n) = A_m(t_0, t_n) \times V(\delta t) \times \Sigma^{-1} \]
Real – time dynamics : example

\[ U(t_0, t_n) = A_{m}(t_0, t_n) \times V(\delta t) \times \Sigma^{-1} \]
Real-time dynamics

Actual system

Principal components
UPO detection
The model
Results

- $X_i$
- Control Value
- Control
- log RMS $y(t_2)-y(t)$
- time
Results
Experimental implementation

- Determine the uncontrolled dynamics: done offline.
- Identify UPOs.
- Didn’t we forget we have a space-time dynamics?
- Actuator: which and where
Solutions comparisons

• Retraction on wind tunnel
  - Pros: similar to Bonhomme.
  - Cons: wind tunnel inertia, probably too heavy on BDL.

• Plasma upstream
  - Pro: benefit from shear layer amplification
  - Cons: transfer function hard to evaluate. Can’t go in negative

• Speaker downstream
  - Pro: close to sensor, benefit from incompressibility, easy to relocate
  - Cons: what are you talking about?
Conclusions & perspectives

• Simulations.
  - real-time control achieved
  - conditions of experiment simulated
  - control is effective

• Experiment.
  - in progress

• Perspectives:
  - simulate a dynamical system obtained from DNS or experiment + pyragas
  - implement other type of model-based control.
Thanks for your attention

From Jcbandlab11 (Youtube, 2010)