Systematic monetary policy and persistence.

Luca Bindelli* Université de Lausanne

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Abstract

The question we wish to address in this paper pertains as to the nature of inflation persistence as modelled in the New Keynesian framework. We estimate a standard, forward looking sticky price model, in which the monetary authority is assumed to commit to an optimal rule. The role of the monetary authority's objective in altering the persistence properties of the model is discussed and an estimate of its quantitative importance is performed. We find that such a model is able to replicate most of the data's moments. Lagged terms in both modelled Phillips and IS curves are found to be either quantitatively not significant or very small, so that 'built in' inertia is not empirically relevant. Commitment policy accounts for a substantial part of output persistence. While the pricing mechanism at the heart of this model helps transfer output persistence into inflation persistence, commitment policy manages to undo this link. Consequently, inflation persistence is reported to be mainly determined by 'cost push' shock inertia alone.

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*Contact address: Université de Lausanne, cwp, bfsh1, 1015 Lausanne (Switzerland); tel: +41216922843; fax: +41216922845; e-mail: luca.bindelli@etu.unil.ch. I wish to thank Philippe Bacchetta, Jean Imbs, Pierre-Yves Yanni as well as participants at the SMYE conference for comments and discussions. Any error remains my own.

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1 Introduction

In two important contributions, Chari, Kehoe and Mc Grattan (2000) and Huang and Liu (2002) conclude that the sticky price mechanism that is at the heart of recent New Keynesian macromodels is hardly able to generate any persistence beyond the exogenously assumed period of price stickiness. They demonstrate that for reasonable parameter values used to calibrate their model, monetary shocks have a limited effect on output in impact and duration. Observed persistence in inflation could then hardly be explained by such models. This result has led researchers to look at mechanisms which could emphasize additional channels through which more persistent responses to monetary shocks might arise. In addressing this issue, some studies have emphasized labour market frictions. The idea backing these studies is to reduce the sensitivity of real marginal costs to output fluctuations.[1]

However, some recent results in Galí and Gertler (1999), Sbordone (2002) and Bindelli (2003) find support in favour of the forward looking, or New Keynesian Phillips curve. Importantly, the Phillips curve implies that the persistence properties of inflation stem almost exclusively from that of the only inflation's driving variable, either marginal cost or output gap depending on underlying assumptions about labour market structure. It is thus possible that once one looks at the supply side only, no 'persistence problem' exists. This is simply because the main driving term itself and/or shocks affecting it are already indeed persistent. Maybe then, it is not about making prices unresponsive to the main driving variable, it is about making the driving variable persistent to start with. The question we wish to address in this paper pertains to the nature of inflation and its main driving term persistence as modeled in the New Keynesian frameworks. More precisely, this study will explore systematic monetary policy behavior and its role in affecting inflation persistence.

The general equilibrium literature cited above explores the impact of random policy shocks i.e. the unsystematic component of central bank's actions and its role in affecting the dynamic properties of inflation and output. However, as Mc Callum (1999) convincingly argues, random monetary shocks account for a very small fraction of total policy instrument variability (with respect to the variability induced by systematic policy behavior). More generally, it is indeed likely that monetary policy behavior is hardly affected by an unsystematic component. Even more importantly, the way given shocks affect the economy is in general intimately linked to the systematic behavior of the monetary authority.

In accordance with this view, a recent strand of literature has integrated endogenous policy making in this class of models. For example, the central bank optimizes its response to shocks on a day to day basis or chooses to behave according to an

[1] For example, Huang and Liu (2002) show that wage staggering is more able at producing persistence. Christiano et al. (2001) confirm this result and show furthermore that the contribution of price staggering to overall persistence is very poor, so that assuming wage staggering performs nearly as well as when both staggering mechanisms are assumed.

instrument rule, whereby it evaluates the current and possible future state of the economy and chooses to act accordingly, generally by choosing the short term interest rate as its instrument.[2] Such studies, as for the previously cited literature, have also generally been confronted with the difficulties purely forward looking models have in explaining the persistence in the data, and have consequently either added lagged terms in the equations reflecting private agent's decision rules, or assumed an inertial central bank objective to start with (for example an interest rate smoothing objective).

In an early analysis, Woodford (1999) has shown that the central bank might be willing to optimally induce some inertia. If the central bank precommits to a rule, its decision procedure will affect present and future private sector expectations. However, an optimal plan will typically not allow a central bank's current decisions to depend exclusively on future events given the current state. Rather, as Woodford (2003) clearly puts it:

"Optimal policy must take account of the advantages of the anticipation of the policy at earlier dates; and for this reason it must generally be history dependent rather than purely forward looking. Past conditions should be taken into account in choosing the current policy setting, because it is desirable that people be able, at the earlier time, to count on the fact that the central bank will subsequently do so."

Accordingly, it is optimal for the monetary authority not only to respond to contemporaneous shocks affecting the economy, but also to lagged ones. This behavior will in turn introduce an additional and independent channel of inertia.

In light of this, it is quite surprising how little work has sought to analyze empirically this type of optimal policy rule in a standard New Keynesian model. We argue that committing to a policy rule might be a good approximation of real central bank behavior when the period of analysis reflects a stable and credible policy regime. Because our benchmark economy will be purely forward looking, the only assumed inherent source of persistence resulting from private sector behavior, arises due to serially correlated shocks to the economy. More precisely, we assume correlated preference and money demand shocks on the demand side and correlated 'cost push' shocks on the supply side. Such choice is motivated because of our willingness to focus our discussion on the role of monetary policy in generating persistence, without specifying any definite form of persistence that would initially arise from private sector behavior or decisions.[3] Importantly however, supply shocks can be derived, as shown by Steinsson (2003) and Ireland (2004), from the model's microfoundations.

Our first goal is then to estimate, in a relevant time period, the standard general equilibrium model described above, in which a central bank chooses to commit, and see wether it can match the broad characteristics of US data. Because persistence might mistakenly be attributed to serially correlated shocks alone, another goal of the paper is concerned with the quantitative importance of 'built in' inertia as measured

[2] Typical instrument rules have mainly focused on the so called 'Taylor rule'.

[3] Except for the assumed pricing mechanism.

by the presence of lagged terms in the structural relationships of the model. Finally, we estimate the importance of policy commitment in affecting persistence.

The forward looking model is shown to be a reasonably good description of the 1987-1999 period. The persistence properties of the data are overall well reproduced, but the model generates slightly lower persistence in inflation than is observed. Modelled output and inflation volatilities are close to their empirical counterpart. Interest rate volatility is however somewhat lower than in the data. Controlling for lagged terms in our model structural equations reveals that backward looking behavior is not important at all if the central bank follows an optimal rule. Moreover, we find that an optimal commitment policy has a important role in generating large and persistent effects on output. However, because this type of policy implies that the central bank is willing to let output stay below its target level for a long period of time in reaction to a supply shock, it will generate a quicker fall in inflation followed by slight (but long lasting) undershooting of the inflation target, so that inflation persistence is slightly lowered. Consequently, inflation persistence is attributed to 'cost push' shocks alone.

In the next section, we briefly describe our framework, how optimal commitment policy might induce inertia, and explore more in detail the persistence generating mechanisms. Section 3 presents our empirical methodology. In section 4, we present the results. Section 5 concludes.

2 A Standard Model

In order to preserve the generality of our argument, we specify a framework that is now fairly standard in the literature. We consider a standard small scale New Keynesian economy in which prices are staggered *à la* Calvo, and the production function is linear in labour. The price setting behavior is derived as the product of optimization by monopolistically competitive firms subject to constraints on the frequency of the price adjustment. Moreover, Calvo's partial adjustment rule stipulates that each period firms are allowed to adjusts their price with a fixed probability $1-\omega$. The central equations of our model economy are as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t + \varepsilon_t^s, \tag{1}$$

$$y_t = E_t y_{t+1} - 1/\sigma(i_{2t} - E_t \pi_{t+1}) + \varepsilon_t^d, \tag{2}$$

$$i_{2t} = i_t - \varepsilon_t^i \tag{3}$$

$$mc_t = (\sigma + \eta) y_t, \tag{4}$$

$$\varepsilon_t^s = \rho^s \varepsilon_{t-1}^s + \xi_t^s,\ \varepsilon_t^d = \rho^d \varepsilon_{t-1}^d + \xi_t^d,\ \varepsilon_t^i = \rho^i \varepsilon_{t-1}^i + \xi_t^i. \tag{5}$$

$\beta < 1$ is a subjective discount rate and $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega} > 0$ is interpretable as a price flexibility parameter. For instance, as the probability of adjusting price increases, so does κ. $\sigma > 0$ measure the inverse of the intertemporal elasticity of substitution of consumption while $\eta > 0$ measures that of labor. π_t is defined as the inflation rate, y_t the output gap, defined as the distance between actual output and the natural level of output i.e. the output that would prevail if prices were fully flexible, mc_t is the deviation of real marginal cost from its steady state level and i_t is the nominal short term interest rate expressed as a deviation from its steady state value.[4] Equation (1) is the Phillips Curve resulting from Calvo's pricing rule; it describes the price adjustment mechanism and is akin to a traditional AS supply curve except for its forward looking component. Equation (2) represents the forward looking IS curve and can be derived from the traditional Euler condition along with the assumption that no investment is present in our closed economy model, so that consumption equals output. Equation (3) describes the relation between the central bank control variable i_t and i_{2t}, the nominal rate relevant to the economy. The latter can deviate from the interest rate set by the monetary authority up to a stochastic component, ε_t^i, which represents a shock to money demand. This type of behavior is widely believed to be consistent with real policy practice. For instance, in recent years, the US monetary policy has been conducted by controlling the Fed fund rate.[5] Under the assumption that labor market is competitive and output is linear in labor, a linear relation between real marginal cost and the output gap, as described by (4), emerges.[6] ε_t^d is a shock to current demand and may be interpreted as a preference shock.[7] Steinsson (2003) and Ireland (2004) provide some theoretical foundations to the presence of a shock term in the Phillips curve. Both present models were the elasticity of demand for each intermediate good is assumed time-varying. In a monopolisic setup, and under the assumption that prices are fully flexible, price will be set equal to a markup above nominal marginal cost.[8] Since the markup is function of demand elasticity, the representative firm faces time varying monopoly power.[9] We

[4] This steady state value is sometimes refered to as being the natural rate of interest, which is the rate compatible with a natural level of output (hence a zero output gap). Throughout our analysis, this rate is assumed to be constant.

[5] See for example Goodfriend (1991) or Erceg and Levin (2003) on the characterization of the Federal Reserve policy.

[6] We avoid the modelisation of the labor market which is somewhat difficult and controversial in order to preserve simplicity of exposition and focus on our primary interest.

[7] Ireland (2004) shows formally how such shock can be derived from a New Keynesian model microfoundations.

[8] In addition, we only consider representative firms, so that all firms will set the same price when they can adjust. Consequently, the real marginal cost is expressed as the inverse of the markup.

[9] Whenever the probability of adjusting prices is lower than one, the actual markup will differ from the desired one.

can thus interpret ε_t^s as a disturbance to the firm's desired markup. Throughout the paper, we refer to ε_t^s as a 'cost push' shock, as defined in Galí *et al.* (1999). Finally, ξ_t^j, where $j = [i, d, s]$, are uncorrelated innovations, normally distributed with mean zero and standard deviation given by $\sigma_j(\xi)$.

2.1 Monetary Policy

The problem of monetary policy is formulated as an optimal response to shocks hitting the economy. Assume furthermore that the monetary authority behaves optimally according to a *targeting* rule. Suppose for instance, that the central bank seeks to minimize the following loss function:

$$W_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i L_{t+i}, \tag{6}$$

and the loss at each period is given by,

$$L_{t+i} = \left[\chi(\pi_{t+i} - \pi^*)^2 + \lambda(y_{t+i} - y^*)^2 \right], \ for \ i \geq 0.$$

λ represents the weight the central bank places on output gap stabilization and χ the weight placed on inflation stabilization. π^* and y^* represent the target variables. We will assume that the target level for inflation is normalized to zero, while that of output is given by the natural level, so that the target for the output gap is also zero. Woodford (2003) shows formally, for a standard sticky price economy such as ours, how the deviations of the expected discounted utility of the representative agent around the steady state level of utility are approximated by the above loss function.[10]

In what follows, two different assumptions about monetary policy behavior will be discussed: discretionary and commitment policy. The fundamental difference between these approaches lies in the central bank's ability to make a credible promise about its future actions. It will turn out, as presented more formally in the next section, that this distinction is crucial for the equilibrium dynamics of the model economy and its persistence properties, and will furthermore help us isolate the overall effect commitment policy has on output and inflation inertia. Forward looking private sector behavior implies that future expected events affect current ones. By not making any commitment about future actions, a discretionary policymaker cannot, by definition, affect private sector expectations. Hence, the effects of shocks last only as long as the shock themselves. On the other side, making a commitment about current and future policy requires the private sector to understand and believe this resolution. Because agents act rationally, the central bank may then take advantage of previously anticipated policy actions by 'assuring' the private sector that his current optimization plan decisions are build upon anterior economic conditions. It is in this sense that

[10] With a positive inflation target (6) can be simply viewed as the representaiton of the central bank's preferences. These may then not be compatible with welfare maximizing objectives.

monetary policy introduces a *history dependence* which might imply more interesting outcomes in terms of shock responses and variables dynamics.

At this stage, two natural questions arise: First, is commitment a realistic policy behavior? Second, under the affirmative, is the central bank objective realistic? To answer the first, recall that in a well known paper, Kydland and Prescott's (1977) have raised the so called *time inconsistency* problem that usually characterizes the commitment approach to policy. Their argument is that if output is inefficiently low and the central bank is unable to make a credible stand on its policy behavior, the authority will systematically tend to inflate in order to reach an output above its potential (non inflationary) level. The resulting discretionary policy will lead to an *inflation bias* whereby the average inflation level will be higher than is optimal without having any effect on the average level of output. In the model presented above, output is maximized at its flexible price level. Since the central bank is assumed to target this very same level of output, no such problem is present in our theoretical setup. Besides, it is now commonly acknowledged by central bankers, that this type of time inconsistency is relatively unimportant in practice.[11]

Concerning the second question, the Federal Reserve Act of 1977 stipulates that the goals of "maximum employment and stable prices" should be pursued by the monetary authorities. Hence, although no precise target is mentioned by the Federal Open Market Committee, we can view this objective as compatible with the one depicted in (6) above. Of course, credibility and reputation are central to a successful commitment policy statement. A suitable period of analysis should thus be reflective of such conditions. As is now well documented, the period of relative instability, beginning in 1979, due to the 'Volcker' disinflation ended in the beginning of 1983. Moreover, we can suppose that some subsequent years were likely to be spent by the Fed at building reputation and credibility in following the above mentioned objectives. The Fed did indeed manage to get stable and low inflation as well as a stable unemployment rate by the mid 80's (after peaking above 10% in early 80's, unemployment fluctuated between 5-7% since 1985 on).[12] Hence, we think a relevant period of analysis shall start by the mid 80's.

[11] For instance Blinder (1997) argues: "...I can assure you that my central bankers friends would not be surprised to learn that academic theories that assume that they seek to push unemployment below the natural rate then deduce that monetary policy will be too inflationary. They would doubtless reply, "Of course. That's why we don't do it."…. It is, I believe, the way parents, teachers, government officials and others solve time-inconsistency problems in the real world every day."

[12] A recent study by Wall and Zoega (2003) suggests that the natural unemployment rate has roughly averaged 5.5% throughout the 1985-2000 period.

2.2 Analytical Solutions

We present below the model solution under both discretionary and commitment policy.[13] Detailed derivations are shown in the appendix.

2.2.1 Discretion

In order to better understand why a central bank might be willing to introduce some inertia in its response to shocks, it is useful to first present the situation in which the monetary authority acts on a discretionary basis. Because the decisions of the central bank are not binding for the future, and thus cannot affect agent's future expectations, discretionary policymaking requires that the optimization takes place period by period taking initial conditions as given.[14] Accordingly, using the law of iterated expectations, the central bank optimization problem can be written as:

$$\min_{\{i_t,\pi_t,y_t\}} E_t \left\{ \begin{array}{c} \frac{1}{2}(\chi\pi_t^2+\lambda y_t^2) \\ +\gamma_t(y_t - y_{t+1} + 1/\sigma(i_t-\pi_{t+1}) + 1/\sigma\varepsilon_t^i - \varepsilon_t^d) \\ +\varphi_t(\pi_t - \beta\pi_{t+1} - \psi y_t - \varepsilon_t^s) \end{array} \right\}, \tag{7}$$

where γ_t and φ_t are the lagragian associated to the constraints, and $\psi = \kappa(\sigma+\eta)$ is used for simplification. As already said, the central bank's control variable is the short term interest rate i_t. This control over i_t enables the monetary authority to set whatever short term rate it wants, given optimal paths for π_t, in order to satisfy equation (2) above and achieve the desired value of y_t. As a consequence, the IS curve imposes no constraints on the central bank's behavior.[15] After combining the relevant first order conditions, we obtain:

$$\pi_t = -\frac{\lambda}{\chi\psi}y_t. \tag{8}$$

This condition describes the optimal targeting rule under discretionary policy. The central bank, by adjusting its instrument, ensures that that the target criterion (8) is satisfied at any time. This rule corresponds to the well known 'lean against the wind' policy, whereby a central bank contracts demand whenever inflation is above target. Replacing for this policy rule in the structural equations yields the equilibrium output and inflation :

$$y_t = e\ \varepsilon_t^s, \text{ and } \ \pi_t = f\ \varepsilon_t^s.$$

where $e = -\frac{\chi\psi}{\lambda[1-\beta\rho^s]+\chi\psi^2} < 0$, $f = \frac{\lambda}{\lambda[1-\beta\rho^s]+\chi\psi^2} > 0$. The equilibrium short term interest rate is

[13] Similar lines of presentation are found in Woodford (2003) and Walsh (2003).

[14] We are ignoring the possibility of reputational equilibria.

[15] This would not be true in case there are restrictions or costs attached to the variation of interest rates.

$$i_t = \frac{\rho^s \lambda + (1-\rho^s)\,\chi\psi\sigma}{\lambda\,[1-\beta\rho^s] + \chi\psi^2}\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d.$$

Note from this equation that the equilibrium nominal interest rate is varying in order to exactly compensate any demand shock (that is, either ε_t^i or ε_t^d).[16] Given that the monetary authority enjoys full credibility, and given that the latter entirely compensate any demand shock, the only reason for inflation and output to deviate from the targeted values is to face a 'cost push' shock which will generate a trade-off in the policy objective and thus allow for a temporary deviation of both inflation and output from targeted values. Equilibrium output and inflation dynamics are thus solely depending on 'cost push' shocks and the only source of persistence pertains exclusively to the shock process itself.[17] Accordingly, the pricing mechanism itself does not play any role in generating persistence. By reacting on a period by period basis, the central bank cannot manipulate private sector's expectations. This explains why neither structural rigidity parameters nor policy objectives parameters do play any role in affecting inflation persistence in this case.

2.2.2 Commitment

In this case, the central bank makes a stand as to its current and future behavior and sticks to it. The minimization problem is given by

$$\min_{\{i_{t+i},\pi_{t+i},y_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} \frac{1}{2}(\chi\pi_{t+i}^2 + \lambda y_{t+i}^2) \\ +\gamma_{t+i}(y_{t+i} - y_{t+1+i} + 1/\sigma(i_{t+i} - \pi_{t+1+i}) + 1/\sigma\varepsilon_{t+i}^i - \varepsilon_{t+i}^d) \\ +\varphi_{t+i}(\pi_{t+i} - \beta\pi_{t+1+i} - \psi y_{t+i} - \varepsilon_{t+i}^s) \end{array} \right\}. \tag{9}$$

and the monetary authority reacts according to the following rule:

$$\pi_t = -\frac{\lambda}{\chi\psi}(y_t - y_{t-1}). \tag{10}$$

This equation reflects the fact that the central bank wishes to implement a targeting rule that links inflation to *variations* in the output gap. This type of policy response is reflective of what we have earlier called 'history dependence' of policy. Whenever, actual inflation is above the target value (here, zero), the output gap will be projected

[16] It is now fairly well documented that such setup is unable to generate the observed low volatility in the policy instrument variable. Söderström *et al.* (2003) have calibrated a New Keynesian model to the US data in which lagged inflation and output appear in the Phillips and the IS curve respectively. They show that under optimal discretionary policy, a high preference for interest rate smoothing is needed to reproduce the low policy instrument volatility.

[17] The impulse response functions are simply given by $IRF^d_{y,\varepsilon^s}(n) = e\,(\rho^s)^n$ and $IRF^d_{\pi,\varepsilon^s}(n) = f\,(\rho^s)^n$.

to decline. Similarly, an inflation rate below target value will require the central bank to let the output gap grow through time. For the central bank to be able to bring exactly the kind of dynamic responses shown, it needs to make sure that the private sector will indeed understand this and furthermore believe in it. Because, agents behave rationally and are forward looking, this will only be possible when actual policy calls on past economics conditions, which in turn dependent on past policy actions. Accordingly, the monetary authority reacts not only to current 'cost push' shocks, but also to past shocks. Furthermore, past output gaps now affect current output gap and inflation. Equilibrium output and inflation are given by:

$$y_t = ay_{t-1} + b\varepsilon_t^s, \text{ and } \pi_t = cy_{t-1} + d\varepsilon_t^s,$$

where $a = \frac{\left[\lambda+\lambda\beta+\chi\psi^2-\left((\lambda+\lambda\beta+\chi\psi^2)^2-4\beta\lambda^2\right)^{-1/2}\right]}{2\lambda\beta} > 0$, $b = -\frac{\chi\psi}{\lambda[1+\beta(1-a-\rho^s)]+\chi\psi^2} < 0$, $c = \frac{\lambda}{\chi\psi}(1-a) > 0$, and $d = \frac{\lambda}{\lambda[1+\beta(1-a-\rho^s)]+\chi\psi^2} > 0$. The equilibrium interest rate is now given by:

$$i_t = a\left[\sigma(a-1)+c\right]y_{t-1} + \left[\sigma b\left(a+\rho^s-1\right)+\left(cb+\rho^s d\right)\right]\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d.$$

Since $a > 0$, inertia in the interest rate, inflation and the output gap is now present. Thus, in the case of commitment, the response to past supply shocks will enable the monetary authority to introduce a separate and independent channel through which persistence might arise. This is true even if there is no inherent source of persistence in the model i.e. if $\rho^s = 0$. Because the central bank lets past shocks affect current inflation, price stickiness has a role to play in determining the persistence properties of inflation.[18]

If we now compute the theoretical impulse responses under commitment to a 'cost push' shock resulting from the solution above, we obtain for the output gap:

$$\begin{aligned} IRF_{y,\varepsilon^s}(n) &= b, \qquad for\ n = 0, \\ &= ba^n + b\sum_{i=1}^{n} a^{n-i}\left(\rho^s\right)^i, \qquad for\ n \geq 1. \end{aligned}$$

The persistence properties of the output gap process are entirely determined by the evolution of a and ρ^s. The impulse response simply tells us that the higher a will be, the higher the inertia in the output gap process. Furthermore, there is no need for 'built in' inertia in the shock process to generate gradually decaying impulse responses. Thus, commitment generates an amplified and more persistent output response. The impulse response for inflation is very similar:

$$\begin{aligned} IRF_{\pi,\varepsilon^s}(n) &= d, \qquad for\ n = 0, \\ &= cba^{n-1} + cb\sum_{i=1}^{n-1} a^{(n-1)-i}\left(\rho^s\right)^i + d\left(\rho^s\right)^n, \qquad for\ n \geq 1. \end{aligned}$$

[18] Technically, both a and c depend on ψ.

As can be seen from figure 1, inflation's impulse response implies an initial positive impact that gradually declines and turns into a long lasting deflationary episode (undershooting of target inflation) before going back to its initial value.[19] If the central bank precommits to a given policy rule, it will keep output below its potential level for several periods after the shock. Private agents understand and believe that the central bank will indeed do so, and thus will incorporate this fact into their current expectations about inflation. Future expected inflation is lower and a slight deflation is induced.[20]

We have seen in this section that a unique rational expectation equilibrium exists under both optimal discretionary and commitment policy. Our assumptions presuppose that the monetary authority is able to enforce either (8) or (10) in order to reach its desired objective without explicitly specifying its implementation through an interest rate setting rule. This issue will however not be considered in our presentation since it is extensively discussed in Woodford (2003). In order, to have an intuition on the way policy affects real variables, note that despite the fact that the central bank uses the short term interest rate as its instrument, output and inflation are likely to respond to variations in the long term rates. Clearly then, the way in which the monetary policy should alter inflation and output dynamics is through an implicit term structure that links short to long term rates. To see this, first rearrange and iterate equation (2) forward to obtain:

$$y_t = -1/\sigma \sum_{j=0}^{\infty} E_t(i_t - \pi_{t+1}).$$

From this equation, we can see that aggregate demand, conditional on the expectation theory of the term structure, not only depends on short term rates, but on long ones too. If the agents believe that moderate adjustments of short rates could have a significant impact on long ones, achieving a given stabilization goal without inducing too much volatility in the short term rates would be possible. Because of policy commitment, a channel through which private sector expectations are affected by current policy actions is introduced (which in turn depend on past economic conditions). Hence, the central bank will be able to act with less strength on interest rates after a given shock, because private agents understand and believe that it can take advantage of its possibility to affect future outcomes. Accordingly, the stabilization

[19] Impulse responses are plotted under both discretion and commitment case. Parameter values are set to: $\chi = 1$, $\psi = 0.2$, $\lambda = 0.2$, $\rho^s = 0.5$, and $\beta = 0.99$. Also, in order to visualize the impact of the sole monetary policy commitment effect, the figures present the impulse responses under the assumption of i.i.d supply shocks (labelled 'MP induced').

[20] On the other hand, inducing a small undershooting of the target inflation rate in the case of discretionary policy is not credible for private agents because after the initial shock, the agents understand that the central bank only reacts to contemporaneous output gap movements and consequently is not willing to induce 'undershooting' in the future. That it why this type of policy is considered as suboptimal with respect to commitment. In the literature, this relative inefficiency has been termed the *stabilization bias*.

of inflation and output achieves a better outcome in terms of (6) i.e. a better output gap-inflation volatility trade-off can be obtained.

Policy Induced Persistence We derive in the appendix the autocorrelation functions for both $y_t = ay_{t-1} + b\varepsilon_t^s$ and $\pi_t = cy_{t-1} + d\varepsilon_t^s$ respectively. We can write the autocorrelogram for the output gap as,

$$\rho_y(k) = \frac{(1-a\rho^s)\left((\rho^s)^k + a\sum_{i=1}^{k-1}(\rho^s)^i a^{k-1-i}\right) + a((\rho^s)^{k+1} + a^{k-1})}{1+a\rho^s}. \tag{11}$$

Using the same method for inflation, we obtain:

$$\begin{aligned}\rho_\pi(k) &= \frac{1}{c^2b^2(1+a\rho^s) + d^2(1-a^2)(1-a\rho^s) + 2bcd\rho^s(1-a^2)} \qquad (12)\\ &\left\{\begin{array}{c} b^2c^2\left[\left((\rho^s)^k + a\sum_{i=1}^{k-1}(\rho^s)^i a^{k-1-i}\right)(1-a\rho^s) + a((\rho^s)^{k+1} + a^{k-1})\right] \\ +d^2(\rho^s)^k(1-a^2)(1-a\rho^s) \\ +bcd\left[((\rho^s)^{k+1} + a^{k-1})(1-a^2) + \sum_{i=1}^{k-1}(\rho^s)^i a^{k-1-i}(1-a^2)(1-a\rho^s)\right]\end{array}\right\}\end{aligned}$$

Note that both output and inflation inertia are independent of the shocks' volatility. What is then the proportion of inertia induced by commitment policy? To address this question, we derive the autocorrelation functions for both y_t and π_t in the limiting case where supply shocks are i.i.d.. Under such an assumption, we can interpret the resulting inertial properties as arising exclusively from monetary policy behavior. Expressions (11) and (12) then simplify considerably as we obtain :

$$\lim_{\rho^s \to 0} \rho_y(k) = a^k \tag{13}$$

and

$$\lim_{\rho^s \to 0} \rho_\pi(k) = \frac{b_0^2c^2a^k + b_0cd_0a^{k-1}(1-a^2)}{c^2b_0^2 + d_0^2(1-a^2)}, \tag{14}$$

where $b_0 < 0$ and $d_0 > 0$ denote respectively b and d with $\rho^s = 0$. We have identified in our model economy the contribution to overall persistence commitment policy would have in case of hypothetical i.i.d. shocks to the economy. In the more general case where shocks are assumed to be serially correlated, it is relatively easy to identify the source of persistence stemming from policy commitment alone. If policy is discretionary, the equilibrium outcome only depends on (supply) shocks, the only source of inertia present in the model. Comparing the persistence properties of this equilibrium with those obtained under commitment (for the same structural parameter values), should thus allow us to identify the contribution of policy to overall persistence.

We have plotted in figure 2 and 3, using the same parameter values set in figure 1 (i.e. the general case where shock are correlated), the impulse responses of output and inflation for different values of ψ and λ. First, consider the case where the central bank precommits and prices become relatively inflexible (figure 2). The higher price rigidity requires output to deviate from its target for a longer period of time. Because a reduced proportion of firms manage to adjust their price in each period, the central bank knows it will take more time to affect prices and hence inflation. Also, it will increase interest rates in a more pronounced way, because, due to the lower adjustment probability, the initial impact on inflation is bigger. The private sector understands that output gap deviations will be below target for a longer period of time and anticipates a smaller shock on impact on output (the policy preference parameters did not change). Accordingly, output persistence increases, and due to the Calvo pricing mechanism, so does inflation persistence. Next, in figure 3, we have pictured an increase in the parameter λ. In this case, the monetary authority not only reduces its response to the initial shock on impact but responds more strongly to variations in the output gap. Policy responses are thus milder, in the sense that interest rate variations are attenuated. This makes the output gap more persistent. In turn, due to the Calvo pricing mechanism, price level changes are reduced in response to smaller expected movements in output, implying relatively more persistence in inflation.

To better understand the mere effect of policy on equilibrium dynamics, we have displayed in figures 4 to 6, both reduced form estimates parameters and theoretical output gap and inflation persistence (defined as the first lag correlation) under the assumption that shock are i.i.d..[21] As we have seen above, the autoregressive parameter a, which also represents output gap persistence, is increasing in λ and decreasing in ψ. Recall, that as lambda approaches zero, there is no more role for output stabilization and consequently no role for responding to past shock. That is why, on impact, a small but positive lambda takes us away from a 'corner solution' and increases output persistence drastically.

The contribution of past output gap movements to inflation variability, as measured by c, is always positive in lambda, for a given level of price rigidity. Price rigidity will generally add to output driven inflation variability, but not always though. As one approaches full price rigidity, the central bank is willing to let output stay below its target level for a longer period of time following a shock. We argued above that this is because firms have a reduced probability of adjusting their price and the central bank knows this will lengthen the time before policy is effective. The deflation period thus lasts longer and this increases output driven inflation variability. On the other side, a lower shock on impact occurs on the output gap process because policy preferences did not change, and this relatively decreases output driven inflation variability. Also, this shock is smaller the higher the preference for output stabilization because the central bank implicitly accepts more inflation volatility. That is why c decreases faster in ψ the higher λ. Accordingly, output driven inflation variability

[21] We set $\rho^s = 0$, $\chi = 1$, $\beta = 0.99$ and $\psi = 0.05$.

increases up to the point where the 'duration effect' is outweighed by the 'magnitude effect' as implied by the presence of monetary policy commitment.

By definition, with i.i.d. shocks, the parameters b_0 and d_0 only govern the initial level impact after a 'cost push' shock. b_0 is decreasing (in absolute value) and d_0 increases whenever the central bank increases its aversion toward output variability or prices become more sticky.

Finally, from figure 6, we see that the first lag theoretical inflation autocorrelation is always negative but increasing in both λ and ψ. The central bank thus induces sufficiently strong negative correlation between output and inflation to undo the positive impact of output serial correlation on inflation persistence. Despite the output process contributes positively to inflation inertia through c, the policy response implying a target 'undershooting' might thus completely eliminate this previous positive correlation, possibly reporting much of observed inflation persistence on 'cost push' shock correlation alone. We now turn to the empirical investigation of the model.

3 Econometric Methodology

The assessment of the model performance and the structural parameter estimation will rely on the approach suggested by Söderlind (1999). Our model, and more generally, any linear rational expectation model under optimal policy, can be integrated into a setup evolving according to:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Bu_t + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0} \end{bmatrix}, \tag{15}$$

where x_{1t} is a vector of n_1 predetermined variables (x_{10} is given), x_{2t} is a vector of n_2 non predetermined variables (forward looking), u_t a vector of k policy instruments and ξ_{t+1} an n_1 vector of innovations to x_{1t}. Our policy objective can be expressed as

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t), \tag{16}$$

where $x_t = [x_{1t}\ x_{2t}]'$, and matrix Q (symmetric) is function of structural parameters. The solution to the above problem can be casted into a state space model which can be directly estimated via maximum likelihood. The procedure is the following: A first guess of the parameter vector is used in the solution algorithm. This will give a system of linear difference equations that can be framed into a state space model in which a Kalman Filter is then used to build up a likelihood function. We then iterate over the entire parameter space to find the parameter vector maximizing the likelihood.[22]

We use data found on the FRED database of the Federal Reserve Bank of St. Louis. It consists of quarterly data for the period from 1987Q4 to 1999Q4. We

[22] The application to our benchmark model economy is briefly discussed in the appendix.

choose this sample because, as argued above, it excludes the disinflationary period in the early 1980's and is characterized by a stable monetary policy regime. The inflation rate is measured as the annualized quarterly change in the GDP deflator and the output gap is the percentage deviation of real GDP from potential GDP as calculated by the Congressional Budget Office. Finally, the interest rate is the annualized 3 month T-bill rate, which is calculated as the average of daily rates (we use the deviation from the unconditional mean in our estimations). All series are seasonally adjusted except for the 3 Month T-bill series (see figure 7).

4 Results

Model Estimation In table 1 below, we have summarized our estimation results under optimal commitment policy for the system of equations as given by (1)-(6).

Table 1: Commitment policy

Parameter	Estimate	St.err
β	0.9773	0.0062
σ	1.9691	0.0046
ω	0.8360	0.0090
η	0.4142	0.0006
ρ^d	0.9471	0.0284
ρ^s	0.6953	0.0245
ρ^i	0.5218	0.0012
$\sigma_d(\xi)$	0.1960	0.0212
$\sigma_s(\xi)$	0.4208	0.0216
$\sigma_i(\xi)$	0.0853	0.0102
λ	0.1827	0.0079
χ	0.9964	0.0029
LogL	-176.38	

All our parameter estimates are found to be significant. Our estimate for β is slightly lower than conventional values but still well within theoretical bounds. As is well known, the literature does not reach conclusion as to the value of $1/\sigma$, but it is generally included between zero and one depending on the studies whereas $1/\eta$ is generally calibrated around 3. In our setup, σ is estimated as being close to two and η near one half. Thus, our estimates can also be considered as compatible with standard calibrated values. χ is estimated close to one, which is a common assumed value in the literature. A value of 0.1827 for λ suggest that the Federal Reserve policy has responded to output gaps movements in a moderate way throughout our period of analysis. This estimate is within usual values found in the literature, who generally range between 0.1 and 0.25. Concerning the autoregressive parameters of the shocks, we find relatively persistent desired markup shocks (root of the AR process is 0.69). Interest rate shocks are moderately persistent (0.52), while preference shocks are very inertial (0.95). The value of ω, which reflects the probability of leaving the price unchanged in the pricing equation (1), implies that the estimated average time between price changes is roughly six quarters. Concerning the structural parameters of the Phillips curve, $\psi = \kappa(\sigma + \eta)$ is estimated to be approximately equal to 0.0855. This figure stands in line with previous partial equilibrium results on the Phillips curve for the US when output gap is used as the main driving variable of inflation.[23]

Turning now to the reduced form solution parameter estimates, we have:

$$y_t = 0.8274\ y_{t-1} - 0.882\ \varepsilon_t^s \tag{17}$$

$$\pi_t = 0.3699\ y_{t-1} + 1.8903\varepsilon_t^s \tag{18}$$

$$i_t = 0.0248\ y_{t-1} + 0.0803\ \varepsilon_t^s + 1.9691\ \ \varepsilon_t^d - \ \varepsilon_t^i. \tag{19}$$

The autoregressive root for the output gap is quite large at roughly 0.83. This is suggestive that the presence of commitment policy indeed strongly affects the inertial properties of output. For the equilibrium inflation rate, we see that the coefficient on lagged output is estimated at 0.37, which implies that, due to the presence of commitment policy, a rather important component of output variability is transferred to inflation. In the same time, the inflation rate reacts strongly to shock variations (coefficient is 1.89), this reflects the strong response of monetary policy to inflation variability following 'cost push' shocks. The equilibrium interest rate is only modestly affected by lagged output gap and 'cost push' shocks, more so by both demand shocks (recall that these are exactly compensated by the central bank). This indicates that the central bank only needs to make modest changes in the interest rate to generate substantial movements in future output.

The Importance of 'Built in' Inertia Before we address the issue of the quantitative contribution of monetary policy to the overall persistence, we wish to discuss an important issue, still matter of considerable debate. 'Built-in' inertia such as habit formation or backward looking pricing behavior for example, is not included in our benchmark setup. Consequently, the only sources of inertia arise from correlated disturbances to the economy. However, precisely because the possibility of other sources of inertia, stemming from private sector behavior, are not present

[23] See Bindelli (2003) for instance. On the contrary, Galí and Gertler (1999) find a negative parameter estimate for the contemporaneous output gap.

in our benchmark forward looking setup, we may falsely attribute variable dynamics to correlated shocks when instead those are driven by other frictions not present in our model economy. For example, several authors have argued that a lagged inflation term should enter the forward looking Phillips curve on the basis that the latter is unable to capture the fact that inflation is highly persistent (see Fuhrer and Moore (1995)). A similar justification has been pointed out for the presence of a lagged output term in the standard IS curve. The contribution of such inertial behavior can be evaluated by specifying an aggregate supply and demand containing lagged terms. To this purpose, we replace (1) and (2) by:

$$\pi_t = \beta\left[\alpha_\pi \pi_{t-1} + (1-\alpha_\pi)E_t\pi_{t+1}\right] + \kappa mc_t + \varepsilon_t^s,$$

$$y_t = \alpha_y y_{t-1} + (1-\alpha_y)E_t y_{t+1} - 1/\sigma(i_{2t} - E_t\pi_{t+1}) + \varepsilon_t^d.$$

We then estimate the resulting model economy, under the assumption of optimal commitment policy. The results are presented in table 2.

Table 2: Commitment policy

Parameter	Estimate	St.err
β	0.9740	0.0062
σ	1.9708	0.0043
ω	0.8294	0.0086
η	0.4144	0.0005
ρ^d	0.9518	0.0276
ρ^s	0.6835	0.0225
ρ^i	0.5215	0.0012
$\sigma_d(\xi)$	0.1962	0.0201
$\sigma_s(\xi)$	0.3996	0.0198
$\sigma_i(\xi)$	0.0846	0.0070
λ	0.1827	0.0100
χ	0.9945	0.0027
α_π	0.0119	0.0130
α_y	0.0485	0.0056
LogL	-176.04	

Strikingly, virtually no backward looking pricing behavior is detected by our specification. α_π is very small and not significantly different from zero. On the other side, α_y is found significant and slightly higher at 0.0485, but still very small. As a consequence, other parameter estimates are left relatively unaffected by the presence of lagged terms in the Phillips and IS curves, suggesting that forward looking behavior is predominant on both demand and supply side. In a similar setup to ours, Ireland (2004) models the central bank behavior as a modified Taylor rule. His results for the post 1980 period also show that lagged terms parameter estimates are not significant.[24] Our evidence thus suggest that frictions such as backward pricing behavior and habit formation should not alter significantly the model variables, in particular their persistence properties. Hence, we will rely on our benchmark, forward looking model economy as a valid representation of the data. Moreover, the inertia present in our economy can be consistently attributed either to monetary policy or to correlated shocks in the economy alone.

In trying to further assess the model performance at replicating volatility and persistence features of the data, we have computed some unconditional moments for inflation, the output gap and the short term interest rate.[25] The results are presented in the table below.

Table 3: Unconditional moments: Commitment[26]

Inflation	$\boldsymbol{\sigma}$	$\boldsymbol{\rho}(1)$	$\boldsymbol{\rho}(2)$	$\boldsymbol{\rho}(3)$
Data	1.02	0.69	0.59	0.64
	(0.15)	(0.08)	(0.09)	(0.08)
Estimated	0.98	0.55	0.26	0.08
Output				
Data	1.65	0.91	0.82	0.72
	(0.16)	(0.08)	(0.08)	(0.07)
Estimated	1.77	0.97	0.90	0.81
Int. rate				
Data	1.50	0.96	0.88	0.76
	(0.19)	(0.07)	(0.08)	(0.08)
Estimated	1.21	0.94	0.89	0.84

The estimated standard deviation for inflation is very close to the data's one. Also, while the first autocorrelation is relatively close to its empirical counterpart,

[24] Söderström *et al.* (2003) argue that backward looking behavior is needed in the Phillips curve, but that the forward looking IS curve is a good approximation of actual behavior.

[25] Note that all moments below and in successive tables could also be directly calculated from our reduced form solution. To this purpose, we need to express the supply shock volatility in terms of its innovation volatility. Since $\varepsilon_t^s = (1 - \rho^s L)\ \xi_t^s = \sum_{i=0}^{\infty} (\rho^s)^i \xi_{t-i}^s$, we have $\sigma_s(\varepsilon) = \frac{\sigma_s(\xi)}{1-(\rho^s)^2}$.

[26] Standard deviation and autocorrelations up to 3 lags are displayed. In parenthesis are displayed the standard errors for the data series. Those are calculated using bootstrapping techniques (200 sample replications).

lying within two standard errors band, further autocorrelation lags are estimated to be too low. Furthermore, the model generates very little third lag autocorrelation in inflation. Output autocorrelations and volatility are well reproduced as they lie generally within one standard error above data's estimated moments. The unconditional moments for the short term interest rate are relatively well reproduced. The estimated volatility is however slightly lower. Accordingly, adding an interest rate change term in the central bank's objective function is not necessary to produce the low volatility found in the data when the central bank commits to a rule. Overall, the model performs relatively well, but has difficulties at replicating the third lag autocorrelation in inflation.

In figure 8 are displayed the estimated impulse responses for inflation, the output gap and the interest rate following a supply shock (using table 1 parameter values). The central bank slightly increases the interest rate for the first 3 to 4 quarters, and drives the output gap further down as it helps reducing the initial impact on inflation. As the central bank slowly decreases the interest rate, inflation expectations are lower and private agents expect a slight but long lasting undershooting of the inflation target since the output gap only smoothly returns to its initial level.[27] Using a smoothing algorithm, we also have estimated the supply shocks that have hit the economy during that period. The series is presented in figure 9. The positive 'cost push' shock period affecting the second quarter of 1990 until end 1991 is probably due to the burst of the war in Kuwait. On the opposite, the end of the period is characterized by negative 'cost push' shocks. We believe that throughout this very same period, increasing industry competitiveness, mostly in technology intensive sectors, may have indeed produced downward shocks to desired markups.

Overall, while the standard New Keynesian model presented here is highly stylized, we believe that this model captures some important features of the data as shown above, notably of persistence properties.

Shocks vs. Policy Induced Persistence Recall from above that if the central bank behaves in a discretionary manner, it will act on a day to day basis, responding to contemporaneous shocks only. The equilibrium outcome then implies no role for policy, neither for price rigidity mechanism, in altering the persistence properties of the model variables. Thus, only exogenous shocks' inertia is transmitted to output, interest rate and inflation. Comparing the moments obtained under such hypothetical outcome will thus help define the commitment policy contribution to persistence. In accordance with this, the estimated parameter vector presented in table 1 is used

[27] It is certainly defensible to contend that inducing a slight but long lasting deflation in response to a supply shock is relatively unlikely to happen in the real world, even though our estimated magnitude of deflation is small. After a supply shock, inflation peaks at roughly -0.2 after 10 quarters. Note that average inflation for the period 1987-1999 is roughly two percent. If we consider this number is sufficiently close to the true target value over the period and the model still is a good approximation in such 'neighborhoods', then the validity of the argument might be attenuated because this would simply reduce inflation to 1.8 instead (i.e. a disinflation would occur).

to solve for the state space system resulting from optimization under discretionary policy. The obtained unconditional moments are displayed in the table below.

Table 4: Unconditional moments: Discretion

Inflation	σ	$\rho(1)$	$\rho(2)$	$\rho(3)$
Data	1.02	0.69	0.59	0.64
	(0.15)	(0.08)	(0.09)	(0.08)
Estimated	1.63	0.69	0.48	0.34
Output				
Data	1.65	0.91	0.82	0.72
	(0.16)	(0.08)	(0.08)	(0.07)
Estimated	0.76	0.69	0.48	0.34
Int.rate				
Data	1.50	0.96	0.88	0.76
	(0.19)	(0.07)	(0.08)	(0.08)
Estimated	1.99	0.79	0.63	0.52

Inflation persistence (first lag correlation) exactly matches that of the data in this case. While data's inflation autocorrelation would be relatively better captured for the second and third lags, model inflation is more volatile and the first lag correlation is higher, about one standard error above data's estimate. On the contrary, output volatility and autocorrelations would be too low. The same comment applies for the interest rate's inertial properties, but in this case however, the problem is more severe. As expected, the interest rate volatility would be too high. Thus, for the estimated structural parameter values under commitment, the main differences appearing between the discretionary policy compared with commitment are: higher inflation variability and persistence, lower output gap volatility and persistence, and finally, higher interest rate variability and lower persistence. Inflation persistence can be reproduced up to 80% of its empirical counterpart (and model discretionary case). Again, while the output gap is a determinant of equilibrium inflation under commitment policy, this very same policy behavior produces, through its ability to affect expectations, a slight reduction in inflation's autocorrelation.

A last question concerns the quantitative impact of such policy on overall persistence when no other source of persistence is present in the model. First, we have computed the theoretical persistence in output according to (13) using the parameter values estimated in the preceding section. We obtain for the first three lags 0.82, 0.68, and 0.57 respectively. The autocorrelations for the output gap in table 1 were respectively 0.91, 0.82, and 0.72. Thus, around 90% of the persistence in output may be reproduced by policy behavior alone. Whereas results in table 1 suggest that inflation is strongly positively correlated (0.69, 0.59, and 0.64), the potential contribution of policy induced persistence to inflation, as measured by (14), is estimated at -0.09, -0.07, and -0.06 respectively. This means that policy commitment actually manages to induces a slight *negative* inflation correlation. Consequently, the only source of positive serial correlation in inflation is supply shock driven.[28]

5 Conclusion

Implementing a commitment policy is welfare enhancing from a theoretical viewpoint. However, we believe that commitment is not only a theoretically desirable feature, but that it is compatible with real life assumed policy goal and behavior by the central banks. Indeed, we have seen that a simple forward looking general equilibrium model can be a reasonably good description of reality when the policy followed by the monetary authority is bound to be a rule. Second, the presence of lagged terms in both Phillips and IS curve is not found to be empirically relevant, so that both output and inflation inertia remain unaltered when we control for these variables. Third, we have seen that a lot of output persistence can be generated through a policy that is concerned with variations in output, but that commitment policy responses also involve two distinct effects on inflation inertia: i) A slight undershooting is generated, implying relatively less inflation persistence, because the central bank manages to reduce inflation faster. ii) A positive effect, which reflects the gradual return of inflation to its target from *below*. Since the former slightly dominates, much of inflation inertia is reported to serially correlated 'cost push' shocks alone.

The timeless perspective in a purely forward looking economy involves however an inflation behavior which might be difficult to reconcile with the data. In fact, a relatively long lasting (but small) disinflation with respect to the targeted value following a supply shock is hardly observed in reality.[29]

A key point in our setup is the assumption that the central bank targets clear and fixed inflation and output goals. While this is a good first approximation, it is likely that policy targets are not clearly pinned down by the private sector. One reason is simply because the Fed does not target precise values. In that respect, the assumed credibility monetary authorities have and the fact that agents have potentially access to all available information in the economy, including the central bank's behavior is probably only a good starting point. In a recent study, Erceg and Levin (2003) explore a model in which the private sector faces a signal extraction problem about the central bank's targeted inflation value.[30] Analyzing the 'Volcker' disinflation period, they show that the 'transparency' of the monetary policy regime also affects inflation persistence properties considerably. Finally, if one is willing to accept that markup

[28] Interestingly, Ireland (2004) finds that the most important contributor to movements in inflation is indeed the 'cost push' shock for the post 1980 period.

[29] A gradual decline to the targeted inflation value is generally observed in most VAR's.

[30] Similarly, Ehrmann and Smets (2003) explore in a small model for the euro area, the implications of incomplete information about potential output for the conduct of monetary policy.

shock autocorrelation is an acceptable feature of the New Keynesian modelling, then the forward looking behavior that characterizes these models could well be a good starting point for policy evaluation analysis. In that case, further research should probably take into account investment decisions both on firms' and consumers' side.

References

[1] Bils, Mark, Peter J. Klenow and Olesky Krystov, 2003, "Sticky Prices and Monetary Policy Shocks.", *Quarterly Review*, Federal Reserve Bank of Minneapolis.

[2] Bindelli, Luca, 2003, "Testing the New Phillips Curve: an Alternative Approach.", *mimeo*, University of Lausanne.

[3] Blinder, Alan S., 1997, "What Central Bankers Could Learn from Academics-and Vice Versa.", *Journal of Economic Perspectives*, 11(2), pp.3-19.

[4] Chari, V.V., Patrick J. Kehoe and Ellen Mc Grattan, 2000, "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica*, 68, pp. 1151-1179.

[5] Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans, 2001, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.", *NBER Working Paper*, n° 8403.

[6] Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective.", *Journal of Economic Literature*, Vol. 37(4), pp. 1661-1707.

[7] Dittmar, Robert, William T. Gavin and Finn E. Kydland, 2002, "Inflation Persistence and Flexible Prices." *Working Paper*, Federal Reserve Bank of St. Louis.

[8] Dotsey, Michael, 1999, "The Importance of Systematic Monetary Policy for Economic Activity.", *Economic Quarterly,* Vol. 85/3, Federal Reserve Bank of Richmond.

[9] Ehrmann, Michael and Franck Smets, 2003, "Uncertain Potential Output: Implications for Monetary Policy.", *Journal of Economic Dynamics and Control*, 27, pp. 1611-1638.

[10] Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin, 2000, "Optimal Monetary Policy with Staggered Contracts.", *Journal of Monetary Economics*, 46, pp.281-313.

[11] Erceg, Christopher J., and Andrew T. Levin, 2003, "Imperfect Credibility and Inflation Persistence.", *Journal of Monetary Economics*, 50, pp.915-944.

[12] Fuhrer, Jeffrey and Geoffrey Moore, 1995. "Inflation Persistence." *Quarterly Journal of Economics,* 110(1), pp.127-159.

[13] Hamilton, James, 1994. "Time Series analysis.", Princeton University Press.

[14] Harvey, Andrew C., 1989, "Forecasting, structural time series models and the Kalman Filter.", Cambridge University Press.

[15] Huang, Kevin X.D., and Zheng Liu, 2002, "Staggered Price Setting, Staggered Wage Setting, and Business Cycle Persistence." *Journal of Monetary Economics*, 49, pp.405-433.

[16] Ireland, Peter, 2004, "Technology Shocks in the New Keynesian Model.", *NBER Working Papers*, n°10309.

[17] Mc Callum, Bennett T., 1999, "Analysis of the Monetary Transmission Mechanism: Methodological Issues.", *NBER Working Paper*, n° 7395.

[18] Mc Callum, Bennett T., 1999, "Role of the Minimal State Variable Criterion in Rational Expectation Models.", *mimeo.*

[19] Mc Callum, Bennett T., and Edward Nelson, 2000, "Timeless Perspective vs. Discretionary Monetary Policy in Forward Looking Models.", *NBER Working Paper*, n° 7915.

[20] Sbordone, Argia, 2002. "Prices and Unit Labor Costs: A New Test of Price Stickiness." *Journal of Monetary Economics, 49, pp. 265-292.*

[21] Söderlind, Paul, 1999, "Solution and Estimation of RE Macromodels with Optimal Policy.", *European Economic Review*, 43, pp.813-823.

[22] Söderströrm, Ulf, Paul Söderlind, and Anders Vredin, 2003, "New-Keynesian Models and Monetary Policy: A Reexamination of the Stylized Facts.", mimeo.

[23] Steinsson, Jón, 2003, "Optimal Monetary Policy in an Economy with Inflation Persistence.", *Journal of Monetary Economics*, 50, pp.1425-1456.

[24] Wall, Howard J., and Gylfi Zoega, 2003, "U.S. Regional Business Cycles and the Natural Rate of Unemployment", *Working Paper*, Federal Reserve Bank of St. Louis.

[25] Walsh, Carl E., 2003, "Monetary Theory and Policy.", MIT Press, 2nd edition.

[26] Woodford, Michael, 1999, "Optimal Monetary Policy Inertia.", *NBER Working Paper*, n° 7261.

[27] Woodford, Michael, 2003, "Interest and Prices: Foundations of a Theory of Monetary Policy.", Princeton University Press.

[28] Yun, Tack, 1996, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles.", *Journal of Monetary Economics*, 37, pp.345-370.

6 Appendix

6.1 Solutions

6.1.1 Discretion

The first order conditions for i_t, π_t and y_t are respectively given by:

$$\begin{aligned} 1/\sigma E_t \gamma_t &= 0, \\ E_t\left(\chi \pi_t + \varphi_t\right) &= 0, \\ E_t\left(\lambda y_t - \psi \varphi_t\right) &= 0. \end{aligned}$$

Notice that the Lagrangian associated with the aggregate demand curve is always equal to zero. As argued in the text, this simply states that equation (2) is irrelevant in determining the equilibrium outcome for π_t and y_t. Combining the last two optimal conditions yields:

$$\pi_t = -\frac{\lambda}{\chi\psi} y_t.$$

The central bank can thus implement the behavior dictated by the first order conditions in an exact manner. Replacing this expression in the Phillips curve, we have:

$$-\frac{\lambda}{\chi\psi} y_t = \beta E_t \left(-\frac{\lambda}{\chi\psi} y_{t+1}\right) + \psi y_t + \varepsilon_t^s,$$

$$(1 + \frac{\chi\psi^2}{\lambda}) y_t = \beta E_t y_{t+1} - \frac{\chi\psi}{\lambda} \varepsilon_t^s.$$

To solve this equation, guess a solution of the form: $y_t = e\ \varepsilon_t^s$. Since the shocks is assumed AR(1), we have that $E_t y_{t+1} = \rho^s\ e\ \varepsilon_t^s$. We then have:

$$e = -\frac{\chi\psi}{\lambda\left[1 - \beta\rho^s\right] + \chi\psi^2}.$$

Equilibrium output and inflation are thus given by:

$$y_t = e\ \varepsilon_t^s,$$

and

$$\pi_{t+i} = f\ \varepsilon_t^s.$$

where $f = \frac{\lambda}{\lambda[1-\beta\rho^s]+\chi\psi^2}$. The short term interest rate is now

$$\begin{aligned}
i_t &= \sigma(E_t y_{t+1} - y_t) + E_t \pi_{t+1} - \varepsilon_t^i + \sigma \varepsilon_t^d, \\
&= \sigma(-\frac{\rho^s \chi \psi}{\lambda\left[1-\beta\rho^s\right] + \chi\psi^2}\varepsilon_t^s + \frac{\chi\psi}{\lambda\left[1-\beta\rho^s\right] + \chi\psi^2}\varepsilon_t^s) + \frac{\rho^s \lambda}{\lambda\left[1-\beta\rho^s\right] + \chi\psi^2}\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d, \\
&= \frac{\rho^s\lambda + (1-\rho^s)\chi\psi\sigma}{\lambda\left[1-\beta\rho^s\right] + \chi\psi^2}\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d.
\end{aligned}$$

6.1.2 Commitment

In this case the first order conditions are given by:

$$\begin{aligned}
1/\sigma E_t \gamma_{t+i} &= 0, \qquad i \geq 0, \\
E_t\left(\chi\pi_{t+i} + \varphi_{t+i} - \varphi_{t-1+i}\right) &= 0, \qquad i \geq 1, \\
E_t\left(\lambda y_{t+i} - \psi\varphi_{t+i}\right) &= 0, \qquad i \geq 0.
\end{aligned}$$

The first order condition for the start up inflation value (when $i = 0$) is $\chi\pi_t + \varphi_t = 0$. This introduces some inconsistency in the commitment approach. To see this, note that at time t, the central bank would set $\pi_t = -\frac{1}{\chi}\varphi_t$ and promises then to set $\pi_{t+i} = -\frac{1}{\chi}(\varphi_{t+i} - \varphi_{t+i-1})$ in the future. But when the central bank arrives in period $t+i$, it will prefer to set $\pi_{t+i} = -\frac{1}{\chi}\varphi_{t+i}$, as its optimization plan would suggest. The *timeless perspective* approach circumvents this problem by assuming that the optimal commitment policy has been chosen in the past and that current values of inflation and output gap satisfy the second first order condition above. This means that we abstract from the start up condition above.[31] To motivate this choice, we think of commitment as a policy regime that is effective, and understood by rational agents as such, for a sufficiently long period of time before initial conditions are set. We perceive this behavior as economically relevant for the issue at stake.[32] By combining the last two equation, we easily obtain:

$$\pi_{t+i} = -\frac{\lambda}{\chi\psi}(y_{t+i} - y_{t-1+i}). \tag{20}$$

Note that this equation holds also in realized values because when period $t+i$ comes, the central bank can observe by assumption both π_{t+i} and y_{t+i}. Using equation (11) back in the Phillips Curve, we obtain an expectational linear difference equation for the output gap of the form:

$$-\frac{\lambda}{\chi\psi}(y_t - y_{t-1}) = \beta E_t\left[-\frac{\lambda}{\chi\psi}(y_{t+1} - y_t)\right] + \psi y_t + \varepsilon_t^s,$$

[31] Hence, throughought the text, we will loosely speak of commitment as a *timeless perspective* policy

[32] Mc Callum and Nelson (2000) argue for instance that commitment is convincing when one is concerned with macroeconomic performance "...within and across [policy] regimes,....".

rearranging, we finally get:

$$(1+\beta+\frac{\chi\psi^2}{\lambda})y_t = \beta E_t y_{t+1} + y_{t-1} - \frac{\chi\psi}{\lambda}\varepsilon_t^s. \tag{21}$$

Now conjecture a solution of the form $y_t = ay_{t-1} + b\varepsilon_t^s$. Then, we have $E_t y_{t+1} = a^2 y_{t-1} + (a+\rho)b\varepsilon_t^s$. Plug this back in (12), the equation becomes:

$$(1+\beta+\frac{\chi\psi^2}{\lambda})(ay_{t-1}+b\varepsilon_t^s) = \beta\left[a^2 y_{t-1} + (a+\rho^s)b\varepsilon_t^s\right] + y_{t-1} - \frac{\chi\psi}{\lambda}\varepsilon_t^s.$$

By rearranging, one finally gets,

$$\left[\beta a^2 - (1+\beta+\frac{\chi\psi^2}{\lambda})a + 1\right] y_{t-1} + \left[\beta(a+\rho^s)b - \frac{\chi\psi}{\lambda} - b(1+\beta+\frac{\chi\psi^2}{\lambda})\right]\varepsilon_t^s = 0.$$

The solution for b to the above equation is easily obtained as one gets,

$$b = -\frac{\chi\psi}{\lambda\left[1+\beta\left(1-a-\rho^s\right)\right]+\chi\psi^2}.$$

Similarly, a unique solution for a is obtained by solving the equation $\beta\lambda a^2 - (\lambda + \lambda\beta + \chi\psi^2)a + \lambda = 0$, and imposing $a < 1$ for stability, which gives:

$$a = \frac{\lambda+\lambda\beta+\chi\psi^2 - \sqrt{\left(\lambda+\lambda\beta+\chi\psi^2\right)^2 - 4\beta\lambda^2}}{2\lambda\beta}.$$

By using the above solutions in our conjectured output gap dynamic equation and plugging it back into (11), we obtain:

$$\pi_t = cy_{t-1} + d\varepsilon_t^s, \tag{22}$$

where $c = \frac{\lambda}{\chi\psi}(1-a)$, and $d = \frac{\lambda}{\lambda[1+\beta(1-a-\rho^s)]+\chi\psi^2}$. Finally,

$$\begin{aligned} i_t &= \sigma(E_t y_{t+1} - y_t) + E_t\pi_{t+1} - \varepsilon_t^i + \sigma\varepsilon_t^d, \\ &= \sigma\left[a^2 y_{t-1} + (a+\rho^s)b\varepsilon_t^s - ay_{t-1} - b\varepsilon_t^s\right] + acy_{t-1} + (cb+\rho^s d)\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d, \\ &= a\left[\sigma(a-1)+c\right]y_{t-1} + \left[\sigma b\left(a+\rho^s-1\right) + (cb+\rho^s d)\right]\varepsilon_t^s - \varepsilon_t^i + \sigma\varepsilon_t^d. \end{aligned}$$

6.2 Autocorrelations

In what follows, we assume covariance stationarity of all variables. $\sigma_y(k)$ denotes $cov(y_t, y_{t+k})$ and $\sigma_{y\varepsilon}(k)$ denotes $cov(y_t, \varepsilon_{t+k})$. Also, to alleviate the notation we use $\rho^s = \rho$.

$$\begin{aligned}
\sigma_y(0) &= a^2\ var(y_t) + b^2 var(\varepsilon_t) + 2ab\ cov(y_{t-1}, \varepsilon_t) \\
&= a^2\sigma_y(0) + b^2\sigma_\varepsilon(0) + 2ab\rho\ cov(y_{t-1}, \varepsilon_{t-1}), \\
\sigma_y(0) &= \frac{1}{1-a^2}\left[b^2\sigma_\varepsilon(0) + 2ab\rho\sigma_{y\varepsilon}(0)\right], \\
&= \frac{b^2}{1-a^2}\left(1 + \frac{2a\rho}{1-a\rho}\right)\sigma_\varepsilon(0) = \frac{b^2}{1-a^2}\left(\frac{1+a\rho}{1-a\rho}\right)\sigma_\varepsilon(0).
\end{aligned}$$

The last equality is obtained by noting that $\sigma_{y\varepsilon}(0) = cov(y_t, \varepsilon_t) = cov(ay_{t-1} + b\varepsilon_t, \varepsilon_t) = b\sigma_\varepsilon(0) + cov(ay_{t-1}, \varepsilon_t) = b\sigma_\varepsilon(0) + cov(ay_{t-1}, \rho\varepsilon_{t-1} + \xi_t) = b\sigma_\varepsilon(0) + a\rho\sigma_{y\varepsilon}(0)$. From our stationarity assumption, we have then $\sigma_{y\varepsilon}(0) = \frac{b\sigma_\varepsilon(0)}{1-a\rho}$. For the covariance we have,

$$\begin{aligned}
\sigma_y(k) &= a^2\sigma_y(k) + b^2\sigma_\varepsilon(k) + ab\left[cov(y_{t-1}, \varepsilon_{t+k}) + cov(y_{t+k-1}, \varepsilon_t)\right] \\
&= a^2\sigma_y(k) + b^2\rho^k\sigma_\varepsilon(0) \\
&\quad + ab\left[\rho^{k+1}cov(y_{t-1}, \varepsilon_{t-1}) + cov(a^{k-1}y_t + b(\sum_{i=1}^{k-1}\rho^i a^{k-1-i})\varepsilon_t, \varepsilon_t)\right],
\end{aligned}$$

thus,

$$\begin{aligned}
\sigma_y(k) &= \frac{1}{1-a^2}\left[\left(b^2\rho^k + ab^2\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\right)\sigma_\varepsilon(0) + ab(\rho^{k+1} + a^{k-1})\sigma_{y\varepsilon}(0)\right], \\
&= \frac{1}{1-a^2}\left[\left(b^2\rho^k + ab^2\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\right)\sigma_\varepsilon(0) + ab(\rho^{k+1} + a^{k-1})\frac{b\sigma_\varepsilon(0)}{1-a\rho}\right], \\
&= \frac{b^2}{1-a^2}\left[\rho^k + a\sum_{i=1}^{k-1}\rho^i a^{k-1-i} + \frac{a(\rho^{k+1} + a^{k-1})}{1-a\rho}\right]\sigma_\varepsilon(0).
\end{aligned}$$

We can write the autocorrelogram for the output gap as,

$$\rho_y(k) = \frac{\sigma_y(k)}{\sigma_y(0)} = \frac{(1-a\rho)\left(\rho^k + a\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\right) + a(\rho^{k+1} + a^{k-1})}{1+a\rho}.$$

Using the same method for inflation, we obtain:

$$\begin{aligned}
\sigma_\pi(0) &= c^2\sigma_y(0) + d^2\sigma_\varepsilon(0) + 2cd\rho\sigma_{y\varepsilon}(0), \\
&= \frac{c^2b^2}{1-a^2}\frac{1+a\rho}{1-a\rho}\sigma_\varepsilon(0) + d^2\sigma_\varepsilon(0) + 2cd\rho\frac{b\sigma_\varepsilon(0)}{1-a\rho}, \\
&= \frac{c^2b^2(1+a\rho) + d^2(1-a^2)(1-a\rho) + 2bcd\rho(1-a^2)}{(1-a^2)(1-a\rho)}\sigma_\varepsilon(0).
\end{aligned}$$

$$
\begin{aligned}
\sigma_\pi(k) &= c^2\sigma_y(k) + d^2\rho^k\sigma_\varepsilon(0) + cd\left[(\rho^{k+1}+a^{k-1})\sigma_{y\varepsilon}(0) + b\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\sigma_\varepsilon(0)\right],\\
&= \frac{b^2c^2}{1-a^2}\left[\rho^k + a\sum_{i=1}^{k-1}\rho^i a^{k-1-i} + \frac{a(\rho^{k+1}+a^{k-1})}{1-a\rho}\right]\sigma_\varepsilon(0)\\
&\quad + d^2\rho^k\sigma_\varepsilon(0) + cd\left[(\rho^{k+1}+a^{k-1})\frac{b\sigma_\varepsilon(0)}{1-a\rho} + b\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\sigma_\varepsilon(0)\right],\\
&= \left[\begin{array}{c} \frac{b^2c^2\left[\left(\rho^k + a\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\right)(1-a\rho)+a(\rho^{k+1}+a^{k-1})\right]}{(1-a^2)(1-a\rho)} \\ +\frac{d^2\rho^k\left(1-a^2\right)(1-a\rho)+bcd\left[(\rho^{k+1}+a^{k-1})\left(1-a^2\right)+\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\left(1-a^2\right)(1-a\rho)\right]}{(1-a^2)(1-a\rho)} \end{array}\right]\sigma_\varepsilon(0),
\end{aligned}
$$

$$
\begin{aligned}
\rho_\pi(k) &= \frac{1}{c^2b^2(1+a\rho)+d^2(1-a^2)(1-a\rho)+2bcd\rho(1-a^2)}\\
&\quad\left\{\begin{array}{c} b^2c^2\left[\left(\rho^k + a\sum_{i=1}^{k-1}\rho^i a^{k-1-i}\right)(1-a\rho)+a(\rho^{k+1}+a^{k-1})\right] \\ +d^2\rho^k(1-a^2)(1-a\rho) \\ +bcd\left[(\rho^{k+1}+a^{k-1})(1-a^2)+\sum_{i=1}^{k-1}\rho^i a^{k-1-i}(1-a^2)(1-a\rho)\right] \end{array}\right\}
\end{aligned}
$$

6.3 The Model in State Space Form

First write:

$$
\begin{aligned}
x_{1t} &= \begin{bmatrix}\pi_{t-1} & \pi_{t-2} & \pi_{t-3} & \pi_{t-4} & y_{t-1} & y_{t-2} & y_{t-3} & y_{t-4} & i_{t-1} & i_{t-2} & i_{t-3} & i_{t-4} & \varepsilon_t^i & \varepsilon_t^s & \varepsilon_t^d\end{bmatrix}',\\
x_{2t} &= \begin{bmatrix}\pi_t & y_t\end{bmatrix}',\\
\xi_{t+1} &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \xi_t^i & \xi_t^s & \xi_t^d\end{bmatrix}',\\
u_t &= i_{t.}
\end{aligned}
$$

The model can then be written as

$$
x_{t+1} = Ax_t + Bu_t + \epsilon_{t,}
$$

where

$$A=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\rho^i&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\rho^s&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\rho^y&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{\beta}&0&\frac{1}{\beta}&-\frac{\kappa}{\beta}(\sigma+\eta)\\
0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{\sigma}&\frac{1}{\beta\sigma}&-1&-\frac{1}{\beta\sigma}&1+\frac{\kappa}{\beta\sigma}(\sigma+\eta)
\end{bmatrix},$$

and

$$B=[\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1/\sigma\ ]',$$

are the matrices containing the structural parameters of the model. We denote Σ_ϵ the covariance matrix of ϵ_t which is diagonal, and $x_{t+1} = [x_{1t+1}\ E_t x_{2t+1}]'$, $\epsilon_t = [\xi_t\ \mathbf{0}_{n2\times 1}]'$.[33]

6.3.1 Solving under optimal policy

To solve the central bank optimization problem, first recall that our central bank loss function is given by

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t x_t' Q x_t,$$

where Q is a symmetric matrix. Since we assume the central bank is able to commit to a constant policy rule, its optimization problem can be stated as the following Lagrangian

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t' Q x_t + 2\phi_{t+1}' [A x_t + B u_t + \epsilon_t - x_{t+1}] \right\}.$$

[33] The additional lags in y, π and i appearing in x_{1t} are needed to calculated the unconditional moments below.

The solution for the commitment case can be expressed in a reduced form as:[34]

$$\begin{bmatrix} x_{1t+1} \\ \phi_{2t+1} \end{bmatrix} = M \begin{bmatrix} x_{1t} \\ \phi_{2t} \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0} \end{bmatrix}, \tag{23}$$

with x_{10} given and $\phi_{20} = \mathbf{0}$, and

$$\begin{bmatrix} x_{2t} \\ u_t \\ \phi_{1t} \end{bmatrix} = C \begin{bmatrix} x_{1t} \\ \phi_{2t} \end{bmatrix}. \tag{24}$$

This state space representation is estimated by transposing it into a Kalman Filter framework used to build up the likelihood function of the data. Write the state space model transition equation as

$$X_{s,t+1} = M \ X_{s,t} + \epsilon_t. \tag{25}$$

The measurement equation is,

$$X_{m,t} = C_1 \ X_{s,t}, \tag{26}$$

where $X_{s,t} = [x_{1t} \quad \phi_{2t}]'$ and $X_{m,t} = [x_{2t} \quad u_t]'$.[35] Note that ϕ_{1t} has disappeared from the measurement equation since it does not add any useful information to the dynamics of the forward looking endogenous variables.[36] At each guess of the parameter vector, the solution algorithm gives an optimal policy rule and a system of linear difference equations for the model variables i.e. our state space model. To obtain unconditional moments of the state and endogenous variables simply write:

$$\Sigma_{X_s} = M \ \Sigma_{X_s} M' + \Sigma_\epsilon,$$

and the solution is:

$$vec(\Sigma_{X_s}) = (I - M \quad M)^{-1} vec(\Sigma_\epsilon).$$

where Σ_{X_s} is the variance covariance matrix of the states. Finally, we have $\Sigma_{X_m} = C_1 \ \Sigma_{X_s} \ C_1'$. A similar solution procedure can be applied under discretionary optimal policy.[37]

[34] See Söderlind (1999) for details.

[35] In our estimated system, we add in the transition and measurement equation two vectors of constants c and d respectively. Also a vector of shocks η_t is added to the measurement equation (with covariance Σ_η).

[36] We thus eliminate the rows in C that are unnecessary. This yields C_1.

[37] See Söderlind (1999) or Söderström *et al.* (2003) for an exposition.

Figure 1: Impulse responses with $\psi = 0.2$ and $\lambda = 0.2$

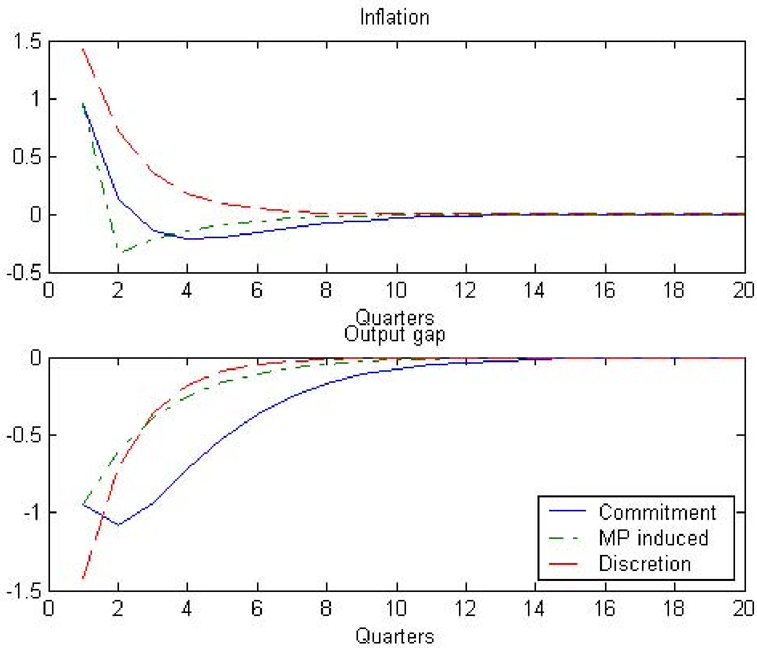

Figure 2: Impulse responses with $\psi = 0.02$ and $\lambda = 0.2$

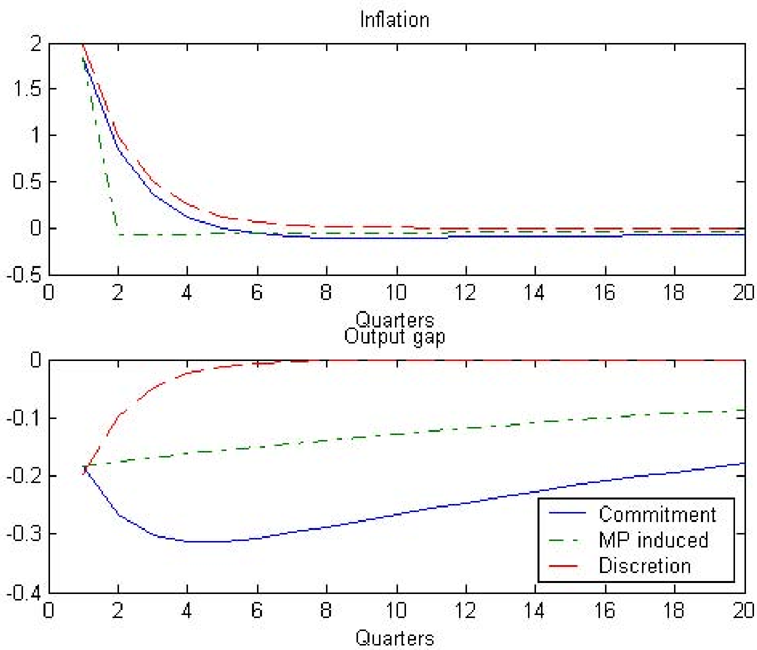

Figure 3: Impulse responses with $\psi = 0.2$ and $\lambda = 0.6$

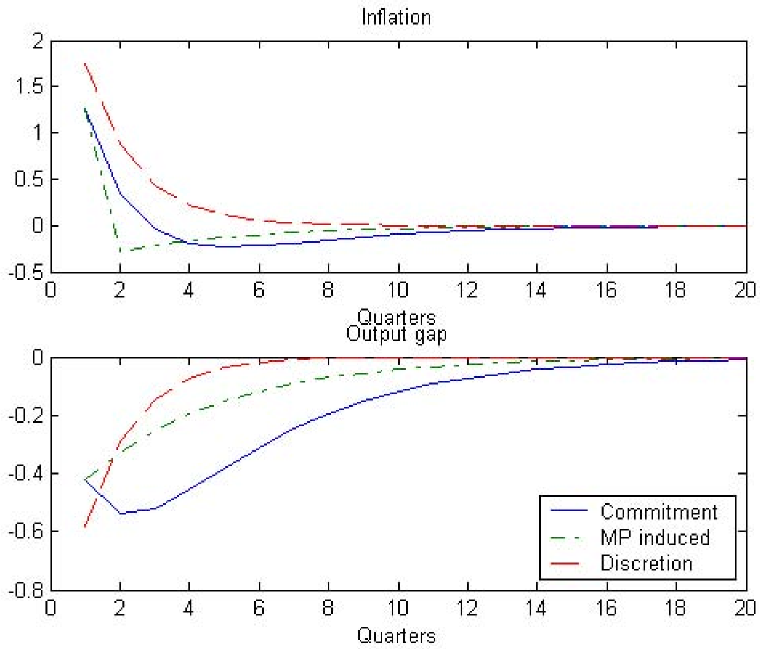

Figure 4: Parameter a (Output Persistence)

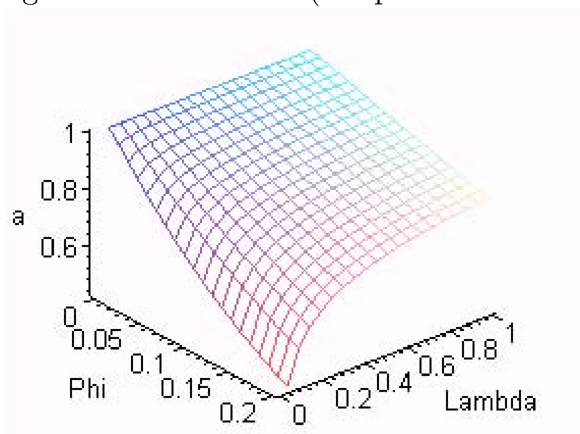

Figure 5: Parameter c

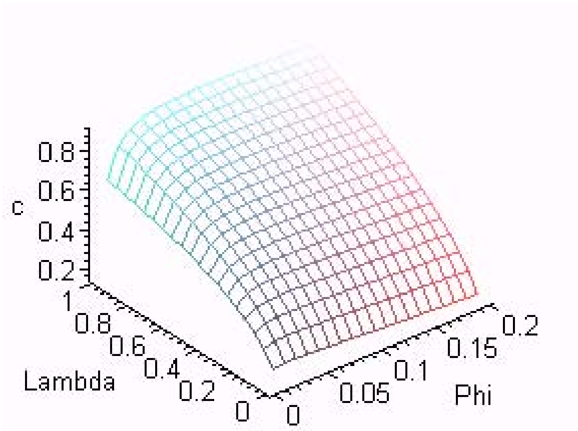

Figure 6: Inflation Persistence

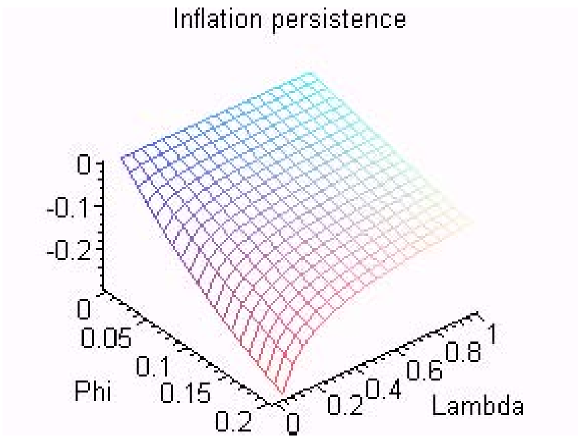

Figure 7: Data series 1987Q4-1999Q4

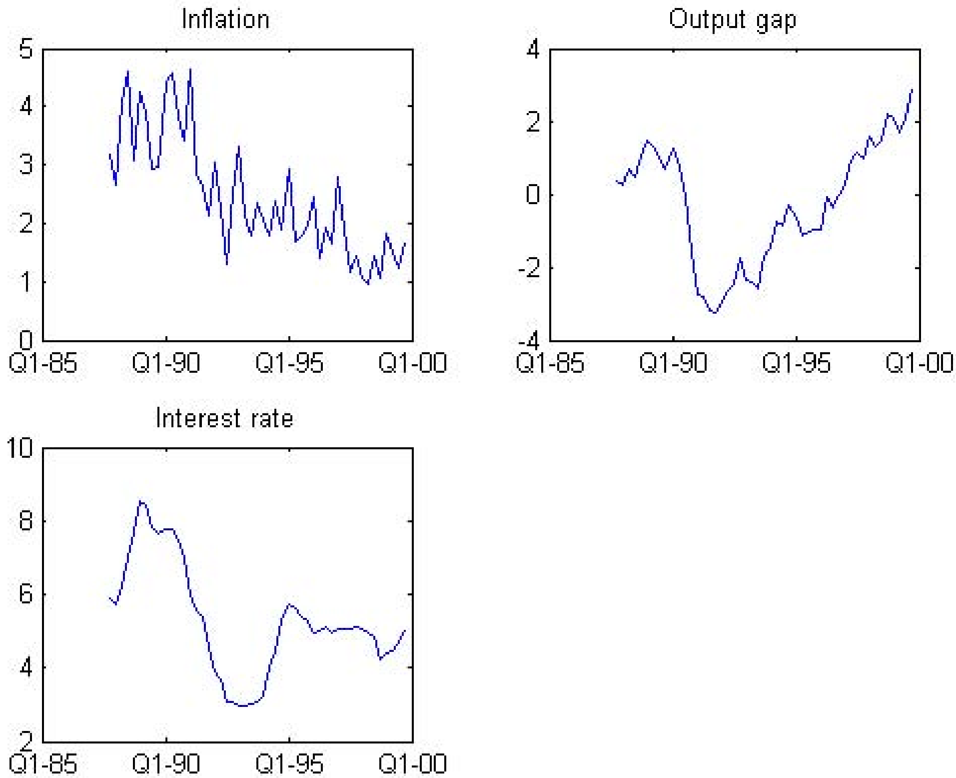

Figure 8: Impulse Responses to a Supply Shock: Commitment Policy

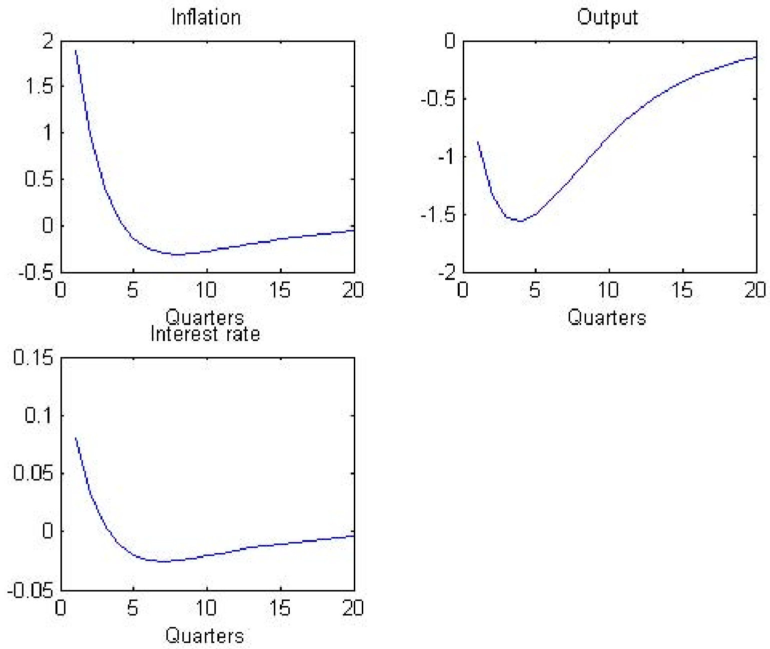

Figure 9: Smoothed Estimate of Supply Shock