Testing for Rational Bubbles with Time Varying Risk Premium and Non-Linear Cointegration: Evidence from the US and French Stock Markets

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Abstract:
This article examines the empirical validity of the present value model with time varying risk premium for the US and French stock markets. We apply a momentum threshold autoregressive (MTAR) procedure (Enders and Granger, 1998; Enders and Siklos, 2001) designed to detect asymmetric short-run adjustments to the long run equilibrium. Our results indicate that without a large conception of fundamental (i.e. earnings instead of dividends) and proxies for the time-varying risk premium, classical tests conclude that the US stock prices exhibit explosive rational bubble (Blanchard and Watson, 1982). However, our findings provide support for the hypothesis that, in the short-run, US stock prices (1871-2002) and French stock prices (1951-2002) exhibit run-ups followed by crashes while, in the long-run, stock prices are attracted by fundamentals.

JEL Classification: G12, C22, C52, E44.

Keywords: Asymmetric Adjustment, Threshold Autoregressive Model, Present Value Model, Cointegration, equity risk premium.

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1.Introduction

The rise and fall in stock prices in recent years and the development of new cointegration procedures (non-linear and fractional cointegration) has drawn renewed attention to the possible existence of rational bubbles. A speculative bubble is usually defined as the difference between the market value of a security and its fundamental value. The standard definition of fundamental value is the summed discounted value of all future cash flows (the present value model). Theoretical literature provides alternative explanations for the existence of speculative bubbles on asset markets. This literature is generally classified into three classes: i) rational bubbles; (ii) irrational bubbles, fads, and noise traders and (iii) inefficiencies that are due to imperfect and heterogeneous information.

The theory of "rational bubbles" (e.g. Blanchard, 1979; Blanchard and Watson, 1982) demonstrates that, even with rational expectations in the sense of Muth (1961), asset price deviations from the fundamental value would be possible. The growth of rational bubbles reflects the presence of self-fulfilling expectations about future increases in the asset price. They would be a feature of a market in which an investor purchases an asset solely in anticipation that it could be resold at a higher price to another investor willing to purchase the asset for the same reason.

There are several problems with this formulation. First, rational bubbles have explosive conditional expectations that imply a negative-bubble cannot exist. Second, the rational bubble can thus never disappear and reappear which seems in contradiction with the successions of booms and crashes observed in stock markets (Diba and Grossman, 1988b). At last but not least, we only know the dynamics of the bubble and not what gives causes it, the bubble exists at the issue of the financial asset.

New models of rational bubble have been developed at the beginning of the 1990's to consider the criticism of Diba and Grossman (1988b): the intrinsic bubbles (Froot and Obstfeld, 1991; Ikeda and Shibata, 1992), and the periodically collapsing bubble (Evans, 1991). These new classes of rational bubbles share a common characteristic: they are consistent with observed stock price dynamics as they diminish periodically instead of diverge continuously.

The "noise trader" approach rejects the assumption of agents homogeneity (e.g. Summers, 1986; De Long et al., 1990a, 1990b, 1991). One category comprises traditional rational investors. Their opinions and trading decisions are based on economic fundamentals and all available information. In the second category, the "noise traders" buys or sells assets based on erroneous beliefs[1]. The interactions between these various actors might induce deviations of asset prices from their fundamental value.

The last class of models focuses on incomplete or imperfect information as the main factor explaining deviations of observed asset prices from the fundamental value. These deviations are due to informational asymmetry and the crash is implied by information cascade or herd behaviors (e.g. Avery and Zemsky, 1998; Bikchandani, Hirsheilfer and Welch, 1992).

Numerous studies have empirically tested the existence of rational bubbles in diverse asset markets: in stock markets (e.g. West, 1987; Craine, 1993), in exchange rate markets (e.g. Meese, 1986; West, 1987), in housing markets (e.g. Abraham and Hendershott, 1996; Kim and Suh, 1993). These studies provide mixed results concerning the evidence of rational bubbles.

The occurrence of price bubbles in experimental financial markets is less controversial. Smith, Suchanek, and Williams (1988) first reported bubbles in experimental financial markets. They investigate an experimental asset market where an asset is traded that pays a dividend in each of the 15 consecutive periods. The dividend payment is identical for all traders and the distribution of dividends is common knowledge to all traders. Participants differ only in their endowments of the number of stocks and the amount of money, but there is no asymmetric information. In this setting trading, the time series of transaction prices in markets is characterized by large upward deviations in prices from fundamental value followed by crashes back to the fundamental value. This finding has been corroborated in many other asset market experiments, with varying designs (e.g. King, Smith, Williams and van Boening, 1993; Porter and Smith, 1995; Noussair, Robin and Ruffieux, 2001; Smith, van Boening and Wellford, 2000)[2].

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[1] "Noise traders falsely believe that they have special information about the future price of the risky asset. They may get their pseudosignals from technical analysts, stockbrokers, or economic consultants and irrationally believe that these signals carry information" (De Long et al., 1990b, p. 706).

[2] King, Smith, Williams, and Van Boening (1993) investigate whether bubbles are moderated by several treatment variables including the ability to short sell, margin purchases, the presence of brokerage fees, equal endowments across traders, a subset of informed traders, limit price change rules, design experience, and experience in the business world.. Porter and Smith (1995) investigated the effect of futures markets and of eliminating all uncertainty about the dividend payment. Though futures markets attenuated bubbles somewhat, certainty in dividends had no effect. The fact that the elimination of uncertainty about dividends does not reduce bubbles reveals that they are not a result of heterogeneous risk attitudes on the part of subjects. Noussair et al. (2001) showed that bubbles and crashes also occur when the fundamental value is constant over time. Lei, Noussair and Plott (2001) show that, even if speculation is prohibited (that is, a subject can only buy or only sell the asset, but subjects are not able to do both in order to reap the capital gains), bubbles occur. The only factor that has been shown to eliminate bubbles in experimental asset markets is previous experience. If all subjects have participated previously in at least two sessions in markets with exactly the same structure, bubbles usually do not occur (Noussair and Ruffieux, 2002).

The empirical investigation of the existence of explosive rational bubble knows two limits. First, Evans (1991) shows that unit root and cointegration tests, which are usually used, are unable to detect periodically collapsing bubbles. This class of bubble generate non-linear series which do not follow an enough explosive process to be detected by traditional unit roots and cointegration tests.

Therefore, apparent evidence of bubbles in the data can always be interpreted as misspecification of the model. Indeed, a major problem with bubble tests is that evidence for bubbles can be reinterpreted in terms of market fundamentals that are unobserved by researchers as argued by Flood and Garber (1980) and Hamilton and Whiteman (1985). Evidence of non-stationarity or excessive fluctuation of residuals between prices and dividends could result from sources other than bubbles as time-varying risk premium.

The aim of this paper is to examine the empirical validity of the present value model with time-varying equity premium for the US and French stock markets. We apply a momentum threshold autoregressive procedure (Enders and Granger, 1998; Enders and Siklos, 2001) designed to detect asymmetric short-run adjustments to the long run equilibrium. Bohl and Siklos (2004) suggest to apply this technique to detect periodically collapsing bubbles defined by Evans (1991)[3]. Additionally, in order to consider additional sources of fluctuation of the fundamental value, we augmented the cointegrating regression by including additional variables as proxies for a time-varying risk premium.

The following section reviews the present value model with time varying discount rate and the conventional testing procedure for explosive rational bubbles based on the unit-root/cointegration approach. Section 3 describes the econometric procedure used. Data description and unit roots tests are provided in Section 4. Section 5 contains empirical results. Section 6 provides some concluding comments.

2. The Present Value Model and Rational Bubbles

The basic framework of our analysis is the present value model ("the discounted cash-flow model"):

[3] Alternative procedures to testing for the presence of speculative bubbles of the form described by Evans (1991) have been suggested in the literature. Hall, Psaradakis and Sola (2002) proposed a Markov-Swithching stationarity test. Taylor and Peel (1998) proposed a residuals-augmented least squares (RALS) Dickey-Fuller test. Van Norden and Schaller (1999) used switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental value.

$$P_t = E_t\left[\sum_{i=1}^{K}(1+R)^{-i} D_{t+i}\right] + E_t\left[(1+R)^{-K} P_{t+K}\right], \tag{1.1}$$

where P_t is the stock price at t, D_t is the dividend paid to the owner of the stock between t and $t+1$, and R is the expected return (or discount rate) composed of the risk free rate and a risk premium. E_t denotes the conditional expectations operator. Under the transversality condition that:

$$\lim_{T\to\infty}(1+R)^{-T} E_t P_{t+T} = 0, \tag{1.2}$$

the unique forward-looking market fundamental solution to (1.1), P_t^f, is given by:

$$P_t^f = \sum_{i=1}^{+\infty} E_t D_{t+i}(1+\gamma)^{-i} . \tag{1.3}$$

If the transversality condition (1.2) does not hold, the general solution to (1.1) has the following form:

$$P_t = P_t^f + B_t, \tag{1.4}$$

where P_t^f is the fundamental value given by (1.3) and B_t is an explosive rational speculative bubble[4] generated by self-fulfilling expectations that satisfies:

$$E_t B_{t+i} = (1+R)^i B_t \tag{1.5}$$

If there is no bubble, i.e. the transversality condition hold, $P_t = P_t^f$. Although both P_t and D_t are nonstationary, then, their linear combination should be stationary and the two series are cointegrated (e.g. Campbell and Shiller, 1987; Diba and Grossman, 1988a; Hamilton and Whiteman, 1985).

[4] The bubble can take various forms, such as a deterministic bubble and a stochastic bubble. See Flood and Hodrick (1990) for further detail.

This approach is typically done by first applying unit-root tests to P_t and D_t to verify their non-stationarity. Then, a cointegrating regression between the stock prices and the dividends is performed to obtain the residual process. Applying the unit-root tests on the residuals then tests the existence of a bubble. If the residual process contains a unit-root, then P_t and D_t are not cointegrated, and we conclude that the stock prices contain an explosive rational bubble. If the residual process from the cointegration regression does not have a unit-root, we reject the null hypothesis and conclude that there is no explosive rational bubble.

In recent years, there have been a number of empirical studies on testing for speculative bubbles in the US stock market. Cointegration tests between stock prices and dividends and unit root test in the dividend price ratio give mixed and ambiguous findings depending on the implemented test, its specification, the sample period and the frequency of the data (e.g. Campbell and Shiller, 1987; West, 1987 ; Diba and Grossman, 1988a ; Froot and Obstfeld, 1991 ; Craine, 1993 ; Lamont, 1998).

However, the capability of this procedure to detect bubbles depends on the assumption that an autoregressive process can characterize the bubble process with a constant coefficient around unity. By conducting a Monte Carlo simulation, Evans (1991) showed that collapsing speculative bubbles are not detected by the traditional cointegration approach based on unit root and stationary tests. One reason for this result is that collapsing bubbles are not conventional unit-root processes, so that the hypothesis of a bubble is not equivalent to the hypothesis of a unit root[5]. These bubbles never completely disappear but they diminish periodically as they reach a threshold.

Therefore, Flood and Garber (1980) and Hamilton and Whiteman (1985) pointed out that speculative bubbles are observationally equivalent to movements in unobserved fundamentals governing dividends and discount factors. Thus, evidence of non-stationarity does not necessarily establish the existence of bubbles. Time-varying risk premium, which are not traditionally consider in bubble tests, might be an alternative source of excessive fluctuation. This is why our empirical analysis is based on the present value model with time-varying risk premium.

Campbell and Shiller (1988a,b) propose a log-linear approximation of the present value framework that enables us to investigate stock price behavior under model of a time-varying discount rate. Under the condition of no rational bubble, (1.1) becomes:

[5] Charemza and Deadman (1995) show that this problem extends to a broader range of bubble processes than those considered by Evans (1991).

$$p_t = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j \left[(1-\rho)d_{t+1+j} - r_{t+1+j}\right]\right], \tag{1.6}$$

where p_t denotes the log of the stock price, d_t the log of the dividends and r_t the log of the time-varying discount rate. ρ and k are linearization parameters defined by $\rho = 1/(1+\exp(d-p))$ and $k = -\log(\rho) - (1-\rho)\log\left(\frac{1}{\rho} - 1\right)$.

Rewriting equation (1.3) in terms of the log dividend-price ratio, and imposing the transversality condition, leads to:

$$d_t - p_t = -\frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j (-\Delta d_{t+1+j} + r_{t+1+j})\right]. \tag{1.7}$$

Under the assumptions that the dividend growth and the discount rate in logarithms follow a stationary process, the log stock price and the log dividends are cointegrated, and the log dividend-price ratio follows a stationary process (Campbell and Shiller, 1988a,b). But if we suppose a time varying-discount rate, the present value model does not generally imply the existence of a stationary relationship between dividends and stock price (Timmermann, 1995).

3. The MTAR Procedure

Standard models of cointegrated variables assume linearity and symmetric adjustment. However, these cointegration tests are misspecified if adjustment is asymmetric. In recent research, Enders and Granger (1998) have extended the Dickey-Fuller (1979) testing procedure to allow the unit root hypothesis to be tested against an alternative of asymmetric stationarity. Consider the Dickey-Fuller test in its simplest form:

$$\Delta\mu_t = \rho\mu_{t-1} + \varepsilon_t, \tag{2.1}$$

where ε_t is a white-noise process. The procedure consists on testing the null hypothesis of ρ = 0. This type of test is used in the Engle and Granger (1987) cointegration test where μ_t are the estimated residuals from a long-run equilibrium relationship. Rejecting the null hypothesis of no cointegration implies that μ_t are stationary with mean zero. This is an implicit symmetric specification since the long-run equilibrium relationship is an attractor that its pull is strictly proportional to the absolute value of μ_{t-1}.

To allow for the possibility of asymmetric stationarity, Enders and Granger (1998) extend (2.1) by drawing upon the threshold autoregressive method of Thong (1990). Following this approach, the resulting generalization of (2.1) is given as:

$$\Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t)\rho_2 \mu_{t-1} + \varepsilon_t \,, \tag{2.2}$$

where I_t is the zero-one Heaviside indicator function. Engle and Granger consider two specifications for this Heaviside indicator function based upon the sign and difference of y_{t-1}. These rival specifications are given as:

$$I_t = \begin{cases} 1, \text{if } \mu_{t-1} \geq 0 \\ 0, \text{if } \mu_{t-1} < 0 \end{cases} \tag{2.3}$$

and:

$$I_t = \begin{cases} 1, \text{if } \Delta\mu_{t-1} \geq 0 \\ 0, \text{if } \Delta\mu_{t-1} < 0 \end{cases} \tag{2.4}$$

Engle and Granger (1998) refer to the model defined by (2.2) and (2.3) as threshold autoregressive (TAR) model, while a model combining (2.2) and (2.4) is referred to as momentum-threshold autoregressive (MTAR) model in that μ_t exhibits more “momentum” in one direction than the other.

Under both models, the unit root hypothesis ($H_0 : \rho_1 = \rho_2 = 0$) is tested using specifically derived critical values provided by Enders and Granger (1998) in an univariate context and Enders and Siklos (2001) in a multivariate context. The statistic testing the null hypothesis of no unit root (cointegration) is noted Φ. Thus, if the unit-root hypothesis is rejected, it is possible to test for symmetric adjustment ($H_0 : \rho_1 = \rho_2$) using a standard F-test.

Equation (2.2) may not be sufficient to capture the dynamic adjustment of $\Delta\mu_t$ toward its long-run equilibrium value. In order to ensure that the errors approximate a white-noise process, the specification (2.2) is augmented with lagged changes in the μ_t sequence such that it becomes:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1-I_t)\rho_2\mu_{t-1} + \sum_{i=1}^{l}\gamma_i\Delta\mu_{t-i} + \varepsilon_t. \quad (2.5)$$

The value of the threshold is generally unknown and needs to be estimated along with ρ_1 and ρ_2. Enders and Siklos suggest using Chan's grid search method to find a consistent estimate of the threshold. The estimated residuals from the estimated cointegration relationship are sorted in ascending order. The largest and smallest 15% of these values are discarded and each of the remaining 70% of estimated residuals are considered as possible thresholds. For each of these possible thresholds, we estimates by OLS an equation in the form of (2.5) and (2.6) or (2.7). The estimated threshold yielding the lowest residual sum of squares is deemed the appropriate estimate of the threshold. The two specifications for the Heaviside indicator became:

$$I_t = \begin{cases} 1, \text{if } \mu_{t-1} \geq \tau \\ 0, \text{if } \mu_{t-1} < \tau \end{cases} \quad (2.6)$$

and

$$I_t = \begin{cases} 1, \text{if } \Delta\mu_{t-1} \geq \tau \\ 0, \text{if } \Delta\mu_{t-1} < \tau \end{cases} \quad (2.7)$$

where τ denotes the value of the threshold. When the threshold is unknown, critical values of Φ are not the same as in the case where τ equals 0. Note that the Engle and Granger test (1987) is a special case of the MTAR model (2.5) and (2.6) or (2.7) in case of a symmetry in the error correction process.

Bohl and Siklos (2003) suggest that the MTAR technique is designed to detect empirically periodically collapsing bubbles (Evans, 1991). This type of process bubble can, after a period of stability, accelerate in growth, then collapse and finally begin again. These characteristics suggest an asymmetry in the behavior of stock prices and then in the behavior of the valuation ratio (earning-price and dividend-price).

4. Data and Integration order of the series

We test for long-run Present Value Model with time-varying risk premium and asymmetric adjustment using data from France and United States. The data set used in the US stock market estimation consists of the Standard and Poor's 500 composite stock market index and corresponding dividends and earnings. Data were obtained from Shiller (2000) and updated from the Standard and Poor's web site. Primary sources for these data are the Cowles Commission (from 1871 through 1925) and the S&P Composite (from 1926 through 2003). They are available at a monthly frequency on the website of Robert Shiller since 1871. However, these monthly data are computed with linear interpolation from annual data since 1871 and from quarterly data since 1926. So we used two samples according to the original frequency: annual data from 1871 through 2002 and quarterly data from 1926:1 to 2003:2.

Data for the French stock market are extracted from Arbulu (1998)[6] (from 1801 through 1918) and from the INSEE BMS (from 1919 through 2002). The dividend series are computed from the dividend yield and stock prices.

In order to consider additional sources of fluctuation as a time-varying risk premium, we also included in the regression: the consumer price inflation and the GDP volatility when series were available in the different samples. Consumer price index from 1871 to 2003 was obtained from Shiller (2000) and updated from the BLS (Consumer Price Index-All Urban Consumers, not seasonally adjusted). The GDP volatility series are computed from rolling standard deviation of quarterly real GDP growth with a 24 quarters window according to the methodology retained by Blanchard and Simon (2001). This series is only available for the United States since 1953:1 (Quarterly GDP from the BEA are available since 1947:1). The Consumer Price Index for France was obtained from Chabert (1949) from 1801 to 1820, Lévy-Leboyer and Bourguignon (1985) from 1821 to 1913 and the INSEE from 1914 to 2002.

The choice of these two variables is justified by preceding estimates. The GDP volatility represents a macroeconomic risk likely to influence the systematic risk[7]. While the negative

[6] We thank Pedro Arbulu for providing us the data.

relationship between stock prices and inflation was largely documented in the empirical literature (e.g. Fama and Schwert, 1977; Blanchard, 1993) the theoretical explanation is controversial. First, this relationship could reflect an inflation-related risk premium on stocks relative to bonds but it is a common knowledge that stocks are a better hedge against inflation than bonds (Siegel, 1998). Modigliani and Cohn (1979) argued that a form of «money illusion» plagues investors. Investors, on the one hand, would use a nominal rate to discount real dividends and, on the other hand, would underestimate the future dividends while omitting to correct the fall of the actual value of the companies' debt. Another channel, proposed by Fama (1981), suggests that higher inflation induces lower expected real economic activity and/or uncertainty over the conduct of future monetary and fiscal policies, leading investors to demand higher risk premium. Geske and Roll (1983) argue that a reduction in real activity leads to an increase in government deficits. The monetization of government deficit leads directly or indirectly to a higher level of inflation. Then, the inverse relationship between stock returns and inflation reflects changes in real activity conditions. We also suggest that low inflation indicates future directions of the monetary policy and induces a lower expected risk free rate[8].

The considered variables are: the log of stock prices, *p*, the log of earnings, *e*, the log of dividends, *d*, the log of dividend-price, *dp*, the log of earning-price, *ep*, the log of the rate of inflation (log CPI_t/CPI_{t-1}), *cpi*, and the log of GDP volatility, *vol*.

To investigate the order of integration of the variables, we use initially the Augmented Dickey-Fuller (Said and Dickey, 1984) and Kwiatkowsky, Phillips, Schmidt and Shin (1992) tests to detect the presence of a unit root in the series. The number of lagged differences of the series that is included in the ADF regressions to eliminate autocorrelation was selected by the Akaike Information Criteria (AIC). For the KPSS test, we used this number of lags. The ADF

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[7] The risk of holding risky assets is composed of a "systematic" and a "specific" risk. Systematic risk is a non-diversifiable risk. It is a market-wide and pervasively influences virtually all security prices. Specific risk (or idiosyncratic risk) involves unexpected events peculiar to a single security or a limited number of securities.

[8] The negative relation between real returns and inflation may also be related to Lucas (1973) signal extraction problem associated with inflation. In Lucas model, agents have problems assessing whether a price increase is the result of higher demand in a specific market or the result of an overall price inflation. This may result in less efficient resource allocation that translates into higher required real return as inflation goes up, possibly through a higher equity premium. Feldstein (1980) develops a market equilibrium model of share valuation and shows that an increase in steady-state inflation lowers share prices because of the way depreciation costs and capital gains are treated in tax codes. Danthine and Donaldson (1986) develop a monetary model with rational expectations in which the stock market does provide full insurance against monetary shocks but not against temporary inflation induced by real shocks. In their model, real shocks have an effect on consumption goods prices without changing the expected stream of dividends (because of dividends stickiness) and thus have an impact on stock market real returns.

test asserts the variable is I(1) in the null hypothesis while the KPSS test formulates the stationary assumption as the null.

However, it is not possible to use the standard approach for testing unit root, given the low power of the test in the presence of structural breaks (Zivot and Andrews, 1992). To test for structural change in log dividend-price and log earning-price, we use procedure suggested by Perron (1997). The Perron test has an advantage over other unit root tests, which allow for structural breaks, by not requiring the end points of the sample to be trimmed. That is, we undertake estimation without assuming any prior knowledge of any potential break dates. The model is estimated over all possible break dates in the data set, and the break date is chosen to maximize the probability of rejection of the unit root hypothesis. The break tested is a change in the intercept.

These test results for the two stock markets and the different samples are reported in Table 1, 2 and 3. At all frequencies and samples, the ADF test cannot reject the unit-root null hypothesis for *p*, *e*, *d*, *ep*, *dp* and *vol* except for the US log earning-price ratio on the sample 1871-2002. For the variable *cpi*, results are mixed. The ADF t-test can reject the unit-root hypothesis for the inflation rate in France and in United States on the sample periods 1927:01-2003:02 and 1871-2002. The KPSS test can reject the stationary null hypothesis for all variables and samples without exception. However, given the well-known low power of the ADF test and in order to proceed at cointegration tests, we consider the inflation rate as I(1) and the log of the earning-price ratio on the sample 1871-2002 as I(1). The ADF and KPSS tests conclude that in differences, all variables are stationary processes with except for Δd on the French sample 1802-2002.

The test of Perron (1997) that test the stationary of the series with break on the intercept concludes that the log dividend-price and the log earning-price are not stationary processes with break except for the 1802-1950 sample in France. This last finding could be interpreted as a break in the risk premium.

These preliminary results suggest that the US stock prices exhibit explosive rational bubble over the three samples (1871-2002, 1926:1-2003:2 and 1953:1-2003:2). For the French stock market, we can conclude to the existence of a rational bubble only for the sample period 1951-2002.

5. Empirical results

Different cointegration specifications are tested. We provide results from classical Engle-Granger cointegration tests and MTAR models. We used first the classical specification for testing cointegration between stock prices and dividends (Diba and Grossman, 1988a):

$$p_t = \alpha_0 d_t + \alpha_1 + \mu_t . \tag{1}$$

In order to consider another measure of the fundamental, since the dividend-price ratio can be affected by corporate financial policy, we consider the following model:

$$p_t = \alpha_0 e_t + \alpha_1 + \mu_t . \tag{2}$$

To integrate additional sources of fluctuation, we also augmented the precedent cointegrating regression by including the *cpi* variable as a risk premiums proxy:

$$p_t = \alpha_0 d_t + \alpha_1 cpi_t + \alpha_2 + \mu_t \tag{3}$$

and

$$p_t = \alpha_0 e_t + \alpha_1 cpi_t + \alpha_2 + \mu_t , \tag{4}$$

If we impose the restriction $\alpha_0 = 1$, in order to consider the log dividend-price and log earning-price ratios, models (3) and (4) become:

$$(d/p)_t = \alpha_1 cpi_t + \alpha_2 + \mu_t \tag{5}$$

and

$$(e/p)_t = \alpha_1 cpi_t + \alpha_2 + \mu_t . \tag{6}$$

For the sample period 1953:1-2003:2, we also augmented the models (3),(4),(5) and (6) by including *vol*. Then, these four models become:

$$p_t = \alpha_0 d_t + \alpha_1 cpi_t + \alpha_2 vol + \alpha_3 + \mu_t \,, \tag{3'}$$

$$p_t = \alpha_0 e_t + \alpha_1 cpi_t + \alpha_2 vol + \alpha_3 + \mu_t \,, \tag{4'}$$

$$(d/p)_t = \alpha_1 cpi_t + \alpha_2 vol + \alpha_3 + \mu_t \,, \tag{5'}$$

$$(e/p)_t = \alpha_1 cpi_t + \alpha_2 vol + \alpha_3 + \mu_t \,. \tag{6'}$$

We used the residuals μ_t of each model to estimate a MTAR model of the form of (2.5) and (2.7) and practice the ADF t-test of the Engle-Granger procedure[9]. The number of lags in the ADF t-test for cointegration was selected by AIC. Lag lengths of the MTAR regression, l, was selected according to the general-to-specific method starting with $l = 8$ and using the 5% significance level. Ljung-Box Q statistics using 4, 8, and 12 lagged autocorrelations enable us to investigate the autocorrelation properties of the residuals in the MTAR regression. The threshold, τ, is consistently estimated via Chan's (1993) method.

Tables 4,5,6,7,8 and 9 show results of Engle-Granger cointegration tests and MTAR for the different samples[10]. For the US sample 1927:1-2003-2, the Engle-Granger test indicates at conventional significance level that all models are not cointegrating relationships, except the model (4) at a 10% significance level. In this last model, the coefficient of *cpi* has the expected sign.

The value of the Φ-statistic indicates that the null hypothesis of no cointegration can be rejected at the 1% significance level for the models (3) and (4) and at a 5% level for the model (6). The F statistic rejects the null hypothesis of symmetric adjustment toward the long-run equilibrium for these three models at conventional significance level. Comparisons of the

[9] The Engle-Granger critical values at the 10%, 5% and 1% significance levels are -3.07, -3.37 and –3,96 for m = 2; -3.45, -3.77 and –4.31 for m = 3; -3.83, -4.11 and –4.73 for m = 4; where m = number of series in the long-run equilibrium relationship.

[10] When there are more than two variables in the model, we verify the number of cointegrating relationships by applying the Johansen (1988, 1991) procedure. In all estimated models, there is never more than one cointegrating relationship. Details of the Johansen test results are available in Appendix.

absolute values of $\hat{\rho}_1$ and $\hat{\rho}_2$ in models (3) and (4) suggest convergence such that the speed of adjustment is faster for negative than for positive discrepancies from the threshold estimated.

For the US sample 1953:1-2003:2, the Engle-Granger test indicates that only the models (4) and (6) are cointegrating relationships. This result indicates that without the large conception of fundamental (i.e earnings instead of dividends) and the proxies for the time-varying risk premium (*cpi* and *vol*), classical tests conclude that the US stock market exhibits explosive rational bubble. The value of the Φ-statistic indicates that the null hypothesis of no cointegration can be rejected at the 1% significance level for the models (4'), (5'), (6') and at a 5% significance level for the model (3). The F statistic rejects the null hypothesis of symmetric adjustment toward the long-run equilibrium at conventional significance level for the four models. It appears that the conventional cointegration tests fail to detect the long-run relationship, while the asymmetric cointegration tests uncover relationships for the model with dividends (3) and (5). These empirical findings provide support for the hypothesis that in the long-run stock prices adhere to the fundamentals.

For the US sample 1871-2002, only models with earnings (2), (4) and (6) appear to be cointegrating relationships according to the Engle-Granger test. The Φ-statistic allows us to soundly reject the null hypothesis of no cointegration at the 1% significance level for these three models. This hypothesis can be rejected at a 10% significance level for the model (3). The F statistic rejects the null hypothesis of symmetric adjustment toward the long-run equilibrium at a 1% significance level for the three models with earnings. These results strongly suggest that in the long-run stock prices adhere to *extend* fundamentals and that the adjustment mechanism is asymmetric. Therefore, according to Bohl and Siklos (2004), comparisons of the absolute values of $\hat{\rho}_1$ and $\hat{\rho}_2$ in models (3), (4) and (6) suggest the existence of short-run stock prices run-up relative to fundamentals followed by a crash.

The empirical results for the French stock market are mixed. On the entire sample, the Engle-Granger test indicates that *d* and *p* are cointegrated at a 1% significance levels. However, the Φ-statistic and the F-statistic enable us to conclude that the log of stock prices and the log of dividends are cointegrated and that the adjustment mechanism is asymmetric. Comparison of the absolute values of $\hat{\rho}_1$ and $\hat{\rho}_2$ of models (3) and (4) suggest convergence such that the speed of adjustment is faster for negative than for positive discrepancies from the threshold estimated.

These two tests indicate that models (3) and (5) are cointegrating relationships at a 1% significance level with asymmetric adjustment at a 1% level of significance. However, the sign of *cpi* in the cointegration vectors are ambiguous. They indicate a positive relationship between stock prices and inflation. For the sample period 1802-1950, the main conclusions were found.

For the 1951-2002 sample period, the Engle-Granger test indicates at a 10% level that models (3) and (5) appear to be cointegrating relationships. The Φ-statistic and the F-statistic indicate that *p*, *d* and *cpi* are cointegrated and that the adjustment mechanism is asymmetric with or without the restriction $\alpha_0 = 1$. These findings illustrate once again the need to integrate additional fundamentals when we test the present value model. As advocated by Bohl and Siklos (2004), the absolute values of $\hat{\rho}_1$ and $\hat{\rho}_2$ suggest than in the sample period 1951-2002, French stock prices exhibit run-ups followed by crashes in the short-run

6. Conclusion

The application of classical cointegration and unit root tests investigating the evidence of rational bubbles provides mixed empirical results (e.g. Campbell and Shiller, 1987; West, 1987; Diba and Grossman, 1988a ; Froot and Obstfeld, 1991 ; Craine, 1993 ; Lamont, 1998). Based on the largest samples with respect to the original frequency of the data, we implement these classical tests and we conclude to the existence of explosive rational bubbles in US stock prices (for the sample periods 1871-2002 and 1926:1-2003:2) and French stock prices (for the sample period 1951-2002).

However, our empirical findings provide support for the hypothesis that in the long-run stock prices and fundamentals are cointegrated. First, we consider another measure of the fundamental. Since dividends can be affected by corporate financial policy, we implement additional unit roots and cointegration tests with earnings. Then, to integrate additional sources of fundamental fluctuations, we also augmented the cointegrating regression by including consumer price inflation (and GDP volatility for the US sample period 1953:1-2003:2) as a risk premium effect. Finally, we implement cointegration tests between stock prices and fundamentals with asymmetric adjustment mechanism. Therefore, we apply the momentum threshold autoregressive (MTAR) procedure developed by Enders and Granger (1998) and Enders and Siklos (2001).

Our results indicate that without the large conception of fundamental (i.e. earnings instead of dividends) and the proxies for the time-varying risk premium, classical tests conclude that the US stock prices exhibit rational bubble. Then, it appears that the conventional cointegration tests fail to detect a long-run relationship between stock prices, dividends and proxies of the risk premium for US sample period 1953:1-2003:2, while the asymmetric cointegration tests uncover them. These results strongly suggest that in the long-run US stock prices adhere to fundamentals and that the adjustment mechanism is asymmetric.

Therefore, Bohl and Siklos (2004) suggest that the MTAR procedure is able to detect periodically collapsing bubbles defined by Evans (1991) and Charemza and Deadman (1995)[11]. We find evidence of this class of rational bubble in French stock prices for the sample period 1951-2002 and US stock prices for the sample period 1871-2002. These findings provide support for the hypothesis that, in the short-run, US and French stock prices exhibit run-ups followed by crashes while, in the long-run, stock prices are attracted by fundamentals.

[11] However, they do not provide Monte Carlo experiments using Evans' data-generating process to gauge the performance of the MTAR procedure.

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Table 1. Unit root and stationary tests (level)

	p	*e*	*d*	*ep*	*dp*	*cpi*	*vol*
US Annual							
1871-2002							
ADF t test	2,26	0,92	1,27	3,24**	-1,04	-8,15***	
l	5	5	4	0	0	0	
KPSS	2,06	2,11	2,53	0,51	4,17	1,46	
US Quarterly							
1927:01-2003:02							
ADF t test	0,66	-0,31	0,17	-1,60	-1,05	-3,07**	
l	7	5	6	7	7	8	
KPSS	3,66	5,02	4,37	0,64	1,95	0,71	
1953:01-2003:02							
ADF t test	-0,47	-0,97	-0,91	-2,02	-1,34	-1,89	-1,31
l	1	1	1	1	1	8	1
KPSS	9,24	9,92	10,20	2,50	3,86	0,44	6,31
French Annual							
1802-2002							
ADF t test	1,38		1,23		-4,86***	-5,89***	
l	0		2		0	1	
KPSS	17,99		5,98		0,97	1,62	
1951-2002							
ADF t test	-1,47		0,53		-2,27	-2,94**	
l	0		4		0	0	
KPSS	4,63		1,15		0,95	0,69	
1802-1950							
ADF t test	0,52		1,07		-4,60**	-5,06***	
l	0		2		2	1	
KPSS	13,02		14,60		1,41	1,71	

For the ADF-test, *,**,*** denotes the significance level of reject H_0 at respectively 10%,5% and 1%.
For the KPSS-test, *,**,*** denotes the significance level of not reject H_0 at respectively 10%,5% and 1%.

Table 2. Unit root and stationary tests (differences)

	Δp	*Δe*	*Δd*	*Δep*	*Δdp*	*Δcpi*	*Δvol*
US Annual							
1871-2002							
ADF t test	-6,14***	-5,65***	-7,10***	-7,24***	-9,51***	-6,40***	
l	4	8	3	4	1	8	
KPSS	0,40***	0,21***	0,23***	0,08***	0,14***	0,06***	
US Quarterly							
1927:01-2003:02							
ADF t test	-7,31***	-6,50***	-6,42***	-8,55***	-8,31***	-8,87***	
l	6	5	7	6	6	7	
KPSS	0,13***	0,08***	0,13***	0,05***	0,05***	0,03***	
1953:01-2003:02							
ADF t test	-12,64***	-6,70***	-3,54***	-10,80***	-12,61***	-7,13***	-10,73***
l	0	4	8	0	0	7	0
KPSS	0,09***	0,04***	0,15***	0,10***	0,11***	0,08***	0,11***
French Annual							
1802-2002							
ADF t test	-12,76***		-8,30***		--13,99***	-10,89***	
l	0		0		0	3	
KPSS	0,44**		0,61		0,02***	0,02***	
1951-2002							
ADF t test	-7,09***		-6,40***		-8,02***	-4,43***	
l	0		1		0	3	
KPSS	0,12***		0,06***		0,08***	0,06***	
1802-1950							
ADF t test	-10,56***		-5,27***		-11,18***	-9,88***	
l	0		5		0	3	
KPSS	0,17***		0,33***		0,08***	0,02***	

Table 3. Perron (1997) Test

	dp			*ep*		
Samples	t	Break date	l	t	Break date	l
US Annual						
1871-2002	-3,85	1995	11	-4,21	1995	10
US Quarterly						
1927:1-200302	-3,64	1994:3	7	-2,81	7	1990:2
1953:1-2003:2	-3,67	1994:3	1	-4,43	1	1972:2
French Annual						
1802-2002	-3,60	1945	12			
1951-2002	-3,50	0	1963			
1802-1950	-6,85***	1853	5			

Table 4. Us Stock market (1927:01-2003:02)

	(1) (*p d*)	(2) (*p e*)	(3) (*p d cpi*)	(4) (*p e cpi*)	(5) (*dp cpi*)	(6) (*ep* cpi)
Engle Granger						
ADF t-test	-2,98	-2,65	-3,11	-3,67*	-1,82	-2,72
l	0	0	0	1	0	0
vector	[-1 1,25 3,02]	[-1 1,07 2,56]	[-1 1,27 -1,63 3,06]	[-1 1,12 –4,54 2,62]	[-1 –0,18 –3,26]	[-1 3,30 –2,77]
MTAR						
τ	-0,02852	-0,01181	-0,01848	-0,4575	0,04441	0,09541
$\hat{\rho}_1$	-0,04 (-1,57)	-0,07 (-2,89)	-0,04 (-1,52)	-0,05 (-1,76)	-0,06 (-2,36)	-0,13 (-3,14)
$\hat{\rho}_2$	-0,10 (-2,79)	-0,01 (-0,29)	-0,13 (-3,99)	-0,17 (-4,12)	-0,01 (-0,79)	-0,043 (-2,03)
Φ	4,78	4,17	8,97***	8,90***	3,09	6,79**
$\rho_1 = \rho_2$	1,97	3,22	4,91**	7,37***	2,42	3,74*
l	5	5	2	7	2	2
AIC	376,33	432,84	383,04	431,65	373,52	426,81
Q(4)	0,20	0,20	2,79	0,14	3,35	2,34
Q(8)	7,04	12,27	12,97	1,21	17,53	21,39
Q(12)	8,07	13,94	15,75	3,96	20,17	27,72

Table 5. Us Stock market (1953:01-2003:02)

	(1) (*p d*)	(2) (*p e*)	(3') (*p d cpi vol*)	(4') (*p e cpi vol*)	(5') (*dp cpi vol*)	(6') (*ep cpi vol*)
Engle Granger						
ADF t-test	-1,56	-2,44	-2,82	-4,18**	-2,86	-4,12**
l	0	1	0	1	0	1
vector	[-1 1,26 3,02]	[-1 1,13 2,43]	[-1 1,04 -5,09 -0,63 1,04]	[-1 0,96 -7,33 –0,65 0,05]	[-1 4,86 0,70 –0,32]	[-1 7,67 0,58 –0,32]
MTAR						
τ	-0,06226	0,00119	0,00943	0,05461	-0,01107	-0,04429
$\hat{\rho}_1$	-0,04 (-2,34)	-0,06 (-2,52)	-0,15 (-4,00)	-0,05 (-0,80)	0,01 (0,18)	-0,16 (-4,34)
$\hat{\rho}_2$	0,03 (0,65)	-0,02 (-0,93)	-0,01 (-0,07)	-0,16 (-4,52)	-0,17 (-4,48	-0,06 (-1,15)
Φ	2,97	3,59	8,02**	10,44***	10,03***	9,97***
$\rho_1 = \rho_2$	2,60	1,23	7,81***	3,19*	11,46***	2,81*
l	1	1	0	1	0	1
AIC	26,81	102,13	43,49	101,92	46,86	102,14
Q(4)	0,14	1,22	3,13	1,16	3,56	1,28
Q(8)	1,96	5,82	4,99	2,37	5,57	2,47
Q(12)	6,23	12,26	12,37	8,44	13,37	8,85

Table 6. US Stock Markets (1871-2002)

	(1) (*p d*)	(2) (*p e*)	(3) (*p d cpi*)	(4) (*p e cpi*)	(5) (*dp cpi*)	(6) (*ep cpi*)
Engle Granger						
ADF t-test	-2,27	-3,35*	-2,87	-4,59***	-1,08	-3,99***
l	0	0	0	0	0	0
vector	[-1 1,16 3,10]	[-1 1,01 2,60]	[-1 1,17 –0,93 3,11]	[-1 1,04 –2,16 2,62]	[-1 –0,20 –3,11]	[-1 1,78 –2,64]
MTAR						
τ	0,02163	0,09799	0,16399	0,26067	-0,16079	-0,25325
$\hat{\rho}_1$	-0,05 (-0,80)	-0,37 (-4,60)	0,025 (0,26)	-0,59 (-4,45)	-0,08 (-2,10)	-0,16 (-2,16)
$\hat{\rho}_2$	-0,15 (-2,41)	-0,04 (-0,52)	-0,20 (-3,52)	-0,16 (-1,77)	0,09 (1,34)	-0,45 (-4,06)
Φ	3,23	10,71***	6,24*	10,29***	3,09	10,55***
$\rho_1 = \rho_2$	1,28	9,48***	4,03**	8,94***	4,97**	4,74**
l	0	0	0	0	0	0
AIC	117,83	235,02	144,57	265,41	111,65	258,29
Q(4)	2,78	2,07	3,82	0,11	4,03	1,62
Q(8)	3,94	7,40	4,14	4,88	5,66	8,70
Q(12)	12,05	11,55	16,38	9,84	12,94	16,05

Table 7. French Stock Market (1802-2002)

	(1) (*p d*)	(3) (*p d cpi*)	(5) (*dp cpi*)
Engle Granger			
ADF t-test	-4,39***	-4,95***	-4,93***
l	0	0	0
vector	[-1 1,04 2,99]	[-1 1,03 1,47 2,99]	[-1 –1,60 –3,11]
MTAR			
τ	-0,17444	-0,17696	0,18972
$\hat{\rho}_1$	-0,17 (-3,45)	-0,17 (-3,49)	-0,33 (-3,90)
$\hat{\rho}_2$	-0,49 (-5,70)	-0,37 (-4,05)	-0,17 (-3,53)
Φ	20,32***	14,26***	13,84***
$\rho_1 = \rho_2$	11,79***	3,65*	3,09*
l	3	0	0
AIC	401,58	433,68	440,50
Q(4)	0,59	1,49	1,39
Q(8)	1,82	3,95	3,37
Q(12)	5,88	8,57	8,29

Table 8. French Stock Market (1802-1950)

	(1) (*p d*)	(3) (*p d cpi*)	(5) (*dp cpi*)
Engle Granger			
ADF t-test	-4,75***	-5,18***	-5,53***
l	2	0	0
vector	[-1 1,13 2,71]	[-1 1,06 1,75 2,89]	[-1 –1,97 –3,07]
MTAR			
τ	0,01930	-0,08148	0,12133
$\hat{\rho}_1$	-0,23 (-2,78)	-0,29 (-3,23)	-0,54 (-4,79)
$\hat{\rho}_2$	-0,59 (-6,35)	-0,52 (-4,32)	-0,27 (-3,47)
Φ	21,29***	12,81***	17,53***
$\rho_1 = \rho_2$	10,77***	2,74*	3,89**
l	4	2	0
AIC	244,59	272,88	285,94
Q(4)	1,64	0,82	2,18
Q(8)	3,71	5,23	7,03
Q(12)	7,46	6,24	10,08

Table 9. French Stock Market (1951-2002)

	(1) (*p d*)	(3) (*p d cpi*)	(5) (*d/p cpi*)
Engle Granger			
ADF t-test	-2,22	-3,66*	-3,63**
l	0	0	0
vector	[-1 1,11 2,32]	[-1 1,05 -6,35 3,13]	[-1 6,61 -3,59]
MTAR			
τ	-0,21283	0,17651	0,04844
$\hat{\rho}_1$	-0,13 (-1,53)	-0,86 (-3,57)	-0,57 (-3,97)
$\hat{\rho}_2$	-0,38 (-2,03)	-0,29 (-2,46)	-0,18 (-1,11)
Φ	3,24	9,40**	8,50**
$\rho_1 = \rho_2$	1,48	4,49**	3,26*
l	0	0	0
Q(4)	3,48	4,12	1,93
Q(8)	5,83	8,99	6,67
Q(12)	15,35	18,90	12,32

Appendix: Johansen cointegration tests results

Table A1. Determination of Cointegration Rank: Us Stock market (1927:01-2003:02)

model	Trace test		Critical values		l	LM(1)	LM(4)
	r=0	r=1	r=0	r=1			
(3)	44,91	17,05	40,84 (1%)	24,74 (1%)	10	9,88 (p=0,36)	19,83 (p=0,02)
(4)	41,17	18,37	34,80 (5%)	19,99 (5%)	10	12,01 (p=0,21)	23,05 (p=0,01)

Table A2. Determination of Cointegration Rank: Us Stock market (1953:01-2003:02)

model	Trace test		Critical values		l	LM(1)	LM(4)
	r=0	r=1	r=0	r=1			
(3')	60,49	31,68	60,42 (1%)	40,84 (1%)	9	18,01 (p=0,32)	10,17 (p=0,86)
(4')	69,10	37,15	53,42 (5%)	34,80 (5%)	10	16,05 (p=0,45)	22,50 (p=0,13)
(5')	25,81	7,78	40,84 (1%)	24,74 (1%)	7	6,22 (p=0,72)	12,94 (p=0,17)
(6')	39,07	9,16	34,80 (5%)	19,99 (5%)	7	5,48 (p=0,79)	13,83 (p=0,13)

Table A3. Determination of Cointegration Rank: US Stock Markets (1871-2002)

model	Trace test		Critical values		l	LM(1)	LM(4)
	r=0	r=1	r=0	r=1			
(3)	67,76	14,81	40,84 (1%)	24,74 (1%)	2	10,85 (p=0,29)	6,08 (p=0,73)
(4)	67,88	21,79	34,80 (5%)	19,99 (5%)	2	5,42 (p=0,80)	9,20 (p=0,42)

Table A4. Determination of Cointegration Rank: French Stock Market (1802-2002)

model	Trace test		Critical values		l	LM(1)	LM(4)
	r=0	r=1	r=0	r=1			
(3)	59,28	16,87	40,84 (1%) 34,80 (5%)	24,74 (1%) 19,99 (5%)	7	8,66 (p=0,47)	9,75 (p=0,37)

Table A5. Determination of Cointegration Rank: French Stock Market (1802-1950)

model	Trace test r=0	Trace test r=1	Critical values r=0	Critical values r=1	l	LM(1)	LM(4)
(3)	53,81	18,72	40,84 (1%) 34,80 (5%)	24,74 (1%) 19,99 (5%)	7	6,93 (p=0,64)	15,22 (p=0,09)

Table A6. Determination of Cointegration Rank: French Stock Market (1951-2002)

model	Trace test r=0	Trace test r=1	Critical values r=0	Critical values r=1	l	LM(1)	LM(4)
(3)	38,99	18,28	40,84 (1%) 34,80 (5%)	24,74 (1%) 19,99 (5%)	4	9,30 (p=0,41)	8,18 (p=0,52)

Note: The number of lags was selected in order to accept the assumption that residuals of the model are a white noise (LM(1) and LM(4) criteria). The determination of the cointegration rank is based on the Trace test. We did not include linear trends in the model but we have constrained intercepts to appear only in cointegration relations.