

# Optimistic Investors, IPOs Cycle, and Real Investment\*

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## Abstract

This paper studies the dynamics of the IPO (Initial Public Offering) market and its effect on real investment decisions in emerging industries. We first propose a model of IPOs cycle based on divergence of opinion among investors and short-sale constraints. Using a real option approach, we show that firms are more likely to go public when the ratio of overvaluation over profits is high, that is after stock market run-ups. Because initial returns increase with the demand from optimistic investors at the time of the offer, the model provides an explanation for the observed positive causality between average initial returns and IPO volume. Second, we discuss the possibility of real overinvestment in new industries. Investing in the industry gives agents an option to sell the project on the stock market at an overvalued price. The IPO market enables then the financing of positive NPV projects which would not be undertaken without overvaluation. It can however also lead to overinvestment in emerging industries.

**JEL Codes:** G10, G31, E44.

**Keywords:** IPO, divergence of opinion, real option, stock market and investment.

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# 1 Introduction

Investment in high technology industries, although representing a small part of GDP, plays a major role in the performance of the economy by promoting innovation. Now, many economists have claimed that having a market-based financial system is the best way of encouraging investment in emerging industries and several studies have shown that the presence of a vibrant stock market was a necessary condition to finance projects characterized by poor collateral value and highly uncertain returns. For example, Carpenter and Petersen (2002) for the US or Bottazzi and Da Rin (2002) for Europe have found that high-tech firms going public were experiencing a substantial increase in capital expenditures and size in the immediate period following their IPO (Initial Public Offering). In addition, the stock market constitutes a very profitable exit way for the venture capitalism industry. As noted by Lerner (1994), venture capitalists generate the main part of their profits from firms sold on the stock market via an IPO<sup>1</sup>. Having an active IPO market is then critical to the existence of a well developed venture capitalism industry (see Black and Gilson, 1998) and so to the adoption of high-tech technologies (Michelacci and Suarez, 2003).

Periods of strong activity on the IPO market are however usually associated with excessively optimistic market sentiment. The recent frenzy over Internet firms is not unique. It has been established for a long time that IPOs underperform in the long run (Ritter, 1991), whereas they initially exhibit high returns, known as initial returns or underpricing. As shown in Figure 1, periods of high IPO volume correspond with periods of high initial returns, a proxy for investors' sentiment. Such periods, called hot issue periods have been repeated in the US several times with varying degrees during the forty past years. Issuers would then take advantage of a window of opportunity by selling stocks to overly optimistic investors. Empirical studies have confirmed this idea, showing that firms in a particular industry were more likely to go public when industry valuations were the highest. Few theoretical models have however studied the dynamics of IPOs cycles in an overvaluation context. Benveniste, Busaba and Wilhelm (2002) as well as Hoffman-Burchardi (2001) propose models where the clustering of IPOs is partly explained by a decrease in the information production costs related to firms' industry prospects<sup>2</sup>. The approach used by these authors, by assuming that secondary market prices reflect the

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<sup>1</sup>Lerner reports that a Venture Economics study finds that on average, an investment of \$1 in a firm that is taken public provides a return to the venture capitalist of \$1.95 beyond the initial investment with an average holding period of 4.2 years.

<sup>2</sup>Underpricing is defined as the difference between the closing price of the first trading day – assumed to reflect the true value of the firm – and the offer price. It is a compensation received by institutional investors for their production of information about the value of the stock and their truthful revelation of this value to the investment bank and the issuer (Benveniste and Spindt, 1989). As the number of IPOs increases, the production of information related to a common factor (namely, the industry factor) decreases and so underpricing, leading more firms to go public.

true value of firms taken public, is nevertheless at odds with the well-documented long run underperformance of IPOs.

The objective of this paper is first to offer a model of IPOs cycle based on heterogeneous beliefs among potential investors in the spirit of Miller (1977). Because of short-sale constraints, market prices are determined by the most optimistic investors. We assume that before the IPO the firm is held by two agents: an entrepreneur and a venture capitalist. Going public enables the entrepreneur to finance the growth of the firm by issuing new shares whereas it allows the venture capitalist to sell his participations a few months after the IPO. With both profits and a number of optimistic investors being stochastic, the optimal timing of the IPO is derived by considering the going public decision as a real option. Draho (2000) has first analyzed IPO timing in a real option framework<sup>3</sup> but his model does not explain why IPOs underperform in the long run nor why they initially exhibit abnormal positive returns. In a second stage, we introduce underpricing by supposing as in Aggarwal, Krigman and Womack (2002) that underpricing the issue generates more demand for the stock on the secondary market. We show that the optimal underpricing increases with the number of optimistic investors present at the time of the offer. As the exercise of the IPO option occurs after positive shocks on the number of optimists, the model provides an explanation for the observed positive causality between initial returns and IPO volume.

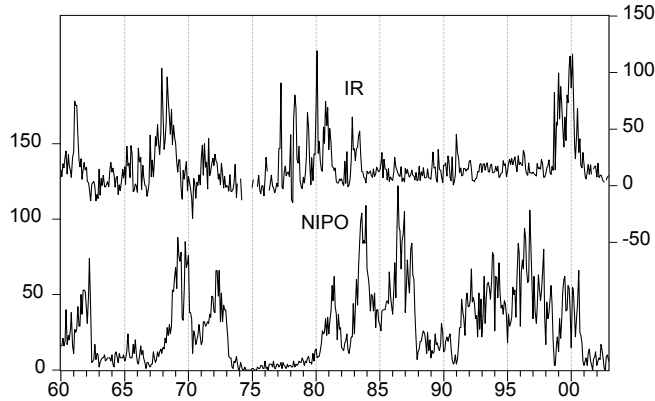
Finally, we discuss the role played by the IPO market in the decision to invest in new industries and the possibility of real overinvestment in such industries. Since projects are sold on the stock market at an overvalued price, one can wonder about the efficiency of the stock market in allocating capital to profitable investments. The link between the stock market and real investment has been extensively studied in economics and empirical works have concluded that the stock market had poor incidence on real investment (see for e.g. Morck, Shleifer and Vishny, 1990; Blanchard, Rhee and Summers, 1993; Bond and Cummins, 2000; among others) so that there was no reason to worry about the consequences of stock market misvaluation on real economy. These studies concern however firms that are generally listed on the stock market for a long time and which may not be financially constrained. Moreover, the approach used does not consider the effects of overvaluation on the decision to undertake a project before going public. We show that overvaluation, which is a characteristic feature of the IPO market, can have beneficial effects on investment in emerging industries up to a certain point. Indeed, when the hot market lasts too long, overinvestment occurs in the industry in vogue on the stock market whereas there is underinvestment in other promising industries not as much popular in the investors' community.

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<sup>3</sup>In Draho's model, investors on the stock market are supposed to be better diversified than the entrepreneur so that this latter discounts the revenues of the firm at a higher rate than do public investors. With profits evolving stochastically, the firm is taken public only when the market valuation is sufficiently larger than the entrepreneur's one. The clustering of IPOs around market peaks would then be the consequence of strategic timing by issuers, waiting for price run-ups before exercising their going public option.

The rest of the paper is organized as follows. Section 2 describes the structure of the model. Section 3 derives the optimal IPO timing. In section 4, we discuss the effects of the going public option on real investment decision. Section 5 concludes.

Figure 1. Monthly US data on average initial returns (IR) in per cent and initial public offerings (NIPO) for the period January 1960-December 2002.



Source: Jay Ritter's web site (<http://bear.cba.ufl.edu/ritter/ipoall.htm>).

## 2 The model

We consider the emergence of a new industry in the economy. To be implemented, each project of this industry requires an investment cost  $I$  and generates a profit flow  $\pi_t$  with an initial profit level  $\pi_0$ . The fundamental value of the project is simply defined as the expected present discounted value of the profit flow and is denoted  $V_t$ . We suppose that entrepreneurs who are at the origin of the project's idea have no funds so that they must find an agent in the economy who accepts to provide financing in return for a participation in the fundamental value of the project. Let us call this agent a venture capitalist<sup>4</sup>. We denote  $Q_S$  the quantity of shares initially composing the capital of the firm. By paying  $I$  at date  $t$ , the venture capitalist receives  $\theta Q_S$  shares with  $0 < \theta < 1$ , that is an equity stake whose expected value is  $\theta V_t$ . Once the investment is undertaken, the project can be taken public on the stock market at any time with no delays to simplify. Going public enables the firm to issue a fixed quantity  $Q$  of new shares whereas it allows

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<sup>4</sup>The contribution of the venture capitalist to the project goes of course beyond the financial investment  $I$  as he will also use his business expertise to achieve the fundamental value  $V$ . This latter characteristic makes the venture capitalist financing a very appropriate one particularly in high-tech industries, in comparison with a common financing.

the venture capitalist to resell his  $\theta Q_S$  shares on the stock market a specified period of time after the IPO, called the lockup period<sup>5</sup>.

The proceeds of the issue are used by the entrepreneur to finance new investments or marketing expenditures in order to grab industry market shares. We suppose that the venture capitalist anticipates that the fundamental value of the project will not be affected by these expenditures and so always prefers selling his  $\theta Q_S$  shares on the secondary market at the lockup expiration date whereas the entrepreneur, more confident about the success of the firm strategy, keeps his stake in the firm for an undetermined period.

## 2.1 Industry description and fundamental valuation

Profits of the project, denoted  $\pi$ , are supposed to follow the geometric Brownian motion:

$$\frac{d\pi_t}{\pi_t} = \alpha_\pi dt + \sigma_\pi dz_\pi, \quad (1)$$

where  $\alpha_\pi < 0$  and  $\sigma_\pi$  are constant parameters and  $dz_\pi$  is the increment of a standard Wiener process. The assumption relative to the drift's sign is commonly used to describe the evolution of profits in high technology industries. As noted by McDonald and Siegel (1986), when a new product is introduced by a firm, other firms introduce similar products and so, because of lagged entry in the industry, profits tend to disappear. This means there is enough competition in the industry, what seems quite realistic for high-tech and communication industries (see for instance Huisman, 2001).

For a given level of profits  $\pi_t$ , the conditional expectation for profits at date  $s$  is

$$E[\pi_s | \pi_t] = \pi_t e^{\alpha_\pi(s-t)}. \quad (2)$$

Agents are risk averse. Discounting expected profits at the appropriate risk-adjusted rate  $\rho > 0$ , the value of the project at date  $t$ , denoted  $V_t$  is

$$V_t = E \left[ \int_t^\infty \pi_s e^{-\rho(s-t)} ds \mid \pi_t \right] = \int_t^\infty \pi_t e^{-(\rho-\alpha_\pi)(s-t)} ds = \frac{\pi_t}{\rho - \alpha_\pi}. \quad (3)$$

Let us define  $\delta_\pi = \rho - \alpha_\pi > 0$  and rewrite (3) as:

$$V_t = \frac{\pi_t}{\delta_\pi}. \quad (4)$$

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<sup>5</sup>In practice, insiders and other pre-IPO shareholders are generally prohibited from selling their own shares at the IPO date particularly when the firm is young and belongs to a new industry. Instead, they have to wait for a fixed period of time which typically lasts six months.

## 2.2 Stock market prices

Following Miller (1977), we consider that stock prices are prone to overvaluation due to both divergence of opinion and short-sale constraints. Indeed, firms going public are generally operating in new industries so that uncertainty about future profits is high. As uncertainty and divergence of opinion are likely to be correlated<sup>6</sup>, a high heterogeneity of beliefs concerning the return of firms taken public is observed. With short-sale constraints, the market price is determined by the most optimistic investors, leading to an overvaluation of the stock. Several papers have provided support for Miller's theory. For example, Boehme, Danielsen and Sorescu (2002) find that shares which are difficult to borrow and are prone to a high dispersion of investors opinion underperform in the long run<sup>7</sup>. Ofek and Richardson (2003) report that Internet stocks were abnormally costly to short whereas they were owned relatively more often by retail investors, a class of investors considered as the more likely to be optimistic.

Here we consider two types of investors. First, rational investors who estimate the project at the fundamental value  $V$  as do the venture capitalist. Next, optimistic investors<sup>8</sup> whose valuations are greater than  $V$ . Rational and optimistic investors use the same geometric Brownian motion (1) in their estimate of the value of the project but optimists anticipate that profits of a particular firm will experience an upward jump with a non-null probability at a future unknown date<sup>9</sup>. They behave as if they were investing in the next Microsoft. This has for consequence to increase the expected trend of the diffusion process of profits and so the valuation of the project. As each optimistic investor has different estimates of the probability of the positive Poisson event and/or of the size of the jump, we observe divergence of opinions even among this class of investors. For simplicity, we assume that each optimistic investor, generally associated to small, unsophisticated investors, can buy only one share. Finally, since we are interested in hot issue periods, it is supposed that the number of optimistic investors is always greater than the number of stocks issued so that the marginal investor is always an optimist and the price is overvalued<sup>10</sup>.

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<sup>6</sup>As Miller notes: "it is implausible to assume that although the future is very uncertain, and forecasts are very difficult to make, that somehow everyone makes identical estimates of the return and risk from every security".

<sup>7</sup>The authors find that there is overvaluation only when the stock is both difficult to sell short and is prone to divergence of opinion. Indeed, stocks with a great dispersion of investor opinion are not necessarily overpriced as soon as the cost of short selling is not prohibitive. Symmetrically, when short-sales are costly but there is little disagreement among investors about the value of the stock, overvaluation does not occur.

<sup>8</sup>One could also consider the existence of pessimistic investors. However, because rational investors (typically associated to institutional investors) have sufficient resources to buy all the stocks at  $V$ , the presence of pessimistic investors will have no effect on the determination of market prices.

<sup>9</sup>Of course this positive event never occurs and that's why these investors are considered as optimistic.

<sup>10</sup>It is common to observe during hot issue periods substantial oversubscription ratios for high-tech IPOs, the level of demand being equal to several times the amount of shares offered.

As a result of dispersion of opinion and short-sale constraints the demand curve slopes down. However, contrary to preceding models considering stock pricing in an heterogeneous beliefs and short-sale constraints framework (see Ljungqvist, Nanda and Singh, 2001 or Aggarwal, Krigman and Womack, 2002), the absolute value of the slope of the demand curve is not supposed to be constant but decreasing with the number of shares. In other words, when the quantity of shares sold on the market increases, the price decreases but to a smaller and smaller extent. This is because as we get closer to the fundamental value, there are more and more potential investors willing to pay for the current price. This seems to be a reasonable hypothesis as the number of potential investors may be distributed normally around the fundamental value rather than uniformly. Thus, the appropriate price curve is not a straight line but an hyperbola (see Figure 2). It is further assumed that as the number of optimistic investors increases, divergence of opinion gets higher.

Before the IPO, the total number of shares is  $Q_S$ , the quantity of secondary shares held by the entrepreneur and the venture capitalist. The initial public offering consists in an increase in the capital of the firm to  $Q + Q_S$ , where  $Q$  denotes the number of primary shares issued at the time of the offer. All profits are paid out as dividends to the shareholders, that is the initial owners of the firm (the entrepreneur and the venture capitalist) and the new shareholders from the IPO. The fundamental component of the stock at date  $t$  is then  $\frac{V_t}{Q+Q_S}$ . The overvalued part of the stock price is a convex decreasing function of the quantity of shares issued and increases with the number of optimistic investors. Let us suppose for simplicity that it is equal to  $\frac{n_t}{Q}$ , where  $n_t$  is the number of optimistic investors at date  $t$ . Overall, with an IPO occurring at  $t$ , the market price at the end of the first trading day is

$$P_t = \frac{V_t}{Q + Q_S} + \frac{n_t}{Q}. \quad (5)$$

The number of optimistic investors,  $n_t$  is supposed to follow a mixed Poisson-Wiener process of the form:

$$\frac{dn_t}{n_t} = \alpha_n dt + \sigma_n dz_n - dq, \quad (6)$$

where  $\alpha_n > 0$  and

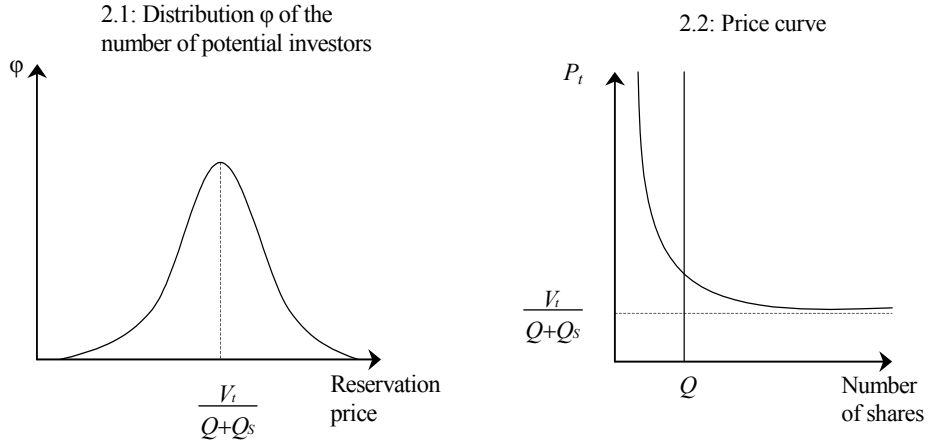
$$dq = \begin{cases} 1 & \text{with probability } \lambda dt, \\ 0 & \text{with probability } 1 - \lambda dt. \end{cases}$$

The positive sign of the drift parameter comes from a contagion effect among unsophisticated investors<sup>11</sup>. The aim of the inclusion of the Poisson process is to

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<sup>11</sup>Individual investors are considered as infrequent traders, explaining partly why they are unsophisticated in comparison with professional investors. At a given date, some of them may not participate on the IPO market but will only enter as time goes by due to a trend effect.

Figure 2. Distribution of the number of potential investors and corresponding price curve.



2.1: the closer the reservation price to the fundamental valuation, the greater the observed number of potential investors. 2.2: with a number of potential investors distributed normally, the price curve slopes down in a convex manner. An increase in the number of optimistic investors will shift the curve outwards.

take into account the possibility of a krach. Indeed, if the Poisson event occurs, the number of optimistic investors falls to zero where it remains forever, as zero is a natural absorbing barrier for geometric Brownian motions. Optimistic investors have definitively disappeared and this is the end of the hot market.

### 3 The timing of the IPO

Several empirical studies have stressed the ability of issuers to time their initial public offering in favorable market valuations conditions (see for instance, Lerner, 1994 or Pagano, Panetta and Zingales, 1998). Following Draho (2000), the IPO timing is here modelled as a real option. For this, we suppose that the IPO is irreversible. Besides, the optimal decision of when to go public is derived from the venture capitalist’s utility. This seems a reasonable assumption since, as noted by Lerner (1994), venture capitalists can use control rights and board seats to insure that the IPO occurs at the time they consider as optimal<sup>12</sup>.

<sup>12</sup>Pfirmann, Wupperfeld and Lerner (1997) cite the case of the contract signed by the venture capitalist Apex Investment Partners and the firm AccessLine: “Apex proposed several contract provisions (...): punitive interest of dividend payments if the firm did not go public; the right for the venture capitalist to fire management if the firm did not go public by a certain date; the ability for the venture capitalist to require AccessLine to repurchase their shares (...)”.

We first assume to simplify that there is no underpricing. The offer price is then equal to the market price observed the day of the offer. The decision to go public will depend on the difference between the value of the shares to the venture capitalist when the firm is private (continuation payoff) and the value of these shares when the firm is taken public (termination payoff).

### 3.1 The basic model

As long as the firm remains private, the venture capitalist gets at date  $t$  a fraction  $\theta$  of the profits flows generated by the project. However if the firm is taken public, profits flows must not only be shared with the entrepreneur but also with the new shareholders. Then, the going public decision entails an indirect cost for the original owners of the firm due to a dilution effect. On the other side, it enables the venture capitalist to sell the  $\theta Q_S$  shares at an overvalued price at the end of the lockup if the market is still bullish. With a lockup period of length  $T$ , the expected market price at the end of the lockup is

$$E[P_{t+T} | \pi_t, n_t] = \frac{E[V_{t+T} | \pi_t]}{Q + Q_S} + \frac{E[n_{t+T} | n_t]}{Q + \theta Q_S}. \quad (7)$$

Notice that the overvalued component of the market price is affected by the sale of the  $\theta Q_S$  secondary shares which appear in the denominator of the second term on the right-hand side of (7). This is corroborated by several empirical studies which showed that stock prices declined around lockup expiration date, particularly for venture-backed IPOs (see Field and Hanka, 2001; Bradley, Jordan, Roten and Yi, 2001; Ofek and Richardson, 2003).

Overall, the termination payoff received by the venture capitalist by going public at  $t$  is

$$\Omega(\pi_t, n_t) = \frac{\theta Q_S}{Q + Q_S} \frac{\pi_t}{\delta_\pi} + \frac{\theta Q_S}{Q + \theta Q_S} n_t e^{-(\delta_n + \lambda)T}, \quad (8)$$

where  $\delta_n = \rho - \alpha_n$  with  $\rho > \alpha_n - \lambda$  (see appendix).

It is composed of two terms. The first one is what we call a fundamental component. It is the sum of the present values of the expected profit flow accruing to the venture capitalist between date  $t$  and  $t + T$  and the expected fundamental part of the price at date  $t + T$  when secondary shares are sold. The second term is the expected discounted overvalued component of the stock price at  $t + T$  times the number  $\theta Q_S$  of secondary shares.

Let  $F(\pi_t, n_t)$  denote the value to the venture capitalist from owning the  $\theta Q_S$  shares at date  $t$ . It is equal to the expected sum of the present value of the cumulated profit flows received until the unknown optimal IPO date  $\tau(\pi^*, n^*)$  and the present value of the

termination payoff  $\Omega(\pi^*, n^*)$ , that is:

$$F(\pi_t, n_t) = E \left[ \int_t^{t+\tau(\pi^*, n^*)} \theta \pi_s e^{-\rho(s-t)} ds + e^{-\rho\tau(\pi^*, n^*)} \Omega(\pi^*, n^*) \mid \pi_t, n_t \right]. \quad (9)$$

The objective of the VC is to find the IPO date  $\tau(\pi^*, n^*)$  that maximizes (9). However, with  $\pi$  and  $n$  evolving stochastically,  $\tau(\pi^*, n^*)$  is a random variable and so cannot be determined *a priori*. Instead, the only way to resolve this optimal stopping problem is to derive the critical values  $\pi^*$  and  $n^*$  that trigger the IPO (we are more precisely looking for a threshold ratio  $\frac{n^*}{\pi^*}$ ).

Over a small time interval  $dt$ , the state variables will have changed to  $(\pi_t + d\pi_t)$  and  $(n_t + dn_t)$ . In the continuation region, the value of the shares can be expressed as the sum of the immediate profit flow and the discounted value of being private, that is:

$$F(\pi_t, n_t) = \theta \pi_t dt + \frac{1}{1 + \rho dt} E [F(\pi_t + d\pi_t, n_t + dn_t) \mid \pi_t, n_t]. \quad (10)$$

The Bellman equation for the optimal IPO timing strategy can then be written as:

$$F(\pi_t, n_t) = \max \left\{ \Omega(\pi_t, n_t), \theta \pi_t dt + \frac{1}{1 + \rho dt} E [F(\pi_t + d\pi_t, n_t + dn_t) \mid \pi_t, n_t] \right\}. \quad (11)$$

Because time horizon is infinite and  $\alpha_\pi, \sigma_\pi, \alpha_n, \sigma_n$  and  $\lambda$  are constants, one can drop the  $t$  subscripts on  $\pi$  and  $n$ . In the continuation region, the second term on the right-hand side of (11) is the larger of the two. Let multiply it by  $(1 + \rho dt)$ , omit terms that go faster than  $dt$  as  $dt \rightarrow 0$  and rearrange, we get:

$$\rho F(\pi, n) dt = \theta \pi dt + E [dF(\pi, n) \mid \pi, n], \quad (12)$$

where

$$dF = F_\pi d\pi + F_n dn + \frac{1}{2} F_{\pi\pi} (d\pi)^2 + \frac{1}{2} F_{nn} (dn)^2 + F_{\pi n} (d\pi dn) + [F(\pi, 0) - F(\pi, n)] dq \quad (13)$$

by application of Itô's Lemma, subscripts denoting derivatives<sup>13</sup>.

Note that if the Poisson event in (6) occurs,  $n$  goes to zero that is the market is cold forever and the firm is never taken public. Then, the value of the firm consists merely of the fundamental value  $\frac{\pi}{\delta_\pi}$ . This yields the following condition:

$$F(\pi, 0) = \theta \frac{\pi}{\delta_\pi}. \quad (14)$$

Finally, taking the expectation of (13) where  $E[dz_\pi] = E[dz_n] = 0$ ,  $E[dq] = \lambda dt$ ,  $E[dz_\pi dq] = E[dz_n dq] = 0$  and  $E[dz_\pi dz_n] = r dt$ , rearranging and dividing by  $dt$ , equation

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<sup>13</sup>Equation (12) is the usual equilibrium relation which states that over the small time interval  $dt$ , the total expected return from holding the shares must be equal to the profit flow  $\theta \pi dt$  plus the expected capital appreciation.

(12) becomes the elliptic partial differential equation:

$$\begin{aligned} & \frac{1}{2}(\sigma_\pi^2 \pi^2 F_{\pi\pi} + 2r\sigma_\pi\sigma_n\pi n F_{\pi n} + \sigma_n^2 n^2 F_{nn}) + (\rho - \delta_\pi)\pi F_\pi \\ & + (\rho - \delta_n)n F_n - (\rho + \lambda)F + \left(\frac{\delta_\pi + \lambda}{\delta_\pi}\right)\theta\pi = 0. \end{aligned} \quad (15)$$

Besides (14) the solution to (15) must satisfy the standard value-matching and two smooth-pasting conditions, respectively:

$$F(\pi, n) = \frac{\theta Q_S}{Q + Q_S} \frac{\pi}{\delta_\pi} + \frac{\theta Q_S}{Q + \theta Q_S} n e^{-(\delta_n + \lambda)T}, \quad (16)$$

$$F_\pi(\pi, n) = \frac{\theta Q_S}{Q + Q_S} \frac{1}{\delta_\pi}, \quad (17)$$

$$F_n(\pi, n) = \frac{\theta Q_S}{Q + \theta Q_S} e^{-(\delta_n + \lambda)T}. \quad (18)$$

Solving this kind of problem generally requires the use of numerical methods. However, following McDonald and Siegel (1986) or Dixit and Pindyck (1994) we can reduce it to one dimension. Indeed, because  $F(\pi, n)$  is homogeneous of degree one in  $(\pi, n)$  we can rewrite the value of the option as:

$$F(\pi, n) = \pi f\left(\frac{n}{\pi}\right) = \pi f(s). \quad (19)$$

The function  $f$  and the critical value of the ratio  $s = \frac{n}{\pi}$  are then obtained using the standard optimal stopping problem method. Intuitively, the firm should go public when the dilution cost is relatively low, that is when  $\pi$  is low or when the number of optimistic investors is relatively high. Overall, IPO occurs when  $s$  is sufficiently high.

**Proposition 1** *With profits of the project and a number of optimistic investors following respectively the stochastic processes (1) and (6), the firm is taken public when the ratio  $s = \frac{n}{\pi}$  is greater than the threshold value*

$$\frac{n^*}{\pi^*} = s^* = \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{Q(Q + \theta Q_S)}{\delta_\pi Q_S(Q + Q_S)} e^{(\delta_n + \lambda)T}, \quad (20)$$

where

$$\varepsilon_1 = \frac{1}{2} - \frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2} + \sqrt{\left[\frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}\right]^2 + \frac{2(\delta_\pi + \lambda)}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}} > 1. \quad (21)$$

The value of the firm to the venture capitalist is

$$F(\pi, n) = A_1 \frac{n^{\varepsilon_1}}{\pi^{\varepsilon_1 - 1}} + \theta \frac{\pi}{\delta_\pi}. \quad (22)$$

$A_1$  is given in appendix.

**Proof:** see appendix.

Note that an increase in the probability of the Poisson event has two opposite effects on the IPO timing. First, a positive one. Indeed, the higher the probability  $\lambda$  of the negative event, the less valuable the option as  $E[dn]$  decreases and so the firm should go public earlier. Second, a negative effect. Since the overvalued part of the proceeds from selling secondary shares is not received for  $T$  units of time, when  $\lambda$  increases, the venture capitalist will wait for a higher threshold ratio to take the firm public.

### 3.2 IPO timing with underpricing

In practice, the offer price is set below the market price of the first trading day so that initial return or underpricing is observed. One can wonder why issuers don't take advantage of the optimistic investors valuations by setting an offer price equal to the expected secondary market price<sup>14</sup>, resulting in a null initial return as assumed in the basic model. Actually, insiders and other pre-IPO shareholders are barred from selling their own shares at the time of the IPO but instead have to wait until the expiration date of the lockup period. Shares sold during the IPO are new ones, issued by the firm, and are then called primary shares whereas shares held by the entrepreneur and the venture capitalist are called secondary shares. In these conditions, the objective of an issuer is not necessarily to maximize the proceeds from the IPO, that is the offer price, but eventually the price at which he expects to sell his secondary shares a few months after the initial offering.

As shown by Aggarwal, Krigman and Womack (2002), buy and hold return from the IPO's first day closing price through 180 days following the offer increases with underpricing in a concave manner, via more recommendations for the stock from research analysts<sup>15</sup>. We then suppose that underpricing the issue enables the issuer to sell secondary shares six months after the IPO at a higher price than he would otherwise get.

**Assumption 1** *The issuer strategically underprices the IPO at date  $t$  in order to increase demand for the stock and to sell secondary shares at the lockup expiration date*

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<sup>14</sup>The definitive offer price is set the day before the IPO and sometimes the morning of the offer day. At this date, the investment bank has collected precise indications of interest from potential investors. In addition, more than 90% of the initial return is achieved at the first trading price. This means that the investment bank and the issuer face little uncertainty concerning the demand for the stock at the time the offer price is set. High initial returns cannot then be interpreted as the result of a non intentional mispricing.

<sup>15</sup>The authors find that research coverage depends positively on underpricing but negatively on underpricing squared.

$t + T$  at a higher price than he would otherwise get. The expected market price at the end of the lockup is given by:

$$E[P_{t+T} | \pi_t, n_t] = \frac{E[V_{t+T} | \pi_t]}{Q + Q_S} + \frac{E[n_{t+T} | n_t]}{Q + \theta Q_S} g(P_t - P0_t), \quad (23)$$

where  $P0_t$  is the offer price and  $g(x) = \gamma x^\nu$  is the momentum generating function with  $0 < \nu < 1$  and  $\gamma$  a positive parameter reflecting the reputation of the investment bank. For a given level of underpricing, prestigious banks have a greater ability to generate optimistic investors' demand and are so associated with a greater  $\gamma$ .

The IPO takes place in the context of the bookbuilding procedure: the issuer and the investment bank collect indications of interest from potential investors, set the offer price the day before the IPO and then allocate shares in a discretionary manner to some institutional investors<sup>16</sup>. These investors are ready to pay any price for the stock as soon as they can resell the shares on the secondary market at a higher one. We suppose that institutional investors resell immediately the shares on the secondary market the day of the IPO (a common practice known as flipping). As shown below, there is always underpricing at equilibrium so that the participation constraint of institutional investors is satisfied.

The determination of the offer price is supposed to result from a negotiation process between the entrepreneur, the venture capitalist and the investment bank. The entrepreneur wants to maximize the overvalued component of the IPO proceeds, that is  $Q(P0_t - V_t)$  in order to finance new investments or marketing expenses. He expects that these expenditures may increase the value of the firm but in a concave manner and then negotiates for a high offer price according to a concave function  $h(x) = x^\mu$ . The objective of the venture capitalist is to maximize the expected overvalued part of the proceeds resulting from the sales at  $t + T$  of his  $\theta Q_S$  secondary shares. Finally, the investment bank, like the venture capitalist, wants to maximize underpricing as it facilitates the placement of shares towards institutional investors<sup>17</sup>. We assume that the bargaining power of the entrepreneur is higher the market is hotter<sup>18</sup>. This is summarized by a non-convex function  $b(n_t) = n_t^\beta$ , increasing in the number of optimistic investors  $n_t$ . The negotiation process is summed up by the following program:

$$\max_{P0_t} N = b(n_t) \times h(Q(P0_t - V_t)) + \theta Q_S (E[P_{t+T}] - \frac{E[V_{t+T}]}{Q + Q_S}) e^{-\rho T} \quad (24)$$

<sup>16</sup>In practice, about 80% of IPO shares are sold on the primary market to institutional investors.

<sup>17</sup>In addition, it is widely recognized that investment banks get substantial profits by allocating underpriced shares to professional investors who in return overpay for commissions (see for instance Loughran and Ritter, 2002).

<sup>18</sup>It may be more difficult for the bank and the venture capitalist to negotiate a low offer price when demand for the stock is very high. The eventual increase in the length of the negotiation process may delay the IPO and so compromises the achievement of the optimal timing strategy of the VC.

which can be rewritten as:

$$\max_{P0_t} N = n_t^\beta (Q(P0_t - V_t))^\mu + \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma (P_t - P0_t)^\nu, \quad (25)$$

with  $\delta_n = \rho - \alpha_n > -\lambda$ .

(25) does not admit any explicit analytical solution. However, it can be shown that for  $0 < \beta \leq 1$ ,  $0 < \mu < 1$  and  $0 < \nu < 1$ , the optimal offer price is always set below  $P_t$  (there is underpricing), but above the fundamental value  $V_t$  (the stock is overpriced from a fundamental value perspective). Moreover, the equilibrium initial return, or underpricing, increases with  $n_t$ ,  $\gamma$  and  $\theta Q_S$  but decreases with  $Q$ .

**Proposition 2** *In the presence of optimistic investors, the offer price is always set below the market price of the first trading day and above the fundamental value. The size of the underpricing increases with the number of optimistic investors present at the time of the offer, the reputation of the investment bank and the number of secondary shares expected to be sold at the lockup expiration date, and decreases with the quantity of primary shares sold at the IPO.*

**Proof:** see appendix.

When the number of optimistic investors  $n_t$  increases, the optimal offer price  $P0_t^*$  increases but to a lesser extent than do the market closing price  $P_t$  so that underpricing  $u_t^*$  is higher. Several empirical studies have found that initial returns were positively related to the demand from retail investors. For example, from a sample of french IPOs, Derrien (2002) finds that the offer price as well as initial return increase with the demand of individual investors<sup>19</sup>. Similarly for the US, using the TAQ database, Zhang (2000) identifies the buying volume generated by small investors during the first trading day of the IPO and finds that underpricing is positively related to this volume. Finally, it is well documented in the IPO literature that initial returns are positively and very significantly explained by stock market returns preceding the date of the offer (see Loughran and Ritter, 2002 or Derrien and Womack, 2003, among others). As evidenced by Derrien (2002), this is because market conditions have an impact on the demand of individual or optimistic investors, even in the few days before the IPO. Overall, high initial returns may be interpreted as a high level of optimists demand.

The link between underpricing and the reputation of the investment bank was first studied by Beatty and Ritter (1986), showing that IPOs managed by the most prestigious banks were the less underpriced. This relation seems to have reversed in the nineties since IPOs associated with prestigious investment banks (here a high parameter  $\gamma$ ) experienced the highest initial returns as documented by Beatty and Welch (1996) or Krigman, Shaw

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<sup>19</sup>There exists in France an IPO procedure commonly used since 1999 which reserves a fraction of offered shares for individual investors. The demand from these investors is hence directly observable.

and Womack (2001). We claim this is due to a greater ability of prestigious banks to generate a strong demand from optimistic investors and then a higher incentive for the issuer to underprice the offer. We could also argue that issuers who expect to sell their own shares at the end of the lockup will be more inclined to engage prestigious investment banks.

Finally, Aggarwal *et al.* (2002) find that insiders with larger holdings are more willing to underprice the IPO and Francis and Hasan (2001) show that venture-backed IPOs are significantly more underpriced than nonventure capital ones. Here, this is because the larger the number of shares to be sold at the end of the lockup, the greater the incentive for the VC to negotiate a low offer price in order to generate high demand.

Let us consider for analytical convenience the case where  $\beta = 1$  and  $\mu = \nu = \frac{1}{2}$ . This will allow us to discuss the IPO timing in a more realistic way than in the basic model. Resolving (25) yields:

$$P0_t^* = V_t + \frac{n_t(Q + \theta Q_S)^2}{Q(Q + \theta Q_S)^2 + (\theta Q_S \gamma e^{-(\delta_n + \lambda)T})^2}. \quad (26)$$

The optimal underpricing is given by:

$$u_t^* = (P_t - P0_t^*) = \frac{n_t (\theta Q_S \gamma e^{-(\delta_n + \lambda)T})^2}{Q((Q + \theta Q_S)^2 + (\theta Q_S \gamma e^{-(\delta_n + \lambda)T})^2)}. \quad (27)$$

Substituting  $P0_t$  for  $P0_t^*$  in the second term on the right-hand side of (23) gives the expected overvalued component of the market price at the lockup expiration date, conditionally on the outcome of the negotiation process:

$$E_u[P_{t+T} | \pi_t, n_t] = \frac{E[V_{t+T} | \pi_t]}{Q + Q_S} + \frac{\theta Q_S (\gamma e^{-(\delta + \lambda)T})^2 n_t^{3/2}}{(Q + \theta Q_S) \sqrt{Q(Q + \theta Q_S)^2 + (\theta Q_S \gamma e^{-(\delta_n + \lambda)T})^2}}. \quad (28)$$

Let  $S_t^e$  denote the expected overvalued component at  $t$  of the stock price at  $t + T$ . We have:

$$S_t^e = a n_t^{3/2}, \quad (29)$$

where

$$a = \frac{\theta Q_S (\gamma e^{-(\delta + \lambda)T})^2}{(Q + \theta Q_S) \sqrt{Q(Q + \theta Q_S)^2 + (\theta Q_S \gamma e^{-(\delta_n + \lambda)T})^2}}. \quad (30)$$

Applying Itô's Lemma to (29), the stochastic process followed by  $S_t^e$  is given by:

$$\frac{dS_t^e}{S_t^e} = \alpha_{S^e} dt + \sigma_{S^e} dz_{S^e} - dq, \quad (31)$$

where  $dz_{S^e} = dz_n$ ,  $\alpha_{S^e} = \frac{3}{2}\alpha_n + \frac{3}{8}\sigma_n^2$  and  $\sigma_{S^e} = \frac{3}{2}\alpha_n$ .

The termination payoff received by the venture capitalist when the issue is underpriced can now be written as:

$$\Omega_u(\pi_t, S_t^e) = \frac{\theta Q_S}{Q + Q_S} \frac{\pi_t}{\delta_\pi} + \theta Q_S S_t^e e^{-\rho T}. \quad (32)$$

Applying the same procedure as in 3.1, we obtain the critical value of the ratio  $s_u = \frac{S^e}{\pi}$  above which the firm is taken public.

**Proposition 3** *With profits of the project and an expected stock price overvaluation following respectively the stochastic processes (1) and (31), the IPO occurs when the ratio  $s_u = \frac{S^e}{\pi}$  is greater than*

$$\frac{S^{e*}}{\pi} = s_u^* = \frac{\zeta_1}{\zeta_1 - 1} \frac{Q}{\delta_\pi Q_S (Q + Q_S)} e^{\rho T}, \quad (33)$$

where

$$\zeta_1 = \frac{1}{2} - \frac{\delta_\pi - \delta_{S^e}}{\sigma_\pi^2 - 2r\sigma_{S^e}\sigma_\pi + \sigma_{S^e}^2} + \sqrt{\left[ \frac{\delta_\pi - \delta_{S^e}}{\sigma_\pi^2 - 2r\sigma_{S^e}\sigma_\pi + \sigma_{S^e}^2} \right]^2 + \frac{2(\delta_\pi + \lambda)}{\sigma_\pi^2 - 2r\sigma_{S^e}\sigma_\pi + \sigma_{S^e}^2}} > 1, \quad (34)$$

$\delta_{S^e} = \rho - \alpha_{S^e}$  and  $E[dz_\pi dz_{S^e}] = rdt$ .

The venture capitalist will take the firm public when the benefit from the IPO, namely the opportunity to sell his shares at an overvalued price  $T$  units of time after the offer, is sufficiently high relatively to the dilution cost. This will occur in particular after a positive shock on  $S^e$  that is on the number of optimistic investors  $n$ .

**Proposition 4** *Holding the profit level constant, firms should go public after positive shocks on the number of optimistic investors.*

Proposition 4 is highly consistent with empirical results. Lowry and Schwert (2002) show that past initial returns positively Granger cause the number of IPOs. More precisely, the Granger F-test rejects with a  $p$ -value below 1% the hypothesis that three lags of monthly average initial returns have no power to predict the number of IPOs. This means that firms are going public when average underpricing is high that is when the number of optimistic investors is high by proposition 2. Since in practice the option of going public cannot be exercised instantaneously because of the offer registration period, a lag is observed between the increase in the number of optimistic investors and the rise in IPO volume<sup>20</sup>.

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<sup>20</sup>Introducing a lag between the going public decision and the date of the IPO would not modify the results of the model. It would only raise the critical value  $s^*$  beyond which the issuer decides to go public.

So far, we have considered the IPO timing decision by assuming that the firm faced competition on the product market but enjoyed a monopolistic position on the IPO market. However, when the market is bullish, other firms in the same industry will be taken public and one might expect that the number of optimistic investors for each issuer gets down, leading to a decrease in initial returns and subsequent IPO volume. Econometric results does not however support this last assertion. As evidenced by Lowry and Schwert’s study, the number of IPOs has no power to predict future initial returns. This could be because of bottlenecks on the offer side of the IPO market, as the investment bank industry cannot instantaneously adjust its size to a sudden increase in the demand for its services and the Security and Exchange Commission is unable to process the registration statements as quickly as in normal periods<sup>21</sup>. As a result, it may be that the number of optimistic investors present the first day of the offer (the variable  $n$  in the model) increases at a higher rate than do the number of IPOs and then, despite a larger quantity of stocks offered to investors, initial returns are still substantial. It would be interesting to introduce such logistic effects in the model. We leave this open for further research.

## 4 IPO option and real investment

Shleifer (2000) points out that “when capital markets are not sufficiently developed to enable the financing of all privately profitable projects, bubbles play an extremely positive social role”. Indeed, “overvaluation may enable firms to finance profitable projects that they could not finance otherwise because of imperfections in the capital markets”. In this section, we discuss the role played by the IPO market in the decision to invest in new industries.

First, let us consider the investment decision of the venture capitalist when there is no IPO market. At date  $t$ , the venture capitalist can pay  $I$  to get  $\theta V_t$  where  $V_t$  is defined by (3). Since profits and so, the project value, are expected to decrease over time, the optimal timing strategy clearly consists in investing immediately, provided that the current value  $V_t$  lies above a given critical level noted  $V^*$ . One can calculate  $V^*$  using the basic investment problem model under uncertainty.

Let  $\Psi(\pi_t)$  denote the termination payoff received by the venture capitalist when investing  $I$  at date  $t$ . We have:

$$\Psi(\pi_t) = \theta \frac{\pi_t}{\delta\pi} - I. \tag{35}$$

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<sup>21</sup>Lowry and Schwert report that high initial returns are associated with longer registration periods in following months. Without considering bottlenecks effects, this finding may seem quite paradoxical as potential issuers interpret high initial returns as a signal of a high level of optimistic demand and so should exercise their IPO option more quickly.

The program of the venture capitalist can be stated as follows:

$$H(\pi_t) = \max \left\{ \Psi(\pi_t), \frac{1}{1 + \rho dt} E [H(\pi_t + d\pi_t) \mid \pi_t] \right\}, \quad (36)$$

where  $H(\pi_t)$  is the value of the investment opportunity without IPO market.

The only return from holding the investment opportunity is its expected appreciation. In the continuation region, the Bellman equation is then:

$$\rho H(\pi_t) dt = E[dH(\pi_t) \mid \pi_t]. \quad (37)$$

Once again, we can drop the  $t$  subscripts on  $\pi$  since the problem is time homogeneous. Expanding  $dH$  using Itô's Lemma, taking its expected value by noting that  $E(dz_\pi) = 0$ ,  $E(dz_\pi^2) = dt$  and dividing by  $dt$ , (37) becomes:

$$\frac{1}{2} \sigma_\pi^2 \pi^2 H''(\pi) + (\rho - \delta_\pi) \pi H'(\pi) - \rho H(\pi) = 0. \quad (38)$$

In order to use the relation (37) when  $\alpha_\pi$  is negative, one must suppose that  $\alpha_\pi$  is not too low in comparison with the variance  $\sigma_\pi^2$  otherwise  $E[dH(\pi) \mid \pi]$  would be negative and so the value of the option, which makes no sense. Indeed, the value of waiting comes from the eventuality of a negative shock on profits in the future. So, the value of the opportunity to invest will be positive if the following hypothesis is satisfied:

**Assumption 2** *The amplitude of shocks over profits is sufficiently large so that the value of waiting is positive in the continuation region. More precisely,  $\alpha_\pi$  and  $\sigma_\pi^2$  are such that the following inequality holds:*

$$\frac{1}{2} \sigma_\pi^2 \pi^2 H''(\pi) + \alpha_\pi \pi H'(\pi) > 0. \quad (39)$$

Note that this assumption is more likely to be true, *ceteris paribus*, in industries where profits are very volatile as this is the case in emerging industries.

**Proposition 5** *When profits are expected to decline over time, the critical value of  $V$  until which it is optimal to invest  $I$  is*

$$V^* = \frac{\xi_1}{\xi_1 - 1} \frac{I}{\theta}, \quad (40)$$

with

$$\xi_1 = \frac{1}{2} - \frac{(\rho - \delta_\pi)}{\sigma_\pi^2} + \sqrt{\left[ \frac{(\rho - \delta_\pi)}{\sigma_\pi^2} - \frac{1}{2} \right]^2 + \frac{2\rho}{\sigma_\pi^2}} > 1. \quad (41)$$

**Proof:** see appendix.

In the present case, the venture capitalist receives only a fraction  $\theta$  of the value of the project. Then, he will invest  $I$  at date  $t$  if  $\theta V_t$  is greater than

$$\theta V^* = \frac{\xi_1}{\xi_1 - 1} I > I. \quad (42)$$

Suppose that initially  $V_0$  is greater than  $V^*$ , then agents will enter the industry until an unknown date  $t^*$  when  $V_{t^*} = V^*$ . Note however that there are still positive NPV projects in the industry which are not undertaken simply because of uncertainty over the evolution of profits.

**Proposition 6** *The higher the variance of the project's returns, the greater the critical value below which no investment will occur.*

Proof follows from derivation of  $\xi_1$  with respect to  $\sigma_\pi^2$ .

From a fundamental value perspective, if  $V$  is initially greater than  $V^*$ , investment in an industry will occur until the critical value  $V^*$  is hit. However, one can imagine industries in the economy where  $V^*$  is so high that only a few project will be financed, despite the presence of other positive NPV projects. From proposition 6, this is more likely to be true in industries where the variance of returns is high, that is in emerging industries.

Having the option to go public and so, the opportunity to sell the project at an overvalued price can now enable the financing of projects which would not be undertaken otherwise. Indeed, in presence of optimistic investors, the value of the firm at date  $t$  is given by (22). Then, the venture capitalist will invest in the industry if

$$I < F(\pi, n) = A_1 \frac{n^{\varepsilon_1}}{\pi^{\varepsilon_1 - 1}} + \theta \frac{\pi}{\delta_\pi}. \quad (43)$$

Overvaluation on the stock market will be beneficial to the economy if it enables the implementation of projects with positive net present values. In the absence of the IPO market, the region over which such projects are not financed is larger, the higher the variance of returns in the industry. But as the variance of returns increases, the evaluation of projects is prone to greater divergence of opinion and so (43) may hold. As long as (43) holds, venture capitalists finance projects and sell them on the stock market.

If all positive NPV projects have been financed and the IPO market is still hot (that is the Poisson event in (6) has not yet occurred), venture capitalists may find optimal to undertake negative present value investments. In this case, overvaluation is harmful for two reasons. First, because it leads to overinvestment in the industry (too many similar projects are financed) and then wasting of capital. Gompers and Lerner (2003) report for example that in the early eighties, nineteen disk drive companies received venture capital financing and that two-thirds of these investments came in 1982 and 1983, a period when valuation of publicly traded computer hardware firms experienced a substantial increase. Whereas the rationality of the scale of investment in the industry was highly

questionable, many of these companies went public, just before the market collapsed a few month later. A similar dynamic was observed in the biotechnology industry in the early nineties. The second reason why overvaluation is harmful is that positive NPV projects in other industries will be unfunded<sup>22</sup>. Indeed, the fundamental or real return from these projects, even if significant, may be lower than the financial return received from the sale of overvalued stocks if the number of optimistic investors  $n$  in (43) is very high, as it was the case for Internet companies in the late nineties.

## 5 Conclusion

In this paper, we studied the effect of the IPO market on the decision to invest in new industries when investors on the stock market agree to disagree about the fundamental value of firms taken public and short sales are constrained. Following Draho (2000), we treated the IPO decision as a real option and showed that firms were more likely to go public when overvaluation was high relatively to profits, that is after price run-ups. Though not explicitly modelled here, we proposed an explanation for the dynamics of IPOs cycles. Early issuers optimally exercise their IPO option when the number of optimistic investors is sufficiently high, resulting in high initial returns. Private firms of the same industry interpret this as a signal of a high optimists demand leading to an increase in subsequent IPO volume. Because of congestion effects on the offer side of the market, the total volume of shares offered increases less quickly than the demand from optimists explaining why average initial returns cause future IPO volume while the reverse is not true.

Finally, stock market bubbles may affect real investment decisions because of a substantial value of the IPO option. Bubbles can be beneficial in the sense that they enable the financing of positive NPV projects which would not be undertaken otherwise because of great uncertainty over profits. On the other side, excessive and durable overvaluation leads to overinvestment, that is investment in negative NPV projects in the industry in vogue on the stock market, whereas some positive NPV investments in other promising industries are not undertaken.

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<sup>22</sup>Gompers and Lerner note that during the IPO cycle of 1998-2000, investments were concentrated in the sectors of Internet and telecommunication. Promising areas like energy technologies, micro manufacturing or advanced materials were not funded as venture capitalists preferred allocating funds to most popular investment areas.

## Appendix

### A.1 Derivation of the termination payoff $\Omega(\pi_t, n_t)$ .

With a total number of shares equal to  $Q + Q_S$ , the discounted value of the expected flow of profits received by the venture capitalist between the IPO date  $t$  and the lockup expiration date  $t + T$  is

$$\frac{\theta Q_S}{Q + Q_S} \frac{\pi_t}{\delta_\pi} (1 - e^{-\delta_\pi T}). \quad (\text{A1})$$

The expected market price at  $t + T$  is

$$E[P_{t+T} | \pi_t, n_t] = \frac{E[V_{t+T} | \pi_t]}{Q + Q_S} + \frac{n_t e^{(\alpha_n - \lambda)T}}{Q + \theta Q_S}, \quad (\text{A2})$$

where

$$E[V_{t+T} | \pi_t] = E \left[ E \left[ \int_{t+T}^{\infty} \pi_s e^{-\rho(s-t-T)} ds | \pi_{t+T} \right] | \pi_t \right] = \frac{\pi_t}{\delta_\pi} e^{\alpha_\pi T}. \quad (\text{A3})$$

Discounting (A2) at the rate  $\rho$  for length  $T$ , multiplying by the number of secondary shares  $\theta Q_S$  and adding to (A1) yields:

$$\Omega(\pi_t, n_t) = \frac{\theta Q_S}{Q + Q_S} \frac{\pi_t}{\delta_\pi} + \frac{\theta Q_S}{Q + \theta Q_S} n_t e^{-(\delta_n + \lambda)T}. \quad (\text{A4})$$

### A.2 Proof to Proposition 1

With  $F(\pi, n) = \pi f\left(\frac{n}{\pi}\right) = \pi f(s)$ , we have:

$$F_n(\pi, n) = f'(s), \quad F_\pi(\pi, n) = f(s) - s f'(s),$$

$$F_{nn}(\pi, n) = \frac{f''(s)}{\pi}, \quad F_{\pi\pi}(\pi, n) = \frac{s^2 f''(s)}{\pi} \quad \text{and} \quad F_{\pi n}(\pi, n) = -\frac{s f''(s)}{\pi}.$$

Substituting these expressions in the partial differential equation (15), dividing by  $\pi$  and rearranging gives the ordinary differential equation for the function  $f(s)$ :

$$\frac{1}{2}(\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2)s^2 f''(s) + (\delta_\pi - \delta_n)s f'(s) - (\delta_\pi + \lambda)f(s) + \left(\frac{\delta_\pi + \lambda}{\delta_\pi}\right)\theta = 0. \quad (\text{A5})$$

The general solution of the homogeneous part of (A5) have the form:

$$f(s) = A_1 s^{\varepsilon_1} + A_2 s^{\varepsilon_2}. \quad (\text{A6})$$

Following Dixit and Pindyck (1994), it can be expressed as:

$$f(s) = As^\varepsilon. \quad (\text{A7})$$

The particular integral to (A5) is  $\frac{\theta}{\delta_\pi}$ . Adding it to (A7) yields:

$$f(s) = As^\varepsilon + \frac{\theta}{\delta_\pi}. \quad (\text{A8})$$

Substituting (A8) in (A5), we get the fundamental quadratic equation:

$$\frac{1}{2}(\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2)\varepsilon(\varepsilon - 1) + (\delta_\pi - \delta_n)\varepsilon - (\delta_\pi + \lambda) = 0, \quad (\text{A9})$$

whose two roots are

$$\varepsilon_1 = \frac{1}{2} - \frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2} + \sqrt{\left[\frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}\right]^2 + \frac{2(\delta_\pi + \lambda)}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}} > 1$$

and

$$\varepsilon_2 = \frac{1}{2} - \frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2} - \sqrt{\left[\frac{\delta_\pi - \delta_n}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}\right]^2 + \frac{2(\delta_\pi + \lambda)}{\sigma_\pi^2 - 2r\sigma_n\sigma_\pi + \sigma_n^2}} < 0.$$

Moreover, note that from  $F(\pi, 0) = \theta \frac{\pi}{\delta_\pi}$ ,  $f$  must satisfy:

$$f(0) = \frac{\theta}{\delta_\pi}. \quad (\text{A10})$$

Because  $\varepsilon_2 < 0$ , we must have  $A_2 = 0$ , otherwise as  $n$  went to zero,  $f(s)$  would tend to infinity, which violates (A10). Finally, applying the value-matching and the smooth-pasting conditions to  $f(s) = A_1s^{\varepsilon_1} + \frac{\theta}{\delta_\pi}$  yields the system:

$$\begin{cases} A_1s^{\varepsilon_1} + \frac{\theta}{\delta_\pi} = \frac{\theta Q_S}{Q + Q_S} \frac{1}{\delta_\pi} + \frac{\theta Q_S}{Q + \theta Q_S} s e^{-(\delta_n + \lambda)T} \\ A_1s^{\varepsilon_1} \frac{\varepsilon_1}{s} = \frac{\theta Q_S}{Q + \theta Q_S} e^{-(\delta_n + \lambda)T}. \end{cases} \quad (\text{A11})$$

Solving (A11) for  $s$  and  $A_1$  gives:

$$s^* = \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{Q(Q + \theta Q_S)}{\delta_\pi Q_S(Q + Q_S)} e^{(\delta_n + \lambda)T} \quad (\text{A12})$$

and

$$A_1 = \theta Q [\delta_\pi (\varepsilon_1 - 1) (Q + Q_S)]^{\varepsilon_1 - 1} \left( \frac{Q_S}{\varepsilon_1 Q (Q + \theta Q_S)} \right)^{\varepsilon_1} e^{-\varepsilon_1 (\delta_n + \lambda)T}. \quad (\text{A13})$$

### A.3 Proof to Proposition 2

The first order condition of (25) is

$$\frac{\partial N}{\partial P0_t} = 0 \Leftrightarrow n_t^\beta Q^\nu \nu (P0_t^* - V_t)^{\nu-1} - \frac{\theta Q_S}{Q + \theta Q_S} \gamma n_t e^{-(\delta_n + \lambda)T} \mu (P_t - P0_t^*)^{\mu-1} = 0. \quad (\text{A14})$$

As  $P0_t^*$  goes to  $V_t$ ,  $\nu(P0_t^* - V_t)^{\nu-1}$  goes to infinity and  $\frac{\partial N}{\partial P0_t} > 0$ . Hence,  $P0_t^* > V_t$ . As  $P0_t^*$  goes to  $P_t$ ,  $\mu(P_t - P0_t^*)^{\mu-1}$  goes to infinity and  $\frac{\partial N}{\partial P0_t} < 0$ . Therefore,  $P0_t^* < P_t$ .

Finally, one can verify that  $\frac{\partial^2 N}{\partial P0_t^2}$  is negative so that  $P0_t^*$  is a maximum.

Comparative statics:

With  $u_t = P_t - P0_t = V_t + \frac{n_t}{Q} - P0_t$ , the program (25) can be rewritten as:

$$\max_{u_t} N = n_t^\beta \left( Q \left( \frac{n_t}{Q} - u_t \right) \right)^\mu + \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma u_t^\nu, \quad (\text{A15})$$

with the first order condition

$$\frac{\partial N}{\partial u_t} = 0 \Leftrightarrow G = -\mu n_t^\beta Q^\mu \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-1} + \nu \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma u_t^{*\nu-1} = 0. \quad (\text{A16})$$

For the parameters values of interest, partial derivations yield:

$$\frac{\partial G}{\partial u_t^*} = \mu(\mu - 1) n_t^\beta Q^\mu \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-2} + \nu(\nu - 1) \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma u_t^{*\nu-2} < 0,$$

$$\begin{aligned} \frac{\partial G}{\partial n_t} &= -\beta \mu n_t^{\beta-1} Q^\mu \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-1} - \mu(\mu - 1) n_t^\beta Q^{\mu-1} \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-2} \\ &\quad + \nu \theta Q_S \frac{e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma u_t^{*\nu-1} > 0, \end{aligned}$$

$$\frac{\partial G}{\partial \gamma} = \nu \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} u_t^{*\nu-1} > 0,$$

$$\frac{\partial G}{\partial \theta Q_S} = \nu \frac{n_t e^{-(\delta_n + \lambda)T}}{Q + \theta Q_S} \gamma u_t^{*\nu-1} - \nu \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{(Q + \theta Q_S)^2} \gamma u_t^{*\nu-1} > 0,$$

$$\begin{aligned} \frac{\partial G}{\partial Q} &= -\mu^2 n_t^\beta Q^{\mu-1} \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-1} + \mu n_t^{\beta+1} Q^{\mu-2} \left( \frac{n_t}{Q} - u_t^* \right)^{\mu-2} (\mu - 1) \\ &\quad - \nu \theta Q_S \frac{n_t e^{-(\delta_n + \lambda)T}}{(Q + \theta Q_S)^2} \gamma u_t^{*\nu-1} < 0. \end{aligned}$$

From implicit derivations, we get:

$$\frac{du_t^*}{dn_t} = -\frac{\partial G/\partial n_t}{\partial G/\partial u_t^*} > 0, \quad \frac{du_t^*}{d\gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial u_t^*} > 0, \quad \frac{du_t^*}{d\theta Q_S} = -\frac{\partial G/\partial \theta Q_S}{\partial G/\partial u_t^*} > 0$$

$$\text{and } \frac{du_t^*}{dQ} = -\frac{\partial G/\partial Q}{\partial G/\partial u_t^*} < 0.$$

#### A.4 Proof to Proposition 5

The general solution  $H(\pi)$  to the second-order differential equation (38) have the form:  $H(\pi) = B_1\pi^{\xi_1} + B_2\pi^{\xi_2}$ . It must satisfy the boundary conditions:

$$H(0) = 0, \tag{A17}$$

$$H(\pi^*) = \theta \frac{\pi^*}{\delta_\pi} - I, \tag{A18}$$

$$H'(\pi^*) = \frac{\theta}{\delta_\pi}. \tag{A19}$$

Condition (A17) states that if  $\pi$  goes to zero, then the option has no value,  $\pi$  staying at zero forever due to its stochastic properties. Equations (A18) and (A19) are respectively the value-matching and the smooth-pasting conditions.

The solution to (38) is

$$H(\pi) = B_1\pi^{\xi_1}, \tag{A20}$$

where

$$\xi_1 = \frac{1}{2} - \frac{\rho - \delta_\pi}{\sigma_\pi^2} + \sqrt{\left[\frac{\rho - \delta_\pi}{\sigma_\pi^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma_\pi^2}} > 1 \tag{A21}$$

is the positive root to the fundamental quadratic equation:

$$\frac{1}{2}\sigma_\pi^2\xi(\xi - 1) + (\rho - \delta_\pi)\xi - \rho = 0. \tag{A22}$$

Applying the value-matching and the smooth-pasting conditions to  $H(\pi) = B_1\pi^{\xi_1}$  yields:

$$\begin{cases} B_1\pi^{*\xi_1} = \theta \frac{\pi^*}{\delta_\pi} - I \\ \xi_1 B_1\pi^{*(\xi_1-1)} = \frac{\theta}{\delta_\pi}. \end{cases} \tag{A23}$$

Resolving (A23) gives:

$$\frac{\pi^*}{\delta_\pi} = V^* = \frac{\xi_1}{\xi_1 - 1} \frac{I}{\theta} \quad (\text{A24})$$

and

$$B_1 = \frac{(\xi_1 - 1)^{\xi_1 - 1}}{\xi_1^{\xi_1}} \left( \frac{\theta}{\delta_\pi} \right)^{\xi_1} I^{1 - \xi_1}. \quad (\text{A25})$$

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