

# Credit Allocation and Monetary shocks in a Model with Heterogeneous Lenders

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*Preliminary Version*

**Abstract:** *The paper presents a model of endogenous credit allocation with heterogeneous lenders. We consider three classes of agents' -firms, individual investors and banks. Banks differ according to their level of capital and monitoring technology. In a setting of moral hazard with limited liability, we stress that firms' ability to obtain external funds is conjointly determined by their own wealth, bank capital, and monitoring technology. We show that small (medium) firms invest with the small (large) bank and pay a high (low) interest rate whereas large firms are financed by the financial market. Moreover, we stress that restrictive monetary policy leads to a contraction in aggregate investment and to a credit reallocation mechanism, between the two banks and the market, similar to a "flight to quality" effect. This restrictive policy has a strong effect not on bank-dependent firms but on small bank-dependent firms.*

JEL: E5, D8, G2

## 1. INTRODUCTION

The liberalization of geographic restrictions on U.S. banking institutions has produced a consolidation of the banking industry. However, assessments of this industry continue to describe it as a composite of banks that differ in size, capital, and location, and suggest that banking system remains highly heterogeneous<sup>1</sup>. This relative persistence of heterogeneity in banking system may be explained by specialization effects in the monitoring of different kinds of borrowers<sup>2</sup>. Indeed, lending to small, informationally opaque borrowers and lending to large, informationally transparent borrowers are two different activities that require the use of distinct monitoring technologies. In particular, it appears that small banks are specialized in financing small businesses while large banks tend to be specialized in financing loans to medium

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<sup>1</sup> See Peek and Rosengren (1998), Basset and Brady (2001).

<sup>2</sup> If small banks have a cost advantage in providing relationship loans to small businesses, consolidation will not lead necessarily to the disappearance of small banks; they will continue to play a vital role at the small end of the lending market (Strahan and Weston (1996)).

or large businesses<sup>3</sup>. This results from the better ability of small banks to meet the credit needs of small businesses as their organizational structures allow them to be better delegated monitors than large banks<sup>4</sup>. Peek and Rosengren (1996) and Berger, Kashyap and Scalise (1995) show a strong link between banking institution size and the supply of credit for small businesses, since, compared to smaller financial institutions, large banks tend to devote proportionally less of their assets to small businesses.

Moreover, as bank lending constitutes an important source of external funds<sup>5</sup>, all shocks to the banking system can have important macroeconomic implications. In the bank lending channel, monetary policy affects firms' investment level when banks have to reduce their loans, and borrowers are bank-dependent<sup>6</sup>. However, a restrictive monetary policy does not influence banks' ability to lend in the same way. Kashyap and Stein (1995), (2000)) show that monetary policy appears to have a more powerful impact on small banks than on large banks while Kishan and Opiela (2000) underline that small undercapitalized banks are the most affected by monetary policy which leads to a slowdown of their loan growth. Since small and undercapitalized banks are the most affected by monetary policy, a restrictive policy will hit their borrowers the hardest.

These facts suggest that banking system heterogeneity strongly affects the ability of firms to obtain external funds. Indeed, the relative share of small and large banks in the banking system may determine the availability of total credit. Similarly, it is important to take this heterogeneity into account in order to evaluate precisely the indirect and asymmetric effect of monetary policy on firms' investment level. In this paper we develop an equilibrium model of endogenous credit allocation with heterogeneous lenders in order to appraise the impact of monetary policy on firms' financing opportunities. The major questions we address in this framework are the following: How is the optimal credit allocation in a heterogeneous banking system determined? In what way does this allocation determine the relative strength of monetary policy across intermediaries and impact on the level of firms' investment?

Our model is built on Holmstrom and Tirole (1997). It adds to this initial contribution concerned with the existence of heterogeneous lenders. Specifically, we assume that our financial intermediaries (banks) differ by their amount of capital and by their size. A first category of intermediaries (named "the small bank") is specialized in

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<sup>3</sup> See Berger, Miller, Petersen, Rajan and Stein (2002), and Berger and Udell (2002).

<sup>4</sup> Because of legal lending limits and problems of diversification, small banks have equally fewer opportunities to make loans to large business than do large banks.

<sup>5</sup> Because of asymmetric information between lenders and borrowers, most small and medium sized businesses find banks the only economical source of debt financing (Gertler and Gilchrist (1994)).

<sup>6</sup> See Kashyap and Stein (2000) for a comprehensive survey on bank lending channel. The data also confirm that monetary policy appears to affect investment of small manufacturing firms more than that of large manufacturers (see Bernanke, Gertler and Gilchrist (1996)). For the broad credit channel see Hubbard (1997).

monitoring small firms whereas a second category (named "the large bank") is specialized in monitoring large businesses. Moreover, the small bank is also the less capitalized. Three main results emerge from our analysis.

First, we stress that firms' ability to obtain external funds is conjointly determined by their own wealth, bank capital, and monitoring technology. We model the optimal borrowing contract for a firm that can choose between financial market, the large bank, and the small bank. Firms are characterized by a different amount of internal funds and have access to an identical fixed size investment project. These projects require external funds in order to be undertaken. External investors are uninformed about the expected profitability of a specific project, which leads to a moral hazard problem between entrepreneurs and financiers as the former can divert project resources towards private uses. According to the financial intermediation literature<sup>7</sup>, banks can alleviate this entrepreneurial moral hazard problem owing to a specific monitoring technology. However, because monitoring is costly for the banks, there is a second moral hazard between intermediaries and investors in the provision of these monitoring services. In order to reduce this problem, banks must invest a part of their own capital in the project it monitors. Thus, both monitoring technology and bank capital must be considered in the banks' ability to lend. In order to finance their investment, we show that firms with high wealth rationally choose the financial market, firms with middle wealth choose to be monitored by the large bank, and firms with low private wealth are monitored by the small bank. Finally, firms without sufficient wealth cannot invest. This result is explained in terms of relative efficiency of banks' monitoring compared to the relative level of banks' capitalization.

Second, we stress that in equilibrium, the cost of borrowing from the large bank (well-capitalized) is lower than the cost of borrowing from the small bank (low-capitalized) whereas the financial market interest rate is the lowest one. This spread in the interest rate between the two banks is compatible with a no-arbitrage condition, as it is linked with equality of banks' payoff.

Our final point is related to the impact of the monetary policy<sup>8</sup> on the level of firms' investment. Restrictive monetary policy leads to a contraction in aggregate investment and to a credit reallocation mechanism, between the two banks and the market, similar to a "flight to quality" effect. This restrictive policy has a strong effect not on bank-dependent firms but on small bank-dependent firms. Small firms are the most affected because they have to be financed by the small bank and cannot substitute other sources of financing.

Few theoretical papers have tried to encompass elements of banking heterogeneity. Concerning bank expertise, Dell'Ariccia and Marquez (2001) have built a model where a lender with an informational advantage competes with a lender with

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<sup>7</sup> See Boot (2000) for a comprehensive survey.

<sup>8</sup> We interpret a restrictive monetary policy as an increase in the riskless interest rate (See Holmstrom and Tirole (1997)).

worse information but with a cost advantage. They examine how credit allocation reacts to changes in the relative cost of funds for the two lenders. Boot and Thakor (2000) study the incidence of increased competition on a bank's choice between different modes of lending and specialization. They define a relationship loan as a loan that permits the bank to use its expertise to improve a borrower's project payoff. They show that this kind of loan is reduced with capital market competition. In a different view, other theoretical papers explore how bank capital plays an important role to mitigate information imperfections (Bernanke and Gertler (1987), or Stein (1998)). These models insist on the role of capital as an imperfect substitute to expertise or as a signal for depositors. Linked with these two approaches, Almazan (2002) has developed a model of banking competition with two banks that differ by their expertise and their amount of capital. In this framework, he focuses on the bank's optimal geographic position. Similarly, Ennis (2001b) studies a model that supports the view that the existence of small banks is justified even in an unregulated environment<sup>9</sup>.

The paper is organized as follows. Section 2 presents the basic incentive model. Section 3 characterizes the optimal contracts. Section 4 describes the optimal choice of financing between entrepreneurs and lenders, and examines the impact of a restrictive monetary policy. Section 5 concludes and gives some macroeconomic implications. Mathematical proofs are contained in the appendix.

## 2. THE MODEL

We consider three classes of agents' -firms, individual investors and banks - and two periods. In the first period, firms need external funds in order to invest in a risky project. They have access to capital market and/or bank loan. During this period, financial contracts are signed between lenders and borrowers and investment decisions are made. In the second period, returns on investment are realized and firms have to pay for their external funds. We assume that all parties are risk neutral and protected by limited liability.

### 2.1. Firms' behaviour

There is a continuum of firms, all of which having access to the same technology. The only difference among firms relies on their private wealth ( $\theta$ ) that is supposed to be uniformly distributed on  $[0,1]$ <sup>10</sup>.

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<sup>9</sup> See equally Winton (1995) for a model in which capital can substitute bank diversification and Winton (2000) for a financial institution's choice between diversification and specialization as the source of competitive advantage for intermediaries.

<sup>10</sup> As in Holmstrom and Tirole (1997), firms' wealth is assumed to be cash. However, private wealth could be any type of asset that can be used as collateral (see Chen (2001)).

In order to undertake a risky project a specific firm must invest an amount of capital equal to 1. We take for granted that firms lack capital. In period 1, a firm with a wealth  $\theta$  needs at the most  $(1 - \theta)$  in external fund to invest. In period 2, the investment generates a stochastic payoff equal to 0 (in the case of project failure) or  $R > 0$  (If project is successful). Project's success probability depends on the behaviour of the entrepreneur who runs the firm. If the entrepreneur undertakes a good action (state  $h$ ) the probability of success is equal to  $p_h$ , whereas a bad action (state  $l$ ) leads to a success probability equal to  $p_l$ , where  $p_h > p_l$ . A moral hazard problem occurs because entrepreneurs enjoy a private benefice equal to  $B > 0$  if they act improperly (bad action) and nothing if they behave properly (good action)<sup>11</sup>. Define  $\gamma$  as the minimum rate of return on investors' capital ( $\gamma$  is exogenously given by monetary policy conditions and is defined as the riskless interest rate), we assume that only the good action leads to an economically viable investment project.

*Assumption 1.* Investment projects have positive-net-value during the second period only if entrepreneurs do not shrink

$$p_h R - \gamma > 0 > p_l R + B - \gamma \quad (1)$$

## 2.2. Individual investors

Individual investors (or uninformed investors) are endowed with a large amount of capital named *uninformed* capital. These investors are not able to reduce hazard moral problem. In order to accept lending to entrepreneurs, uninformed investors must receive at least the rate of return  $\gamma$  on their capital. In case of project's failure, none of the parties is paid. In case of success, investment's payoff is shared between entrepreneurs and investors. Define  $R_f$  and  $R_i$  respectively as the firm and the investors' payoff if investment succeeds, we have  $R = R_f + R_i$ . Incentive condition for investor to finance a risky project requires that the expected payoff of the project ( $p_h R_i$ ) is at least equal to the opportunity cost of the loaned funds ( $I_i \gamma$ ),  $I_i$  being the amount lent by an investor

$$p_h R_i \geq I_i \gamma \quad (2)$$

Moreover, according to (1), investors will finance a project only if entrepreneur prefers to be diligent. On the opposite case, the project is not economically viable and condition (2) is never fulfilled. An entrepreneur behaves properly (undertakes the good

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<sup>11</sup>  $B$  may also be interpreted as an opportunity cost for entrepreneurs if they manage their firm properly.

action) only if the return linked with the good action is higher than the return linked with the bad action, which requires that

$$p_h R_f \geq p_l R_f + B \text{ which leads to } R_f \geq \frac{B}{\Delta p} \quad (3)$$

Equation (3) defines the minimum amount an entrepreneur must receive in order to choose the good action when it borrows uninformed capital.

### 2.3. Financial sector

The financial sector consists of two segments: the one of large banks and the one of small banks. Size is defined according to the amount of asset held by banks. In our framework, banks' capital is the only asset held by banks and banks' size is defined by their respective amount of capital. A multiplicity of identical banks competes on each segment whereas we assume there is no possibility for a bank of a given segment to go to the other<sup>12</sup>. Banks aggregation on each segment lead to a structure with two representative banks, named small bank and large bank and indexed by  $j \in \{S, L\}$ . The small bank is also the one with the lower amount of capital.

*Assumption 2.* Bank  $S$  (small bank) is endowed with an amount  $K_S$  of capital while bank  $L$  (large bank) is endowed with an amount  $K_L$ , with  $K_L > K_S$  ■

The function of bank is to monitor firms and consequently to alleviate the moral hazard problem. We suppose that bank's monitoring reduces firm's opportunity cost (or private benefice) from  $B$  to  $d$  (with  $B > d > 0$ ). This means that the minimum amount an entrepreneur must receive in order to choose the good action decreases. Denote by  $R_{fj}$  entrepreneurs' minimum expected return in order to be diligent when investment project is monitored by bank  $j$ . Firms incentive condition requires that

$$p_h R_{fj} \geq p_l R_{fj} + d \text{ and } R_{fj} \geq \frac{d}{\Delta p} \quad (4)$$

In order to monitor firms' projects, banks have access to a specific (and costly) monitoring technology that differs according to their size<sup>13</sup>. Following Stein (2002) we suppose that efforts to coordinate lending in large institutions could lead to standardized credit policies based on easily observable, verifiable, and transmittable data. This kind

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<sup>12</sup> Because of banks competition, the equilibrium price of bank capital is determined by the market (see also Boot and Thakor (2000) and Almazan (2002) for other models of banks competition).

<sup>13</sup> Specifically, small banks have an informational advantage in evaluating and monitoring loan quality of opaque small firms (Nakamura (1994)). For an empirical investigation see Jayaratne and Wolken (1999) and McNulty, Akhigbe and Verbrugge (2001).

of information could be named "hard information" as it is based on relatively objective ratio, such as collateral ratio or credit scoring<sup>14</sup>. According to this approach, it becomes possible to conclude that large banks are disadvantaged in the lending relationship for small opaque firms because this kind of lending often requires "soft information". Soft information is not easily observed and verified by others. Consequently, soft information is difficult to transmit through the communication channels of large organizations.

We model these two sorts of specialization by providing respectively the large bank (bank  $L$ ) and the small bank (bank  $S$ ) with a cost advantage in monitoring large business (small business). Moreover, following the existing literature, we assume that banks monitoring costs are decreasing functions of firms' wealth. Consequently, the lower the wealth of a borrower the higher the monitoring cost for a bank to reduce moral hazard problem.

*Assumption 3.* Define  $c_j(\theta) > 0$  as the cost for Banks  $j$  (with  $j \in \{S, L\}$ ) to monitor a firm  $\theta$ .  $c_j(\theta)$  is monotonous and continuous on  $[0,1]$  and has the following properties:

1-  $0 < c_s(0) < c_L(0) \rightarrow +\infty$ ,  $c_s(1) > c_L(1) = 0$  : large bank (small bank) has a cost advantage in monitoring highly (weakly) capitalized firms.

2-  $\frac{dc_j(\theta)}{d\theta} = c'_j(\theta) < 0$  and  $|c'_L(\theta)| > |c'_s(\theta)|$ : the cost of monitoring is decreasing for the two banks along with firm's net wealth.

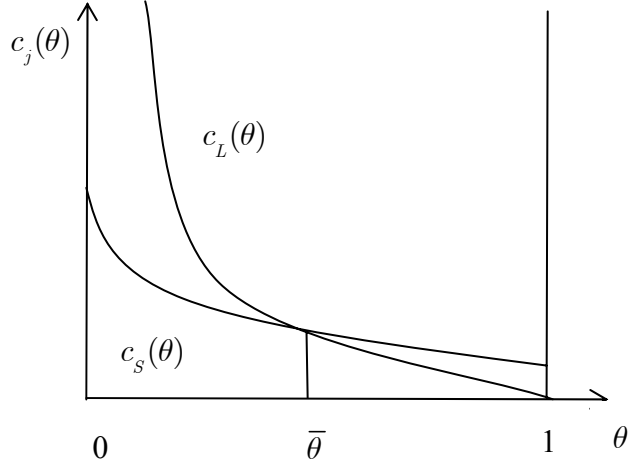
3-  $\frac{d^2c_j(\theta)}{d\theta^2} = c''_j(\theta) > 0$ : banks operate with decreasing return to scale in the monitoring technology.

4- Cost functions are integrable on  $[0,1]$  and  $C_j(\theta) = \int c_j(\theta) d\theta > 0$  with  $C_j(\theta)$  an increasing function of  $\theta$ .  $C_j(\theta)$  represent the total cost of monitoring support by bank  $j$  ■

Under these conditions there exist a critical wealth value ( $\bar{\theta} \in [0,1]$ ) for which the cost of the two banks are identical ( $c_s(\bar{\theta}) = c_L(\bar{\theta})$ ). A possible graphical representation of the two cost functions is given below.

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<sup>14</sup> For a distinction between hard and soft information see also Berger and Udell (2002) and Stein (2002). Tirole (1986) and Laffont and Rochet (1997) use the distinction between hard and soft information in order to expose the condition of collusion in small and large organisations.



We assume, following Holmstrom and Tirole (1997) and Almazan (2002) that all projects monitor by the banks are perfectly correlated. In that case, there is no way for bank diversification and bank capital becomes a key element in our framework because it appears as a signal of bank willingness to monitor<sup>15</sup>. In fact, a moral hazard problem arises since monitoring is costly for banks and not verifiable by uninformed investor. To make monitoring credible for the financial market, a bank must invest a part of its own capital in financing project it monitors. Denoting  $R_{bj}$  as the expected payoff received by bank  $j$  in payment of monitoring activity. The payoff of the project is now shared between three parties: firms, banks and individuals investors,  $R = R_{ff} + R_{bj} + R_{ij}$ . A bank funds a project only if it receives at least the opportunity cost of the funds. Defines  $\beta_j$  and  $I_{bj}$  respectively as the cost of the capital of the bank  $j$  and the amount lent by the bank, we have

$$p_h R_{bj} \geq \beta_j I_{bj} \quad (5)$$

Finally, as monitoring is costly, banks will monitor a firm only if the payoff linked with monitoring is higher than the payoff obtained when there is no monitoring. Consequently, the minimum amount a bank must received in order to monitor is given by

$$p_h R_{bj} - c_j(\theta) \geq p_l R_{bj} \text{ which leads to } R_{bj} \geq \frac{c_j(\theta)}{\Delta p} \quad (6)$$

We can now focus attention on the nature of the optimal contract between firms, banks and uninformed investors.

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<sup>15</sup> Without some degree of correlation, banks would not need to put any capital in a project (Diamond (1984), Williamson (1986) and Ennis (2001a)). To simplify our analysis, we assume perfect correlation even if only a certain degree of correlation is necessary in order for capital to play a role (Winton (1995)).

### 3. OPTIMAL CONTRACTS

A firm who needs external funds must contract with a lender. The optimal contract between firms and lenders must specify how much each party should invest and how much it should be paid in case of success.

#### 3.1. Optimal contract with direct finance

Consider a firm  $\theta$  that wishes to finance its investment's project on the financial market. According to incentive constraints, an optimal contract is the solution of the following problem

$$\max_{M, M'} R_f(\theta) \quad (7)$$

Subject to constraints

$$I_i(\theta) + \theta \geq 1 \quad (8)$$

$$R_f(\theta) + R_i(\theta) \leq R \quad (9)$$

$$p_h R_i(\theta) \geq \gamma I_i(\theta) \quad (10)$$

$$R_f(\theta) \geq \frac{B}{\Delta p} \quad (11)$$

$M \equiv [I_i(\theta)]$  represents the amount of uninformed capital invested in the project whereas  $M' \equiv [R_i(\theta), R_f(\theta)]$  is the allocation of the payoff between uninformed investor and entrepreneur if the project succeeds.

We can interpret the constraints of program (7). Eq. (8) describes how the financing of investment is shared between both parties. Eq. (9) shows how the project's payoff must be shared between agents. Eq. (10) ensures that uninformed investors' expected outcome must be at least equal to the market value of the funds. If this equation is not enforced, uninformed investors are not encouraged to invest their funds in the risky project. Finally, eq. (11) guarantees that the entrepreneur behaves fairly since its minimum payoff must be at least equal to the opportunity cost of being diligent (cf. *supra*). The optimal contract defines the amount invested by uninformed investor and the payoff it receives in equilibrium. Using (7) and (10) we have

$$I_i^*(\theta) = 1 - \theta \quad R_i^*(\theta) = (1 - \theta) \frac{\gamma}{p_h}$$

Optimal contract requires that: i) firm invests its entire "cash" in its project whereas uninformed investors finance the difference, ii) uninformed investors payoff is just equal to the opportunity cost of the funds.

The equilibrium payoff for the firm is calculated by substituting the equilibrium value in (9)

$$R_f^*(\theta) = R - R_i^*(\theta) = R - (1 - \theta) \frac{\gamma}{p_h} \quad (12)$$

It is obvious that the equilibrium firms' payoff when investment is financed by uninformed investor (12) is an increasing function of firms' net wealth. Substituting (11) in (12) and binding the resulting equation, we determine the last firm able to realise the incentive constraint

$$\left[ R - \frac{B}{\Delta p} \right] \geq (1 - \theta) \frac{\gamma}{p_h} \text{ and } \theta_M^* = 1 - \frac{p_h}{\gamma} \left[ R - \frac{B}{\Delta p} \right] \quad (13)$$

with  $\theta_M^* \in [0;1]$ .  $\theta_M^* \leq 1$  is always true because  $R \geq \frac{B}{\Delta p}$  (*assumption 1*).  $\theta_M^* \geq 0$  requires that  $p_h R - \gamma < p_h \frac{B}{\Delta p}$  which means that the total surplus of a project ( $p_h R - \gamma$ ) is less than the minimum amount a firm must be paid to behave diligently<sup>16</sup>.

LEMMA 1. Firms with a wealth  $\theta \geq \theta_M^*$  can ensure to uninformed agents the minimum incentive payoff and will resort to finance only. Firms with a wealth  $\theta < \theta_M^*$  don't invest using financial market ■

### 3.2. Optimal contract with indirect finance

When a firm has an insufficient wealth, it cannot raise funds in financial market. Because of their specific monitoring technology banks can help these firms to obtain external funds. Indeed, we have shown that monitoring activity reduces firm's opportunity cost of being diligent by reducing private benefit from  $B$  to  $d$ . The total surplus available for uninformed investor increases and the incentive condition is fulfilled for firms with wealth lower than  $\theta_M^*$ . In this context, we assume that investors deposit their money at the banks, which invest, along with its own funds, in the firms it monitors. Therefore, in the case of indirect finance, there are three parties to the financial contract: the firm, one of the two banks, and the uninformed investors. We assume that firms take as granted both the rate of return on bank capital ( $\beta_j$ ), and the cost of uninformed capital ( $\gamma$ ).

Consider a firm  $\theta$  who wants to have its project financed by bank  $j$ . The optimal contract between the three parties is solution of the following optimization problem

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<sup>16</sup> See Holmstrom and Tirole (1997).

$$\max_{H, H'} R_{fj}(\theta) \quad (14)$$

Subject to constraints

$$I_{bj}(\theta) + I_{ij}(\theta) + \theta \geq 1 \quad (15)$$

$$R_{ij}(\theta) + R_{bj}(\theta) + R_{fj}(\theta) \leq R \quad (16)$$

$$p_h R_{ij}(\theta) \geq \gamma I_{ij}(\theta) \quad (17)$$

$$p_h R_{bj}(\theta) \geq \beta_j I_{bj}(\theta) \quad (18)$$

$$R_{bj} \geq \frac{c_j(\theta)}{\Delta p} \quad (19)$$

$$R_{fj}(\theta) \geq \frac{d}{\Delta p} \quad (20)$$

Where  $H \equiv [I_{bj}(\theta), I_{ij}(\theta)]$  represents the amount invested in the project by the bank (large or small) and by the uninformed investor.  $H' \equiv [R_{bj}(\theta), R_{ij}(\theta), R_{fj}(\theta)]$  denotes the payoff, of bank, uninformed investors, and the firm, if the project is successful. Interpretation of program (14) is straightforward. Eq. (15) refers to the respective amounts invested by bank, uninformed investors and firm. Eq. (16) describes how the cash flow must be shared among all. Eq. (17) states the participation constraint for uninformed investors if they finance a part of the investment with the bank. Identically, knowing that  $\beta_j$  is the rate of return on banks informed capital, eq. (18) stresses that bank's expected payoff must cover the cost of its informed capital. Eq. (19) determines the minimum amount the bank must be paid in order to monitor. Finally, eq. (20) still describes the minimum return a firm has to receive. Binding equations (15) to (19) and substituting (17), (18) (19) into (15) gives the optimal contract between a  $\theta$  firm, the bank and the uninformed investors.

LEMMA 2. The optimal financial contract is defined by the amount of funds provided by the bank, by the uninformed investor, and by their corresponding shares in the project's payoff.

$$I_{bj}^*(\theta) = \frac{p_h c_j(\theta)}{\beta_j \Delta p}, \quad R_{bj}^*(\theta) = \frac{c_j(\theta)}{\Delta p} \quad (21)$$

$$I_{ij}^*(\theta) = (1 - \theta) - \frac{p_h c_j(\theta)}{\beta_j \Delta p}, \quad R_{ij}^*(\theta) = \left( (1 - \theta) - \frac{p_h c_j(\theta)}{\beta_j \Delta p} \right) \frac{\gamma}{p_h} \quad (22)$$

Three observations emerge from lemma 2:

i)- Optimal contract requires that firms invest all their wealth in the project whereas bank and uninformed investors put up the balance (respectively  $I_{bj}$  and  $I_{ij}$ ).

ii)- Banks' payoff is just equal to the minimum amount required to enforce monitoring and uninformed investors just receive the opportunity cost of the funds.

iii) According to the rate of return on informed capital, firms' demand of informed capital just allows to pay banks at the minimum incentive payoff.

The cost of informed capital is higher than the cost of uninformed capital and firms submitted to monitoring exactly demand the minimum level of bank capital<sup>17</sup>. This level is decreasing with firm's wealth, as the cost of monitoring is a decreasing function of firms' net wealth. Consequently, the relative share of uninformed capital in investment project, compared to informed capital, increases with firms' wealth as  $I_{ij}^*(\theta) = (1 - \theta) - I_{bj}^*(\theta)$ . In equilibrium, uninformed investors provide this amount because there is a sufficient quantity of informed capital ensuring that the hazard moral problem is alleviated.

The equilibrium firm's payoff when it finances a share of its investment thanks to a bank and uninformed investors is given by

$$R_{fj}^*(\theta) = R - R_{bj}^* - R_{ij}^* = R - \frac{c_j(\theta)}{\Delta p} - \left( (1 - \theta) - \frac{p_h c_j(\theta)}{\beta_j \Delta p} \right) \frac{\gamma}{p_h} \quad (23)$$

which is an increasing function of firms net wealth.

#### 4. OPTIMAL CREDIT ALLOCATION AND MONETARY POLICY

Knowing the optimal contract, entrepreneurs compare the financial conditions offered by all lenders (financial market and the two banks) before choosing the best one. We will show that firms' choice between uninformed and informed capital is endogenously determined according to their own wealth and according to bank monitoring technology and level of capital. Moreover, this endogenous allocation allows us to determine the level of aggregate investment in the economy.

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<sup>17</sup>  $\beta_j$  must be high enough to make banks prefer monitoring rather than investing their capital on the open market where they would earn a rate of return equal to  $\gamma$ . The minimum acceptable rate of return on informed capital is determined, for both banks, by  $p_h R_{bj}^* - c_j(\theta) = \gamma I_{bj}^*$ . Substituting equilibrium values in this equation leads to  $p_h \frac{c_j(\theta)}{\Delta p} - c_j(\theta) = \gamma \frac{p_h c_j(\theta)}{\Delta p \beta_j}$  with  $\Delta p = p_h - p_l$  and

$$\beta_j(\min) = \frac{p_h}{p_l} \gamma > \gamma.$$

#### 4.1. Endogenous choice between market, the small bank, and the large bank

Knowing the financial conditions offered by the market and the two banks, a firm  $\theta$  must choose rationally the contract that allows it to achieve a maximal payoff. This choice is described in proposition 1.

PROPOSITION 1. *We have  $1 > \theta_M^* > \theta_c^* > \theta_m^* > 0$  such that firms with net wealth lower than  $\theta_m^*$  are not financed at all, firms with net wealth between  $\theta_m^*$  and  $\theta_c^*$  (small firms) are financed by the small bank and uninformed investors; firms with net wealth between  $\theta_c^*$  and  $\theta_M^*$  (middle firms) are financed by the large bank and uninformed investors. Large firms with a wealth between  $\theta_M^*$  and 1 resort to financial market only (uninformed investors). The total amount of investment undertaken in the economy is given by  $[1 - \theta_m^*]$ .*

PROOF : See Appendix B.

The intuition behind proposition 1 is the following. Whatever its wealth, the payoff a firm can achieve on financial market is always higher than the payoff associated to a mix between bank and market (see Appendix A). Consequently, firms with net wealth higher than  $\theta_M^*$  will finance their investment with uninformed capital only (*via* financial market). This means they do not need to be monitored by a bank as they can credibly provide the minimal payoff to uninformed investors. We can turn our attention on a firm with a wealth lower than  $\theta_M^*$ . Such a firm must be monitored in order to obtain external finance. However, it can resort to large or small banks. A firm will choose to be monitored by the large bank as long as its equilibrium payoff is higher than the one it can achieve if the small bank monitors it. This condition is fulfilled while

$$R_{fL}^* \geq R_{fS}^* \text{ or, put differently } p_h \left[ \frac{c_S(\theta)}{\Delta p} - \frac{c_L(\theta)}{\Delta p} \right] \geq [I_{bS}^* - I_{bL}^*] \gamma \quad (24)$$

The left part of equation (24) reflects the difference in the banking rent a firm must pay. It is an indicator of the large bank cost advantage compared to the small bank in monitoring a  $\theta$  firm. The right part expresses the opportunity cost, in term of uninformed capital, if the firm uses the large bank rather than the small bank. A firm will choose the large bank whenever the cost advantage is higher than the opportunity cost. Consequently, there is a marginal firm ( $\theta_c^*$ ), which is indifferent between the two banks. For this indifferent firm it must be

$$R_{fL}^*(\theta_c^*) = R_{fS}^*(\theta_c^*) \quad (25)$$

It comes from this equation that  $\theta_c^*$  can be lower or higher than  $\bar{\theta}$  (the value for which large and small banks monitoring costs are equal). Particularly, under our hypothesis, we have  $\bar{\theta} > \theta_c^*$  (see Annexe C-b). This allows us to understand the importance of monitoring and capital in the model. Monitoring cost advantage is not sufficient to explain a firm's choice between the two banks. We must also consider the two banks relative amount and prices of capital. This trade off between monitoring cost advantage and capital drives the arbitrage condition.

Firms with wealth ( $\theta < \theta_c^*$ ) will resort to the small bank. When monitoring takes place, the incentive condition requires that the minimum payoff a firm must receive in order to behave diligently is equal to  $\frac{d}{\Delta p}$ . Thus, we must find the last firm ( $\theta_m^*$ ) for which this incentive condition is fulfilled. ( $\theta_m^*$ ) is determined by the realisation of the following condition

$$R_{fS}^*(\theta_m^*) = R - \frac{c_S(\theta_m^*)}{\Delta p} - \left( (1 - \theta_m^*) - \frac{p_h c_S(\theta_m^*)}{\beta_S \Delta p} \right) \frac{\gamma}{p_h} = \frac{d}{\Delta p} \quad (26)$$

Finally, knowing the respective shares of the financial market, the large bank, and the small bank in the financing, we can compute the equilibrium price of informed capital. In this perspective, we must turn our attention to bank's behaviour. As monitoring is costly and the rate of return on informed capital is higher than the one on uninformed capital, banks are encouraged to finance risky projects. Idle capital is incompatible with profit maximizing behaviour in perfect competition. Thus, small bank and large bank devote all their capital in the financing of the risky project. The equilibrium rate of return for the two banks ( $\beta_L^*, \beta_S^*$ ) is given by the following two conditions

$$\int_{\theta_c}^{\theta_M} I_{bL}^*(\theta) d\theta = K_L \quad (27)$$

$$\int_{\theta_m}^{\theta_c} I_{bS}^*(\theta) d\theta = K_S \quad (28)$$

Formally, equilibrium is fully defined by the quadruplet  $(\beta_L^*, \beta_S^*, \theta_c^*, \theta_m^*)$ , which is the solution of the system (25) to (28). Once  $(\theta_c^*)$  and  $(\theta_m^*)$  is obtained, equilibrium rates of return are given by

$$\beta_L^* = \frac{p_h}{K_L \Delta p} [C_L(\theta_M^*) - C_L(\theta_c^*)] \quad (29)$$

$$\beta_s^* = \frac{p_h}{K_s \Delta p} [C_s(\theta_c^*) - C_s(\theta_m^*)] \quad (30)$$

where  $[C_L(\theta_m^*) - C_L(\theta_c^*)]$  and  $[C_s(\theta_c^*) - C_s(\theta_m^*)]$  respectively accounts for the total cost of funds for the large and the small bank.

PROPOSITION 2. *The cost of small bank capital is higher than the cost of large bank capital. As the rate of return on uninformed capital is the less expensive we have  $\gamma < \beta_L^* < \beta_S^*$ .*

PROOF : See Appendix C.

It results from proposition 2 that the small bank's capital equilibrium rate of return is higher than the large bank one. This situation is allowed by the impossibility for a large bank to become a small bank. Moreover, even if  $\beta_s^* > \beta_L^*$  small firms (with wealth lower than  $\theta_c^*$ ) prefer addressing to the small bank, because the cost of monitoring is lower. Consequently, for these firms, the payoff associated to small bank is higher than the one corresponding to large bank.

Figure 1 is a graphical representation of the equilibrium formally exposes in propositions 1 and 2.

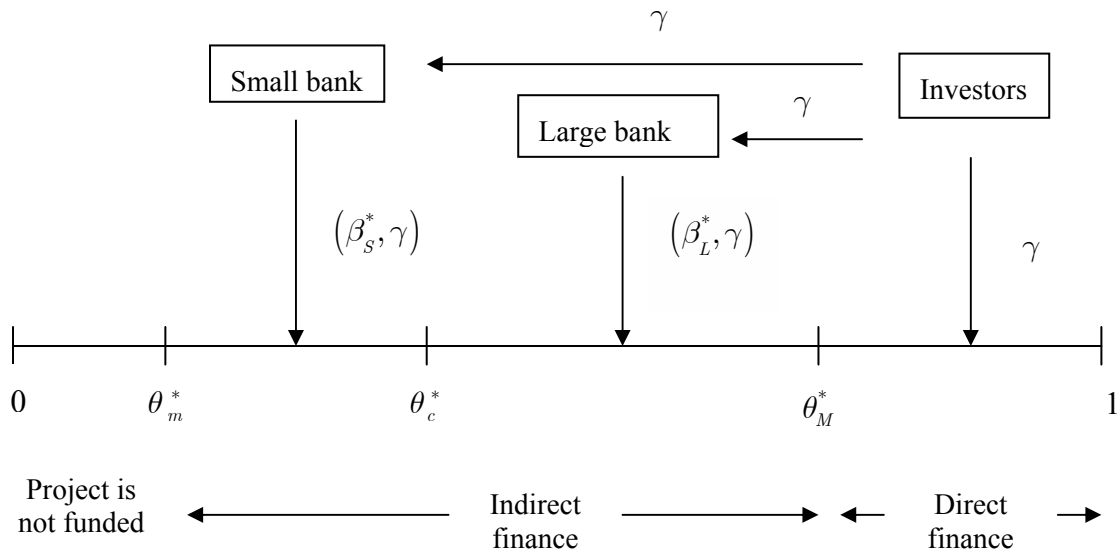


Figure -1-

## 4.2. Monetary shock

The bank-lending channel operates if a monetary shock forces banks to curtail lending and if borrowers cannot find alternative sources of financing. Our model aims at improving this analysis; showing how bank credit is reallocated when a restrictive monetary shock hurt the economy in a context of heterogeneous banking system. Consider a change in  $\gamma$  we relate to a change in monetary policy conditions.

PROPOSITION 3. *An increase in the riskless rate implies a decrease in global level's investment as  $\frac{d\theta_m^*}{d\gamma} > 0$  and  $[1 - \theta_m^*]$  is reduced. Moreover, there is a reallocation mechanism leading to a "flight to quality" effect as small and large bank tend to finance some wealthier borrowers  $\left(\frac{d\theta_m^*}{d\gamma} > 0\right), \left(\frac{d\theta_c^*}{d\gamma} > 0\right), \left(\frac{d\theta_M^*}{d\gamma} > 0\right)$ .*

PROOF : Appendix D.

Proposition 3 portrays the global effect of monetary shock on aggregate investment and on the level of firm's investment. Let's first have a look to the reallocation effect. As the opportunity cost of uninformed capital rises, firms with a wealth close to  $\theta_M^*$  cannot ensure the minimum incentive payoff to uninformed investors. These firms cannot invest *via* direct finance any more. They must switch toward the large bank in order to fulfil incentive conditions and credibly convince uninformed investors to invest. At the same time, according to eq. (24), the rise in  $\gamma$  implies an increase in the opportunity cost of uninformed capital whereas monitoring costs remain fixed for a firm of given wealth. Thus, firms close to  $\theta_c^*$  choose to switch toward the small bank in order to obtain a better payoff. As a result, the heterogeneous banking system allows medium firms to find alternative sources of financing. This compositional change reduces the impact of a negative monetary shock on these borrowers. Finally, firms close to  $\theta_m^*$  cannot ensure to uninformed investors that they will behave properly and are supplanted of the credit market. These borrowers are precisely the most opaque because they are small bank-dependent and are not able to signal to other lenders. Simultaneously, the relative optimal repartition of external funds between informed and uninformed capital is modified. As the banks payoff remains fixed, an increase in  $\gamma$  leads to a bank capital relatively less expensive than uninformed capital. To finance their investment, firms use more informed capital than before. As banks are capital constrained, the number of projects a bank can finance fall. These two

effects lead to an increase of the global wealth to obtain external funds similar to a flight to quality mechanism<sup>18</sup>.

Usual results on bank lending channel argue that the tightening of monetary conditions has a stronger effect on bank dependent firms<sup>19</sup>. Our approach completes this channel. Indeed, proposition 3 emphasizes the asymmetric impact of monetary shock on small firms. If capital and monitoring technology are divided not equally among banks, monetary policy leads to reallocations effects and to a flight to quality. We show that monetary shock has a strong effect not on bank-dependent firms but on small bank-dependent firms. Small firms are the most affected because they exclusively financed by the small bank and cannot substitute for other sources of financing.

## 5. CONCLUDING REMARKS

Our model of credit allocation with heterogeneous lenders analyses how monetary policy influences the distribution of credit to small firms. We have characterized a credit market equilibrium in which firms with high wealth prefer market lending, those with middle wealth get large bank lending, and those with little wealth prefer small bank lending. The remainder of firms are unable to invest. In accordance with the empirical literature, we show that small bank interest rate is higher than large bank's one<sup>20</sup>. Moreover, this analysis sheds light on an asymmetric impact of monetary shock. The general contraction of firms' investment is explained by an optimal credit reallocation and a flight to quality mechanism. On the one hand, small borrowers will suffer disproportionately from a restrictive monetary shock because they are at the small-end of the credit market. On the other hand, medium firms are less constrained because they could find some alternative external funds. These points suggest that small bank specialization is important to allow some opaque borrowers to obtain finance. Compare to large intermediaries, these small banks must be higher capitalized in order to better absorb monetary shocks. Moreover, it appears that banking consolidation by reducing the quantity of small banks could affect the availability of small business credit<sup>21</sup>. More generally, our results underline that a same monetary policy must have a differential impact on aggregate investment according to the structure of the financial system. This point is important in order to evaluate the conduct of monetary policy in the EMU<sup>22</sup>. It

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<sup>18</sup> Our definition of this mechanism is somewhat different from those described in the literature. In the standard flight to quality literature, there is a decline in the share of credit of borrowers with high agency costs to borrowers with greater access to credit market (Bernanke, Gertler and Gilchrist (1996)).

<sup>19</sup> See Besanko and Kanatas (1993) and Repullo and Suarez (2000).

<sup>20</sup> See Hubbard, Kuttner and Palia (1999), and Berger and Udell (1996) for empirical investigation.

<sup>21</sup> See Sapienza (2002), Berger, Rosen and Udell (2001), and Berger, Saunders, Scalise and Udell (1998) among others.

<sup>22</sup> See Kashyap and Stein (1997) and Cechetti (1999).

suggests that a unified monetary policy will have an asymmetric impact as long as European financial system remains diversified.

This model is highly stylized in order to capture the essential mechanisms of credit allocation and some important points are missing. A first caveat concerns the exogenous level of bank capital. In particular, we do not model the liability side of bank's balance sheets. Banks may refinance on markets. It has been suggested that banks with more transparent portfolios may have cheaper access to funds and may therefore be favoured by regulators. In such case, this possibility could mitigate our results. A second caveat concerns the nature of firm's wealth. Indeed, we do not taking into account the use of real asset as collateral. Actually, a complete model should integrate the impact of the interest rate on real assets. We leave these extensions for future research.

## APPENDIX

APPENDIX A: Firms payoff achieves on capital market is always higher than the one achieves with bank capital.

$$R - (1 - \theta) \frac{\gamma}{p_h} > R - \frac{c_j(\theta)}{\Delta p} - \left( (1 - \theta) - \frac{p_h c_j(\theta)}{\beta_j \Delta p} \right) \frac{\gamma}{p_h} \text{ which lead to}$$

$$\frac{c_j(\theta)}{\Delta p} \left( 1 - \frac{\gamma}{\beta_j} \right) > 0 \text{ as } c_j(\theta) > 0 \text{ and } \beta_j > \gamma \quad \blacksquare$$

APPENDIX B: Proof of PROPOSITION 1.

We must impose restrictions on model's parameters in order to maintaining our exogenous values in plausible range.

B1:  $p_h \left[ R - \frac{c_s(0)}{\Delta p} \right] > \gamma > p_h \left[ R - \frac{d}{\Delta p} \right]$ . Firms with no wealth cannot be financed by the small bank.

B2:  $R_f^*(\theta_c^*) < \frac{B}{\Delta p}$ . Incentive constraint (11) is not realised by the marginal firm  $\theta_c^*$ .

B3:  $p_h \left[ R - \frac{c_s(\theta_c^*) + d}{\Delta p} \right] > (1 - \theta_c^*) \gamma$ . Incentive constraints are realised for all parties if the marginal firm  $\theta_c^*$  is monitored by the small bank.

The proof of proposition 1 consists in three parts.

1- Existence and uniqueness of  $\theta_c^*$ .

Consider the following equation

$$g(\theta) = R_{fl}^*(\theta) - R_{fs}^*(\theta) = \frac{c_s(\theta)}{\Delta p} - \frac{c_L(\theta)}{\Delta p} + \left( \frac{p_h c_L(\theta)}{\beta_L \Delta p} - \frac{p_h c_s(\theta)}{\beta_s \Delta p} \right) \frac{\gamma}{p_h} \quad \text{A-2}$$

We search for  $\theta_c^*$  such that  $g(\theta_c^*) = 0$ .  $R_{fl}^*$  and  $R_{fs}^*$  are continuous on  $[0,1]$ .

Moreover,  $\frac{\partial R_{fs}^*(\theta)}{\partial \theta} = -\frac{c'_s(\theta)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] + \frac{\gamma}{p_h} > 0$  and according to assumption 3 we

have  $\frac{\partial^2 R_{fs}^*(\theta)}{\partial \theta^2} = -\frac{c''_s(\theta)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] < 0$  which ensures there is no inflexion point.

Therefore,  $R_{fl}^*$  and  $R_{fs}^*$  are continuous increasing concave function of  $\theta$ .

Let's analyse now the behaviour of firm's rent on the limit of the wealth interval  $[0,1]$ .

$$\lim_{\theta \rightarrow 0} R_{e0}^*(\theta) = R - \frac{c_0(0)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_0} \right] - \frac{\gamma}{p_h} > 0 \text{ under restriction B1.}$$

$$\lim_{\theta \rightarrow 0} R_{e1}^*(\theta) = R - \frac{c_1(0)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_1} \right] - \frac{\gamma}{p_h} \rightarrow -\infty \text{ as } c_L(0) \rightarrow +\infty \text{ and } \beta_L > \gamma.$$

$$\lim_{\theta \rightarrow 1} R_{fs}^*(\theta) = R - \frac{c_s(1)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] > 0 \text{ as } R \geq R_{bs} \geq \frac{c_s(\theta)}{\Delta p}$$

$$\lim_{\theta \rightarrow 1} R_{fl}^*(\theta) = R - \frac{c_L(1)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_L} \right] \rightarrow R \text{ as } c_L(1) = 0.$$

We conclude that as  $\lim_{\theta \rightarrow 0} R_{fs}^*(\theta) > \lim_{\theta \rightarrow 0} R_{fl}^*(\theta)$  and  $\lim_{\theta \rightarrow 1} R_{fl}^*(\theta) > \lim_{\theta \rightarrow 1} R_{fs}^*(\theta)$  there is at least one value on  $[0,1]$  (named  $\theta_c^*$ ) for which  $R_{fl}^*(\theta_c^*) = R_{fs}^*(\theta_c^*)$  and  $g(\theta_c^*) = 0$ .

Moreover, as the two functions are increasing in  $\theta$  and there is no inflexion point, we have  $\frac{\partial R_{fl}^*(\theta)}{\partial \theta} > \frac{\partial R_{fs}^*(\theta)}{\partial \theta}$ . This leads to the conclusion that the equilibrium is

$$\text{unique as } \frac{\partial g(\theta)}{\partial \theta} = g'(\theta) = \frac{c'_s(\theta)}{\Delta p} - \frac{c'_L(\theta)}{\Delta p} + \left( \frac{p_h c'_L(\theta)}{\beta_L \Delta p} - \frac{p_h c'_s(\theta)}{\beta_s \Delta p} \right) \frac{\gamma}{p_h} > 0 \quad \text{A-3}$$

Finally, according to  $g'(\theta) > 0$ , all firms with wealth  $\theta > \theta_c^*$  will choose to be monitored by the large bank, the other preferring the small bank.

### 2- Existence and uniqueness of $\theta_m^*$ .

We search for the last firm that are able to be financed by the small bank. Consider the following equation:

$$f(\theta) = R - \frac{c_s(\theta)}{\Delta p} - \left( (1-\theta) - \frac{p_h c_s(\theta)}{\beta_s \Delta p} \right) \frac{\gamma}{p_h} - \frac{d}{\Delta p} \quad \text{A-4}$$

We search the value  $\theta_m^*$  for which  $f(\theta_m^*) = 0$ . Let's study first the behaviour of function  $f(\cdot)$  on 0 and 1.

$$\lim_{\theta \rightarrow 0} f(\theta) = R - \frac{c_s(0)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] - \frac{\gamma}{p_h} - \frac{d}{\Delta p} < 0 \text{ under restriction B1.}$$

$$\lim_{\theta \rightarrow 1} f(\theta) = R - \frac{c_s(1)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] - \frac{d}{\Delta p} > 0 \text{ under restriction B3.}$$

$$\text{Moreover, we have } \frac{\partial f(\theta)}{\partial \theta} = -\frac{c'_s(0)}{\Delta p} \left[ 1 - \frac{\gamma}{\beta_s} \right] + \frac{\gamma}{p_h} > 0.$$

Consequently, there is an unique value,  $\theta_m^*$ , for which  $f(\theta_m^*) = 0$ . This value gives us the characteristic of the last firm that are able to be financed by the small bank.

### 3- Proof that $\theta_M^* > \theta_c^* > \theta_m^*$

a- According to B2,  $R_f^*(\theta_c^*) = R - (1 - \theta_c^*) \frac{\gamma}{p_h} < \frac{B}{\Delta p} = R_f^*(\theta_M^*)$ . As firms' payoff is an increasing function of  $\theta$ , it means that  $\theta_M^* > \theta_c^*$ .

b- According to B3,  $R_{fs}^*(\theta_c^*) > \frac{d}{\Delta p} = R_{fs}^*(\theta_m^*)$ . As firms' payoff is an increasing function of  $\theta$ , it means that  $\theta_c^* > \theta_m^*$ . Consequently, it results that  $\theta_M^* > \theta_c^* > \theta_m^*$  ■

APPENDIX C: Proof of PROPOSITION 2.

a- Equilibrium interest rate are given by (29) and (30). Consequently, in order to show that  $\beta_s^* > \beta_L^*$  we have to prove that the following condition is realized

$$\frac{K_L}{K_S} > \frac{[C_L(\theta_M^*) - C_L(\theta_c^*)]}{[C_S(\theta_c^*) - C_S(\theta_m^*)]} \quad \text{A-5}$$

Compute  $g(\theta)$  for equilibrium interest rates gives using (29) and (30) gives

$$g(\theta) = \frac{c_s(\theta)}{\Delta p} - \frac{c_L(\theta)}{\Delta p} + \left[ \frac{K_L c_L(\theta)}{[C_L(\theta_M^*) - C_L(\theta_c^*)]} - \frac{K_S c_S(\theta)}{[C_S(\theta_c^*) - C_S(\theta_m^*)]} \right] \frac{\gamma}{p_h}$$

Taking the derivative of this function leads to

$$g'(\theta) = \frac{c'_s(\theta)}{\Delta p} \left[ 1 - \frac{\Delta p \gamma K_S}{p_h [C_S(\theta_c^*) - C_S(\theta_m^*)]} \right] - \frac{c'_L(\theta)}{\Delta p} \left[ 1 - \frac{\Delta p \gamma K_L}{p_h [C_L(\theta_M^*) - C_L(\theta_c^*)]} \right]$$

According to A-3 we know that  $g'(\theta) > 0$  consequently we have

$$\frac{c'_s(\theta)}{\Delta p} \left[ 1 - \frac{\Delta p \gamma K_S}{p_h [C_S(\theta_c^*) - C_S(\theta_m^*)]} \right] > \frac{c'_L(\theta)}{\Delta p} \left[ 1 - \frac{\Delta p \gamma K_L}{p_h [C_L(\theta_M^*) - C_L(\theta_c^*)]} \right] \text{ which leads}$$

$$\text{to } \frac{\left[ 1 - \frac{\Delta p \gamma K_S}{p_h [C_S(\theta_c^*) - C_S(\theta_m^*)]} \right]}{\left[ 1 - \frac{\Delta p \gamma K_L}{p_h [C_L(\theta_M^*) - C_L(\theta_c^*)]} \right]} > \frac{c'_L(\theta)}{c'_s(\theta)} > 1 \text{ as } \frac{c'_L(\theta)}{c'_s(\theta)} > 1 \text{ (see assumption 3).}$$

This condition is realised if and only if

$$\left[ \frac{\Delta p \gamma K_L}{p_h [C_L(\theta_M^*) - C_L(\theta_c^*)]} \right] > \left[ \frac{\Delta p \gamma K_S}{p_h [C_S(\theta_c^*) - C_S(\theta_m^*)]} \right] \text{ which implies that}$$

$$\frac{K_L}{K_S} > \frac{[C_L(\theta_M^*) - C_L(\theta_c^*)]}{[C_S(\theta_c^*) - C_S(\theta_m^*)]}$$

Thus, condition A-5 is fulfilled and  $\beta_s^* > \beta_L^*$  ■

b-  $\bar{\theta}$  is financed by the large bank if  $g(\bar{\theta}) > 0$  (which means that  $\bar{\theta} > \theta_c^*$ ). We have

$$g(\bar{\theta}) = \frac{c_s(\bar{\theta})}{\Delta p} - \frac{c_L(\bar{\theta})}{\Delta p} + \left( \frac{p_h c_L(\bar{\theta})}{\beta_L \Delta p} - \frac{p_h c_s(\bar{\theta})}{\beta_s \Delta p} \right) \frac{\gamma}{p_h} \quad \text{A-6}$$

Knowing that  $c_s(\bar{\theta}) = c_L(\bar{\theta})$  A-6 becomes  $g(\bar{\theta}) = c_L(\bar{\theta}) \left( \frac{p_h}{\beta_L \Delta p} - \frac{p_h}{\beta_s \Delta p} \right) \frac{\gamma}{p_h} > 0$

when  $\beta_s > \beta_L$  which is always realized ■

#### APPENDIX D : Proof of PROPOSITION 3.

Taking the total derivative of equations (13), (25), (26), (29) and (30) leads to

$$\begin{aligned} d\beta_s &= \left[ \frac{\beta_s^* c_s(\theta_c^*)}{C_s(\theta_c^*) - C_s(\theta_m^*)} \right] d\theta_c - \left[ \frac{\beta_s^* c_s(\theta_m^*)}{C_s(\theta_c^*) - C_s(\theta_m^*)} \right] d\theta_m - \left[ \frac{\Delta p \beta_s^{*2}}{p_h (C_s(\theta_c^*) - C_s(\theta_m^*))} \right] dK_s \\ d\beta_L &= \left[ \frac{\beta_L^* c_L(\theta_M^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)} \right] d\theta_M - \left[ \frac{\beta_L^* c_L(\theta_c^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)} \right] d\theta_c - \left[ \frac{\Delta p \beta_L^{*2}}{p_h (C_L(\theta_M^*) - C_L(\theta_c^*))} \right] dK_L \\ d\theta_c &= \left[ \frac{c_L(\theta_c^*) \gamma}{g'(\theta_c^*) \Delta p \beta_L^{*2}} \right] d\beta_L - \left[ \frac{c_s(\theta_c^*) \gamma}{g'(\theta_c^*) \Delta p \beta_s^{*2}} \right] d\beta_s - \left[ \left( \frac{1}{g'(\theta_c^*)} \right) \left( \frac{c_L(\theta_c^*)}{\Delta p \beta_L^*} - \frac{c_s(\theta_c^*)}{\Delta p \beta_s^*} \right) \right] d\gamma \\ d\theta_m &= \left[ \frac{c_s(\theta_m^*) \gamma}{f'(\theta_m^*) \Delta p \beta_s^{*2}} \right] d\beta_s - \left[ \left( \frac{1}{f'(\theta_m^*)} \right) \left( \frac{c_s(\theta_m^*)}{\Delta p \beta_s^*} - \frac{(1 - \theta_m^*)}{p_h} \right) \right] d\gamma \\ d\theta_M &= \left[ \left( R - \frac{B}{\Delta p} \right) \frac{p_h}{\gamma^2} \right] d\gamma \end{aligned}$$

Knowing that  $dK_s = dK_L = 0$ , we can simplify this system

$$d\beta_s = A_1 d\theta_c - A_2 d\theta_m \quad \text{A-7}$$

$$d\beta_L = B_1 d\theta_M - B_2 d\theta_c \quad \text{A-8}$$

$$d\theta_c = \frac{1}{g'(\theta_c^*)} [C_1 d\beta_L - C_2 d\beta_s - C_3 d\gamma] \quad \text{A-9}$$

$$d\theta_m = \frac{1}{f'(\theta_m^*)} [D_1 d\beta_s - D_2 d\gamma] \quad \text{A-10}$$

$$d\theta_M = E d\gamma \quad \text{A-11}$$

$$\text{with } (A_1, A_2) \equiv \left( \frac{\beta_S^* c_S(\theta_c^*)}{C_S(\theta_c^*) - C_S(\theta_m^*)}, \frac{\beta_S^* c_S(\theta_m^*)}{C_S(\theta_c^*) - C_S(\theta_m^*)} \right)$$

$$(B_1, B_2) \equiv \left( \frac{\beta_L^* c_L(\theta_M^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)}, \frac{\beta_L^* c_L(\theta_c^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)} \right)$$

$$(C_1, C_2, C_3) \equiv \left( \frac{c_L(\theta_c^*)\gamma}{\Delta p \beta_L^{*2}}, \frac{c_S(\theta_c^*)\gamma}{\Delta p \beta_S^{*2}}, \frac{c_L(\theta_c^*)}{\Delta p \beta_L^*} - \frac{c_S(\theta_c^*)}{\Delta p \beta_S^*} \right)$$

$$(D_1, D_2) \equiv \left( \frac{c_S(\theta_m^*)\gamma}{\Delta p \beta_S^{*2}}, \frac{c_S(\theta_m^*)}{\Delta p \beta_S^*} - \frac{(1 - \theta_m^*)}{p_h} \right)$$

$$E \equiv \left( R - \frac{B}{\Delta p} \right) \frac{p_h}{\gamma^2}$$

and  $A_1, A_2, B_1, B_2, C_1, C_2, C_3, D_1, E > 0$  and  $D_2 < 0$ .

From A-11, it is straightforward that  $\frac{d\theta_M}{d\gamma} > 0$ . Substituting A-10 in A-7, A-11 in

A-8, A-7 and A-11 in A-9, we obtain

$$d\theta_c \left[ g'(\theta_c^*) + B_2 C_1 + \frac{A_1 C_2}{1 + \frac{A_2 D_1}{f'(\theta_m^*)}} \right] = d\gamma \left[ B_1 C_1 E - C_3 - \frac{A_2 C_2 D_2}{f'(\theta_m^*) + A_2 D_1} \right] \quad \text{A.12}$$

with  $g'(\theta_c^*) + B_2 C_1 + \frac{A_1 C_2}{1 + \frac{A_2 D_1}{f'(\theta_m^*)}} > 0$  and  $\frac{A_2 C_2 D_2}{f'(\theta_m^*) + A_2 D_1} < 0$ .  $\frac{d\theta_c^*}{d\gamma} > 0$  as long

as  $B_1 C_1 E - C_3 > 0$ .

$$B_1 C_1 E - C_3 = \left[ \frac{\beta_L^* c_L(\theta_M^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)} \right] \left[ \frac{c_L(\theta_c^*)\gamma}{\Delta p \beta_L^{*2}} \right] \left[ \left( R - \frac{B}{\Delta p} \right) \frac{p_h}{\gamma^2} \right] + \frac{c_S(\theta_c^*)}{\Delta p \beta_S^*} - \frac{c_L(\theta_c^*)}{\Delta p \beta_L^*}$$

Simplifying this expression, we obtain

$$\left[ \frac{\beta_L^* c_L(\theta_M^*)}{C_L(\theta_M^*) - C_L(\theta_c^*)} \right] \left[ \frac{c_L(\theta_c^*)\gamma}{\Delta p \beta_L^{*2}} \right] \left[ \left( R - \frac{B}{\Delta p} \right) \frac{p_h}{\gamma^2} \right] + \frac{c_S(\theta_c^*)}{\Delta p \beta_S^*} > \frac{c_L(\theta_c^*)}{\Delta p \beta_L^*}$$

Substituting (29) and (30) in this equation,  $B_1 C_1 E - C_3 > 0$  if and only if

$$\frac{K_S}{K_L} > \frac{(C_S(\theta_c^*) - C_S(\theta_m^*))c_L(\theta_c^*)}{(C_L(\theta_M^*) - C_L(\theta_c^*))c_S(\theta_c^*)} - \frac{(p_h(R\Delta p - B)c_L(\theta_M^*))}{(C_L(\theta_M^*) - C_L(\theta_c^*))^2 \gamma \Delta p c_S(\theta_c^*)} \quad \text{B.4}$$

We assume that this condition is fulfilled. Thus,  $\frac{d\theta_c}{d\gamma} > 0$ .

Finally, we have

$$d\theta_m \left[ f'(\theta_m^*) + \frac{A_2 D_1}{1 + \frac{A_1 C_2}{g'(\theta_c^*) + B_2 C_1}} \right] = d\gamma \left[ \frac{A_1 D_1 (B_1 C_1 E - C_3)}{g'(\theta_c^*) + B_2 C_1 + A_1 C_2} - D_2 \right]$$

with  $f'(\theta_m^*) + \frac{A_2 D_1}{1 + \frac{A_1 C_2}{g'(\theta_c^*) + B_2 C_1}} > 0$ ,  $B_1 C_1 E - C_3 > 0$  (B.4), and  $D_2 < 0$

Consequently,  $\frac{d\theta_m}{d\gamma} > 0$  ■

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