Endogenous money in an elementary search model: intrinsic properties versus bootstrap

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1 Introduction

In the basic model of search-theoretic approach to money ([3])[1], as in most sophisticated models which have followed, the quantity of money is given from outside. That starting point is unsatisfactory and misleading. It is responsible, as we shall see, for an exaggerated emphasis put on bootstrap effects in the existence of monetary equilibrium. Moreover, assuming an exogenous quantity of money contradicts the now widely held opinion according to which monetary authorities have not the power to directly determine the quantity of money but only to indirectly control it by manipulating some variables, namely the rate of interest. Monetary models should account for the important 'stylised fact' that money is issued at agents' initiative under the constrainst of rules fixed by a non competitive authority. The purpose of this paper is to propose an elemetary version of such a model.

In our model money is issued according to a very simple principle which is nothing but a generalisation of pure gold standard. In a pure metallic system, mintage consists in coining privately produced gold, making it useless for any purpose other than circulation. The gold producer has thus a choice between exchanging gold in the market or bringing it to the Mint. A condition of indifference between the two alternatives determines which quantity of gold is coined and which enters the market as a commodity[2]. Elements to be taken into account by gold producers are the cost of production, the cost of coinage (neglected here) and the seignorage imposed by monetary authority.

[1] I am grateful to Andrès Alvarez, Vincent Bignon, Richard Dutu, Bertrand Gobillard, Régis Breton and Stefania Vanacore for their helpful remarks on a preliminary version. The usual disclaimer applies.

[2] In standard theory the quantity of coined gold is given by the equation of exchange, the price of gold (and therefore the value of money) being determined as for any other commodity through market equilibrium conditions (see [4], p. 140).

Let us generalize this view, assuming that the alternative o¤ered to gold producers is now available to any producer. A producer has now to choose between two alternatives: either to enter the market with the hope to meet someone and to exchange or to ask monetary authority to transform his good into an homogenous means of exchange at an additional cost s. Such a parable is not very far from what e¤ectively happens in our economies where means of payment are issued by banks against a liability and possibly a collateral. Debtors have to produce and sell something in order to pay back banks. At stationary equilibrium loans and repayments are equal. Repayment thus requires an activity of production (which implies a cost) and the payment of interests. Banks do not buy commodities, no more than Mint buys the gold it coins, but production of gold or of a commodity is at the starting point of money issuance (in a more complete model money could be issued against a …nancial asset as well). The fact that search models are not well-suited to accomodate credit must not worry us too much. Here it is as if banks were easily located and if monetization of commodities could replace credit. As a matter of fact, bank money obeys a logic which is not so much di¤erent from that of gold standard, once it has been understood that coins are distinct from gold and that cheques are distinct from capital[3].

Brian Peterson ([5]) has recently described a metallic system in the spirit of gold standard. His model is however restricted to a strict metallic money and gold (or money) may be consumed or sold abroad (his model is intended to study problems quite di¤erent from ours). Our purpose di¤ers also from that of Yiting Li ([7]) who determines the conditions under which bank money can circulate, a certain quantity if outside money being given. The model presented here is nothing but that of Kiyotaki & Wright completed by an endogenous procedure of issuing money. Such a model determines an equilibrium quantity of money and allows an embryonic study of monetary policy where the e¤ects of variations of s on welfare are made explicit.

Two main results are the following:

1. There exist three barter equilibria in which a zero quantity of money is associated to three di¤erent degrees of money acceptation. The …rst is trivial: nobody resorts to the Mint when money is never accepted. But this may also be true when a positive fraction of agents accepting money (second equilibrium) and it may even be true when everybody is ready to sell his good for money (third equilibrium)! In other words, an unanimous acceptation of money is not a su¢cient condition for the existence of a monetary equilibrium, contrary to what most search models with exogenous quantity of money seem to pretend.

2. The existence of a monetary equilibrium depends, for s given, on the absolute level of specialisation and not only on a comparison between that level and the degree of money acceptation in the economy. Con…dence in money,

[3] Tooke's interpretation of gold standard may help the modern reader to grasp this point. For a less terse justi…cation of the view adopted here, see [1].

i. e. a positive degree of money acceptation, is a necessary condition of monetary equilibrium but not a sufficient one. An additional condition must be satisfied which concerns the intrinsic properties of the economy, namely preferences and technique. Money is not mainly a bootstrap effect.

A complementary result is relative to welfare and monetary policy; the fact that monetary equilibrium domines equilibrium with a zero quantity of money boils down to the proposition that an economy with a degree of specialisation beyond a certain threshold is better than those that are short of that limit; that monetary equilibrium dominates barter equilbirium can be proved without imposing the rather paradoxical condition that a same quantity of money exists in both cases; here, monetary equilibrium dominance comes from the fact that the quantities of money for which it would cease to be true cannot be reached through the endogenous money creation. A by-product of the study of welfare is a rudimentary study of monetary policy.

2 The model

The economy is described as is usual in search-models à la Kiyotaki-Wright with exogenous prices :

- There is a continuum $[0;1]$ of indivisible nondurable goods and of infinite-lived agents with an uniform rate of depreciation of time r; each agent can produce only a unit of a fraction x of the goods and can consume only a unit of a fraction $y = x$ of the goods; agents never produce goods for their own consumption; production takes place at a uniform cost c and consumption provides a uniform utility u

- A monetary authority transforms at agents initiative and at a cost s the produced good into a durable means of exchange without any intrinsic utility; this means of exchange is called money; storage capacity of agents is limited to one unit of money; a money holder cannot produce

- Any agent may produce : (i) either for exchange (production takes places whenever an agent meets another with whom a transaction is possible) (ii) or for the Mint, in which case the agent becomes a money holder

- The fraction of agents who accept money depends on the strategies chosen by others; it is endogenously determined

- Trade occurs in a bilateral random matching process, where agents meet according to a Poisson process with parameter α

Let N be the proportion of agents who accept money and M that who mint their production. At equilibrium M N must hold. Let V_{0M}, V_m and V_0 be, respectively, the value of an agent minting his good, holding a unit of money and producing for the market.

Bellman equations in a stationary state are:

$$\begin{aligned} rV_{0M} &= V_m - V_0 - c - s \\ rV_m &= \alpha N(1 - M)x(u + V_0 - V_m) \\ rV_0 &= (\alpha(1 - M)x^2(u - c) + \max[0; \alpha Mx(V_m - c - V_0)] \end{aligned} \quad (1)$$

First equation says that the flow return to an agent bringing his good to the Mint is equal to the gain from switching from production to money holding minus a production cost c and an additional cost s imposed by the Mint. The second one says that the flow return to a money holder is the probability of using money in exchange $N(1 - M)x$ (i. e. to meet a non-holder of money accepting money and producing a good suitable for consumption) times the gain in exchange. The last equation equals the flow return to a producer to a sum two terms: the first one is the gain resulting from a barter $(u - c)$ with another producer times the probability of such a meeting $\alpha(1 - M)x^2$, the second term is zero or the positive gain from accepting money and becoming a money holder times the probability of meeting a money holder interested in the good produced.

Two decisions matter for producers: (i) whether to accept or not money in exchange; it depends on the sign of $V_m - c - V_0$ (ii) whether to produce for exchange or for the Mint; it depends on a comparison between $V_m - V_0 - c - s$ and $(\alpha(1 - M)x^2(u - c) + \max[0; \alpha Mx(V_m - c - V_0)]$. Three cases are to be examined, according to the sign of $V_m - c - V_0$.

<1> Let first consider $V_m - V_0 - c < 0$. Nobody would choose to go to the Mint since $s \geq 0$ implies $rV_{0M} < 0$. As a consequence, equilibrium values for N and M are $M^* = 0$ and $N^* = 0$. The economy is populated only by producers who do not accept money. Their value is $V_0 = \frac{\alpha x^2(u-c)}{r}$.

<2> A second barter equilibrium exists for $V_m - V_0 - c = 0$. The relevant system of equations, which allows to make explicit the indifference condition between accepting or not accepting money, is (with a time normalization $\alpha x = 1$):

$$\begin{aligned} rV_m &= (1 - N)(u + V_0 - V_m) \\ rV_0 &= (1 - M)x(u - c) + M(V_m - c - V_0) \end{aligned} \quad (2)$$

Solution of (2) is:

$$\begin{aligned} V_0 &= \frac{N(1-M)(u-c)[(1-M)x+M\} + r(1-Mx)(u-c) - Mrc}{r(r+M+N(1-M))} \\ V_m &= \frac{N(1-M)}{r(r+M+N(1-M))}[u(r + M(1 - x)) + x(u - c) - Mc(1 - x)\} \end{aligned} \quad (3)$$

Reporting the values given by (3) in $V_m - c - V_0 = 0$, yields an equation whose solution is a relation between N and M:

$$N = \frac{x(u - c) - cr - x(u - c)M}{(u - c)(1 - M)} \quad (4)$$

But when $V_m - c - V_0 = 0$, it is easy to check from system (1) that $rV_0 > rV_{0M}$. Agents have never interest to bring their production to the Mint whatever the degree of money acceptation may be. Here, again, we have $M^{**} = 0$, although $N^{**} = \frac{x(u-c)-cr}{u-c}$ which is positive if $r < \frac{x(u-c)}{c}$, that is if $V_0 = \frac{x(u-c)}{r} > c$. That money is accepted by a fraction of agents $N^{**} > 0$ does not prevent the existence of a barter equilibrium. When money is issued at agents initiative, money acceptation, although a necessary condition for going to the Mint, is not a sufficient one. Barter and money equilibria differ less in the degree of money acceptation than in the degree of recourse to the Mint. Barter equilibrium occurs when nobody has an incitment to go to the Mint ($M = 0$), which is the case here despite the fact that $N^{**} > 0$.

<3> The third case is $V_m - c - V_0 > 0$. In this case every agent is ready to accept money in exchange for his good: $N^{***} = 1$. But, as we know, this does not means that everybody has an interest in going to the Mint. It exists an indifference condition between producing for the Mint or for the market which can be derived from the first and last equations of system (1).

Agents have an incitment to produce for the Mint when :

$$(1 - M)x(u - c) + M(V_m - c - V_0) \qquad V_m - c - s - V_0 \tag{5}$$

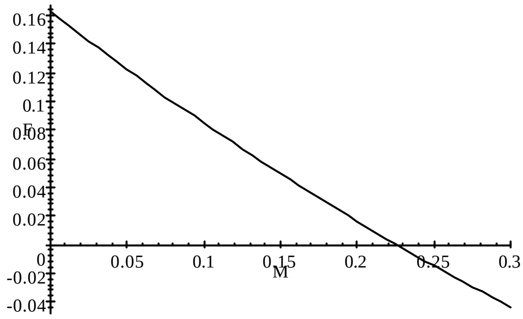
Indifference condition results from reporting in (5) values of V_m and V_0 given by (3), with $N = 1$:

$$F \equiv \gamma M^2 - (2\gamma - \beta)M + (\gamma - \beta) - (1 + r)s = 0 \tag{6}$$

where $\gamma = (1 - x)(u - c) > 0$ et $\beta = (1 + r)x(u - c) + rc > 0$.

It is easy to check that $F(1) < 0$ and that, if $\gamma > \beta$, $F'(0) < 0$ and $F(0) > 0$. If $\gamma > \beta$ then the equilibrium quantity of money, which satisfies (6) is positive. The graph of $F(M)$ cut both axis in their positive part.

Condition (6) is drawn below for the following value of parameters: $\alpha = 10$ $x = .1$ $u = 1$ $c = .5$ $r = .05$ (which means $\gamma = .45$, $\beta = .0775$) and $s = .2$:



F is equal to zero for a positive value of M equal to M^{***}. A fraction M^{***} of the agents chooses to resort to the Mint if a fraction $1 - M^{***}$ decides to produce for the market. Conversely, choosing to produce for the Mint is the best response for a fraction M^{***} to the strategy followed by the fraction $1 - M^{***}$. This non-symmetric pure strategy equilibrium is unique[4].

Condition $\gamma > \beta$ requires:

$$x < \overline{x} = \frac{u - (c+s)(1+r)}{(u-c)(2+r)} \tag{7}$$

Money effectively exists only if goods have a degree of liquidity, measured by x, inferior to a determinate threshold $\overline{x}$. Beyond a fixed level of specialization in tastes and production ($x < \overline{x}$) the liquidity of goods is low enough to incite a positive fraction of agents to bring their production to the Mint. The critical level $\overline{x}$ beyond which no monetary equilibrium can exist does not depend on expectations or on confidence but uniquely on intrinsic properties of the economy such preferences u nd r or technique c. 'Confidence', if one decides to label so the condition of money acceptation, is necessary but not sufficient to push agents to resort to the monetary institution.

Besides, the maximum quantity of money is given for $s = 0$. It is $\overline{M} = \frac{\gamma - \beta}{\gamma} = 1 - \frac{(1+r)x}{1-x} - \frac{rc}{(1-x)(u-c)}$[5].

Economically meaningful solution of (6) is (the other is greater than 1):

$$M^{*} = 1 - \frac{\beta}{2\gamma} - \frac{\sqrt{\beta^2 + 4\gamma(1+r)s}}{2\gamma} \tag{8}$$

Equilibrium quantity of money is a decreasing function of both s, which is rather natural, and x.

[4] For a comparison between symmetric mixed strategy equilibria and asymmetric pure strategy equilibria, see [6].

[5] By analogy with a metallic currency system, the possibility of melting money must be examined. Assume that 'melting' money implies a specific (utility) cost $a \geq 0$. This alternative will not be chosen if the flow return associate to it is less than that given by holding one unit of money. Such a condition is:

$$(1 - M)(u + V_0 - V_m) \geq x(V_0 - V_m + u - a)$$

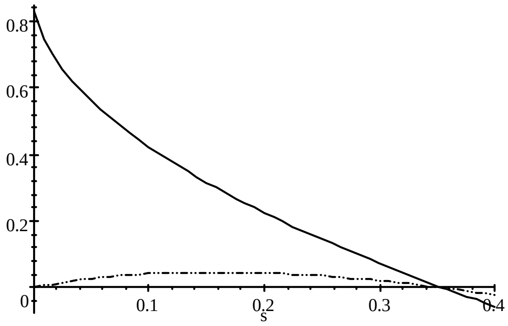
where x is the probability that 'melting' money gives a good with a positive utility. This is equivalent to:

$$G \equiv \gamma M^2 - [\gamma(1-x) + \lambda]M + \lambda(1-x) - (1+r)ax \geq 0$$

where $\lambda = -(x(u-c) + ur) < 0$. For $a = 0$, melting is preferred if $M \geq 1 - x$. In other terms, 'melting' money would be chosen, in the most favorable case, only for amounts of money $(1 - x)$ greater than the maximum quantity $(1 - x - \frac{rc}{(1-x)(u-c)})$. The possibility of 'melting' can thus be neglected thereafter.

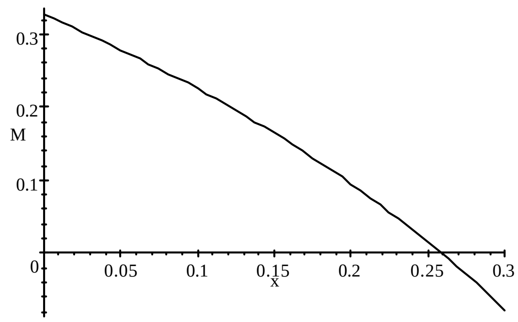
By manipulating mintage conditions the monetary authority influences the equilibrium quantity of money. The relation between M and s is given by (8). It is drawn below (solid line) for the values of parameters adopted above. Equilibrium quantity of money is $M^{***} = .22542$ for $s = .2$ and $M^{***} = .42332$ for $s = .1$. For $s \geq .35476$, agents do not produce for the Mint so that a monetary equilibrium ceases to exist. There is thus a third barter equilibrium in this model ($M^{***} = 0$) if the cost of minting (or rate of interest) is above a determinate threshhold $\bar{s}$. In this case, all agents produce and have a value equal to $\frac{x(u-c)}{r}$, as in the other barter equilibria showed above.

Monetary authority can get a once-for-all amount of utility sM which depends on s as (8) makes it clear. The dotted curve shows how sM varies with s:



A maximum gain is obtained for $s = .15802$, which corresponds to an equilibrium quantity $M^{***} = .3006$ and to a mintage cost $.047501$.

Further examination of (8) shows that M^{***} is also a decreasing function of x, which appears in numerator with a negative sign and in denominators under the form $(1 - x)$. For example, when $s = 0$, we have $M^{***} = \overline{M}$. The graph below represents $M^{***}(x)$ for $s = .2$. For all values of x strictly inferior to $\bar{x} = \frac{u-(c+s)(1+r)}{(u-c)(2+r)} = \frac{.265}{1.025} \simeq .2585$, there exists a monetary equilibrium whereas only a barter equilibrium is possible otherwise.



More generally, it must be underlined that the existence of a monetary equilibrium depends on the absolute degree of specialization x and no longer, as it is the case in search models with an exogenous quantity of money, on a comparison between goods liquidity and the presupposed degree of money acceptation (parameter μ in Kiyotaki & Wright model). Barter equilibrium exists for $x \geq \overline{x} = \frac{u-(c+s)(1+r)}{(u-c)(2+r)}$ although a positive fraction N of agents, possibly 1, accepts money. For $x < \overline{x} = \frac{u-(c+s)(1+r)}{(u-c)(2+r)}$, a monetary equilibrium occurs and $M^{***} = 1 - \frac{\beta}{2\gamma} - \frac{\sqrt{\beta^2+4\gamma(1+r)s}}{2\gamma}$.

A recapitulation of equilibria is given in table below:

$V_m - V_0 - c < 0$	$N^{*} = 0 \quad M^{*} = 0$
$V_m - V_0 - c = 0$	$N^{**} = \frac{x(u-c)-cr}{u-c} \quad M^{**} = 0$
$V_m - V_0 - c > 0 \quad s < \overline{s}$	$N^{***} = 1 \quad M^{***} = 1 - \frac{\beta}{2\gamma} - \frac{\sqrt{\beta^2+4\gamma(1+r)s}}{2\gamma}$
$V_m - V_0 - c > 0 \quad s = \overline{s}$ or $x \geq \overline{x}$	$N^{***} = 1 \quad M^{***} = 0$

3 Welfare

In search models with an exogenously given quantity of money, comparison between barter and monetary equilibria, on the one hand, and study of the influence of the quantity of money on welfare, on the other, are distinct tasks. The first one, paradoxically enough, consists in comparing a monetary economy with a barter one, endowed with a positive quantity if money but not using it. It generally concludes to the superiority of money over barter for a given quantity of money. The second one, independently, evaluates the extent to which the two effects of an increase in the quantity of money balance each other: a negative one due to eviction from production and a positive one due to facilitation of transaction. It generally concludes to the existence of an optimal quantity of money for which the two effects exactly offset each other.

The same conclusions apply here but they result from an unique study comparing welfare associated to equilibria having a common acceptation of money, namely $N^{***} = 1$, but with different monetary policies or with different degrees of specialization (the two most interesting determinants of the equilibrium quantity of money).

Total value of the economy is:

$$W = MV_m + (1 - M)V_0 = V_0(M) + M[V_m(M) - V_0(M)] \tag{9}$$

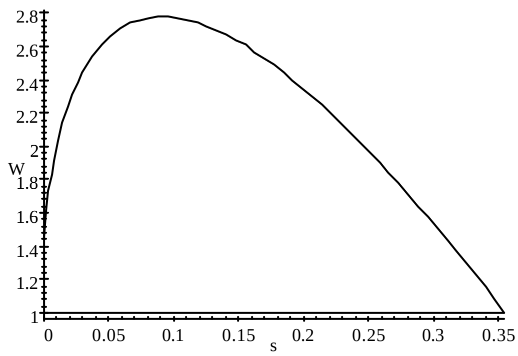
Reporting in (9) the values of V_0 and V_m given by (3) for $N = 1$, yields:

$$W(M^{***}) = \frac{1}{r}[-(u - c)(1 - x)M^2 + (u - c)(1 - 2x)M + x(u - c) \tag{10}$$

Two extreme values are $W(0) = \frac{x(u-c)}{r}$ and $W(1) = 0$[6]. W derivative is $W'(M) = \frac{1}{r}[-2(u-c)(1-x)M + (u-c)(1-2x)]$. It is positive for $M^{\pm\pm\pm} = 0$ and, more generally, for $M^{\pm\pm\pm} < \frac{1-2x}{2(1-x)}$. $W'(M)$ is negative for greater values of $M^{\pm\pm\pm}$. The optimal quantity of money therefore is $M^* = \frac{1-2x}{2(1-x)}$.

Let consider the question from the point of view of monetary policy (for x given). In the numerical example above, the optimal monetary policy consists in fixing $s^* \simeq .09129$ which corresponds to an optimal quantity of money equal to $M^* \simeq .4444$ and a maximum value for the economy $W^* = 2.7778$. It is to be noted that a policy aiming at a maximum global seignorage leads to $s = .15802$ a too high level in terms of welfare: W would be only 2.5916 with $M^{\pm\pm\pm} = .3006$.

Value of economy varies with s according to the graph below:



In order to show that monetary equilibria dominate barter, it must be proved that the quantities of money for which $W(M) < W(0) = \frac{x(u-c)}{r}$, cannot be effective. This is easy to establish. From (10), it results that $W(M) < W(0)$ holds only if $M^{\pm\pm\pm} > 1 - \frac{x}{1-x}$. But it has been shown above that the maximum equilibrium quantity of money is $1 - \frac{(1+r)x}{1-x} - \frac{rc}{(1-x)(u-c)}$. Condition $M^{\pm\pm\pm} > 1 - \frac{x}{1-x}$ is never satisfied since $1 - \frac{(1+r)x}{1-x} - \frac{rc}{(1-x)(u-c)} < 1 - \frac{x}{1-x}$.

However, if a Paretian criterion were adopted, it would be necessary to study the influence of M specifically on the value of money holders and of producers. For a large range of values, manipulation of s (and M) modifies V_0 and V_m in the same direction. But this is not true for all values of s. From (3) two different optimal values of M, M_0^* and M_m^* respectively, correspond to maxima of V_0 and V_m: $M_i^* = \frac{\gamma-\beta}{2\gamma}$ and $M_m^* = \frac{\gamma-(x(u-c)+ur)}{2\gamma}$. Thus:

$$M_i^* - M_m^* = \frac{r}{2} \tag{11}$$

When r tends to zero, M_i^* and M_m^* tend to $M^* = \frac{1-2x}{2(1-x)}M$.

[6] It is worth noting that barter value is $\frac{x(u-c)}{r}$ and not $\frac{x(1-M)(u-c)}{r}$ as in Kiyotaki & Wright model.

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