Trends and cycles in UK stock prices

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Abstract

This paper analyses the forecastable component of UK stock prices over the last century. We start by using the present-value model, in its log-linear form, in order to decompose the 'Beveridge-Nelson' cycle in stock prices into the share of the cycle due to forecastable dividend growth on the one hand, and that due to time-varying discount rates on the other. We then use a permanent/transitory decomposition so as to measure the dynamic effects of dividend and discount-rate innovations over the dividend yield, stock prices and stock returns. The overall conclusion of the paper is that, although dividends are found to be substantially cyclical, they play a minor rôle in the forecastability of stock price movements, virtually all of which being explained by volatile expectations about future returns.

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1 Introduction

That stock prices contain a sizeable forecastable component, and thus depart substantially from the so-called 'random-walk' model of stock-price behaviour, is now widely accepted amongst finance specialists. Early studies, essentially based on returns' auto-regressions and variance-ratio tests, seemed at best limited to establishing the forecastability of stock returns over several year's horizons (eg Poterba and Summers (1988), Fama and French (1988a), Culter *et al.* (1990)). However, multivariate estimates have unambiguously revealed that yearly returns are strongly forecastable on the basis of the 'great' financial ratios, such as price/dividend ratios, price/earning ratios, or dividend payout ratios (see, amongst others, Campbell and Shiller (1987, 1998), Fama and French (1988b)) and Cochrane (1994)).

The close connection between the forecastable component of a time-series and its 'transitory', or 'cyclical', component was highlighted by the seminal contribution of Beveridge and Nelson (1981). More specifically, they identify the trend of a series with the value it is expected to reach in the infinite future, corrected for the cumulated increase in the series due to the drift. If we denote $\{X_t\}_{t=0}^{\infty}$ as the time series under study and $\gamma \equiv E(\Delta x)$ as the average growth rate of the series, then the trend is constructed as follows:

$$\tau_t^x \equiv \lim_{n\to\infty} E_t\left(x_{t+n} - n\gamma\right)$$

With this definition, the dynamics of the Beveridge-Nelson trend is entirely unforcastable on the basis of information available at date t, that is, the trend follows a random walk (with drift γ). In contrast, the Beveridge-Nelson cycle, $c_t^x = x_t - \tau_t^x$, encompasses all that is forecastable in the behaviour of the series, at all horizons:

$$E_t\left(c_{t+i}^x - c_t^x\right) + \gamma i = E_t\left(x_{t+j} - x_t\right) \quad , i = 0, ...\infty$$

Although Beveridge and Nelson (1981) have confined the application of their trend-extraction technique to univariate time-series, the methodology has been generalised to multivariate forecastability by Stock and Watson (1988), and subsequently applied to US macroeconomic time-series by King *et al.* (1991) and Canova (1988). In the present paper,we shall follow these authors in identifying the cycle in stock prices with their forecastable component, such as estimated by a bivariate VAR.

The observation that stock prices are significantly forecastable, that is, contain a Beveridge and Nelson-type cycle, raises several questions. At the theoretical level, one may wonder whether this component merely reflects rational market valuations under changing preferences or investment opportunities, or if they are indicative of the fact that market psychology, bubbles or other 'animal-spirits' play an important rôle in the determination of stock-price movements (see, amongst others, Summers (1986), Poterba and Summers (1988) and Schleifer (2000) on this view). At a more empirical level, one may ask oneself what the sources of the forecastability of stock price movements are; that is, whether it primarily reflects changing dividend prospects, or wether volatile expectations in future returns are the cause.

In some cases, the Beveridge-Nelson trend/cycle decomposition gives an unambiguous answer to the latter question. For example, if dividends approximately follow a random walk, then they are entirely unpredictable, so the Beveridge-Nelson cycle in stock prices unambiguously 'picks' the time-varying *discount rates*' contribution to the forecastability of prices and returns. Conversely, if constant expected returns are assumed, then the cycle in prices directly reflects the extent to which *dividend growth* is forecastable. The issue becomes, however, more complicated when both dividends and discount rates are suspected to be cyclical. In such a situation, the cycle in stock prices comes from the forecastable component of both variables, so that the Beveridge-Nelson cycle becomes uninformative about the *sources* of the forecastability of stock prices (*ie* beyond offering an accurate measurement of its *size*).

The purpose of the present paper is to generalise the Beveridge-Nelson trend/cycle decomposition of stock prices for the case where both dividend growth and discount rates are persistently time-varying, and to decompose the contribution of each of these factors into the forecastability of price movements. The methodology is then illustrated through an application to the long-run behaviour of a broad UK stock-price index, for which both dividend growth and discount rates are found to be significantly cyclical.

We start by noting that, under constant discount rates, the cycle in stock prices is entirely explained by the corresponding cycle in dividends. This allows a definition of the 'dividend cycle' as the cycle in stock prices that would prevail if discount rates were constant; that is, if the forecastability of stock-price movements were entirely explained by fluctuations in dividend growth. Similarly, the 'discount-rate cycle' is defined as the cycle in stock prices that would prevail if dividend growth was entirely unpredictable; that is, if dividends followed a pure random walk. These definitions yield a simple additive decomposition of the level of stock prices, into the price trend, the 'dividend cycle' and the 'discount rate cycle'. The sizes of the two cycles provide a direct measure of the sources of the forecastability of stock-price movements. Applying this decomposition to UK data, we find that, although dividend growth is strongly forecastable, its fluctuations have a negligible impact on the cycle in stock prices, virtually all of which being explained by volatile expectations about future returns.

We then go on to extend the analysis to the effects of dividends and discount rate *innovations* on stock prices. We show that, for the dividend/price ratio to stay stationary in the long-run (that is, for prices and dividends to be cointegrated), discount-rate innovations cannot move the level of dividends permanently, and are constrained to have only transitory effects on prices and dividends. This provides a 'long-run restriction' which, together with a normalisation commonly used in the VAR literature, allows one to analyse the dynamic effect of dividend and discount-rate innovations on the dividend/price ratio, stock prices and stock returns. Here again, we find that, although dividends are substantially cyclical, the impact of a dividend innovation on the short- and medium-run fluctuations of those variables, is very small when compared to that of a discount rate innovation.

Other researchers have studied the historical volatility of UK stock prices in some detail. Bulkley and Tonks (1989), applying Shiller's (1981) original excess volatility test, reject the joint hypothesis of Market Efficiency/constant discount-factor, which suggests that discount rates are indeed time-varying. Cuthbertson *et al.* (1997) apply the VAR methodology of Campbell and Shiller (1987, 1988) to test several asset-pricing models (constant discount rate, constant safe rate, constant risk premium, CAPM), and again find that discount rates are more variable than any of these models predict. Rather than focusing on the volatility of prices, returns or the dividend/price ratio, the present contribution aims to re-assess the evidence by asking *how much* do forecastable dividend growth and time-varying discount factors contribute to the forecastability of stock-price movements, and to measure the impact of their innovations on the dynamic of the variables under study. Therefore, rather than assuming, and then testing, a specific asset-pricing model, we shall basically think of the discount rate as a *residual*, whose measure and influence can be infered from the data.

The paper is organised as follows. Section 2 uses the log-linear present value model to analytically decompose the level of stock-prices into the price trend and the two cyclical components. Section 3 applies this methodology to the long-run behaviour of the UK stock market, using a VAR to compute expectations of future dividend/price ratios and dividend growth. Section 4 identifies dividend and discount-rate innovations, and uses impulse-response functions and variance decompositions in order to assess their dynamic effects on stock prices, stock returns, and the dividend/price ratio. Section 5 concludes.

2 Theoretical framework

2.1 The log-linear present value formula

The present section describes how to use the present value model, in its log-linear formulation, in order to first extract, and then decompose, the forecastable component of stock prices.

Let Q_t denote the real price of a stock at date t, D_t the income received from holding the stock during period t, and R_{t+1} the ex post return on holding stocks from date t to date $t+1$. Then, by definition, we have:

$$R_{t+1} = \frac{Q_{t+1} + D_{t+1}}{Q_t} \tag{1}$$

The present-value model that one obtains by solving (1) for Q_t is non-linear, and hence rather untractable in applied work. One can, however, take a log-linear approximation to eq. (1) along the lines described by Campbell and Shiller (1988) and Cuthbertson *et al.* (1997). More specifically, if one assumes that the dividend/price ratio is stationnary, so that dividends and prices grow at the same average growth rates, Campbell and Shiller (1988) show that equation (1) approximately implies (see appendix A1 for the detailed derivation):

$$r_{t+1} - r^* = (\lambda_t - \lambda^*) - \rho(\lambda_{t+1} - \lambda^*) + (\Delta d_{t+1} - g) \quad ,$$

where $r_t = \log(R_{t+1})$ denotes log-returns, $\lambda_t = d_t - q_t$ the log-dividend/price ratio, $\rho = (1+g)/R^*$ a constant close to but inferior to 1, and where $\lambda^* = E(\lambda_t)$ and $R^* = E(R_t)$ are the mean of the log dividend/price ratio and stock returns, respectively. Solving the latter equation for λ_t, iterating forward and taking the expectation operator at date t on both sides of the resulting equation, one obtains the following *log-linear present value model* for the dividend/price ratio (see appendix A1 again):

$$d_t - q_t = \lambda^* + \sum_{j=0}^{\infty} \rho^j E_t(r_{t+1+j} - r^*) - \sum_{j=0}^{\infty} \rho^j E_t(\Delta d_{t+1+j} - g) \tag{2}$$

In other words, the dividend/price ratio at date t is an increasing function of current, $E_t(r_{t+1})$, and expected future discount rates, $E_t(r_{t+1+j})$ $(j = 1, ...\infty)$, and a decreasing function of current dividends, d_t, and expected future dividend growth, $E_t(\Delta d_{t+1+j})$ $(j = 1, ...\infty)$. Future dividends and discount rates have decreasing weights in the present value formula, since $\rho < 1$. We have assumed that the dividend/price ratio was stationary, and hence both the discount rate and the growth of dividends must be assumed to be stationary variables. Stationarity of the dividend/price ratio means that price may be cointegrated, that is, a pair of integrated series driven by a unique stochastic trend. As many stock price and dividend series appear to have a unit-root (section 3 confirms that such is the case for UK data), we shall interpret the present-value model as featuring this property.

2.2 Trend and cycle in stock prices

We start by constructing the trend in stock prices, in its relation to the trend in dividends. The Beveridge-Nelson trend in prices is given by:

$$\tau_t^q = \lim_{n\to\infty} E_t\left(q_{t+n} - ng\right) = q_t + \sum_{j=1}^{\infty} E_t\left(\Delta q_{t+j} - g\right) \tag{3}$$

The VAR that is estimated in the next section does not include stock returns as a dependent variable, so it is not possible to use eq. (3) directly to compute the price trend. Note, however, that the trend in price and the trend in dividends are related by a straightforward, yet economically meaningful, relationship. Indeed, denoting $\tau_t^d = \lim_{n\to\infty} E_t\left(d_{t+n} - ng\right)$ as the Beveridge-Nelson trend in dividends, one can see that:

$$\tau_t^d - \tau_t^q = \lim_{n\to\infty} E_t\left(\lambda_{t+n}\right) = \lambda^*$$

Hence, the trend in prices is defined by the expected *long-run level of dividends*, from which it can be computed:

$$\tau_t^q = -\lambda^* + \tau_t^d = -\lambda^* + d_t + \sum_{j=1}^{\infty} E_t\left(\Delta d_{t+j} - g\right) \tag{4}$$

As emphasised in the introduction, the trend in prices encompasses the unpredictable component of stock price movements; that is, the price trend follows a pure random walk with drift. We indeed have:

$$E_t\left(\Delta\tau_{t+1}^q - g\right) = 0$$

Finally, the *cycle* in stock prices, that is, their forecastable component, is obtained by simply removing the price trend from the current price:

$$c_t^q = q_t - \tau_t^q$$

Since the Beveridge-Nelson cycle isolates the forecastable component of the series, its size provides a measure of the degree by which prices depart from the so-called *random-walk model* (which, literaly, says prices contain no forecastable component). If discount rates were constant, then transitory stock-price fluctuations would be entirely accounted for by the forecastability of dividend growth, whose impact on stock prices would be accurately measured by the Beveridge-Nelson cycle in prices. This configuration corresponds to Samuelson's (1965) *martingale model*, which features constant discount rates, together with unrestricted dividend growth. Alternatively, if dividends approximately followed a pure random-walk, then the Beveridge-Nelson cycle would measure the extent to which *time variations in discount rates* render prices and returns forecastable. In other words, if only one component of the right hand-side of the present-value model (eq.(2)) is cyclical, then it entirely accounts for the forecastable component in stock prices, and hence is adequately picked by the Beveridge-Nelson cycle. As emphasised in the introduction, there is no reason to assume that such is the case as, in general, both discount rates and dividend growth are forecastable.

2.3 The 'dividend cycle' and the 'discount rate' cycle

We now decompose the cycle in stock prices into two forecastable components, measuring the contributions of the forecastability in dividend growth and that of time varying discount rates to the overall cycle.

Let us first define the *dividend cycle*, denoted c_t^d, as the Beveridge-Nelson cycle in prices that would obtain if discount rates were constant, that is, if $E_t\left(r_{t+1}-r^*\right)$ were equal to zero at all time. When such is the case, the present value model reduces itself to:

$$q_t = -\lambda^* + d_t + \sum_{j=0}^{\infty} \rho^j E_t\left(\Delta d_{t+1+j} - g\right) \tag{5}$$

The dividend cycle is then obtained by removing the trend (eq. (4)) from the current price, as is given by eq (5). One finds:

$$c_t^d = -\sum_{j=1}^{\infty} \left(1-\rho^j\right) E_t\left(\Delta d_{t+1+j} - g\right) \tag{6}$$

Similarly, we define the *discount rate cycle* as the cycle in stock prices that would prevail if dividend growth was unpredictable, that is, if $E_t\left(\Delta d_{t+1} - g\right) = 0$ for all t. When such is the case, the present-value model reduces to:

$$q_t = -\lambda^* + d_t - \sum_{j=0}^{\infty} \rho^j E_t\left(r_{t+1+j} - r^*\right) \tag{7}$$

If dividends follow a pure random walk, the price trend is simply given by $-\lambda^* + d_t$ (see eq. (4)). Therefore, the 'discount rate cycle' is given by:

$$c_t^r = -\sum_{j=0}^{\infty} \rho^j E_t\left(r_{t+1+j} - r^*\right) \tag{8}$$

By solving the present value formula (eq. (2)) for q_t, it is easy to check that the decomposition obtained is additive, so that we have:

$$q_t = \tau_t^q + c_t^d + c_t^r \tag{9}$$

From equations (3) and (6), the price trend and the dividend cycle can be computed on the basis of expectations of future dividend growth, which in turn can be infered from a VAR (see next section). The discount rate cycle cannot be directly observed, but is recovered using eq. (9).

3 An application to the UK stock market, 1918-2001

This section applies the cycle decomposition described above to UK stock prices and dividends. Our database includes nominal stock prices and related dividends from the Barclays value-weighted stock index, over the period 1918-2001. Dividends and prices are recorded in December, so that prices incorporate information about dividends paid out during the entire year. Nominal values are divided by the Barclays cost of living index, to obtain real prices and dividends. All the data is available in Barclays Capital (2002), where further information about its construction is provided.

Panel A of Table 1 gathers some summary statistics about the variables under study. Panel B shows the parameters' estimates that enter the log-linear present-value model. Two observations already suggest that time variations in discount rates may play an important rôle in stock-price fluctuations. Firstly, price growth and the dividend/price ratio appear substantially more volatile than dividend growth. Secondly, the contemporaneous correlation between price growth and dividend growth is very small, indicating that the latter has virtually no influence on the level of stock prices *in the short run.*

The log-linear present value model is derived under the assumption that dividend growth, stock returns, and the dividend/price ratio are stationary variables. This may occur either because dividends and prices share the same deterministic trend, or because prices and dividends are cointegrated. To verify that the theoretical framework presented in section 2 is congruent to the data we use, we applied unit-root and cointegration tests to the variables involved in the present-value model (see Table 2). ADF tests do not reject the null hypothesis that the levels of dividends (d_t) and stock prices (q_t) contain a unit-root, but reject the null at the 1 percent level for their differences, indicating that both variables are $I(1)$. The null hypothesis of non-stationarity of the log dividend/price ratio (δ_t) is rejected at the 5 percent level. This evidence for cointegration between prices and dividends, although weak at first sight, is confirmed by the Johansen Trace statistic, which rejects the null hypothesis of no cointegration between d_t and q_t at the 1 percent level.We interpret these tests as evidence that the data is compatible with the assumption that d_t and q_t are CO(1,1), and that their cointegrating vector is $(1, -1)$.

TABLE 1. STATISTICS AND PARAMETERS (1918-2001)

A. SUMMARY STATISTICS

Variable	Mean	St. Dev.	Correlation matrix		
	$\overline{x}$	σ_x	Δd_t	Δq_t	$d_t - q_t$
Δd_t	0,013	0,128	1,000	-0,024	0,263
Δq_t	0,018	0,216		1,000	-0,415
$d_t - q_t$	-3,030	0,275			1,000

B. PARAMETER VALUES

Div. growth	Div. Yield	Weights	Disc. rate
$g = E\left(\Delta d_t\right)$	$\Lambda^* = E\left(\Lambda_t\right)$	$\rho = \frac{1}{1+\Lambda^*}$	$r^* = \frac{1+g}{\rho} - 1$
1,29%	4,83%	0,9539	6,18%

Panel A reports the mean, standard deviation and contemporaneous correlations of dividend growth (Δd_t), returns (Δq_t), and the log dividend/ price ratio (d_t-q_t). Panel B shows the parameters used in the present value formula, that is, the average dividend growth (g), the average dividend yield (Λ^*), the weights (ρ), and the average discount rate (r^*).

TABLE 2. UNIT-ROOT AND COINTEGRATION TESTS (1918-2001)

Augmented Dickey-Fuller					Trace
d_t	q_t	Δd_t	Δq_t	δ_t	$Z_t = \left(\begin{array}{cc} d_t & q_t \end{array} \right)$
-1.61	-1.87	-6.17**	-6.78**	-2.97*	25.24**

* significant at the 5% level (-2.90 for the ADF test, 15.34 for the Trace test).

** significant at the 1% level (-3.51 for the ADF test, 19.69 for the Trace test).

For theJohansen Trace test, the null hypothesis is that the Π in the VAR $\Delta Z_t = \Pi Z_{t-1} + \epsilon_t$ has rank zero; rejection indicates that d_t and q_t are cointegrated.

Table 3. OLS Estimates from a bivariate VAR (1918-2001)

Dependent variables	δ_{t-1}	Δd_{t-1}	δ_{t-2}	Δd_{t-2}	R^2	F-prob	Q^*
	Explanatory variables						
δ_t	0.714	0.150	-0.074	0.069	0.457	0.000	1.332
	(0.200)	(0.321)	(0.216)	(0.164)			
Δd_t	-0.108	0.421	0.096	-0.168	0.317	0.000	3.814
	(0.036)	(0.061)	(0.036)	(0.051)			

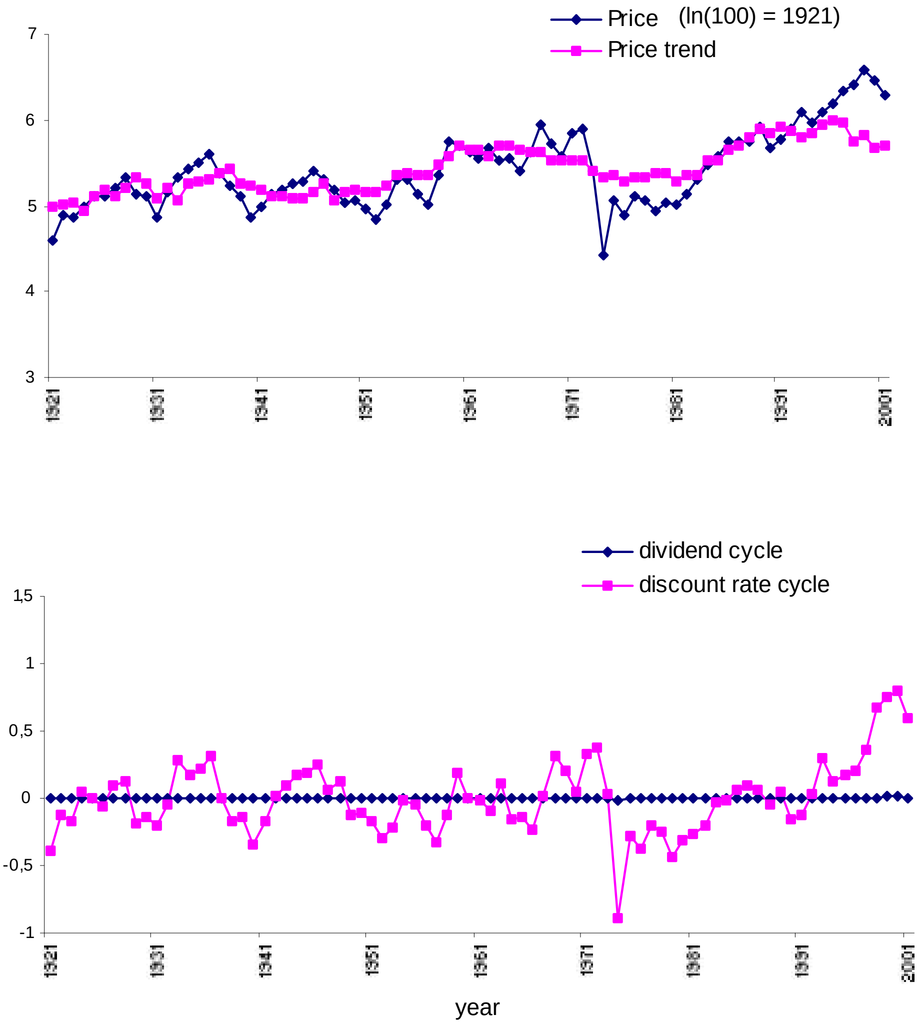
Heteroscedasticity-consistent standard errors are shown below each coefficient. 'F-prob' is the percent probability value of an F-test for the joint significance of the explanatory variable. Q^* is the Ljung-Box statistics for up to 4th-order residuals' autocorrelation.

Expectations of future dividend growth and future dividend/price ratios, which are necessary to compute the trend and the two cycle components, are obtained using a bivariate VAR, similar to that estimated by Campbell and Shiller (1988, 1989) using US data, and by Cuthbertson *et al.* (1997) using UK data. The coefficient estimates and associated standard errors are shown in figure 3. The VAR includes two lags and is applied to de-meaned series. For both equations, the Box-Ljung statistic is low, showing no obvious sign of mis-specification due to residuals' auto-correlation.

Two important features of the dynamic behaviour of the variables are worth noting, as they will play an important rôle in the subsequent analysis. The first equation of the VAR shows that departures of the dividend/price ratio from its long-run equilibrium value, are highly persistent, suggesting the presence of a significant cycle in either of the two right-hand variables of the present value model (equation (2)). The second equation reveals that dividend growth is indeed cyclical and hence predictable, especially on the basis of information on *past* dividend growth. That coefficients on the second equation are jointly significant is also indicative that the data rejects the hypothesis that dividends follow a random walk. Therefore, the cycle decomposition that we sketched out in the previous section applies in a non-trivial way, since one cannot trace trend deviations of stock prices to changing discount factors only.

Appendix A2 shows how the VAR coefficients should be used to compute the price trend, the dividend cycle and the discount rate cycle. These are represented at figure 1. The first panel displays the price series against its trend (as computed from equation (4)),

Figure 1: Trend and cycles in UK stock prices



indicating the size of the forecastable component of stock prices. The second panel shows the corresponding 'dividend cycle' and 'discount rate' cycle, as measured by equation (6) and (8) in section 2. The variances and autocorrelation functions of each cycle are summarised in table 3.

TABLE 4. PROPERTIES OF THE CYCLES

	St. deviation	Autocorrelations				
	$\sigma(\%)$	r_1	r_2	r_3	r_4	r_5
c_t	0.271	0.702	0.460	0.266	0.155	0.043
c_t^r	0.267	0.702	0.461	0.267	0.156	0.043
c_t^d	0.004	0.678	0.357	0.148	0.102	-0.003

c_t denotes the overall cycle, c_t^r the discount-rate cycle, and $c_t^d$the dividend cycle.

The results are rather nambiguous. The dividend cycle is almost flat; it has a very small variance, about 70 times smaller than that of the overall cycle. Therefore, dividends fluctuations have virtually no impact on the transitory components of stock prices, when compared to the forecastability of stock-price movements stemming from time-varying discount rates. Moreover, this occurs in spite of the fact that dividends *are* significantly forecastable, but this forecastability is completely overwhelmed by that coming from the discount rates. As a result, prices behave as if dividend followed a pure random walk.

4 Impulse-response analysis

The previous section allowed us to isolate the relative contributions of forecastable dividends and time-varying discount rateson the level of (detrended) stock prices. We now extend the analysis to carry out an innovation accounting exercise. More specifically, we ask the following question: how much do dividends and discount rate innovations, possibly labelled as *dividend shocks* and *discount-rate shocks*, account for the observed variability of stock prices, stock returns and the dividend/price ratio ? As is well known, recovering underlying sources of perturbations from a VAR, requires the imposition of some identifying restrictions. We first describe the identification technique that we use, and then go on to present the corresponding impulse-response functions and variance decompositions.

4.1 A permanent/transitory decomposition

The empirical analysis of the previous section showed that dividends and prices were cointegrated processes; that is, a pair of integrated processes driven by a unique stochastic trend (τ_t^q). The orthogonalisation of shocks that we use follows naturally from this common trend relationship. More specifically, we allow only one type of shock to move the common trend in prices and dividends, and constrain the other one to have only transitory effects on those variables. This decomposition yields a long-run restriction similar to the one initially used by Blanchard and Quah (1989). Similar identification restrictions were also used to identify the permanent and transitory components of a set of co-integrated variables, estimated through a vector error-correction model (see King *et al.* (1991), Levtchenlova and Pagan (1998) and Gonzalo and Ng (2001)). Although we use a VAR rather than a VECM, we show in appendix A3 that our long-run restriction is formally equivalent to the one that one would obtain by permanent/transitory decomposing a VECM including stock returns and dividend growth (like that estimated by Cochrane (1994) using US data).

Let us denote X_t as the vector of de-meaned dividend/price ratio and dividend growth, respectively. Inverting the VAR of the previous section then yields the following moving-average representation:

$$X_t = B(L)\,\epsilon_t \quad ,$$

where $B(0) = I$ and $\text{cov}(\epsilon) = \Sigma$. If we denote $\eta_t = \left(\eta_t^1, \eta_t^2\right)$ as the vector of orthogonalised shocks, then one can also represent the dynamic of X_t as:

$$X_t = C(L)\,\eta_t \quad , \tag{10}$$

where $\text{cov}(\eta) =$. We identify the effect of each individual component of η_t by means of two restrictions. The first one is a normalisation of orthogonalised shocks such that $\text{var}\left(\eta_t^1\right) = \text{var}\left(\eta_t^2\right) = 1$. The second constrains one of the shocks, η_t^1 say, not to move the stochastic trend in prices and dividends in the long-run. This implies that the cumulative effect of an η_t^1 shock over the long-run *level* of dividends is zero, which writes as:

$$C_{21}(1) = 0$$

It is shown in appendix A3 that these two restrictions allow us to identify the dynamic effect of each orthogonalised shock on X_t, that is, to identify $C(L)$.

4.2 Interpretation of orthogonalised shocks

So far, the orthogonalised shocks have been defined using a set of formal restrictions on their sizes and long-run multipliers, and no economic meaning has been attached to them. To give an economic interpretation of these shocks in terms of 'structural' disturbances, one needs a specific economic model featuring exactly two such shocks.

The present value model introduced in the first section provides such a model. Solving it for prices yields:

$$q_t = -\lambda^* + d_t + \sum_{j=0}^{\infty} \rho^j E_t \left(\Delta d_{t+1+j} - g\right) - \sum_{j=0}^{\infty} \rho^j E_t \left(r_{t+1+j} - r^*\right)$$

Note that discount rates enter this equation as a sequence of stationary residuals, making the present-value model hold at all time, that is, the discount rate is a residual. With this extensive definition, prices are determined by exactly two variables, namely dividends and discount rates. Innovations to each of these two variables, labelled as 'dividend shocks' and 'discount-rate shocks', provide a theoretical model with two shocks that helps interpret the impulse-responses.

Since movements of the common stochastic trend driving stock prices and dividends are entirely driven by changes in dividends, it is natural to interpret the shock that moves the trend in eq. (10) as a 'dividend shock'. By contrast, we shall refer to the shock that only has transitory effects on both prices and dividends as a 'discount rate shock'. Although this interpretation remains debatable, it has the advantage of not constraining the contemporaneous effect of either shocks on the dividend/price ratio, or dividend growth. Since contemporaneous feedbacks from dividends to discount rates are a priori likely to occur in annual data, we find the identification of dividends and discount-rate shocks on the basis of a long-run restriction more appealing than on a basis of a standard Cholesky decomposition (like that used by Cochrane (1994), for instance).

4.3 The dynamic effect of dividend and discount-rate disturbances

Equation (10) allows the study of the effect of dividend and discount-rate disturbances on the dividend/price ratio and dividend growth. Their effect on stock prices and returns is obtained by differencing the first equation in (10), and then 'removing' the effect of dividend growth using the second equation in (10) (the procedure is described in appendix A3).

Figure 2 depicts the dynamic effect of each type of disturbance on the dividend/price ratio, stock prices and stock returns. These are responses to normalised unit shock, and provide information about the persisence and relative impact of each shock on these variables.

The impulse-response functions confirm the predominent rôle of discount-rates in the fluctuations of stock prices. For instance, the contemporaneous responses of prices and returns to a discount-rate shock are about three times as large as the corresponding responses to a dividend shock. The contrast is even more striking if one looks at the responses of the dividend/price ratio, which is left almost unchanged after a dividend shock, despite the auto-correlation pattern of dividend growth. Discount-rate shocks are also quite persistent, with a half-life of about two years. Finally, a negative discount-rate shock is associated with high returns, followed by about five years of negative returns due to trend-reversion.

The share of forecast error variance explained by transitory 'discount-rate' and permanent 'dividend' shocks at various horizons is calculated in table 5. As could be expected from the observation of impulse-response functions, transitory shocks are largely predominent in generating forecast errors of future dividend/price ratios and future returns. Note that this estimate of the influence of transitory shocks over the variance of stock returns is substantially higher than that obtained by Cochrane (1994) on US data. By contrast, transitory shocks are found to be about half as persistent in UK data as they are in US data (which feature a half-life for transitory shocks of about 5 years, see Cochrane (1994)).

Figure 2: Impulse-response analysis

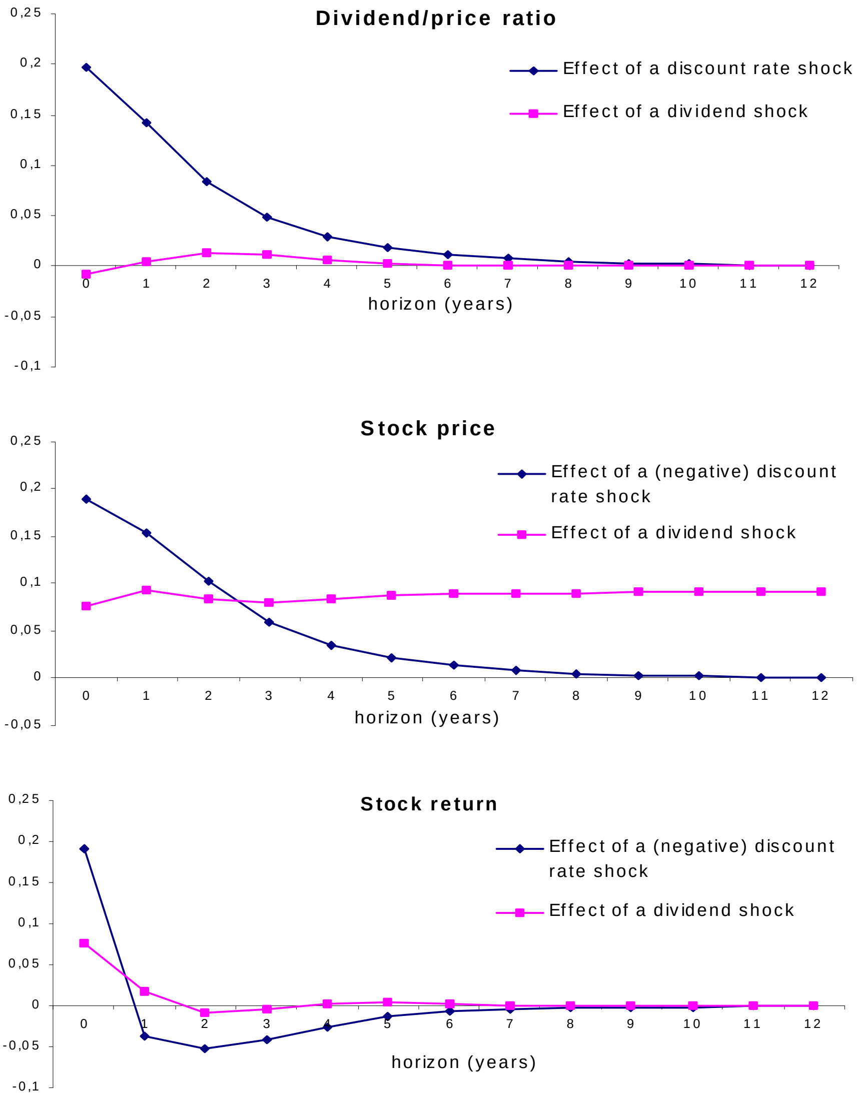


Table 5. Variance Decomposition

h	$\mathrm{var}\left(\delta_t - E_{t-h}\left(\delta_t\right)\right)$		$\mathrm{var}\left(\Delta q_t - E_{t-h}\left(\Delta q_t\right)\right)$	
	η_t^1	η_t^2	η_t^1	η_t^2
1	0.998	0.002	0.861	0.139
2	0.998	0.002	0.860	0.140
4	0.994	0.006	0.871	0.129
∞	0.993	0.007	0.873	0.127

η_t^1 is the transitory 'discount-rate' shock, and η_t^2 is the permanent 'dividend' shock. The table reports the share of each shock in the forecast error variance of the dividend/price ratio and the stock return.

5 Conclusion

Whereas there is little ambiguity about the fact that dividends govern the long-run value of stock market indexes, there is much less knowledge of what forces lead stock-price fluctuations in the short- and medium-run. This paper has addressed this issue by confronting two methods for mesuring the importance of transitoiry movements s in UK stock prices, and assessing the relative rôles of dividends and discount factor in those movements. The overall conclusion of the paper is that mean-reverting, but persistent, deviations of the discount rate from its long-run average *are* the dominant force behind stock-price fluctuations in the medium-run, *eventhough dividends do not follow a random walk and display a clear cyclical pattern.* We found that not only do discount rates explain virtually all of the predictable component of stock prices, as the cycle decomposition of section 2 showed, they also explain most of the variations in ex post returns and the dividend/price ratio, as the impulse-response analysis of section 2 illustrated.

Whether these observed large swings in stock prices and discount rates can be reconciled with the joint hypothesis of market efficiency, together with the specification of a 'reasonable' asset-pricing model, remains an open issue, from which we have stayed away by defining the discount rate as a mere residual, rather than the outcome of a explicit behavioural model. However, an accurate measurement of the contribution of time-varying discount rates to the volatility and forecastability of stock price movements might be usefull in helping one to choose amongst the competing families of asset-pricing models (*ie* equilibrium asset-pricing versus fad-based models) available in the litterature.

Appendix

A1. Le log-linear present value formula

This appendix derives the log-linear present value model initially proposed by Campbell and Shiller (1988). Let us first decompose the equation for ex post returns as follows:

$$R_{t+1} = (Q_{t+1}/Q_t) + (D_{t+1}/D_t) \times (D_t/Q_t) \tag{11}$$

Since prices and dividends may not be trend-stationary, one cannot log-linearise (11) around their average *levels*. However, under the assumption that prices and dividends are CO(1,1), and that their cointegrating vector is $(1, -1)$, it is possible to approximate (11) around the average growth factor common to prices and dividends, $1+g$. This operation yields:

$$R_{t+1} - R^* = (Q_{t+1}/Q_t - 1 - g) + (1+g)(\Lambda_t - \Lambda^*) + \Lambda^* (D_{t+1}/D_t - 1 - g)$$

where $R^* \equiv E(R_t)$, $\Lambda_t \equiv D_t/Q_t$ and $\Lambda^* \equiv E(\Lambda_t) = (R^* - 1 - g)/(1+g)$. This equality can usefully rewritten in terms of proportional deviations from trend:

$$\begin{aligned} \frac{R_{t+1} - R^*}{R^*} &= \left(\frac{1+g}{R^*}\right)\left(\frac{Q_{t+1}/Q_t - 1 - g}{1+g}\right) \\ &+ \left(\frac{(1+g)\Lambda^*}{R^*}\right)\left(\frac{\Lambda_t - \Lambda^*}{\Lambda^*}\right) + \left(\frac{(1+g)\Lambda^*}{R^*}\right)\left(\frac{D_{t+1}/D_t - 1 - g}{1+g}\right) \end{aligned}$$

Now, using the fact that, for all $X_t - X^*$ 'small', one has $(X_t - X^*)/X^* \simeq \log X_t - \log X^*$, the previous equation is approximately identical to:

$$\begin{aligned} r_{t+1} - r^* &= \rho(\Delta q_{t+1} - g) + (1-\rho)(\lambda_t - \lambda^*) + (1-\rho)(\Delta d_{t+1} - g) \\ &= (\lambda_t - \lambda^*) - \rho(\lambda_{t+1} - \lambda^*) + (\Delta d_{t+1} - g) \quad , \end{aligned}$$

where lower case letters denote natural logarithms of the corresponding capital letters, and where $\rho = (1+g)/R^*$. Solving the latter equation for $\lambda_t - \lambda^*$, and then iterating it forwards under the no-bubble condition $\lim_{j\to\infty} \rho^j \lambda_{t+j} = 0$, one obtains:

$$\begin{aligned} \lambda_t - \lambda^* &= \rho(\lambda_{t+1} - \lambda^*) + (r_{t+1} - r^*) - (\Delta d_{t+1} - g) \\ &= \sum_{j=0}^{\infty} \rho^j (r_{t+1+j} - r^*) - \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+1+j} - g) \end{aligned}$$

Taking time-t expectations on both sides of the last equation and replacing λ_t by its value yields (2) in the body of the paper.

A2. Calculation of the price trend, the discount-rate cycle and the dividend cycle

The VAR that is estimated can be written as:

$$X_t = \sum_{i=1}^{2} A_i X_{t-i} + \epsilon_t \quad , \tag{12}$$

where $X_t = \left[\begin{array}{cc} \lambda_t - E(\lambda_t) & \Delta d_t - E(\Delta d_t) \end{array} \right]'$ and $\text{cov}(\epsilon) = \Sigma$. The first steps consists of rewritting (12) in stacked form. Namely, if we define the matrix Π and the vector Y_t as follows:

$$\Pi = \left[\begin{array}{cc} A_1 & A_2 \\ I & 0 \end{array} \right], \quad Y_t = \left[\begin{array}{c} X_t \\ X_{t-1} \end{array} \right] \quad ,$$

then the evolution of the dynamic system is entirely described par the equation:

$$\begin{aligned} Y_{t+k} &= \Pi Y_{t+k-1} + \varepsilon_{t+k} \\ &= \Pi^k Y_t + \sum_{i=0}^{k-1} \Pi^i \varepsilon_{t+k-i} \end{aligned}$$

From eq. (4) in section 2, the price trend at date t is given by:

$$\begin{aligned} \tau_t^q &= -\lambda^* + d_t + \sum_{j=1}^{\infty} E_t(\Delta d_{t+j} - g) \\ &= -\lambda^* + d_t + \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right] \sum_{k=1}^{\infty} E_t(Y_{t+k}) \\ &= -\lambda^* + d_t + \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right] \Pi (I - \Pi)^{-1} Y_t \end{aligned}$$

Let us denote $\left[\pi_{ij}^k\right] \equiv \Pi^k$, which reads as "$\pi_{ij}^k$ is the element of Π^k that lies on its i-th row and its j-th column". From eq. (4) in section 2, the dividend cycle can then be computed as follows:

$$\begin{aligned} c_t^d &= -\sum_{j=1}^{\infty} \left(1 - \rho^j\right) E_t(\Delta d_{t+1+j} - g) \\ &= -\sum_{k=1}^{\infty} \left(1 - \rho^k\right) \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right] E_t(Y_{t+1+k}) \\ &= -\left[\begin{array}{cccc} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{array} \right] Y_t \quad , \end{aligned}$$

where $\varphi_j = \sum_{k=1}^{\infty} \left(1 - \rho^k\right) \pi_{2j}^{k+1}$. Finally, the 'discount-rate' cycle at date t is simply given by:

$$c_t^r = q_t - \tau_t^q - c_t^d$$

A3. Shock orthogonalisation

First, let us invert the VAR to obtain the moving-average representation of the reduced form model:

$$X_t = B\left(L\right)\epsilon_t \tag{13}$$

where $B\left(L\right) \equiv \left(I - A_1 L - A_2 L^2\right)^{-1}$ and $\text{cov}\left(\epsilon\right) = \Sigma$. The orthogonalisation of shocks starts by defining a set of 'structural' shocks $\left(\eta_t^1, \eta_t^2\right)$ whose *joint* effect on X_t must be similar to the joint effect of a $\left(\epsilon_t^1, \epsilon_t^2\right)$ shock, but whose *individual* effect on each component of X_t depends on the identifying restrictions applied. Before applying identifying restrictions, let us rewrite eq. (13) as follows:

$$\begin{aligned} X_t &= B\left(L\right)\epsilon_t \\ &= B\left(L\right)C\left(0\right)C\left(0\right)^{-1}\epsilon_t \\ &= C\left(L\right)\eta_t \end{aligned} \tag{14}$$

where $\text{cov}\left(\eta\right) \equiv \quad$, $C\left(L\right) \equiv B\left(L\right)C\left(0\right)$, and $\eta_t \equiv C\left(0\right)^{-1}\epsilon_t$. Note that by construction we must have $\text{var}\left(C\left(0\right)\eta_t\right) = \text{var}\left(\epsilon_t\right)$, that is:

$$C\left(0\right) \quad C\left(0\right)' = \Sigma$$

The latter matrix equation generates three equations for seven unknowns (the three elements of   and the four elements of $C\left(0\right)$). The normalisation $= I$ adds three equations to the system, and the identifying restriction $C_{21}\left(1\right) = 1$ one equation, which allows to compute $C\left(0\right)$. Finally, from the definition of $C\left(L\right)$ we have $C\left(0\right) = B\left(0\right)C\left(0\right)$, and hence:

$$C_j = B_j C\left(0\right)$$

. This last equation allows to simulate the whole polynomial $C\left(L\right)$, and to compute the dynamic effects of each η_t^j shock $(j = 1, 2)$ on the dividend/price ratio, λ_t, and dividend growth, Δd_t. There effects on stock returns, Δq_t, is computed as follows:

$$\begin{aligned} \Delta q_t - E\left(\Delta q_t\right) &= \Delta d_t - E\left(\Delta d_t\right) - \left(1 - L\right)\left(\lambda_t - E\left(\lambda_t\right)\right) \\ &= \left(\begin{bmatrix} 0 & 1 \end{bmatrix} C\left(L\right) - \left(1 - L\right)\begin{bmatrix} 1 & 0 \end{bmatrix} C\left(L\right)\right)\eta_t \\ &= \begin{bmatrix} C_{21}\left(L\right) - \left(1 - L\right)C_{11}\left(L\right) & C_{22}\left(L\right) - \left(1 - L\right)C_{12}\left(L\right) \end{bmatrix}\eta_t \end{aligned}$$

If we define the vector X_t' as $\begin{bmatrix} \Delta q_t - E\left(\Delta q_t\right) & \Delta d_t - E\left(\Delta d_t\right) \end{bmatrix}'$, we can write its dynamics as

$$X_t' = C^*\left(L\right)\eta_t \quad ,$$

where

$$C^*(L) = \begin{bmatrix} C_{21}(L) - (1-L)\,C_{11}(L) & C_{22}(L) - (1-L)\,C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix}$$

Note that, due to the assumption that d_t and q_t are cointegrated, the matrix of long-run multipliers $C^*(1)$ has rank 1. Under our identifying restriction, it is given by:

$$C^*(1) = \begin{bmatrix} 0 & C_{22}(1) \\ 0 & C_{22}(1) \end{bmatrix},$$

which says that a η_t^1 shock has no long-run effect on either dividends or prices, whereas the long-run responses of prices and dividends to an η_t^2 shock are identical. Henceforth, our long-run restriction is formally analogous to the 'permanent/transitory' decomposition that one would obtain by indentifying the moving-average representation of a vector error-correction model (see King *et al.* (1991), Gonzalo and Ng (2001)).

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