

# A NOTE ON SOME MINSKYAN MODELS OF FINANCIAL INSTABILITY

Sébastien CHARLES\*

University of Paris 8

Department of Economics (LED-EPEH)

2, rue de la liberté

93526 Saint-Denis, France

## Abstract

*Today, numerous Post-Keynesian authors use Minsky's financial instability hypothesis in order to elaborate models that give rise to financial crises or endogenous cycles. But these matters are often extremely complicated. Besides, other minskyan contributions which are analytically tractable suffer, unfortunately, from few flaws which make them inconsistent. Therefore, we propose an introductory model of accumulation and debt which corrects these errors and shows the appearance of financial fragility in a simple way.*

**Keywords :** Minsky, Accumulation, Debt, Financial fragility

**JEL Classification :** E 12, E 21, E 22, E 44

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\* Address for correspondence : CHARLES Sébastien, 115 Bd Jean Jaurès, 92110 Clichy (France). Tel : 01 47 31 04 09. Email : sebcharles92@yahoo.fr.

## 1. Introduction

Post-keynesians macroeconomics based on the framework of Minsky (1975, 1986) has become a prolific field of research since the original contribution by Taylor and O'Connell (1985). Indeed, several articles (cf., for instance, Semmler, 1987; Franke and Semmler, 1989; Delli Gatti and Gallegati, 1994*a* and 1994*b*; Keen, 1995; Asada, 2001 and 2002) offer enlightening precisions about the destabilizing role of external financing on the real sector. Nevertheless, these latters are often very sophisticated and not always analytically tractable.

The goal of this paper is double. First, we show that there remains some simple minskyan models but, unfortunately, they suffer from few flaws. Second, we elaborate an introductory model with two properties : (1) It corrects these errors and (2) remains analytically tractable.

## 2. Some difficulties with simple minskyan models

Several minskyan models give rise to very interesting dynamics however they can not be solved, due to their non-linearity. Moreover, authors are often compelled to simulate their models to find valuable economic results (cf. Asada, 2002; Delli Gatti and Gallegati, 1994*a* and *b*; Keen, 1995; Semmler, 1987). Consequently, few things can be said without a computer. Some other approaches (especially Jarsulic, 1990; but also Arena and Raybaut, 1998 and 2001), showing the destabilizing effect of debt in macrodynamic growth models, have the advantage of mathematical simplicity. However, they seem to suffer from several unjustifiable or incoherent assumptions.

First, Jarsulic (1990, pp. 93-96) assumes the existence of a Keynesian saving equation with a constant propensity to save on income :  $S = sY$ , where  $S$  is saving,  $Y$  is the level of national income and  $0 < s < 1$ . This formulation implies that there exists a unique propensity to save

on all kind of income, i.e. wages and profits. This point is inconsistent with a previous assumption (p. 93) about the existence of a corporate retention ratio, which is nothing else than a firms' propensity to save on profits. So, there exists a distinction between wages and profits which is in contradiction with the assumption of a unique propensity to save  $s$ .

Second, Jarsulic (1990), but also Arena and Raybaut (1998, 2001), presents a puzzling equation for the evolution of debt. This specification has two implications : it does not respect the normal accounting method and contradicts the chronological logic of decisions. Indeed, it assumes that firms determine retained earnings (and the payment of dividends) without paying back financial charges  $rD$  :

$$\dot{D} = I - (s_f P - rD) \quad \dot{D} = \gamma(s_f P - rD)$$

with  $D$  the level of debt,  $r$  the rate of interest,  $I$  investment,  $P$  the flow of profits,  $\gamma$  a positive coefficients and  $0 < s_f < 1$  the propensity to save on profits. We can not make any sense of such an assumption. Indeed, we think a correct proposition would be to state that investment which is not financed by retained earnings,  $s_f(P - rD)$ , must be financed by issuing debt, so :<sup>1</sup>

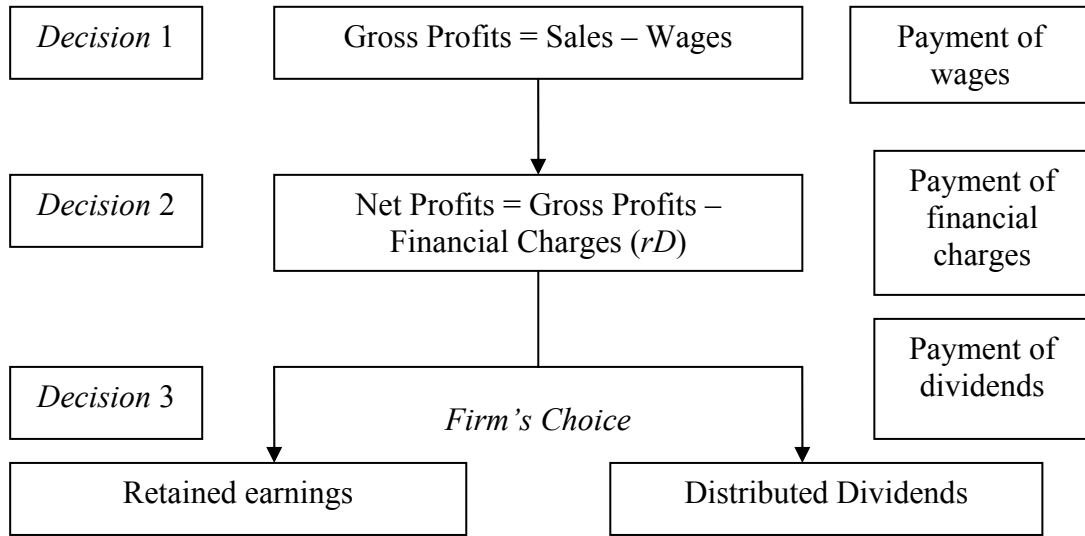
$$\dot{D} = I - s_f(P - rD)$$

First, firms pay back financial charges and, second, they apply a retention ratio on remaining profits (cf. table 1 for the standard chronology of decisions). Consequently, the next section proposes a simple model of accumulation with debt which takes our two remarks into account. Besides, it represents a valuable introduction to more sophisticated minskyan models since it is of simple use and involves easy computations.

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<sup>1</sup> Cf., for example, Asada (2001, 2002).

Table 1 : Chronology of firms' decisions



### 3. A model of accumulation with debt

We consider a closed economy, in which the price level and the rate of interest are constant. We also assume that workers do not save, so their consumption expenditure is equal to their wage income :  $C_W = W$  (cf. the balance sheets in the appendix). Moreover, there is no issue of new shares. So, our saving function is :

$$S = s_f(\Pi - rD) + s_c[(1 - s_f)(\Pi - rD) + rD] \quad (1)$$

The rate of internal retention on net profits by firms is  $s_f$ ,  $s_c$  is the capitalists' propensity to save on dividends and interest received. Assuming equilibrium on the goods market and using equation (1), we define net cash flows as :

$$s_f(\pi - rd) = s_f \frac{g - s_c rd}{s_F} \quad (2)$$

where  $\pi = P/K$  is the profit rate;  $g = I/K$  the rate of accumulation;  $d = D/K$  the debt to capital ratio and  $1 > s_F = s_f + s_c (1 - s_f) > s_f, s_c > 0$ . As for the evolution of the rate of accumulation, we use a standard adjustment equation :

$$\dot{g} = \delta(g^d - g) \quad \delta > 0 \quad (3)$$

The rate of accumulation moves according to the difference between the desired rate arising from firms  $g^d$  and the effective rate. Everything else being constant, whenever the desired rate of accumulation is above the current rate, this drives up the actual rate. To keep things simple, we assume the desired accumulation depends on net retained profits :  $g^d = \gamma s_f (\pi - rd) + b$ ,  $\gamma$  is a coefficient relating the importance of net cash flows for investment decisions and  $b$  a positive parameter. Moreover, each important variable ( $s_f, s_c, r, d$  and  $\pi$ ) has now a role to play. Substituting (2) into  $g^d$  and the resulting expression into (3) gives :

$$\dot{g} = \delta \left[ \gamma s_f (g - s_c rd) / s_F + b - g \right] \quad (4)$$

Henceforth, we can proceed to the analysis of the debt ratio, differentiating  $d$  with respect to time gives :  $\dot{d} = \dot{D} / K - dg$ . We specify the evolution of debt according to remarks of section 1. In this specification, we know that investment which is not financed by net profits must be financed by issuing debt :  $\dot{D} = I - s_f (P - rD)$ . After some simple substitutions, we obtain :

$$\dot{d} = g - (s_f / s_F)(g - s_c rd) - dg \quad (5)$$

The behavior of system (4)-(5), despite its non-linear form, can be found by the construction of a phase diagram. In equilibrium, isoclines are :  $d(\dot{g} = 0) = [(\gamma s_f - s_F)g + bs_F] / \gamma s_f s_c r$  and  $d(\dot{d} = 0) = (s_F - s_f)g / (s_F g - s_f s_c r)$ , with the following forms :

$$\partial d / \partial g(\dot{g} = 0) = \frac{\gamma s_f - s_F}{\gamma s_f s_c r} < 0$$

$$\partial d / \partial g(\dot{d} = 0) = -\frac{(s_F - s_f)s_f s_c}{(s_F g - s_f s_c r)^2} < 0 \quad \partial^2 d / \partial g^2(\dot{d} = 0) = \frac{2(s_F - s_f)s_f^2 s_c}{(s_F g - s_f s_c r)^3} > 0$$

with  $\gamma s_f - s_F < 0$  for plausible value of coefficients and  $s_F g - s_f s_c r > 0$  to assure the existence of debt. The phase diagram is represented in figure 1; it shows the coexistence of two equilibrium points. These results can also be found algebraically by studying the Jacobian matrix associated with system (4)-(5) :

$$J = \begin{bmatrix} \delta[(\gamma s_f - s_F)/s_F] & -\delta \gamma s_f s_c r \\ 1 - (s_f/s_F) - d^* & (s_f s_c r - s_F g^*)/s_F \end{bmatrix} = \begin{bmatrix} - & - \\ -/+ & - \end{bmatrix}$$

Assuming  $a_{21} > 0$ , it comes  $Det(J) > 0$ ; in this case equilibrium is stable since  $Tr(J) < 0$ . If  $a_{21} < 0$  we may have  $Det(J) > 0$  and equilibrium is stable again or  $Det(J) < 0$  and equilibrium is a saddle point.  $Det(J) < 0$  implies that the following inequality holds :

$$g^*(\gamma s_f - s_F) > s_f s_c r [\gamma(1 - d^*) - 1]$$

This condition remains true provided that the rate of accumulation is small and the debt ratio is great. Now, a further step is to know the impact of the rate of interest on equilibrium ( $E_1$ ,  $E_2$ ) by computing the partial derivatives of  $d$  with regard to  $r$  (cf. Appendix). Results are drawn in figure 2. This model shares a common specificity with Jarsulic (1990) : it shows that a rise in interest rate increases fragility since it moves the stable point  $E_1$  and the unstable point  $E_2$  closer together. A slight shock may now shift a stable economy towards an unstable one with a great debt ratio; this kind of dynamics is drawn in figure 2 with a positive shock on

the level of debt (point  $a$ ). Moreover, the previous results are obtained in the scope of a simple and coherent macrodynamic growth model.

Figure 1 : Phase diagram for accumulation and debt

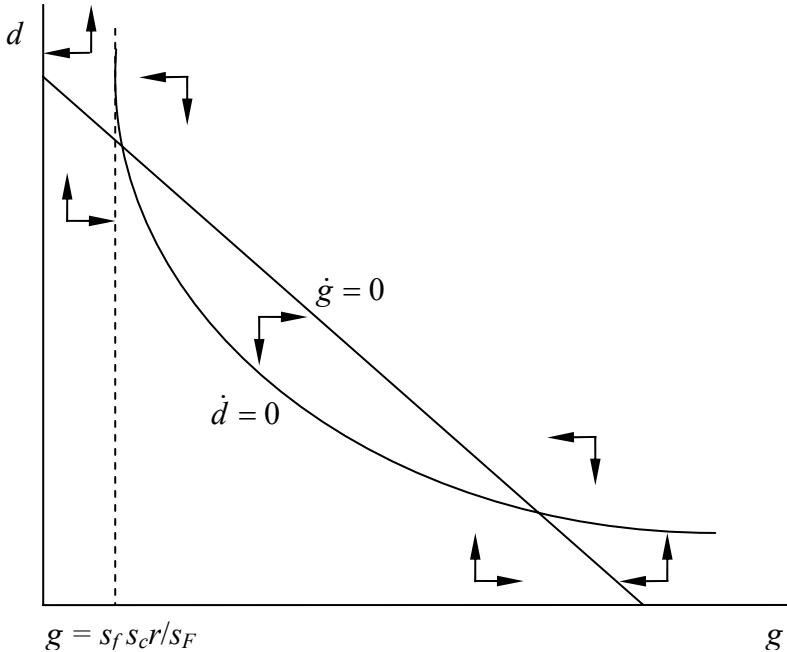
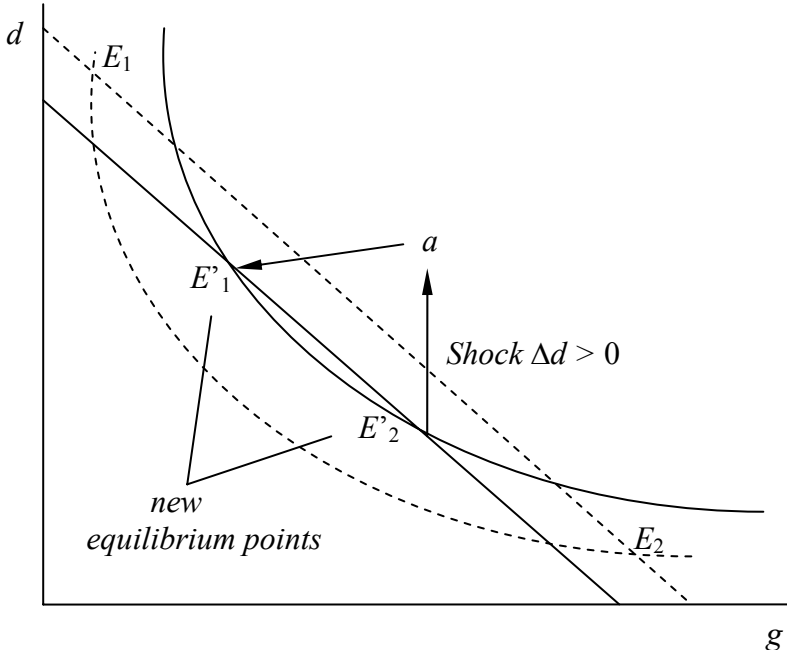


Figure 2 : A shock on  $d$  after a rise in the rate of interest



## 4. Conclusion

Our theoretical framework shows that it is possible to elaborate a tractable Minskyan model which represents a good introduction to more complex contributions. Above all, we have corrected some errors of other simple models, which made them inconsistent, by adopting a quasi-complete saving function. Moreover, this model keeps the destabilizing effect of interest rate on equilibrium which is an essential aspect of Minsky's conclusions (cf., for example, Minsky, 1977 and 1986).

Further extensions should be considered. We think that introducing explicitly the role of demand and the destabilizing effect of market shares would represent useful developments. However, these points are beyond the scope of the present paper.

## Appendix

$$\partial d / \partial r (\dot{g} = 0) = - \frac{(\gamma s_f - s_F)g + bs_F}{\gamma s_f s_c r^2} < 0$$

$$\partial d / \partial r (\dot{d} = 0) = - \frac{(s_F - s_f)g s_f s_c}{(s_F g - s_f s_c r)^2} > 0$$

We immediately see that  $\partial d / \partial r (\dot{g} = 0)$  is negative since the numerator  $(\gamma s_f - s_F)g + bs_F$  must be positive for the existence of a positive level of debt.

## The balance sheets of the flows

<i>Workers</i>	<i>Capitalists</i>
$C_W$	$W$
$C_C$	$(1 - s_f)(P - rD)$
$S_C$	$rD$
<i>Firms</i>	
$\dot{K}$	$s_f(P - rD)$
	$\Delta D$

$W$  represents wages totally spent in consumption –  $C_W$  – by workers.  $S_C = s_c Y_C$  is capitalists’ saving and  $C_C = c_c Y_C$  is capitalists’ consumption expenditure which come from their global income (interest payments and dividends) :  $Y_C = rD + (1 - s_f)(\Pi - rD)$ . By summing the balance sheets, we find Walras’ law :

$$I - s_f(\Pi - rD) - s_c[(1 - s_f)(\Pi - rD) + rD] + S_C - \Delta D = 0$$

$$(I - S) + (S_C - \Delta D) = 0$$

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