# Exploring the Currency board mechanics:
# A basic formal model

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**Abstract:** Over the last years the stability and efficiency of Currency Boards (CB) are in the centre of economic discussions. In this article we present a basic theoretical model, derived from the consumer behaviour, which allows exploring the possibilities of "CB resistance" to various types of shocks. Drawing inspiration from money in the utility function models build by Calvo (1981) and Blanchard and Fischer (1989) we examine two cases of inter-temporal maximisation of utility, where a consumer: (i) uses one asset (national currency (banknotes)) and (ii) is in the conditions of monetary substitution, i.e. uses two assets (national and foreign banknotes). CB stability in the conditions of self-fulfilling exchange rate crisis is also discussed.

**New JEL classification:** E42

**Keywords:** currency board, exchange rate and monetary regime

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## I. Introduction

Over the last years the stability and efficiency of Currency Boards (CB) are in the centre of economic discussions. This was contributed by both the CB crisis in Argentina and the disputes about the compatibility of CB in the Baltic countries and Bulgaria with the standard procedure for integration into the Euro area. As regards the accession countries the various viewpoints on CB operation logically lead to very different practical recommendations – a transition to a floating exchange rate and inflationary targeting, or to a bilateral or even to unilateral euroization.

Despite the various organisational forms and the greater flexibility, in principle, of modern CB, they manifest definite similarities with the orthodox CB, which in turn could serve as a kind of a benchmark[1]. The *orthodox* CB is a monetary regime, where the currency in circulation is fully covered by foreign reserves under a legally fixed exchange rate. A CB does not hold domestic assets in its balance sheet; no monetary policy is pursued and no lender-of-last resort function employed. The CB proponents rely on the operation of an automatic mechanism, under which the national money supply follows in a long run the balance of payments movement. They also rely on utmost flexibility of interest rates, prices, and salaries. It also assumes that the monetary behaviour of economic agents is (supposedly) stable and follows a definite long-term trajectory. Credibility is another major CB advantage, which is often stressed.

And while CB empirical studies, individually or as part of the other types of foreign exchange regimes, are increasing, (Ghosh and al., 2000, Hanke, 2002, Freytag, 2002, Vinhas de Souza, 2002), CB theoretical models are still rare, (for a exception see Chang and Velasco, 2001, Morin, 2002). Usually, the theoretical explanations of CB functioning and stability are explored in three directions: (i) as a similarity with the monetary approach to the balance of payments dynamics, hence with the gold standard (Hanke and Schuler, 1994)[2], (ii) as an extreme form of a fixed exchange rate regime (Tsang and Ma, 2002) and (iii) as a special form of *ex ante* credibility (Ho, 2001, Carlson and Valev (2001)). With a few exceptions the

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[1] On deviations from modern CB from their orthodox form see Nenovsky and al. (2002), Nenovsky and Hristov (2002).
[2] See the model on gold standard crises proposed by Dornbusch and Frenkel (1984).

literature lacks formal models of CB, which could allow for the simulation of various types of shocks.

In this article we present a basic *formal* model, derived from the consumer behaviour, which allows exploring the possibilities of “CB resistance” to various types of shocks. Drawing inspiration from the MIU models of Calvo (1981) and Blanchard and Fischer (1989) we examine two cases of inter-temporal maximisation of utility, where a consumer: (i) uses one asset (national currency (banknotes)) and (ii) is in the conditions of monetary substitution, i.e. uses two assets (national and foreign banknotes). The model is as abstract as possible – with only CB and consumers involved. No financial intermediation (the banking system) is included. In that particular case reserve money coincide with money supply, and the balance of payments - with the current account. Under this extremely abstract model shocks are reduced to: (i) shocks on income – exogenous and (ii) shocks resulting from consumer behaviour (shifts in consumer preferences, shift in money demand, or shift in the structure of money portfolio – national or foreign currency). The depth of abstraction allows analysing CB stability in its primary form – as a “CB – consumer (household)” relationship. Subsequently, this model could gradually be brought closer to reality by including money multiplier, the banking system (plus asymmetric information), capital and interest rate movements.

The structure of the article follows the logic underlying the construction of the model. The first chapter includes a basis model of maximised consumption in the environment of a single asset and income shock simulation. The second chapter gives a broader analysis by including currency substitution and simulation of different types of shocks. Part three discusses the CB stability in the conditions of self-fulfilling exchange rate crisis. Finally, in addition to summarising the results of the model, we also point out its limitations, as well as the directions of its further sophistication and improvement.

## II. Benchmark: an economy with local money as the only asset

We assume that the economy is small and produces a tradable good, so that the law of one price applies. Production ($y$) is at its (exogenous) full employment level. The only form of money considered is banknotes, which are printed only by the central bank.

Only residents hold local currency. Consumption ($c$) and real money demand ($m$) are derived from the behaviour of a representative household:

$$\text{Max } \int_0^{\infty} u(c_t, m_t)\, e^{-dt} dt \qquad (1)$$

$$\text{s.t.: } \dot{m}_t = y - c_t - p_t m_t + g_t \qquad \text{and } m_t \geq 0. \qquad (2)$$

where $m = M/P$ denote real cash balances, $p = \dot{P}/P$ is the rate of inflation, $g$ a transfer received from the government, $d > 0$ is the rate of time preference and $t$ is the time index. If we assume that the utility function is separable ($u_{cm} = 0$), and specifically $u(c_t, m_t) = \ln c_t + h \ln m_t$, the first order conditions yield:

$$\dot{c} = \frac{1}{u_{cc}}\left(-u_m + \left(p_t + d\right) u_c\right) = \frac{h c_t^2}{m_t} - \left(p_t + d\right) c_t \qquad (3)$$

$$\lim_{t \to \infty} u_c m_t e^{-dt} = 0 \qquad \text{(transversality condition)}$$

We complete the model with the following equilibrium conditions and assumptions:

- Goods market equilibrium: $y = c_t + x_t$ (4)
- Purchasing power parity: $P_t = S_t P_t^*$ (5)
- Foreign exchange market: $\dot{R}_t = P_t^* x_t$ (6)
- Central bank balance sheet: $M_t = V_t + D_t$ (7)

  where : $V_t = \int_0^t S_q \dot{R}_q dq + S_0 R_0$ (8)
- Government's budget constraint: $g_t = \dot{D}_t / P_t$ (9)

where $x_t$ denotes the trade surplus (net exports), $S_t$ is the local currency price of the foreign currency, $P_t^*$ is the foreign currency price level that increases at rate $p^*$. $R_t$ is the stock of foreign currency reserves held by the central bank: equation (6) shows that reserves are accumulated whenever the central bank intervenes in the foreign exchange market to compensate for any current account surplus. $V_t$ denotes the local currency value of foreign reserves in the central bank's balance sheet: equation (8) shows that reserves are recorded at their purchase value. $D_t$ is the stock (non interest bearing) public debt held by the central bank. Equation (7) shows the two sources of money creation/destruction: foreign exchange reserve management and government finance (seigniorage).

Since there is only one form of money in the model, namely banknotes printed by the central bank, there is a close relationship between monetary and exchange-rate regimes, which can even be considered equivalent. Three exchange rate regimes are considered: (i) flexible, (ii) fixed and (iii) currency board. We assume that the small country chooses one of these regimes on its own, outside any kind of international agreement, especially for the latter two, either multilateral (such as the Bretton Woods agreement or the European Monetary System) or bilateral (with the country issuing the currency chosen as reserve currency).

### 2.1- The flexible exchange rate regime

The flexible exchange rate regime fulfils two conditions.

- The central bank does not intervene on the foreign exchange market:

$$\dot{R}_t = 0 \tag{10}$$

  Foreign currency reserves are constant at some level $\hat{R}$, and their local currency value in the central bank's accounts are constant at $\hat{V} \equiv S_0 \hat{R}$.

- The money supply is exogenous (monetary financing of government transfers is possible): $\dot{M}_t / P_t = \dot{D}_t / P_t = g_t$.

  We will assume a constant money growth at rate $\dot{M}_t / M_t = m \geq 0$ which implies, $g_t = m\, m_t$

The flexible exchange rate regime is in principle compatible with nominal exchange rate bubbles. Because of the specific form of the utility function we assume, such phenomena are incompatible with optimisation, and therefore ruled out (see Blanchard and Fischer, 1989 p. 243).

Under **flexible exchange rates**, the model is summarized with the following set of equations:

$$\frac{\dot{c}_t}{c_t} = \frac{h c_t}{m_t} - \left(p_t + d\right) \qquad (11)$$

$$\dot{m}_t = y - c_t - p_t m_t + g_t \qquad (12)$$

$$y = c_t \qquad (13)$$

$$g_t = m\, m_t \qquad (14)$$

The endogenous variables are *c*, *m*, *p*, *g*. The exogenous variables are *y*, *m*. Equation (13) combines the equilibrium conditions on the goods market and the foreign exchange market with $\dot{R}_t$ = 0. It implies a stationary consumption. Plugging it into equation (11) yields:

$$p_t = \frac{h\, y}{m_t} - d \qquad (15)$$

Substitute (14) and (15) into (12) to get:

$$\dot{m}_t = (m + d)m_t - h\, y \qquad (16)$$

The transversality condition requires that *m* does not diverge. According to (16), it must be constant:

$$m = \frac{h\, y}{m + d}$$

Hence, from (5): $p = m$.

With a flexible exchange rate, the model exhibits "degenerate" dynamics, in the sense that nominal and real prices and consumption jump instantaneously to their equilibrium level.

## 2.2- The fixed exchange rate regime

The fixed exchange rate regime meets the following three conditions.

- The nominal exchange rate is constant: $S_t = \bar{S}$ and $p_t = p_t^*$;
- The central bank operates on the foreign exchange market to compensate for excess foreign currency demand, $\dot{R}_t = P_t^* x_t$, if its reserves are larger than a floor level equal to $\hat{R}$, which we call the exhaustion level. In other words, the fixed exchange rate regime incorporates an escape clause.

- There are no *a priori* restrictions on the monetary financing of government transfers.

In a **fixed exchange rate** regime, we have:

$$\frac{\dot{c}_t}{c_t} = \frac{h c_t}{m_t} - (d+p^*) \qquad (17)$$

$$\dot{m}_t = y - c_t - p^* m_t + g_t \qquad (18)$$

$$\dot{R}_t = \bar{S} P_t (y - c_t) \qquad (19)$$

$$M_t = \bar{S} R_t + D_t \qquad (20)$$

$$g_t = \dot{D}_t / P_t \qquad (21)$$

where the constant nominal exchange rate condition, $p_t = p^*$, has been taken into account.

Equation (20) is the central bank's balance sheet, using (8) and assuming $S_0 = \bar{S}$. The endogenous variables are *c, m, g, R, D*. It is important to note that nominal reserves, *R*, are a state variable, and should be constant in equilibrium. The exogenous variables are $y$, $p^*$, $\bar{S}$. It should be noted that equations (19), (20) and (21) imply equation (18): as such, the system is under-determined. This reflects the degree of freedom that exists in monetary policy outside steady state, namely the possibility of temporarily sterilizing balance of payment disequilibria, i.e. offsetting the effect of a foreign currency reserve change on the supply of money.

The equilibrium ($\dot{c}_t = \dot{m}_t = \dot{R}_t = 0$) is characterized by:

$$c^\# = y \qquad (22)$$

$$m^\# = \frac{h}{p^* + d}\, y \qquad (23)$$

$$g^\# = p^* m^\# = \frac{p^* h}{p^* + d}\, y \qquad (24)$$

In equilibrium, seigniorage is limited by the foreign rate of inflation: *g* must be equal to $p^* m$ in order for reserves to be constant. According to the household's budget constraint, in steady state, i.e. with constant real cash balances, a larger amount of government transfers would imply higher consumption, a trade deficit, continuously decreasing reserves: reserves would eventually become exhausted (note that with only one asset in the model, no early speculative attack of the first generation type of balance of payment crisis can happen). Contrary to a fixed exchange rate regime, a currency board regime institutionally forbids monetary financing of government transfers, and so resolves the incompatibility between "high" seigniorage and a constant nominal exchange rate, and prevents any balance of payment crisis due to inconsistent monetary policy. Under the previously stated conditions, the fixed exchange rate and currency board regimes differ only in this respect. Put differently, a currency board is a "guaranteed independent" central bank.

Outside steady state, the government in a fixed exchange rate regime could pursue a "constant reserves" strategy, that is use transfers $g$ to keep consumption equal to output, so as to avoid any trade surplus or deficit. The "equilibrium" described above in equations (22) to (24) would apply at any time.

### 2.3- The currency board regime

The currency board regime puts more institutional restraint on the money creation process than the fixed exchange rate regime. It has the following properties:

- The nominal exchange rate is constant, fixed by law: $S_t = \bar{S}$ and $p_t = p^*$;
- The central bank intervenes on the foreign exchange market. In fact, the intervention takes place in an automatic way under a currency board. We maintain the assumption of an escape clause i.e. of a positive exhaustion level of reserves $\hat{R}$, and we discuss it later: $\dot{R}_t = P_t^* x_t$, if $R_t \geq \hat{R}$;
- The money stock is fully backed by foreign currency reserves, $M_t = \bar{S} R_t$, there is no monetary financing of government transfers: $g_t = 0$ and $\dot{D}_t / P_t = 0$.

In a **currency board** regime, the economy evolves according to:

$$\frac{\dot{c}_t}{c_t} = \frac{h c_t}{m_t} - (d + p^*) \qquad (25)$$

$$\dot{m}_t = y - c_t - p^* m_t \qquad (26)$$

$$\dot{R}_t = \bar{S}\, P_t(y - c_t) \tag{27}$$

$$m_t = \bar{S}\, R_t/P_t \tag{28}$$

where the properties stated above have been taken into account. The endogenous variables are $c$, $m$, $R$. Equations (27) and (28) imply equation (26). The exogenous variables are $y$, $p^*$, $\bar{S}$ . The steady state equilibrium is given by:

$$c^{**} = \frac{p^* + d}{(1+h)p^* + d}\, y \tag{29}$$

$$m^{**} = \frac{h}{(1+h)p^* + d}\, y \tag{30}$$

In steady state, constant real cash balances imply that the nominal quantity of money increases at the same rate as the rate of inflation. So do foreign exchange reserves.

**2.4- Comparative statics**

When comparing steady states in the three regimes, the following points are worth noting:

Firstly, the steady state under flexible exchange rates is independent from foreign inflation, whereas it is not the case under the currency board and the fixed exchange rate regimes. This is not surprising since the model is dichotomous, and a nominal variable, the exchange rate is fixed in the latter regimes.

Secondly, steady state consumption is at the same level in the fixed and flexible exchange rate regimes. If $p^*$ is strictly positive, consumption is lower in the currency board regime. Assuming $m = p^* > 0$ in the flexible exchange rate regime, for comparison purposes, steady state real balances are at the same level in the fixed and flexible exchange rate regimes, and at a lower level in the currency board regime.

Hence the following proposition:

*When steady state inflation performance is strictly positive and the same in the three regimes, steady state household utility is lower in the currency board regime.*

This proposition results from key differences between the fixed and flexible exchange rate regimes on the one hand and the currency board on the other hand, which do not appear when foreign inflation is set equal to zero, as is often done (see table 1): the central bank "produces" inflation by collecting seigniorage in the fixed or flexible exchange rate regime ($g = p^{*}m$), and repays it lump sum to the representative household, whereas the currency board does not ($g = 0$).

| Table 1: seigniorage and inflation tax across exchange rate regimes | | |
|---|---|---|
| *Exchange rate regime* | *Seigniorage repaid to households* | *Inflation tax paid to The rest of the world* |
| *Flexible* | Yes | No |
| *Fixed* | Yes | Yes |
| *Currency board* | No | Yes |

Moreover, the fixed exchange rate and the currency board regimes compel the small country to pay an inflation tax to the rest of the world. It is compensated for by seigniorage repaid to the representative household in the fixed exchange rate regime, but not in the currency board regime.

It should be noted that this proposition is counterintuitive from the traditional perspective, according to which the choice of exchange rate regime is irrelevant for efficiency (see Helpman, 1981).

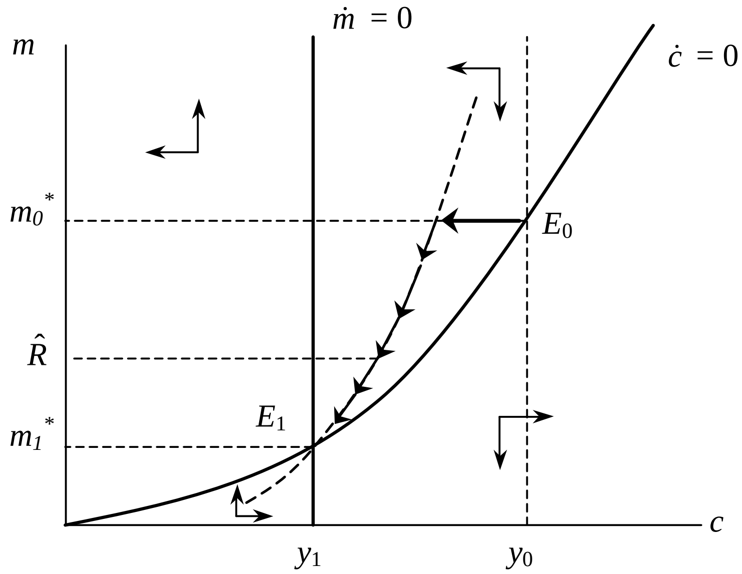
Thirdly, unlike the central bank in the fixed exchange rate regime, the currency board cannot support any nominal exchange rate. Assuming the economy is in steady state in a fixed exchange rate or a currency board regime, its foreign currency reserves are determined as shown in table 2.

The nominal exchange rate is supposed to be supportable if the equilibrium level of foreign currency reserves (i.e. the amount of foreign currency banknotes held by the central bank) is higher than the exhaustion level $\hat{R}$.

In the fixed exchange rate regime, the equilibrium level of reserves depends on the amount of credit to the government $D$, as shown in table 2. Therefore, any nominal exchange rate can be supported by the central bank, by adjusting $D$ so as to keep $R$ above $\hat{R}$. In the currency board regime, the equilibrium level of reserves is exogenously determined, so that it is out of the currency board's control.

| Table 2: foreign currency reserves determination | | |
|---|---|---|
| | *fixed exchange rate* | *currency board* |
| *Domestic price level:* | $P_t = \bar{S}\ P_t^*$ | $P_t = \bar{S}\ P_t^*$ |
| *Equilibrium real cash balances:* | $m^\# = \dfrac{h}{p^* + d}\ y$ | $m^{**} = \dfrac{h}{(1+h)p^* + d}\ y$ |
| *Central bank balance sheet:* | $M_t = \bar{S}\ R_t + D_t$ | $M_t = \bar{S}\ R_t$ |
| *Foreign currency reserves:* | $R_t = P_t^*\ m^\# - D_t/\bar{S}$ | $R_t = P_t^*\ m^{**}$ |

For instance, a permanent negative output shock induces fall of equilibrium real cash balances. The new equilibrium level should be reached by the country losing foreign reserves, but the exhaustion level could well be reached beforehand, depending on the size of the shock. Figure 1 plots the adjustment path, assuming foreign inflation is nil ($p^* = 0$). On impact, consumption jumps so as to put the economy on the saddle path, but stays above its new equilibrium level ($y_1$). There is a trade deficit and the country loses reserves.




Figure°1: Exogenous fall in $y$ in a currency board regime (with $p^* = 0$).

Hence the proposition:

> *Unlike the central bank in the fixed exchange rate regime, the currency board cannot support any nominal exchange rate.*

The moral of this very simple model is that, since a currency board is a very rigid system, and it does not have seigniorage as an additional source of money creation, compared to the fixed exchange rate regime, it has two drawbacks: a welfare cost in equilibrium in case of a positive foreign inflation, and the necessity to choose carefully the level of the nominal exchange rate, because of a poorer ability to support any given exchange rate.

However, this very simple model is not suited to the analysis of a major argument in favour of currency boards: credibility. In order for credibility problems to materialize, there must be another asset that households can use to as an alternative to local money in case of anticipated inflation or currency depreciation. This is the subject of the next section of the paper, where we introduce currency substitutability and substitution.

## III. A currency board in a economy with currency substitution

### 3.1. Incorporating currency substitution in the model

Let us now consider the case of a small open economy in which two monies are used, namely the local currency and a foreign currency. In such an economy, speculative attacks on a fixed exchange rate regime or a currency board are feasible (Krugman, 1979) and credibility problems can materialize, in the form of anticipated regime changes. We study the conditions under which a currency board can resist speculative attacks. We keep the model simple, and introduce only one other asset in households' portfolio, namely foreign currency banknotes. Two kinds of crisis are analysed, triggered either by an exogenous negative shock on output, or by an expected regime switch.

The representative household's programme is the following:

$$\text{Max } \int_0^{\infty} U(c_t, m_t, f_t) e^{-dt} dt$$

$$\text{s.c. : } \quad \dot{a}_t = y - c_t - p_t m_t + g_t \qquad (31)$$

$$a_t = m_t + f_t \qquad (32)$$

There are two assets, local currency (*m*) and foreign currency (*f*).
In order to simplify the analysis, we specify that the utility function is as follows:

$$U(c_t, m_t, f_t) = u(c_t) + h(m_t, f_t) \qquad (33)$$

with: $\quad u(c_t) = ln\ c_t \qquad (34)$

and $\quad h(m_t, f_t) = h\ ln\ m_t + (1 - h)\ ln\ f_t \qquad (35)$

The first order conditions give consumption and money demand functions, with variables *c*, *m* et *f* verifying:

$$\dot{c} = \frac{1}{u_{cc}}\left(-h_f + d u_c\right) \qquad (36)$$

$$h_m - h_f = p_t\ u_c \qquad (37)$$

$$\dot{m}_t + \dot{f}_t = y - c_t - p_t m_t + g_t \qquad (38)$$

The macroeconomic model is completed with the following assumptions and conditions:

- Goods market equilibrium: $\quad y_t = c_t + x_t \qquad (4)$
- Purchasing power parity (PPP): $\quad P_t = s_t P^*_t \qquad (5)$
- Foreign exchange market (flow) equilibrium: $\quad \dot{f}_t + \dot{R}_t = P^*_t x_t \qquad (39)$
- Central bank balance sheet: $\quad M_t = V_t + D_t \qquad (7)$

  where : $\quad V_t = \int_0^t S_q \dot{R}_q dq + S_0 R_0 \qquad (8)$
- Government's budget constraint: $\quad g_t = \dot{D}_t / P_t \qquad (9)$

We focus in this section on the ability of the currency board to resist exogenous shocks, namely a shock on output and a shock on expectations. Therefore, we are more interested in the dynamics of the system rather than its steady state, and we introduce further simplifying assumptions.

- there are no public transfers: $g_t = \dot{D}_t / P_t = 0$. This rules out balance of payment crises due to inconsistent policies (such as providing the government with too high a level of seigniorage in a fixed exchange rate regime).
- foreign inflation is nil. This eliminates the equilibrium welfare bias of the currency board that we pointed out in the previous section.

We study to types of shocks that may produce a balance of payments crisis: (i) a negative shock on output and (ii) an anticipated regime switch (treated in part IV).

### 3.2. Adjustment under flexible exchange rates after a negative shock on output:

Taking into account the specific form of the utility function, and assuming a constant quantity of money ($M_t = \bar{M}$), $c$, $m$, $f$ and $P$ evolve according to:

$$P = \bar{M}/m \qquad \text{hence: } p = -\frac{\dot{m}}{m} \tag{41}$$

$$\dot{c} = -d\,c + (1-h)\frac{c^2}{f} \tag{42}$$

$$\dot{f} = y - c \tag{43}$$

$$\frac{h}{m} - \frac{1-h}{f} = \frac{p}{c} \tag{44}$$

Equations (41) and (44) yield: $\frac{\dot{m}}{m} = (1-h)\frac{c}{f} - h\frac{c}{m}$ (45)

The stationary equilibrium is given by :

$$c^* = y \tag{46}$$

$$f^* = (1-h)y/d \tag{47}$$

$$m^* = [h/(1-h)]\, f^* = h\, y/d \tag{48}$$

$$P^* = \bar{M}/m^* \tag{49}$$

A decrease in $y$ leads to a decrease in the equilibrium levels of consumption and real money balances. The degree of currency substitution ($f/a$) is determined only by preferences in equilibrium, and unaffected by the output shock. The local currency depreciates (the price level increases).

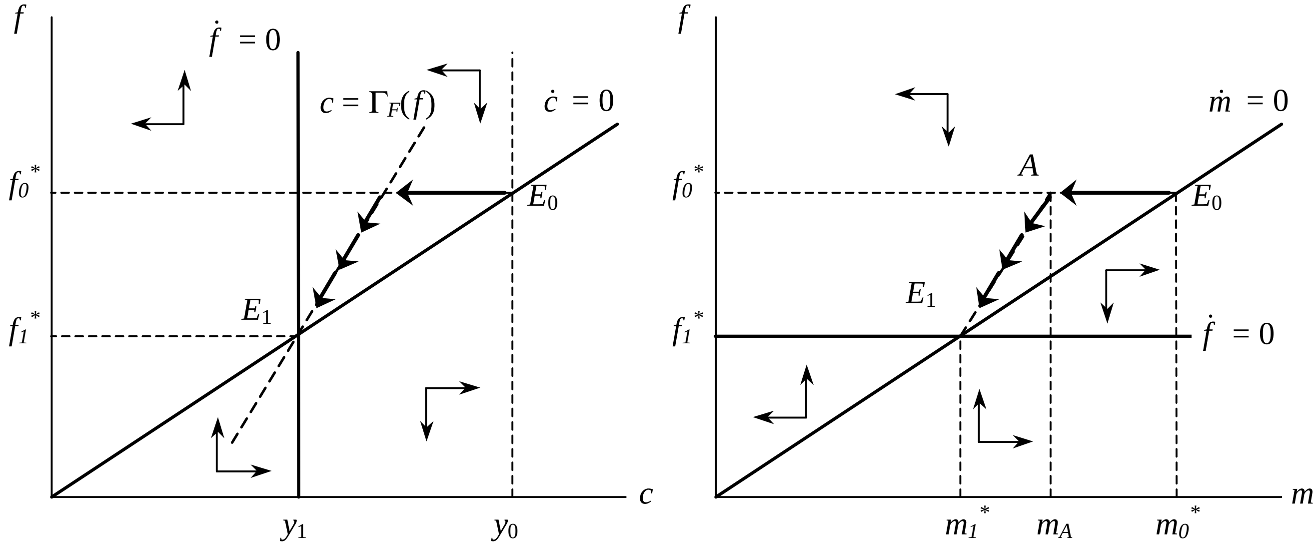



Figure 2a: paths of $f$ and $c$

Figure 2b: paths of $f$ and $m$

**Figure 2 : exogenous decrease in *y* under flexible exchange rates with currency substitution**

The joint evolution of $c$ and $f$ is described by an autonomous system. It is represented on figure 2a.

The convergent path implies a relationship between $c$ and $f$, that we note:

$$c = \Gamma_F\left(f\right) \tag{50}$$

Linearizing $\Gamma_F()$ in the neighbourhood of the steady state gives:

$$(c - c^*) = k_F\,(f - f^*) \tag{51}$$

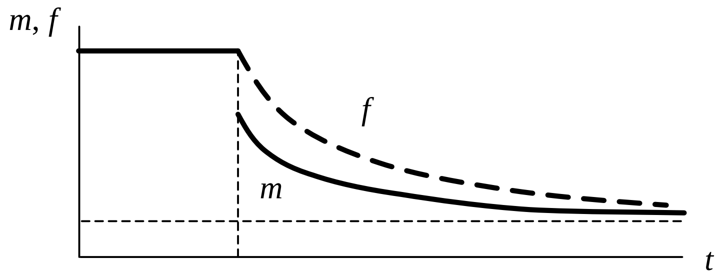
$$\text{where } k_F = \frac{d}{2}\left(\sqrt{\frac{5-h}{1-h}} - 1\right) > d \tag{52}$$

From (45) and (50), the dynamics of local currency real balances, $m$, is then given by:

$$\frac{\dot{m}}{m} = (1-h)\frac{\Gamma_F(f)}{f} - h\frac{\Gamma_F(f)}{m} \qquad (53)$$

The joint evolution of *m* and *f* is represented on figure 2b.

The time path of *m* and *f* is qualitatively illustrated in figure 3: the instant the sock happens, local currency depreciates, *m* jumps so as to converge towards steady state.




Figure°3 : time path of *m* and *f*

**3.3. A negative shock on output in a currency board regime:**

In a currency board regime, variables *c*, *m*, *f* move according to:

$$\dot{c} = -d\,c + (1-h)\frac{c^2}{f} \qquad (42)$$

$$\frac{h}{m} - \frac{1-h}{f} = 0 \qquad \text{hence: } m = \frac{h}{(1-h)} f \qquad (54)$$

$$\dot{m} + \dot{f} = y - c \qquad (55)$$

From (54) and (55): $$\dot{f} = (1-h)(y-c) \qquad (56)$$

and $$\dot{m} = h\,(y-c) \qquad (57)$$

Both types of banknotes are held in fixed proportions, given by preferences, at any instant. Moreover, and any time, households can swap foreign currency notes for home currency notes.

The steady state is the same as under flexible exchange rates (equations 46 to 48):

$c^* = y$

$f^* = (1-h)y/d$

$$m^{*} = [h/(1-h)]\, f^{*} = h\, y/d$$

Taking into account portfolio choices (*m* proportional to *f* at any time, equation 54), the joint evolution of *c* and *f* is given, as under flexible exchange rates, by an autonomous subsystem.

The relationship between *c* and *f* on the convergent path is noted:

$$c = \Gamma_{CB}(f). \qquad (58)$$

The evolution of the currency board system is illustrated in figure 4.

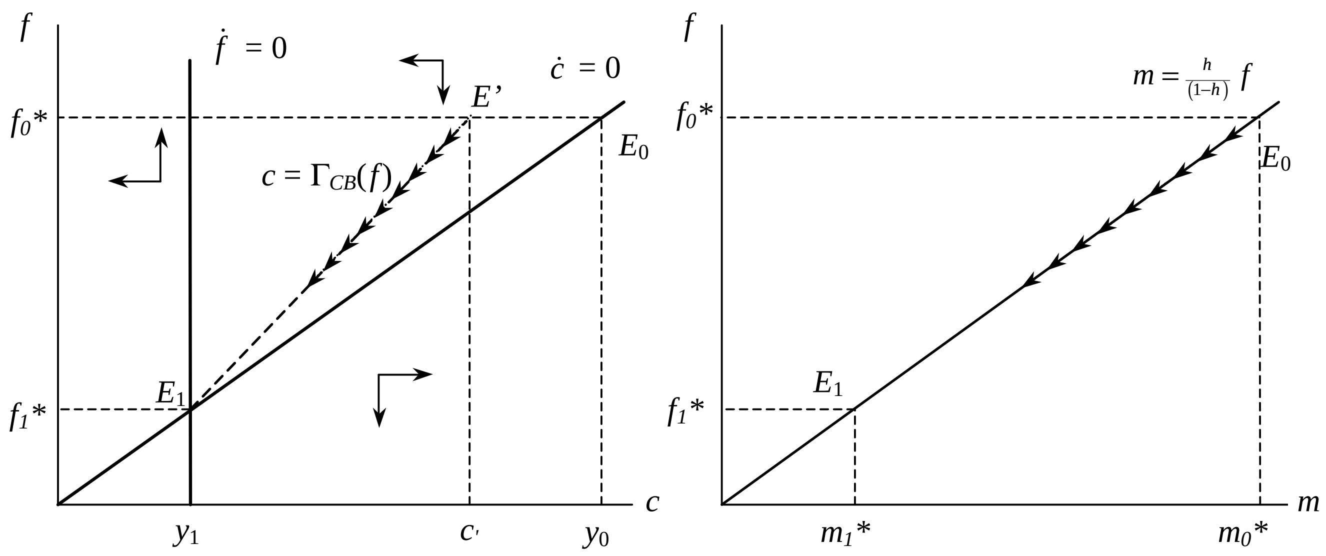



Figure 4a: paths of *f* and *c*

Figure 4b: paths of *f* and *m*

**Figure 4 : exogenous decrease in *y* under a currency board regime with currency substitution**

During the adjustment process, the currency board looses foreign reserves (*m* decreases) and, as in the no currency substitution case, reserve can be exhausted and reach their floor level $\hat{R}$. If need be, households can now choose between to assets, which makes a speculative attack possible, if a devaluation is expected.

Figure 5 illustrates the failure of the currency board. We assume that the economy is affected by a forceful shock, that requires abandoning the fixed exchange rate: the equilibrium level of local currency real cash balances ($m_1^*$) is lower than the floor level ($\hat{m}$) assumed to allow to defend the exchange rate.

Linearizing $\Gamma_{CB}()$ in the neighbourhood of the steady state gives:

$$(c - c^{*}) = k_{CB}\,(f - f^{*}), \quad (59)$$

$$\text{with } k_{CB} = \frac{2d}{(1-h)\left(1+\sqrt{5}\right)} > k_F \quad (60)$$

Just after the shock, the economy evolves according to the “currency board dynamics”: consumption falls (jump from $E_0$ to $E'$), cash balances in local and foreign money ($f$ and $m$) start diminishing (the economy moves along $E_0A$).

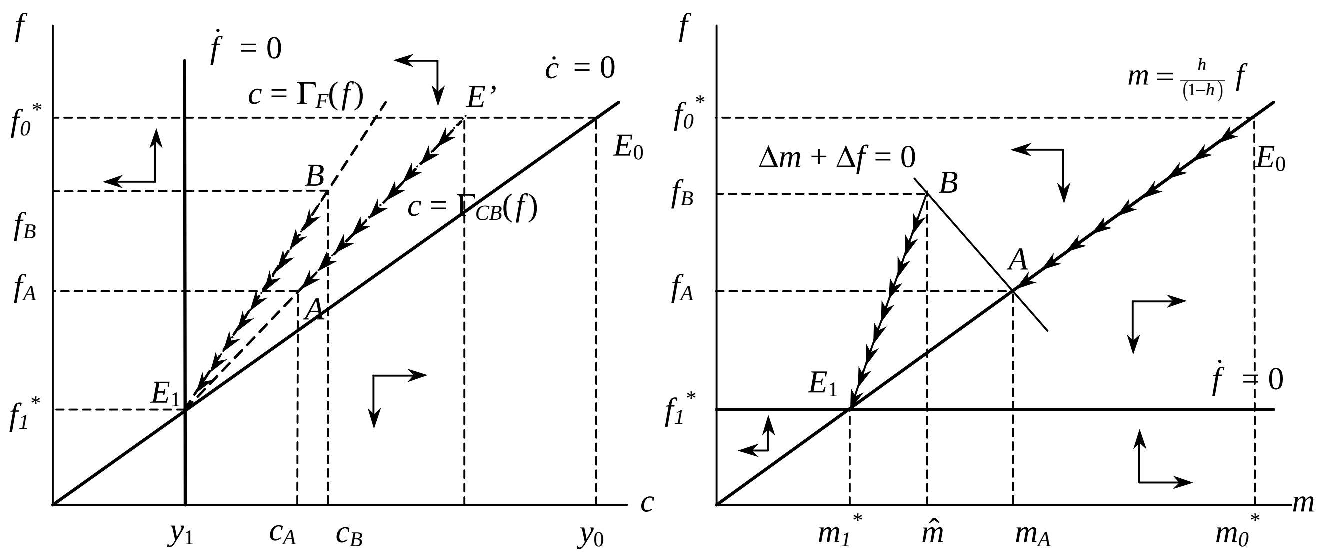



Figure 5a: paths of $f$ and $c$ Figure 5b : paths of $f$ and $m$

***Figure 5 : the currency board failure***

Since the move to the flexible exchange rate regime is perfectly anticipated, the exchange rate must not jump when it happens (Krugman,1979). Let us denote $t$ the time at which the attack on the currency board happens. In figure 5, the economy is at point $A$ at $t_{-}$ and at point $B$ at $t_{+}$. We have:

- $m_{t+} = \hat{m}$ (reserve exhaustion means their falling to $\hat{m}$ )
- $f_{t+} = f_{t-} + (m_{t-} - \hat{m})$ (reserves are transferred from the currency board’s assets to the household assets).

- The point with coordinates ($m_{t+}$, $f_{t+}$) is on a path that converges to the stationary equilibrium (as $BE_1$) on figure 5a.

The latter property determines the timing of the attack: it is the instant when reserves reach some level $m_{t-} = m_A$, the exhaustion of which increases household-held foreign currency balances to an amount $f_{t+}$ that puts the economy, described by ($m_{t+}$, $f_{t+}$), on a convergent path.

The timing of the attack is determined as follows.
The stationary equilibrium is the same before and after the speculative attack.
We note $g$, $m$ and $j$ - the deviations from steady state of $c$, $m$ and $f$ respectively. Trajectories are linearized around steady state.

Before the attack, we have $m = \frac{h}{1-h} j$ and $\dot{j} = -k_{CB} j$ , hence: $m_t = m_0 e^{-k_{CE} t}$ et $m_{t-} = m_0 e^{-k_{CB} t}$ . At the time of the attack: $j_{t+} - j_{t-} = m_{t-} - m_{t+}$ where $m_{t-} = \frac{h}{1-h} j_{t-}$ and $m_{t+} = \frac{h}{1-h} \frac{d}{d+k_F} j_{t+}$. So, $t$ verifies : $m_0 e^{-k_{CB} t} = h\left(1 + \frac{1-h}{h} \frac{k_F + d}{d}\right) m_{t+}$.

Hence, if $m_{t+} = \hat{m} - m^* > 0$: $t = \frac{1}{(1-h) k_{CB}} \left( \ln\left[ h + (1-h)\left(1 + \frac{k_F}{d}\right)\right] + \ln\left[ m_0 / m_{t+} \right] \right)$, or :

$$t = \frac{1}{(1-h) k_{CB}} \left( \ln\left[ h + (1-h)\left(1 + \frac{k_F}{d}\right)\right] - \ln\left[ 1 - (m_0 - \hat{m}) / (m_0 - m^*) \right] \right) \qquad (61)$$

Equation (61) shows that the more violent the shock sustained by the economy (the larger $m_0 - m^*$), or the smaller the initial reserves (the smaller $m_0 - \hat{m}$ ), the earlier the attack (the smaller $t$).

We show now that the speculative attack produces an instantaneous increase in consumption, as in figure 5a.

After the attack (for t>$t$): $\dot{m} = dm - \frac{hd}{1-h} j$ ; $\dot{j} = -k_F j$ ; $g = k_F j$ .

Solving for $m$ and $j$ yields:

$$j_t = e^{-k_F (t - t)} j_{t+}$$

$$m_t = \frac{h}{1-h}\frac{d}{d+k_F}e^{-k_F(t-t)}j_{t+} + e^{d(t-t)}\left(m_{t+} - \frac{h}{1-h}\frac{d}{d+k_F}j_{t+}\right)$$

The convergence of $m$ towards $m^*$, i.e. that of $m$ towards 0, implies: $m_{t+} = \frac{h}{1-h}\frac{d}{d+k_F}j_{t+}$. Hence, since $g = k_F j$ :

$$g_{t+} = k_F \frac{d+k_F}{d}\frac{1-h}{h}m_{t+} \qquad (62)$$

Before the attack (for t<$t$) : $m = \frac{h}{1-h}j$ ; $g = k_{CB}j$ .

Hence: $g_{t-} = k_{CB}\frac{1-h}{h}m_{t-}$ .

Since the speculative attacks implies: $j_{t+} - j_{t-} = m_{t-} - m_{t+}$, we finally have:

$$g_{t-} = k_{CB}(1-h)\left(1+\frac{k_F+d}{d}\frac{1-h}{h}\right)m_{t+} \qquad (63)$$

Comparing (62) and (63), and using the definition of $k_F$ in (52) and $k_{CB}$ in (60), it is easy to show that: $g_{t+} > g_{t-}$.

The obvious implication of currency substitution is that it makes speculative attacks possible, which are the sign of credibility problems for the exchange rate regime, either fixed or, as analysed here, currency board. Bearing in mind the previous section model, we can discuss whether the government should forbid the use of foreign currencies, in order to protect the currency board. Anyway, if it is not officially authorized, the foreign currency can always be used on the black market.

The exhaustion level of reserves ( $\hat{R}$ ) plays a crucial role in the analysis. It should be clear that it needs not be objective: it should be interpreted as “expected”, that is as the amount of foreign currency reserve below which private agents expect the government to give up defending the exchange rate.

Two implications can then be drawn for the currency board. First, the larger the external shocks sustained by the economy, the more precarious the currency board, and the more necessary the commitment of the currency board to defend the currency until reserves are really exhausted (the commitment to an official $\hat{R}$ equal to 0). Second, the morals of the case studied here can possibly be taken as an argument in favour of an external support of the currency board by international institutions in face of temporary shocks.

## IV. Can the Currency Board stand a “self-fulfilling” exchange rate crisis?

We now study the effects of a pure shock on expectations. At time 0, households expect that the currency board will be abandoned at some later date, and switched to a flexible exchange rate regime, with the money growth rate increasing from $\dot{M}/M = 0$ to $\dot{M}/M = m > 0$.

### 4.1- The dynamics of the flexible exchange rate regime with $m > 0$ :

We have the same system as equations (41) to (44) with equation (41) modified to take into account that $m > 0$:

$$p = m - \frac{\dot{m}}{m} \tag{41m}$$

$$\dot{c} = -d\,c + (1-h)\frac{c^2}{f} \tag{42}$$

$$\dot{f} = y - c \tag{43}$$

$$\frac{h}{m} - \frac{1-h}{f} = \frac{p}{c} \tag{44}$$

The joint evolution of $c$ and $f$, given by equations (42) and (43), is independent of $m$. The convergent trajectory is as in the previous section, $c = \Gamma_F(f)$ or, after linearizing around the steady state:

$$(c - c^*) = k_F\,(f - f^*) \tag{51}$$

Only the time path of $m$ changes from the previous scenario. We now have, instead of (45):

$$\frac{\dot{m}}{m} = m + (1-h)\frac{c}{f} - h\,\frac{c}{m} \tag{45m}$$

After substituting for $c = \Gamma_F(f)$, it can be written:

$$\dot{m} = \left[ m + (1-h)\frac{\Gamma_F(f)}{f} \right] m - h\,\Gamma_F(f) \tag{53m}$$

The stationary equilibrium is given by:

$$c^* = y \tag{46}$$

$$f^* = (1-h)y/d \tag{47}$$

$$m^* = h\,y/(m+d) \tag{48m}$$

$$p^* = m \tag{49m}$$

We then have:

$$\dot{m} = 0 \qquad \text{if and only if } m = \frac{h}{1-h+m\frac{f}{\Gamma_F(f)}} f < \frac{h}{(1-h)} f \;.$$

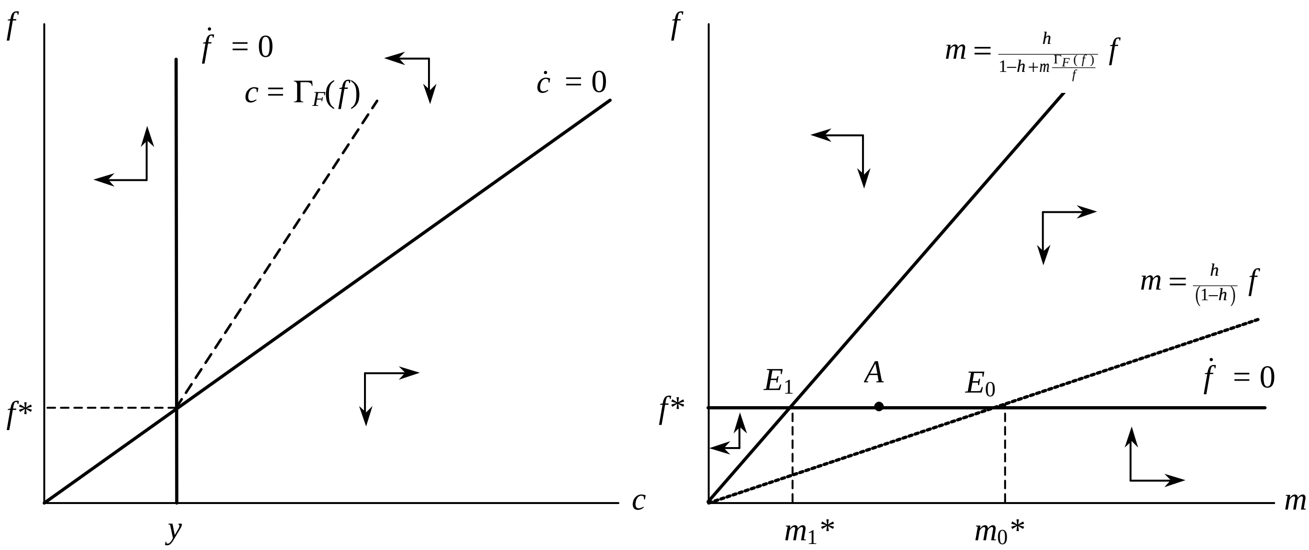



Figure 6a : paths of *f* and *c*

Figure 6b : paths of *f* and *m*

***Figure 6 : unexpected increase in m under flexible exchange rates***

An unexpected increase in *m* produces the following effects, illustrated in figure 6. Just after the shock, real local money balances, equal to $m_0^*$, are higher than their equilibrium level, and on a divergent path, since $\dot{m} = (m-p)m$ with $m > 0$ and $p = 0$, whereas as foreign currency real balances and consumption are at their (unchanged) equilibrium level. An instantaneous depreciation of the local currency happens (jump in *P*), that produces the instantaneous jump of *m* to its equilibrium level (given *M*): the economy jumps from point $E_0$ to point $E_1$.

If the increase in $m$ is anticipated at $t = 0$ for $t = T$, we have the following effects. The steady state values and the time paths of $c$ and $f$ are left unchanged: these variables stay at their equilibrium level. Only $m$ is affected. Its trajectory is given by:

$$\dot{m} = d\,(m - m_0 *) \text{ from } t = 0 \text{ to } t = T \text{ (since } \dot{M}/M = 0\text{);}$$

If we eliminate divergent paths (exchange rate bubbles), this trajectory leads to $m = m_1*$ at $t = T$. We then have:

$$m_0 = e^{-dT} m_1 * + \left(1 - e^{-dT}\right) m_0 * \equiv m_A$$

At time $t = 0$, the exchange rate jumps so that $m$ jumps to $m_A$. It keeps on increasing from $t = 0$ to $t = T$. On figure 6, the economy jumps from point $E_0$ to point $A$, at $t = 0$, and then diverges away from $E_0$ to reach point $E_1$ at time $t = T$.

## 4.2. The "self-fulfilling" crisis under a currency board regime

Assume now that the economy is at a stationary equilibrium under a currency board regime, and that, at $t$=0, households expect the end of the regime for date $t = T > 0$, and a switch at that time to a flexible exchange rate regime with $\dot{M}/M = \mathrm{m} > 0$.

The initial stationary equilibrium is described by: $c = c^*$ ; $f = f^*$ ; $m = m_0^*$.
The expected final equilibrium has only a different value for $m$ : $m_1^* < m_0^*$.

Assume that households expect the regime switch to exhaust foreign reserves, that is: $m_1^* < \hat{m}$. The transition has the following characteristics.

- The exchange rate must not jump at time $T$, since the transition is perfectly anticipated, so that the economy reaches, at time $T$, the convergent path that leads to the final equilibrium ($c = c^*$ ; $f = f^*$ ; $m = m_1^*$).
- As long as the currency board operates, households can swap local for foreign money at any time, but the optimum portfolio has constant shares of foreign and home currencies, $f^* = (1–h)y/d$ and $m^* = h\,y/d = m_0^*$. Therefore, the currency

swap must lead to the end of the currency board and the switch to a flexible exchange rate regime.

- Since we assume that reserves are expected to become exhausted, the currency board must "blow up" as soon as household form their expectations ($t$ = 0).
- The economy jumps instantaneously to a flexible exchange rate regime with $\dot{M}/M = 0$, until $t = T$. The nominal exchange rate possibly jumps at $t$ = 0.

The crisis is illustrated in figure 7.

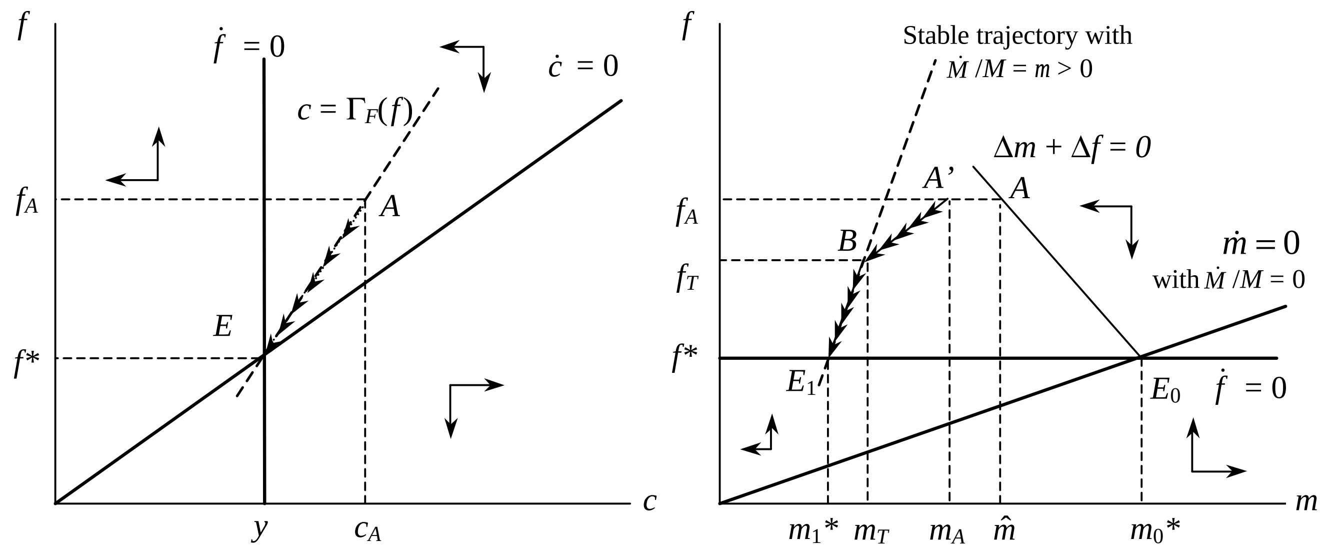



Figure 7a : paths of *f* and *c*

Figure 7b : paths of *f* and *m*

***Figure 7 : switch from a currency board to a flexible exchange rate regime with* m*>0.***

At time $t$=0, an attack exhausts reserve (jump from point $E_0$ to point *A* in figure 7b). Local currency is possibly devalued at the same time (jump from point *A* to point *A'*). Along *A'B*, the economy is in a flexible exchange rate regime with $\dot{M}/M = 0$. Real foreign cash balances are higher than desired, and consumers get rid of the excess balances by consuming more than current income, which produces a trade deficit. The local currency depreciates continuously, hence the continuous decrease in *m*, so as to keep the currency portfolio in compliance with the stable path in a flexible exchange rate regime with $\dot{M}/M = 0$.

Point *B* is reached at time $t = T$.

The currency board cannot defend the nominal exchange rate since, while consumers have too much foreign currency, they do not stand ready to exchange it for local currency with the board. At point *B*, $m$ is still higher than its equilibrium level. Rather than going to the currency board's counter, they go to the goods market to get rid of excess foreign currency cash.

Eventually, it does not mater whether or not the local money growth rate increases to $m > 0$ at time $t = T$. In this sense, it must be noted that the crisis is not strictly speaking self-fulfilling. The simple fact that it has been expected has caused the end of the currency board, and the depreciation of the local currency.

To get this result, we have assume a very unfavourable conjunction of expectations. Households are supposed to expect a large increase in the local money growth rate ($m$) after the regime switch at date $T$, so that equilibrium real cash balances fall below the reserve exhaustion level, $m_1^* < \hat{m}$. What could justify such expectations apart from sheer sunspots? A large public debt to be monetized or a deep public deficit to be financed by money creation (which are not explicitly modelled here), and some doubt about the credibility of the institutional arrangement (see McCallum 1995, 1996). There must also be a level of foreign currency reserves $\hat{m}$, not necessarily objective, as mentioned earlier, under which the currency board is expected to be no longer defendable.

The lesson for the currency board is that it does not bring credibility *per se*. It must be set up as part of a broader institutional reform that must be consistent as a whole.

## V. Conclusion and remarks

In this paper we presented an abstract formal MIU model of orthodox CB allowing us to examine the underlying resilience of this monetary regime to various types of shocks. Although simplified, the model (presented consecutively in two stages - with national money as the only asset, and through integration of money substitution) makes it possible to draw conclusions on the comparative efficiency of the flexible exchange rate, the fixed exchange rate and the CB regimes. For instance, within the one-asset model we found that when steady state inflation performance is strictly positive and the same in the three regimes, steady state household utility is lower in the currency board regime, and that it is necessary to choose very carefully the level of the nominal exchange rate in the currency board regime, because of a poorer ability to support any given exchange rate.

Integrating money substitution in the model adds a new source of instability in the utility function for - along with the possibilities for switch from consumption to national money and vice versa, two other pairs emerge: consumption-foreign money and foreign money - national money. Within the model with money substitution the reactions of the CB are presented, i.e. its resistance to both exogenous shocks (on income) and reversal in consumers' expectations about CB stability. The sheer expectation of exiting the CB and transition to some form of flexible exchange rate "dooms" the CB to failure. This theoretical case has direct practical implications for the Euro-accession countries. According to the standard procedure for euro zone membership, they must go through the ERM II, which entails exit of the CB[3]. In this line of reasoning, any "anticipated" CB exit, and without support from the ECB, could bring about a preliminary CB crisis and spontaneous euroization of these countries. This case could also be taken as an argument in favour of an external support for a CB by international institutions in face of temporary shocks.

In general, our model is a *positive* description of the dynamics of orthodox CBs. In our view, an important advantage of the model is the possibility to make different *normative* prescriptions for the operation of the CB on the basis of the results. Thus instability and volatility of the CB can be interpreted both in favour of transition to a flexible exchange rate and in favour of full dollarization or euroization, or as the need of external support to the CB (e.g. the ECB).

Our model has a number of fundamental and technical limitations and we are aware of them. While technical limitations can be overcome, the fundamental ones are rather a matter of choice. Therefore we will consider them in brief. *First*, integrating money in the utility and assuming $u_{cm} = 0$ has its shortcomings, which are known. In

[3] Although the European Commission pointed out that the CB is principally in compliance with ERM II, the question how this transformation will take place from the legal point of view is not answered, and there is no clear strategic plan different from the standard procedure.

general, the function of money as a medium of exchange is underestimated and the focus is placed on money as an asset. Alternative possibilities for base models integrating money can be the cash in advance model or the overlapping generation model, which also have their limitations (Lavigne and Pollin, 1997). *Second*, because our model does not include banks, it disregards an essential difference and compatibility between exchange rate regime and monetary regime (Schuler, 1999). Actually, the CB differs significantly from the fixed exchange rate regime because of the institutional characteristics of money supply.

Some of the simplifications of the model can be overcome by bringing it closer to reality. This can be done by integrating consecutively new elements into the CB model. The model can develop in the following directions: (i) differentiation between tradable and nontradable sectors, which would affect seriously inflation and real exchange rate dynamics, but not necessarily steady states, (ii) integrating capital flows into the model (i.e. to overcome balance of payments and current account overlapping) and hence the importance of interest rates, (iii) integrating the banking system and the money multiplier, i.e. lifting the limitation for reserve money and money stock overlapping. A new extension is possible within the banking system, taking into account the micro-economic behavior of the banking system and the whole range of problems relating to asymmetric information. These new segments in the model have dual function: on the one hand, they provide opportunity for balancing and overcoming the rigidity of the initial model, on the other hand they create possibilities for new sources of instability in the CB mechanism.

A further step, parallel to the theoretical sophistication, would be empirical testing and simulation of shocks in the model with empirical data. VAR models provide appropriate tools to examine mutual reactions to certain behavioural functions (e.g. (y/c), (c/m), (c/f), (m/f)), as well as their reaction to a number of exogenous shocks.

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