Trading mechanisms and market efficiency

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Preliminary version

1 Introduction

Market microstructure analysis represents a growing part of financial literature. This literature studies the process by which investor's latent demands are ultimately translated into prices and volumes. Research in microstructure covers a very wide range of topics. According to Madhavan (2000), it can be divided into four main categories: price formation and price discovery, market structure and design issues, information and disclosure, informational issues arising from the interface of market microstructure.

The purpose of this article falls into the second area. We focus on the role of the institutional environment. The institutional approach examines how trading mechanisms affect the price equilibrium. This papers departs from the existing literature, since the goal of the paper is not to study investor's strategic choices (see for example Ho and Stoll (1983), Biais (1993), Biais, Foucault, Salanie (1998), Foucault (1999)). Our approach is quite different since we study how the trading mechanisms affect the possibility of exchanges and thus market liquidity.

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We define market liquidity as allocative efficiency. A market is liquid if any mutually desirable exchange may be executed. This defines allocative efficiency of a trading mechanism and corresponds to a Pareto improvement criterium. We thus examine whether or not any Pareto improving exchange should be realized according to the coordination or trading mechanism. A coordination mechanism failure appears if a Pareto improving exchange is prevented by the trading mechanism or an institutional constraint.

To address this question, we shall use an unusual concept coming from decision science, namely reservation buying and selling prices. Those concepts have been applied to finance by Biais (1993) for a specified utility function (constant absolute risk aversion) and by Eeckhoudt and Roger (1999) and Roger (2000) in the general case. It allows us to define any Pareto improving exchange on a market with many investors.

We develop a positive analysis. We compare market liquidity of the two existing centralized trading mechanisms: price driven and order driven financial markets[1]. A market is said to be a *price driven market* if a market maker is quoting prices to buy and sell at which he or she is willing to trade (e.g. London Stock Exchange). We consider two major institutional constraints on the market maker's behavior. The first one obliges him to post unitary prices (linearity constraint). The second one obliges him to commit to buy or sell any quantity at this price (liquidity constraint). This last constraint implies that he cannot price discriminate between investors. We also analyse the impact of a regulatory minimum trading quantity size. We show that both linearity and liquidity constraint reduce market liquidity whereas the minimum trading quantity size can increase market liquidity. Conversely, in *order driven markets* investors post unitary buying or selling prices and corresponding quantities and liquidity is ensured by the order book (e.g. Euronext markets)[2]. We show that the fact that investors post unitary buying or selling prices reduces market liquidity. However, market liquidity is higher on order driven driven markets than on price driven markets.

[1] We do not consider fragmented markets such as NASDAQ, SEAQ, or the Treasury bonds market where trades are the outcome of bilateral negotiations. See Biais (1993) for the comparison of fragmented and centralized markets.

[2] There exists mix systems. For example, on the NYSE market makers post bid ask prices and have order books.

The paper is organized as follows. The model is spelled out in section one. We present the properties of reservation buying and selling prices and we define all Pareto improving exchanges. Section 2 deals with price driven financial markets and section 3 deals with order driven financial markets.

2 The model

The aim of the paper is to study the optimality of the main existing market mechanisms, namely price driven financial markets and order driven financial markets. Hence, we need to characterize mutually desirable or Pareto improving exchanges. We depart from the traditional approach on this subject by using the tools of decision science. We shall compute reservation prices defined by willingness to pay or willingness to accept. It allows us to exhibit the range of all mutually desirable exchanges between agents. In this section, we first present the buying and selling prices properties. Then, we define all mutually desirable exchanges.

2.1 Buying and selling prices

Let us consider a risk-free asset B and a risky asset X. The rate of return on the risk free asset is $r > 0$. The revenue of the risky asset is a random variable X which takes positive values with $\mu = E(X) > 1 + r$. We consider risk averse investors. Each one, noted i exhibits a strictly increasing and strictly concave utility function U_i. Each investor holds an initial portfolio composed of a quantity $\bar{\alpha}_i^B$ $(\bar{\alpha}_i^X)$ of risk-free (risky) asset which revenue is:

$$R_i\left(\bar{\alpha}_i^B, \bar{\alpha}_i^X\right) = (1+r)\,\bar{\alpha}_i^B + \bar{\alpha}_i^X X$$

Buying and selling prices of investors is defined in such a way that his utility does not increase (decrease) after transactions. This defines reservation prices. Hence, agent i selling price $s_i(\alpha)$ for a quantity α of risky asset is given by:

$$U_i(R_i\left(\bar{\alpha}_i^B, \bar{\alpha}_i^X\right)) = U_i\left[(1+r)\,\bar{\alpha}_i^B + \left(\bar{\alpha}_i^X - \alpha\right) X + s_i(\alpha)\right] \quad (1)$$

and agent i buying price $b_i(\alpha)$ for the same quantity α is given by:

$$U_i(R_i\left(\bar{\alpha}_i^B, \bar{\alpha}_i^X\right)) = U_i\left[(1+r)\,\bar{\alpha}_i^B + \left(\bar{\alpha}_i^X + \alpha\right) X - b_i(\alpha)\right] \quad (2)$$

The following lemma gives the results on the properties of bid and ask reservation prices from Eeckhoudt and Roger (1999) in the expected utility case and Roger (2000) in the RDEU case.

Lemma 1 (Eeckhoudt and Roger (1999) and Roger (2000))
(i) Buying and selling prices are increasing function of the trading quantity:

$$\frac{ds_i(\alpha)}{d\alpha} > 0; \frac{db_i(\alpha)}{d\alpha} > 0$$

(ii) Buying and selling prices are non linear function of the trading quantity. Under expected utility or rank dependent expected utility, buying and selling prices are respectively concave and convex:

$$\frac{d^2 s_i(\alpha)}{d\alpha^2} > 0; \frac{d^2 b_i(\alpha)}{d\alpha^2} < 0$$

(iii) Buying and selling prices are decreasing function of the inventory:

$$\frac{ds_i(\alpha)}{d\bar{\alpha}_i^X} < 0; \frac{db_i(\alpha)}{d\bar{\alpha}_i^X} < 0$$

2.2 Exchange

We consider in this paper a centralized exchange mechanism, *i.e.* agents meet on a clearing house in order to realize their exchanges. We consider an economy with 3 agents. Naturally, all our results can easily be generalized to n agents. An exchange takes place when one buys a quantity $\alpha_1 + \alpha_2$ $(\alpha_1 \geq 0, \alpha_2 \geq 0)$ sold by the two others or one sells a quantity $\alpha_1 + \alpha_2$ that the two others buy. We obtain:

$$\begin{aligned} b_i(\alpha_1 + \alpha_2) &\geq s_j(\alpha_1) + s_k(\alpha_2) \\ or\ s_i(\alpha_1 + \alpha_2) &\leq b_j(\alpha_1) + b_k(\alpha_2),\ \forall i \neq j \neq k \end{aligned} \quad (3)$$

This defines a mutually desirable or Pareto improving exchange because it increases the utility of at least one of the three agents without decreasing utility of the two others. Hence, an efficient trading mechanism has to allow to realize all mutually desirable exchanges. In the rest of the paper, we shall compare the allocative efficiency or liquidity of the main existing market mechanisms. We examine successively the price driven financial markets and order driven financial markets.

3 Price driven financial markets

In price driven financial markets, one or several market makers act as coordination mechanisms. A large literature has paid attention to market maker's behavior and to the bid-ask spread quoted (see for example O'Hara (1995), Lyons (2000) or Madahavan (2000)). A part of this literature considers the market maker not only as an intermediary but as a simple market participant, or trader, who is willing to alter is own portfolio away from desired holdings to accommodate the trading desires of other traders (see Stoll (1978), Amihud and Mendelson (1980) or Biais (1993) among others). Risk aversion is an important parameter in the setting of buying and selling prices. This implies an "inventory effect", which leads to a direct relationship between the position of the market maker in the risky asset, and the prices he proposes to the traders. In this section, we complete this analysis by taking into account institutional constraints, which may involve inefficiencies. The first set of constraints is related to unitary bid-ask prices and the liquidity constraint. It appears when we take into account the attitude towards risk of the market maker. The second set of constraints is related to non discriminatory bid-ask prices between agents. It appears even when the market maker is a simple intermediary of exchanges. The two following sections examine successively those two sets of constraints.

3.1 Linear bid - ask prices of the market maker

In this section, bid and ask prices proposed by the market maker to the traders are such that he never looses money by realizing a transaction. To our knowledge, existing litterature does not take into account two major institutional constraints on the market maker's behavior. The first one obliges him to post unitary prices (linearity constraint). The second one obliges him to commit to buy or sell any quantity at this price (liquidity constraint). However, this constraint is limited to a *maximum quoted trading size, i.e.* the maximum quantity the market maker has to commit to buy at this price.

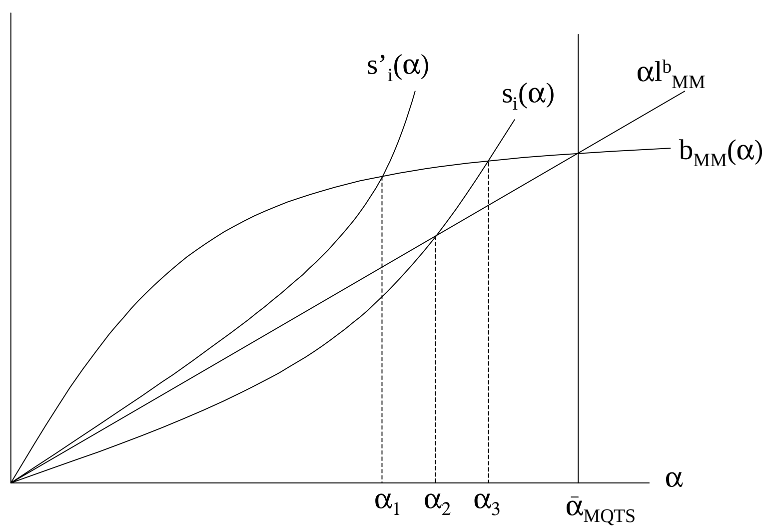
Those both institutional constraints affect the market maker's behavior and hence, realizable exchanges. We shall show that they can conduct to inefficiencies. We consider only bid prices of the market maker. All our arguments apply symmetrically to ask prices.

We denote $\bar{\alpha}_{MQTS}$ the *maximum quoted trading size.* The market maker posts a linear bid l^b_{MM} such that he never loose money whatever the traded quantity α. Its bid price is defined by $\alpha l^b_{MM} \leq b_{MM}(\alpha)\ \forall \alpha \in [0, \bar{\alpha}_{MQTS}]$. Otherwise, he could have to buy a quantity at a higher price than its buying price, *i.e.* his willingness to pay. From lemma 1, we know that buying price is a concave function of the trading quantity. Thus, the optimal linear bid price of the market maker is defined by $\alpha_{MQTS} l^b_{MM} = b_{MM}(\bar{\alpha}_{MQTS})$.

Proposition 2 *An exchange can be Pareto improving and realizable under non linear pricing but non realizable under linear bid price.*

Proof. We just have to find that there exists at least a quantity α such that $b_{MM}(\alpha) > s_i(\alpha)$ and $s_i(\alpha) > \alpha l^b_{MM}$. Since we have the linear bid price such that $\alpha l^b_{MM} < b_{MM}(\alpha)$ for $\alpha \in (0, \bar{\alpha}_{MQTS})$, we can find a selling price $s_i(\alpha)$ which verifies $\alpha l^b_{MM} < s_i(\alpha) < b_{MM}(\alpha)$. ■

The following figure shows graphically the proposition.



If the selling price of agent i is $s_i(\alpha)$, then using non linear prices, the market maker buys to agent i every quantity between 0 and α_3. All those exchanges are Pareto improving and are realizable using non linear prices. Under institutional linearity and liquidity constraints, only exchanges between 0 and α_2 are realizable. Thus, all exchanges between α_2 and α_3 are realizable using non linear prices but not under the both institutional constraints. Inefficiencies are worse off if the selling price of agent i is $s_i'(\alpha)$ instead of $s_i(\alpha)$. Every quantity between 0 and α_1 is exchanged using non linear prices, since $b_{MM}(\alpha) > s_i'(\alpha)$ $\forall\alpha \in [0, \alpha_1]$. Under both linearity and liquidity constraints, no exchange is executed since $s_i'(\alpha) > \alpha l_{MM}^b$ $\forall\alpha \geq 0$. Institutional constraints cancel all exchanges possibilities.

3.2 Market maker as a coordination mechanism

We now consider the market maker as an intermediary of exchanges, *i.e.* an agent who matches sellers and buyers. Thus, we do not consider anymore the attitude towards risk of the market maker. We shall consider exchanges between three agents which are coordinated by a market maker, *i.e.* a centralized mechanism. We show that non discriminatory bid-ask prices can may prohibit Pareto improving exchanges.

Let us consider the following case: agent i wants to sell a quantity $\alpha_1 + \alpha_2$ to agents j and k who are willing to buy respectively α_1 and α_2. We assume that this exchange is Pareto improving, which writes:

$$s_i(\alpha_1 + \alpha_2) \leq b_j(\alpha_1) + b_k(\alpha_2)$$

We assume that the market maker knows perfectly all buying and selling prices. As an intermediary of exchanges, he could buy the quantity $\alpha_1 + \alpha_2$ to agent i and sell α_1 to agent j and α_2 to agent k. But he faces the institutional constraint of non discriminatory bid-ask prices. He posts one bid price at which he buys the asset and one ask price at which he sells it. In our setting, he should post a unitary bid price greater or equal to $s_i(\alpha_1+\alpha_2)/(\alpha_1+\alpha_2)$. Institutionally, he can post only one ask price which must be induce both buyers j and k to buy the global quantity. To be incentive, his ask price has to be less than $\min\{b_j(\alpha_1)/\alpha_1, b_k(\alpha_2)/\alpha_2\}$. Symmetrically, this result holds if agent i buys $\alpha_1+\alpha_2$ to agent j and k. We derive the following proposition.

Proposition 3 *A market maker coordination mechanism prevents Pareto im-*

proving exchanges if:

$$s_i(\alpha_1+\alpha_2) \leq b_j(\alpha_1)+b_k(\alpha_2)\ \forall i \neq j \neq k$$

and

$$\min\left\{\frac{b_j(\alpha_1)}{\alpha_1},\frac{b_k(\alpha_2)}{\alpha_2}\right\} < \frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$$

or

$$b_i(\alpha_1+\alpha_2) \geq s_j(\alpha_1)+s_k(\alpha_2),\ \forall i \neq j \neq k$$

and

$$\max\left\{\frac{s_j(\alpha_1)}{\alpha_1},\frac{s_k(\alpha_2)}{\alpha_2}\right\} > \frac{b_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$$

Remark that inefficiency only comes from non discriminatory bid-ask prices, and not from unitary prices.

Let us consider the following numerical example: $\alpha_1 = 80$, $\alpha_2 = 20$, $b_j(80) = 880$, $b_k(20) = 180$, $s_i(100) = 1000$. An exchange in which i sells 100 units of asset and j and k buy respectively 80 and 20 units is Pareto improving. Indeed, the seller has a willingness to sell equal to 1000 and the sum of the two buyer's willingness to pay is equal to $880 + 180 = 1060$. The market maker posts a unitary bid price greater or equal than 10. However he can not price discriminate between the two buyers. In order to sell the 100 units, he has to post a unitary ask price equal to $\min\{b_j(80)/80, b_k(20)/20\} = \min\{11, 9\} = 9$. The market maker should realize losses by coordinating such an exchange, since its bid price (10) is higher than the ask price (9) which clears the market. This coordination mechanism failure would disappear if the market maker could price discriminate between the two buyers. Indeed, posting an ask price equal to 9 for agent k and equal to 11 for agent j allows the exchange.

4 Order driven financial markets

We now examine order driven financial markets such as in the Euronext market. Liquidity is ensured by the order book. Investors can submit either limit orders or market orders. A limit order specifies a limit price and a quantity. For a buy limit order, the limit price is the maximum price that a buyer will pay ; for a sell limit order, the limit price is the minimum price that a seller will obtain.

All those prices are unitary and linear prices. Market orders are orders to buy or sell a given quantity at any price. When a market order arrives, it is matched with the best orders on the opposite side of the book and one or more trades occur.

In this section, we insulate an inefficiency of the mechanism even in the most favorable environment. We assume that investors have no strategic behavior for their bids. We limit the analysis to truthfull revelating mechanism in which each investor posts its reservation quote by using a unique limit order. Buyers (sellers) post a unitary buying (selling) price and a trading quantity. All orders are stored in the book and exchanges are realized. Matchings and corresponding trades are computed according to price priority. If possible, an order is matched with the best orders on the opposite side of the book and one or more trades occurs. If a trade occurs, the lowest selling price order is matched with the highest buying price order and the traded quantity corresponds to the minimum of the quantities in the two orders.

We derive the following proposition.

Proposition 4 *An order flow coordination mechanism in which investors post their reservation quotes through limit orders may involve at least a partial exchange.*

Proof. Let us consider again the following case: agent i wants to sell a quantity $\alpha_1 + \alpha_2$ to agents j and k who are willing to buy respectively α_1 and α_2. We assume that this exchange is Pareto improving which means that $s_i(\alpha_1+\alpha_2) \leq b_j(\alpha_1)+b_k(\alpha_2)$. We assume that $b_j(\alpha_1)/\alpha_1$ is the highest buying price of existing limit orders and $b_k(\alpha_2)/\alpha_2$ is the second highest one, i.e. $b_j(\alpha_1)/\alpha_1 > b_k(\alpha_2)/\alpha_2$. Agent i posts the following selling price $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$.

There are two cases:

i- if $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} \in [(b_k(\alpha_2)/\alpha_2; b_j(\alpha_1)/\alpha_1)]$ then there is a partial exchange, i.e. agent i sells only the quantity α_1.

ii- if $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} < b_k(\alpha_2)/\alpha_2$ then agent i can sell the quantity $\alpha_1 + \alpha_2$.

The case of no exchange would involve the following condition $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} > b_j(\alpha_1)/\alpha_1$ which is inconsistent with the pareto improving exchange assumption. Indeed, we have $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} > b_j(\alpha_1)/\alpha_1$ and $\frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} > b_j(\alpha_2)/\alpha_2$ wich is equivalent to $\alpha_1 \frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} + \alpha_2 \frac{s_i(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2} > b_j(\alpha_1) + b_j(\alpha_2)$ which contradicts the pareto improving exchange assumption. ■

When there is a partial exchange, we can notice that agent i sells the quantity α_1 at a price which is higher than his reservation selling price for the quantity α_1 because of the convexity of $s_i(.)$.

Let us consider the previous example. The limit buying orders are $b_j(80)/80 = 11$ and $b_k(20)/20 = 9$. Agent i posts the selling price $s_i(100)/100 = 10$. He can only realize a partial exchange because only 80 will be matched.

Remark 5 *In our settings, we consider that investors post simultaneously their reservation quotes through limit orders. If investors post sequentialy their orders, the follower may fragment its order in order to price discriminate. In our previous example, let us assume that the two buyers post firstly their reservation buying prices,* $b_j(\alpha_1)/\alpha_1$ *and* $b_k(\alpha_2)/\alpha_2$*. If the seller observes those limit orders he can either post two fragmented limit orders* $(b_j(\alpha_1)/\alpha_1, \alpha_1)$ *and* $(b_k(\alpha_2)/\alpha_2, \alpha_2)$ *or equivalently a market order for* $\alpha_1 + \alpha_2$*. In both cases, all Pareto improving exchanges are executed.*

Finally, propositions (3) and (4) allows us to conclude :

Proposition 6 *Under the assumption that investors post their reservation quotes, order driven financial markets are more liquid than price driven markets.*

5 Conclusion

This paper compares efficiency and liquidity of trading mechanisms on financial markets using reservation buying and selling prices. We show that institutional constraints, such as linearity and non discrimination may involve inefficencies by preventing Pareto improving exchanges and thus reduce market liquidity. However such inefficiencies are more important on price driven financial markets than on order driven financial markets. The main restriction of this paper is that we assume that investors post their reservation quotes. Further research should relax this assumption by endogeneizing bidders strategies.

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