

Endogenous Asset-Price Fluctuations in a Credit-Financed Economy

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Abstract

This paper develops a stylised production-based asset-pricing model, in which firms must bridge the gap between current investment spending and future receipts by resorting to bank credit. This credit constraint has two dynamic implications. Firstly, it prevents firms from perfectly smoothing dividends over time. Secondly, it opens the dynamics of investment to self-fulfilling fluctuations in aggregate demand. Extrinsic uncertainty, together with imperfect dividend smoothing, may make asset prices highly volatile, and asset returns predictable, two widely documented features of stock markets. Formally, dynamics of the model is shown to be equivalent to that of a ‘classical’ monetary overlapping generation model [Gale, *J. Econ. Theory*, 1973] with production.

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1 Introduction

Over the past twenty five years, the interest in investigating the economic forces behind asset price fluctuations has triggered a growing body of literature focusing on the interrelations between finance and macroeconomics. Guided by the logic of arbitrage, according to which aggregate expected returns can only depend on non diversifiable risk factors (Ross [1976]), empirical research has focused on seeking which set of macroeconomic state variables could provide a sufficient statistics for explaining the behaviour of asset prices and returns (e.g. Chen, Roll and Ross [1986]). From the theoretical point of view, the need to ground these factors on sound and parcimonious foundations naturally led to the extension of equilibrium business cycles models to the pricing of financial securities.

Following the tradition opened by Lucas [1978] and Brock [1982], most of these aggregative models have retained exogenous ‘technology’ shocks as their basic source of aggregate risk (e.g. Balvers *et al.* [1990], Lettau [1998]). Although it was probably a natural route to take in the first place, it is now widely acknowledged that simple consumption-based asset pricing models, as well as real business cycle (RBC) models, perform rather poorly when trying to account for some important cross-sectional and times-series properties of asset data (see, Rouwenhorst [1995], and Campbell [2001]). In particular, two time-series anomalies have retained special attention. Firstly, asset prices seem to be lot more volatile than conventional asset pricing models predict (this was early showed by Grossman and Shiller [1981]); Secondly, asset returns appear to be predictable, which is apparently at odds with the random walk theory of asset prices (e.g. Poterba and Summers [1988], Fama and French [1988]; see also Campbell [2001]). A quite natural line of research to pursue is then to question the fact that technology shocks are the only relevant source of asset-price fluctuations, and to investigate what alternative sources of macroeconomic disturbances may affect financial markets.

In this paper, we study the effect of transitory ‘demand’ shocks on the dynamics of asset prices. How can demand shock be formulated in dynamic models? Recent developments in macroeconomics has emphasised three types of such disturbance. Firstly, monetary shocks, such as those occuring in Keynesian models with sticky

prices, as they provide a source of randomness alternative to technology shocks, are often interpreted as representing such demand shocks (this follows Blanchard and Quah [1991]). Secondly, basic RBC models can be extended to incorporate government spendings shocks (e.g. Christiano and Eichenbaum [1992]). Thirdly, endogenous fluctuations due to self-fulfilling expectations, that occurs in models displaying steady state indeterminacy, can often explicitly be interpreted as aggregate demand shocks, with the attractive property that they are transitory *by construction* (e.g. Woodford [1991], Farmer and Guo [1994]). The model below follows this third approach but, although we find it more satisfactory than the others, it is clear that any sources of aggregate demand disturbances constitute a additional source of macroeconomic risk that, together with technology shocks, should contribute to the determination of asset prices.

The purpose of this paper is, henceforth, twofold. Firstly, we construct an (admittedly stylised) dynamic model displaying steady state indeterminacy and trend-reverting, endogenous fluctuations in aggregate demand. Such fluctuations are known to arise in a number of monetary models, which our model is an example of. The crucial departure from the basic neoclassical growth model that is introduced, is that current investment by firms must be paid for by resorting to bank credit, which firms are able to repay only once their production is sold. In other words, the economy is ‘credit-financed’. This credit constraint breaks the optimal choice of the representative firm into an infinite sequence of two-period maximisation problems, in a fashion very similar to Woodford [1986, 1988], and Barinci and Chéron [2001]. A crucial difference to these papers, however, is that transactions are not carried out with the fiat money that cash-in-advance models and ‘Samuelson’ overlapping generations models rely on; instead, they are carried out with deposit money created by banks on their clients’ accounts as a counterpart of the credit that is extended to them. As will become clear in the following, this feature renders the steady-state of the model generically indeterminate and subject to self-fulfilling expectations in aggregate demand, without (and unlike standard CIA and Samuelson OLG models) requiring unrealistically strong income effects, nor relying on agents’ heterogeneity. The operating mechanism behind such endogenous fluctuations can be summarised

as follows; when firms have nominal claims held on them, variation in nominal prices affect their (expected) real debt and interest burden, and hence their optimal investment policy. As steady-state indeterminacy allows nominal prices to vary (despite the absence of any source ‘fundamental’ disturbances), so do investment, consumption and output. In brief, the model is shown to be observationally equivalent to a ‘classical’ monetary overlapping generation model (Gale [1973]) with production, which is known to display indeterminacy and sunspot fluctuations with much looser assumptions on preferences and technology than ‘Samuelson’ economies do (Davila [1994], Challe [2001]).

Secondly, we go on investigating the asset price implications of these endogenous fluctuations in aggregate demand. To address this issue, the optimal consumption problem of a representative agent is disaggregated into the optimal share purchase of the representative consumer on the one hand, and the optimal dividend policy of the representative firm on the other. While little is changed on the side of consumers (with respect to standard consumption-based capital asset pricing models), the credit constraint affecting firms’ decisions is shown to substantially alter the firms’ dividend policy by preventing them from perfectly smoothing dividends over time. Moreover, as dividends are highly sensitive to the ‘sunspot-driven’ past and current investment flows, so are stock prices and returns. Imperfect smoothing of dividends on the part of firms, together with their sensitivity to extrinsic uncertainty, are shown to make asset prices potentially highly volatile, and ex post returns predictable.

The rest of the paper is organised as follows. Section 2 introduces the basic model. Section 3 derives the dynamics of output, dividends, stock prices and stock returns around the steady state, and shows that they are all driven by self-fulfilling fluctuations in aggregate investment. The relation between the time-series behaviour of asset prices implied by the model and the existing literature on major asset price anomalies is also discussed. Section 4 derives a class of global stationary sunspot equilibria, allowing for ‘wide’ variations in aggregate variables and recurrent cyclical patterns in asset prices. Section 5 concludes briefly.

2 The Model

The economy is populated by two types of infinitely lived agents in large number, called ‘shareholders’ and ‘firms’. The only resource of shareholders are dividends, which they are entitled to by holding shares (available in fixed supply, normalised to 1). We assume that firms produce specific ‘coloured’ goods, all of which being perfect substitutes from the point of view of both shareholders and firms, except for the fact that no firm can use its own production as an input. Hence, although all firms are identical, they need to trade to acquire input goods from other firms. The production technology is identical across firms, and is represented by the C^2 functions $y_{t+1} = f(k_t)$, where y_t and k_t denote output and capital at date t , respectively. As will become clear below, the time lag in production is crucial, as it may force firms to go into debt to finance the delay between current spendings and future receipts from sales.¹ We assume that credit, when required by firms, is offered unelastically by an altruistic ‘central bank’ in fixed amount 1, at zero nominal interest rate. This oversimplifying assumption allows us to focus on the effect of nominal prices on the real interest. The utility function has the standard regularity property, and capital is assumed to depreciate entirely after production.² In addition, we make the following assumptions:

- i) $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$
- ii) $\lim_{k \rightarrow \infty} f'(k) = 0$, $\lim_{k \rightarrow 0} f'(k) = +\infty$,
- iii) $\varepsilon(k) = -f''(k)k/f'(k) < 1$.

2.1 Shareholders

The shareholders’ problem is straightforward. Each share represents a perfectly divisible claim on one firm’s output, and each shareholder chooses their current share holdings, s_t , so as to maximise the expected lifetime utility of their current

¹This motivation for firms to go into debt is reminiscent of Keynes [1937] ‘finance’ motive.

²Henceforth, the model might be best interpreted as representing an economy in which firms produce ‘intermediate goods’ that are used as both input goods and consumption goods.

and future consumption flows, $\{c_t\}_{t=0}^{\infty}$. Hence, they solve:

$$\max E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right) \quad (2.1)$$

subject to

$$c_t + s_t q_t \leq s_{t-1} (q_t + d_t) \quad (2.2)$$

$$c_t, q_t, s_t > 0 \text{ for all } t \quad (2.3)$$

where q_t denotes the price of one share in terms of the consumption good. As consumption is entirely provided for by dividends ($c_t = d_t$ for all t), the first-order condition associated with the consumer problem is :

$$E_t (q_t u'(d_t) - \beta u'(d_{t+1}) (q_{t+1} + d_{t+1})) = 0 \quad (2.4)$$

Solving (2.4) for q_t , iterating forwards and imposing an appropriate transversality condition to rule out rational bubbles, one recovers the familiar ‘Lucas’ [1978] asset pricing equation :

$$q_t = E_t \left(\sum_{i=1}^{\infty} \beta^i \frac{u'(d_{t+i})}{u'(d_t)} d_{t+i} \right) \quad (2.5)$$

2.2 Firms

We assume that firms enter period t with an amount of nominal debt m_{t-1} , that was contracted at date $t - 1$. They acquire purchasing power by borrowing an additional amount m_t of money from the bank (which issues it in constant amounts normalised to 1). This quantity circulates across firms in period t as firms are buying investment goods, and is then used by them to pay off m_{t-1} . The sequence of events is summarised in figure 1.

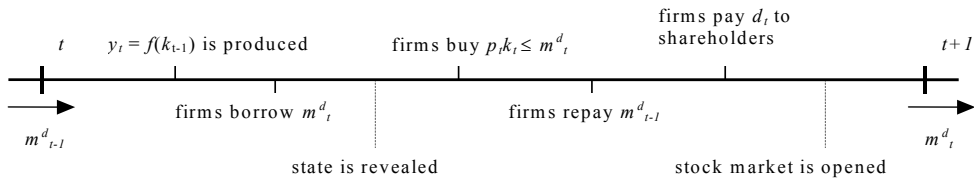


Figure 1: Time line

Importantly, this borrowing scheme requires that firms are held nominal claims on by the bank at date 0. To ensure that such is the case, it is assumed that firms are ‘given’ an arbitrarily small quantity of input goods at $t = 0$ from the bank, against the duty to repay 1 unit of money in period 1. From time $t = 1$ onwards, firms maximise the discounted sum of dividends and solve :

$$\max E_0 \left(\sum_{t=0}^{\infty} \frac{d_t}{\prod_{i=0}^t R_i} \right) \quad (2.6)$$

subject to

$$p_t (d_t + k_t) + m_{t-1} + b_t \leq p_t y_t + m_t + b_{t-1} (1 + i_{t-1}) \quad (2.7)$$

$$p_t k_t \leq m_t \quad (2.8)$$

$$\lim_{n \rightarrow \infty} (b_t - m_t) / p_t > -\infty \quad (2.9)$$

$$k_t, d_t, p_t > 0 \quad (2.10)$$

where $\prod_{i=0}^t R_i$ denotes the rate at which t periods ahead dividends are discounted. As in any production-based asset pricing models, $\{R_t\}_{t=1}^{\infty}$ is endogenously determined by the representative household’s marginal rates of substitution, $\beta u'(d_{t+1}) / u'(d_t)$ (see Brock [1982], Balvers *et al.* [1990])³. Equation (2.7) is the budget constraint of the firm at date t , where firms are allowed to freely issue or buy nominal bonds at the nominal interest rate i_t (these bonds cancel each other in the aggregate). Equation (2.8) is an additional finance constraint that forces firms’ spending to be paid for with (deposit) money. Finally, equation (2.9), which asymptotically constrains the real wealth held by firms, rules out Ponzi schemes. The first-order conditions associated with the firm’s programme are :

$$E_t (f'(k_t) / R_{t+1}) < 1 \quad (2.11)$$

$$E_t (f'(k_t) - p_t / p_{t+1}) = 0 \quad (2.12)$$

$$E_t ((1 + i_t) (p_t / p_{t+1}) / R_{t+1}) = 1 \quad (2.13)$$

If there were no finance constraint like (2.8), then all transactions would be carried out in real terms ; (2.11) would then hold with equality, and expected returns on assets would be equal to the marginal productivity of capital.

³This can be verified by reaggregating the optimal choice of firms and consumers into the optimal choice of a representative agent.

Equation (2.11) holds with strict inequality whenever the finance constraint (2.8) is binding. It will be the case as long as firms are willing to carry out investment spendings and they are not willing to hold idle balances (i.e. when the real interest rate on bonds is positive), which is always true by eq. (2.13). Equation (2.12) defines the optimal investment spendings of a firm. In brief, when (2.8) is binding, dividends are given by $d_{t+1} = f(k_t) - (p_t k_t - b_t(1 + i_t) - b_{t+1}) / p_{t+1}$, so that the intratemporal optimality condition yields (2.12), whereas the intertemporal optimality condition yields (2.13).

There is a crucial difference between the above programme and the agents' optimal labour supply of cash-in-advance economies. While the latter formally reduces a representative agent economy to a 'Samuelson' overlapping generation model (eg Woodford [1986], Huo [1987]), our model replicates the dynamics of a 'classical' economy similar to that introduced by Gale [1973] in the case of pure exchange economies. In our case, the dynamic implication of this structure can be understood by solving the intratemporal optimality condition for k_t :

$$k_t = f'^{-1}(E_t(p_t/p_{t+1})) \quad (2.14)$$

As $f(\cdot)$ is increasing and concave, $f'^{-1}(\cdot)$ is a decreasing function of the real interest factor p_t/p_{t+1} , so that an increase in the real interest rate lowers investment and next period output. This is intuitive: when firms have nominal claims held on them, a decrease in the price level increases the real debt burden (i.e. the gross real interest rate); firms' optimality condition then requires the level of investment to be lower for marginal productivity to match this higher real interest rate. This is, however, in sharp contrast to dynamic models where agents hold nominal claims (e.g. Wilson [1979], Azariadis [1981]); in this case an increase in the real interest rate induce individuals to work more (so that more output is produced) in order to accumulate more assets, unless the income effect is assumed to be very strong⁴.

⁴The assumption of a 'backward bending' labor-supply curve does not, however, necessarily apply to cash-in-advanced models with heterogenous agents (see Woodford [1986, 1988], and Barincci and Chéron [2001]).

2.3 Equilibrium

An equilibrium is defined as a sequence $\{s_t, m_t, k_t, d_t, q_t, i_t\}_{t=0}^{\infty}$ such that (2.4) and (2.11) to (2.13) hold, and such that all markets clear :

$$s_t = 1 \quad (\text{equity market}) \quad (2.15)$$

$$m_t = 1 \quad (\text{money market}) \quad (2.16)$$

$$d_{t+1} + k_{t+1} = f(k_t) \quad (\text{good market}) \quad (2.17)$$

Using equations (2.8) and (2.12) allow us to summarise the aggregate dynamics of the economy with the following difference equation:

$$E_t(k_{t+1}) = F(k_t) \quad (2.18)$$

where $F(k_t) \equiv k_t f'(k_t)$. A perfect foresight equilibrium of this economy is any sequence $\{k_t\}_{t=0}^{\infty}$ such that $k_{t+1} = F(k_t)$; and a steady state is any $k^* \in [0, \widehat{k}]$ verifying $k^* = F(k^*)$, where \widehat{k} is such that $\widehat{k} = f(\widehat{k})$ (existence and uniqueness of \widehat{k} follow from assumptions i-iii). Given the regularity assumptions imposed on $f(\cdot)$, it is straightforward that there are exactly two such k^* (see also figure 2 in the next section). The first one, $k^* = 0$ (and hence $d^* = q^* = 0$) corresponds to a situation in which firms roll over debts from one period to the next, but in which they do not use money in transactions, so that no investment takes place at all (hence the qualification of ‘no-trade equilibrium’ in figure 2 below). It formally corresponds to the ‘balanced’ steady state of the classical case of the monetary overlapping generations model (see Gale [1973, p.16]). The second (‘golden-rule’) steady state is the deterministic solution in which the nominal price of goods is positive and constant. It is defined by :

$$\begin{aligned} k^* &= f'^{-1}(1) & d^* &= f(k^*) - k^* \\ p^* &= m/k^* & q^* &= \beta(f(k^*) - k^*) / (1 - \beta) \end{aligned}$$

Whether there are non deterministic solutions to (2.18) depends, crucially, on the local determinacy of each steady state. As $\lim_{t \rightarrow 0} F(k_t) = \infty > 1$, the ‘balanced’ (or ‘low’) steady state is determinate, and hence locally unique. We now turn to the behaviour of k_t around the golden-rule (or ‘high’) steady state.

3 Endogenous Fluctuations around the Steady State

3.1 Self-fulfilling Fluctuation in Aggregate Demand

The existence of stationary sunspot equilibria in dynamic models can be studied by taking their linear approximation around the steady state and by comparing their number of stable roots and their number of predetermined variables (Farmer [1999]). The model described in the previous section is one-dimensional and has no predetermined variable, as k_1 is endogeneously determined by $E_1(p_1/p_2)$, which in turn depends on k_1 and k_2 (as $p_t = 1/k_t$). Similarly, k_2 is determined by $E_2(p_2/p_3)$, and so on. In this case, we can not exclude that extraneous uncertainty affects the distribution of $\{k_t\}$. To see how the shape of $F(\cdot)$ constraints the parameters of the invariant distribution of $\{k_t\}$, imagine that there is a sequence of belief shocks $\{\xi_t\}_{t=0}^{\infty}$, with yet unspecified mean and variance, that moves the dynamic path of k_t away from any perfect foresight equilibrium:

$$k_{t+1} - F(k_t) = \xi_{t+1} \quad (3.1)$$

where $\{\xi_t\}_{t=0}^{\infty}$ is a stationary stochastic process with small bounded support $[\underline{\xi}, \bar{\xi}]$, mean $E(\xi)$ and variance σ_{ξ}^2 . First, taking time t expectation of eq. (3.1) imposes $E_t(\xi_{t+1}) = 0$, so that $\{\xi_t\}_{t=0}^{\infty}$ is constrained to be a white noise process with mean zero. This identifies the unconditional mean of $\{\xi_t\}_{t=0}^{\infty}$ ($E(\xi_t) = 0$) and of $\{k_t\}_{t=0}^{\infty}$ ($E(k_t) = k^*$). Second, if (3.1) is to define a stationary distribution for $\{k_t\}$, one can take the unconditional variance of the latter equation to obtain:

$$\text{var}(k_t) = \text{var}(F(k_{t+1})) + \sigma_{\xi}^2$$

Recalling that, around the unconditional mean x^* of any stationary random process $\{x_t\}$, and for any continuous real-valued function $g(x_t)$, one can approximate $\text{var}(g(x))$ by $(g'(x^*))^2 \text{var}(x)$, the previous equation is approximately equivalent to:

$$\left(1 - F'(k^*)^2\right) \text{var}(k_t) = \sigma_{\xi}^2$$

As $\text{var}(k)$ and σ_{ξ}^2 are non-negative, they are both constrained to be zero if $F'(k^*)^2 \geq 1$. Hence, a necessary and sufficient condition for a non trivial stochastic

process $\{\xi_t\}_{t=0}^\infty$ verifying eq.(3.1) to exist, is $F'(k^*)^2 < 1$. Or, equivalently :

$$|1 - \varepsilon| < 1 \quad (3.2)$$

Where $\varepsilon \equiv \varepsilon(k^*) = -f''(k^*)k^*/f'(k^*)$, so that $\text{var}(k_t) = \sigma_\xi^2/\varepsilon(2 - \varepsilon) > 0$. Unsurprisingly, condition (3.2), which is always fulfilled under assumptions i-iii, is exactly that under which the golden-rule steady-state is indeterminate. To get an insight into why interminacy occur in this model, suppose that p_t lies below p^* and k_t lies above k^* , in such a way that (2.8) holds with equality. Will then (p_t, k_t) return to (p^*, k^*) as $t \rightarrow \infty$? The answer is yes : from (2.14), for $k_t > k^*$ it must be the case that $p_t/p_{t+1} < 1$ (ie the expected real interest rate must be low), so that p_t is expected to increase, which remains true as long as p_t has not reached p^* . Conversely, $p_t > p^*$ and $k_t < k^*$ requires $p_t/p_{t+1} > 1$ (the real interest rate is high), so that p_t must decrease to eventually get arbitrarily close to p^* . The same reasoning applies to any (p_t, k_t) close enough to (p^*, k^*) . The process $\{\xi_t\}_{t=0}^\infty$ then governs ‘jumps’ that occur in each period from one perfect foresight path to another. The multiplicity of perfect foresight equilibria defined by (2.18) is depicted in figure 2.

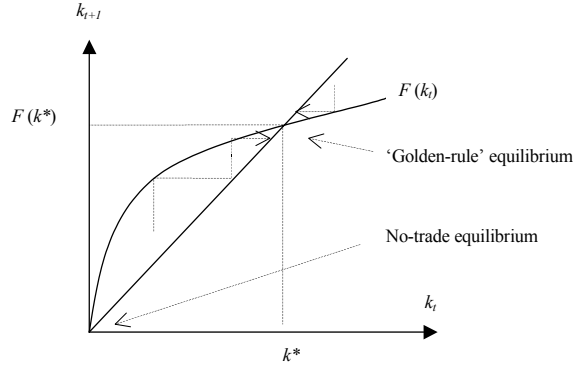


Figure 2: Steady state indeterminacy

To obtain a time series representation of the variables, it is convenient to consider the linear approximation to (3.1) around k^* . The law of motion of capital, dividend and output can then be approximated by the following AR(1), ARMA(1,1) and

AR(1) processes, respectively (where L denotes the lag operator):

$$k_t = k^* + \xi_t / (1 - (1 - \varepsilon)L) \quad (3.3)$$

$$d_t = d^* + k_{t-1} - k_t = d^* - (1 - L) \xi_t / (1 - (1 - \varepsilon)L) \quad (3.4)$$

$$y_t = f(k^*) - k^* + k_{t-1} = f(k^*) + L\xi_t / (1 - (1 - \varepsilon)L) \quad (3.5)$$

Hence, all variables are driven by fluctuations in aggregate investment, $\{k_t\}$, which in turn is driven by the sunspot process $\{\xi_t\}$. We now turn to the behavior of asset prices and expected returns.

3.2 Asset Prices

Denoting $\sigma = -d^* u''(d^*) / u'(d^*)$ as the coefficient of absolute risk aversion, evaluated at the steady-state, the linear approximation to eq. (2.5) around d^* is given by:

$$q_t = \frac{\beta\sigma}{1 - \beta} d_t + \beta(1 - \sigma) E_t \left(\sum_{i=0}^{\infty} \beta^i d_{t+1+i} \right) \quad (3.6)$$

Eq. (3.6) is familiar. If $\sigma = 1$, the substitution and the income effects exactly balance each other, so that current asset prices only depend on current dividends ($q_t = \beta d_t / (1 - \beta)$). If the substitution effect dominates ($\sigma < 1$), then the future growth dividends increase asset prices, as households are willing to delay consumption by holding more shares, whereas they are willing to consume more today, and hence buy less shares, if the income effect dominates ($\sigma > 1$).

To determine the price of shares at date t , one needs to compute the expected flow of future dividends conditional on information available at date t . As the n step ahead forecast of k_t is $E_t[k_{t+n}] = k^* + (1 + \varepsilon_k)^n (k_t - k^*)$ (see eq. (3.3)), one can compute:

$$E_t \left(\sum_{i=0}^{\infty} \beta^i d_{t+1+i} \right) = \frac{d^*}{1 - \beta} + \frac{\varepsilon}{1 - \beta(1 - \varepsilon)} (k_t - k^*) \quad (3.7)$$

where $\varepsilon / (1 - \beta(1 - \varepsilon)) > 0$. Equation (3.7) summarises the dynamic effects of demand shocks over dividends. Equations (3.4) and (3.7) reveal that a demand shock has two effect on the flow of dividend. For instance, a positive demand lowers current dividends as they are crowded out by current investment (see eq. (3.4)). On the other hand, future output is higher, which pushes up next period dividends,

as well as the entire dividend flow thanks to the persistence of demand shocks (eq. (3.7)). The effect is reversed with a negative demand shocks: dividends are pushed up contemporaneously but, *ceteris paribus*, below average in all future periods. The overall effect on asset prices remains, however, ambiguous, as it heavily depends on σ .

Finally, plugging (3.4) and (3.7) into (3.6), asset prices are found to follow the following ARMA(1,1) process :

$$q_t = q^* + \frac{\beta\sigma}{1-\beta}(k_{t-1} - k_t) + \frac{\beta(1-\sigma)}{1-\beta(1-\varepsilon)}(k_t - k^*) \quad (3.8)$$

$$= q^* + \varphi(L)\zeta_t \quad (3.9)$$

where

$$\varphi(L) = \frac{\beta(a + \sigma L)}{(1-\beta)(1-\varepsilon L)} \quad \text{and} \quad a = \frac{\varepsilon(1-\beta)(1-\sigma)}{1-\beta(1-\varepsilon)} - \sigma$$

Thus, self-fulfilling fluctuations in aggregate demand cause asset prices to be volatile despite the absence of fundamental disturbances⁵. Moreover, the persistence of demand shocks makes output, dividends and asset prices mean-reverting. Mean reversion in asset prices means that they are expected to go down when they are above average, and to go up when they are below. This feature of asset-price movements is equivalent to saying that asset returns contain a forecastable component (on average, ex post returns tend to be low when they have been high, and high when they have been low).

Another way to look at asset price movements is to compute the expected return on the market portfolio, that discounts the dividends on any asset. If we denote $R_{t+1} \equiv (q_{t+1} + d_{t+1})/q_t$ as the (gross) returns from holding shares, the linearisation of (2.5) around d^* yields:

$$\begin{aligned} E_t(R_{t+1}) &= \frac{1}{\beta} + \frac{\sigma E_t(\Delta d_{t+1})}{\beta d^*} \\ &= \frac{1}{\beta} + \frac{\sigma(1-\varepsilon)(1-L)}{\beta d^*(1-(1-\varepsilon)L)} \zeta_t \end{aligned}$$

As $1-\varepsilon \in (0,1)$, the mean reverting component in asset prices can be traced back to the positive autocorrelation in expected returns. Predictability of ex post

⁵Note that asset prices are ‘excessively’ volatile in the macroeconomic sense of the word, and not in the financial sense of the word, as prices are entirely driven by fluctuations in dividends.

returns, due to positively auto-correlated movements in expected returns, is a widely documented feature of many speculative markets (e.g. Poterba and Summers [1988], Fama and French [1988], Cutler, Poterba and Summers [1991]) and has been a matter of considerable controversy amongst financial econometricians. Some, in the wake of Shiller [1990] and Poterba and Summers [1988], have argued that returns predictability was more likely to be explained by the ‘irrational’ animal spirits of speculators, than by movements in fundamentals (dividends and discount rates). Proponents of the Efficient Market Hypothesis, on the other hand, submit that changes in fundamentals (i.e. preferences and investment opportunities) unobserved by the econometrician may generate enough variation in expected returns to rationalise asset price movements (e.g. Fama and French [1988] and Cochrane [1991]). Although the mean reverting behavior of asset prices in the above model can ultimately be traced back to transitory fluctuations in aggregate output, so that asset prices are always ‘correctly’ priced (i.e. the Efficient Market Hypothesis is not invalidated), the model above strikes a balance between the two approaches by rooting the dynamics of asset prices into the ‘rational’ animal spirits of entrepreneurs, rather than into changes in fundamentals.

4 Global Sunspot Equilibria

4.1 Stationary SSE

The previous section focused on ‘small’ fluctuations of the variables around the golden-rule steady state. As the type of indeterminacy that occurs in the present model is of global nature, it would be interesting to know whether large fluctuations in investment and asset prices may occur and, if so, what are the dynamic patterns of those variables. As is common in the literature, we shall focus on a particularly simple finite-state markov process with two states, a and b . In other words, we are looking for a set of matrices $\Pi = [\pi^{ss'}]$, $\pi^{ss'} \in [0, 1]$ and $(s, s') = \{a, b\}^2$ (where $\pi^{ss'}$ denotes the probability of being in state s' in the next period conditional on being in state s in the current period), associated capital levels $\{k^a, k^b\}$, and nominal prices $\{p^a, p^b\}$, such that $\sum_{s'} \pi^{ss'} k^{s'} = F(k^s)$. The equilibrium condition on the market

for goods, that must hold for each sequence of state $\{s, s'\}$ now writes as:

$$d^{ss'} + k^{s'} = f(k^s), \quad (s, s') \in \{a, b\}^2 \quad (4.1)$$

where

$$k^s = f'^{-1} \left(\sum_{s'} \pi^{ss'} \left(p^s / p^{s'} \right) \right) \quad (4.2)$$

$$d^{ss'} = f(k^s) - \left(p^s / p^{s'} \right) k^s \quad (4.3)$$

The existence of two-states markov-switching equilibria can be investigated by verifying whether different sequences $\{s, s'\}$ yield different values of $k^{s'}$ and $d^{ss'}$ in eq. (4.1). Note that there is a straightforward interpretation of the latter: $f(k^s)$ denotes the level of aggregate supply of goods at, say, date t , that depends only on the state that prevailed in the previous period. Hence, sunspot equilibria may occur if, given s , different values of s' may yield different values of $d^{ss'}$ and $k^{s'}$ while keeping $d^{ss'} + k^{s'}$ unaffected by s' . In other words, s' must change the composition of firms' spending at date t , while keeping its volume unchanged (and equal to $f(k^s)$). Thus, eq. (4.1) has multiple solutions if the following two equations have multiple roots:

$$H(s = a) = d^{ab} + k^b - d^{aa} - k^a = 0$$

$$\tilde{H}(s = b) = d^{ba} + k^a - d^{bb} + k^b = 0$$

From eq. (4.3), and if we define $\omega = p^a / p^b$, we can rewrite the function $H(\cdot)$ as:

$$H(\omega) = f'^{-1} \left(\sum_{s'} \pi^{bs'} \left(p^b / p^{s'} \right) \right) - \omega f'^{-1} \left(\sum_{s'} \pi^{as'} \left(p^a / p^{s'} \right) \right) \quad (4.4)$$

So that $H(\omega) = k^b - \omega k^a$ and $\tilde{H}(\omega) = -H(\omega) / \omega$. Hence, any root of $H(\cdot)$ is also a root of $\tilde{H}(\cdot)$. $\omega = 1$ (i.e. $p^a = p^b$), the deterministic allocation, is one of these common roots. Although $H(0) = 0$, we can rule out $\omega = 0$ as a common root of $[H(\omega), \tilde{H}(\omega)]$ for $\lim_{\omega \rightarrow 0} \tilde{H}(\omega) = f'^{-1}(\pi^{aa}) > 0$. Hence, as in Spear [1984] and Azariadis and Guesnerie [1986], the roots of $H(\omega) = 0$ can be investigated by studying the asymptotic behavior of $H(\omega)$, as well as its derivative evaluated at

$\omega = 1$ (the deterministic allocation). One can compute

$$\left. \frac{\partial H(\omega)}{\partial \omega} \right|_{\omega=0} = -f'^{-1}(\pi^{aa}) < 0 \quad (4.5)$$

$$\left. \frac{\partial H(\omega)}{\partial \omega} \right|_{\omega=1} = -(\pi^{ab} + \pi^{ba}) \left(\left. \frac{\partial f'^{-1}(\omega)}{\partial \omega} \right|_{\omega=1} \right) - f'^{-1}(1) \quad (4.6)$$

$$\lim_{\omega \rightarrow \infty} H(\omega) = f'^{-1}(\pi^{bb}) > 0 \quad (4.7)$$

Eq (4.5) indicates that $\exists \omega \in (0, 1)$ such that $H(\omega) < 0$. Together with (4.7), this implies that $H(\omega)$ is an odd function that intersects the horizontal axis at least once (at $\omega = 1$). Thus, a sufficient condition for $H(\omega)$ to have (at least) three roots is that (4.6) be inferior to one, which is the case for the set of matrices verifying

$$\text{tr}(\Pi) > 2 - \varepsilon \in (1, 2] \quad (4.8)$$

A change of $\text{tr}(\Pi)$ from $\text{tr}(\Pi) < 2 - \varepsilon$ to $\text{tr}(\Pi) > 2$ causes $H(\omega)$ to bifurcate, as is shown in figure 3. The set of matrices for which condition (4.8) holds is depicted in figure 4.

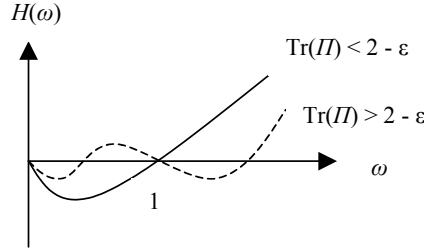


Figure 3: Bifurcation of $H(\omega)$

Contrary to the level of investment, which is entirely determined by the current sunspot state, say s' , dividends, and hence asset prices, depend on both past and current levels of investment (as $d^{ss'} = f(k^s) - k^{s'}$), and hence on both s and s' . This means that asset prices oscillate between four states $\tilde{s} \in \{aa, bb, ab, ba\}$, with transition probabilities

$$Q = \begin{bmatrix} \pi^{aa} & 0 & 1 - \pi^{aa} & 0 \\ 0 & \pi^{bb} & 0 & 1 - \pi^{bb} \\ 0 & \pi^{bb} & 0 & 1 - \pi^{bb} \\ \pi^{aa} & 0 & 1 - \pi^{aa} & 0 \end{bmatrix}$$

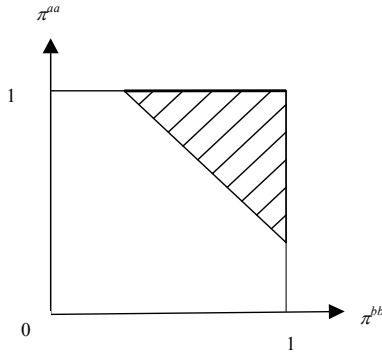


Figure 4: Global stationary sunspot equilibria

In other words, asset prices display recurrent cyclical patterns comparatively richer than those of investment and output. The ordering of dividends and asset prices with respect to the different states of matrix Q , depends largely on Π , $f(\cdot)$ and $u(\cdot)$, so that a wide multiplicity of dynamics is possible

4.2 Sunspots with ‘memory’

Interestingly, the trace condition (4.8) does not formally rule out the possibility of sunspots with ‘memory’ (the terminology is from Weil [1987]) similar to those studied by Blanchard and Watson [1982] and Weil [1987]. As our model displays such a possibility, we mention it briefly. Such processes have the following form:

$$M = \begin{bmatrix} \pi & 1 - \pi \\ 0 & 1 \end{bmatrix}$$

In Weil’s paper, for example, this transition matrix defines the probability that fiat money will be positively valued tomorrow, conditional on whether it is valued today or not. Let us define k^{**} and k as the levels of capital that correspond to the absorbing state of matrix M , and to the transitory state of M , respectively. In this case, the condition $\sum_{s'} \pi^{ss'} k^{s'} = F(k^s)$ becomes

$$F(k^{**}) - k^{**} = 0 \tag{4.9}$$

$$F(k) - \pi k - (1 - \pi) k^{**} = 0 \tag{4.10}$$

Equation (4.9) has two solutions, $k^{**} = k^*$ and $k^{**} = 0$ (in other words, the absorbing state must be one of the steady states). Let us start with the first case,

$k^{**} = k^*$, which constitutes a particular family of solutions to (4.8). Defining $G(k) \equiv kf'(k) - \pi k - (1 - \pi)k^*$, we are now looking for $(\pi, k) \in (0, 1) \times (0, \widehat{k})$ such that :

$$G(k) = 0 \quad (4.11)$$

$$\pi - 1 + \varepsilon > 0 \quad (4.12)$$

$$f(k) - k^* > 0 \quad (4.13)$$

As in the previous section, knowing whether $G(k) = 0$ has other roots than k^* can be checked by looking at the asymptotic behaviour of $G(k)$, as well as at its derivative evaluated at k^* . Given condition (4.12), we have

$$\lim_{k \rightarrow 0} G(k) = -(1 - \pi)k^* < 0, \quad \lim_{k \rightarrow \infty} G(k) = -\infty \quad (4.14)$$

$$\left. \frac{\partial G(k)}{\partial k} \right|_{k=k^*} = 1 - \varepsilon - \pi < 0 \quad (4.15)$$

eq. (4.14) and (4.15) imply that there is at least one $k \in (0, k^*)$ such that $G(k) = 0$ (see figure 5). Such sunspot equilibria, if they exist (i.e. if inequality (4.13) holds⁶) correspond to a situation in which the economy ‘catches up’ from a low level of investment, to settle on the golden-rule steady state forever. The other case, in which the economy has a constant probability of collapsing forever ($k^{**} = 0$), is straightforward. In this case, eq. (4.10) becomes $kf'(k) - \pi k = 0$, so that $k = f'^{-1}(\pi) > k^*$; the level of capital in the temporary states increases with the probability that the economy collapses.

5 Concluding Remarks

In this paper, we have studied the implication for asset prices of a model economy in which firms need to go into debt vis-à-vis a bank, before they can carry out investment. In spite of the absence of any source of fundamental disturbance (such as technology shocks or government spending shocks), the set of stochastic equilibria triggering ‘endogenous’ asset-price fluctuations has been found to be rather

⁶This implies that any k such that $G(k) = 0$ must also verify $f(k)/k > (f'(k) - \pi)/(1 - \pi)$. Although we are not sure that this condition always holds, we found a number of examples in which it does.

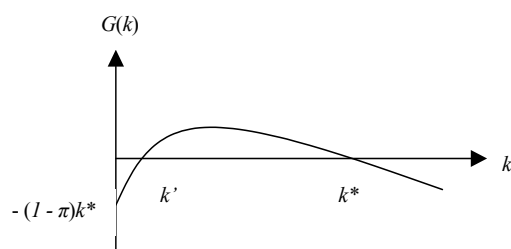


Figure 5: $G(k)$

large. This includes reverting fluctuations of capital, output and asset prices around their long run equilibrium values, regime changes characterised by wide oscillations between high and low levels of investment and associated recurrent cycles in asset values, as well as catching up and market collapses corresponding to a permanent move from one particular temporary state to one of the steady states. Although the model in this paper was very stylised and not directly liable to a quantitative evaluation, it leads us to believe that more attention should be paid to models displaying indeterminacy and sunspot fluctuations when searching for sources of aggregate risk determining the price of, and returns of, financial assets.

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