Strong and Weak Currencies in a Search-Theoretic Model of Money

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Abstract

We construct a one-country search theoretic model of currency competition to study under which conditions a foreign strong currency can circulate alongside a domestic currency. Each competing money is defined by a couple (γ_i, θ_i) where γ_i and θ_i, respectively, account for the rate of return and the bargaining power of money i. We discuss in this framework what strong and weak currencies may be. We show that the weakening of the domestic currency's bargaining power can favour the circulation of a strong currency by increasing the transactional utility of this foreign money - bad money attracts good money. It is then more profitable to use the strong currency in exchange rather than to use it as a store of value. Conversely we show that the weakening of the domestic money's rate of return can prevent the circulation of the foreign money by reinforcing its store of value's role- bad money drives out good money. Finally we express the general relationship between the value of money 2 and the value of money 1.

JEL Classification: E31 E42 F33

1. Introduction

The goal of this paper is to go further in the understanding of currency competition and currency substitution. Most related articles refer to the portfolio theory and set the problem in terms of the real rate of return on money. See Girton and Roper[1981] for a complete view on this topic. The explanation that is then given for the dollarization of many developping countries centres on inflation, real interest rates or risk premium[1].

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[1] We will speak of dollarization for the use in daily exchange of one foreign strong currency instead of the domestic money, yet dollar is not the unique strong currency circulating outside its country.

The application of search theory to monetary economics provides the opportunity to go back over this problem and to give greater place to the transaction role of money in the explanation of currency substitution. Some recent papers are beginning to explore this issue. Whereas most of them postulate additional frictions to the search process, like government intervention or asymetric information, we choose to focus on the trading pattern of monies. Apart from its quantity M_i, each money will be defined by γ_i and θ_i which respectively account for the rate of return and the bargaining power of money i. This allows us to discuss what strong and weak currencies may be in this framework. This also provides the opportunity to study the link between the transaction pattern of the monies, their sphere of activity and their prices. This modelling is motivated by the fact that in many dual currency economies monies differ not only in their real rate of return but also in their bargaining power. To do so we extend the work of Rupert, Schindler and Wright (2001) to multiple currency environement.

Our main results are the following : 1) The weakening of the bargaining power of an already circulating money favours the circulation of a strong currency by increasing the opportunity cost of using it merely as a store of value, and so decreases its real rate of return which was preventing it from circulating. 2) The weakening of the rate of return of an already circulating money produces the opposite effect since it makes the strong currency relatively more useful as a store of value than as a medium of exchange. 3) We can draw two opposite relationships between the value of money 2 and the value of money 1, depending on the maximum value of the producer's surplus. In the first relationship there are some values of money 1 for which money 2 does not circulate. Conversely, in the second case, money 2 will circulate whatever the price of money 1, but cannot hold intermediate values.

The rest of the paper is organized as follows. Section 2 briefly reviews previous literature on search theoretic models of multiple currencies. Section 3 presents the basic structure of the model. Section 4 tackles the determination of each money's value. Section 5 uses the results of the previous section to discuss stylized facts of dual currency economies like dollarization or Gresham's law. In section 6 we try to draw a general relationship between the value of money 2 and the value of money 1. Section 7 concludes. Section 8 includes diagrams.

2. Related literature

Multiple currency economies has been a growing operative field of random matching models[2]. Previous works either focus on two-country, two money models, or are confined to one country with two monies. Belong to the first set Matsuyama, Matsui and Wright (1993), Zhou (1997), Trejos and Wright (2001), Curtis and Waller (2001) and Ravikumar and Wallace (2001). These authors all need to build up some kind of asymetry between traders of the two countries. Thus each money can play a specific role according to the kind of meeting in which it is used. For example, in Matsuyama and alii (1993) or Trejos and Wright (2001), meetings between people

[2] For an introduction to dual monetary economies in search models see Craig and Waller (1999).

from the same country happen more frequently than meetings between people from different countries. Zhou (1997) defines people from one country by a shock preference. Waller and Curtis (2001) study how currency restrictions and government transaction policy affect the values of currencies in this environment. Ravikumar and Wallace (2001) show that a uniform currency eliminates some inferior equilibria - in the sense of low output levels - due to the coexistence of home and foreign money.

A second set of articles tackles the problem in a one country model. Some of them use indivisible goods models to study the coexistence of two monies, one of them being illegal or about to be replaced by a new one. Belong to this first subset Kullti (1996), Green and Weber (1996), Lotz and Rocheteau (2001). Curtis and Waller (2000) and Lotz (2001) add endogeneous determination of prices through bargaining. The main result of these texts is to show that one old or illegal currency may still circulate despite penalty as long as it reduces search frictions. Besides, Curtis and Waller show that the value of the legal currency depends on the value of the illegal one. Another group of articles gets down to the puzzling issue of Gresham's law. Here we find Velde, Weber and Wright (1999) and Li (2001). The first paper introduces asymetric information on the quality of the competing monies and shows that Gresham's law is one possible outcome of their model. Li (2001) uses government transaction policy to show that Gresham's law may be one possible equilibrium as soon as the government favours the weak currency through exchange rate policy. This last paper develops previous results of Li and Wright (1998) by introducing the possibility of currency trading in the model : '*government policy can affect private agent's incentives to such an extent that the market exchange ratio contradicts what implied by the intrinsic content of coins'*. Finally Camera, Craig and Waller (2001) focus on the dollarization process and how government policy acts upon the extent of dollarization. In this last model money is divisible since agents are allowed to hold currency portfolios. Kocherlakota and Krueger (2000) demonstrate that when agents have heterogeneous preferences over goods it may be socially beneficial to have multiple currencies in so far as buyers can more efficiently signal their preferences over goods to sellers.

In this paper we extend the work of Li (2001) and Velde, Weber and Wright (1999) on Gresham's law. We do not add new frictions to the basic search framework, so there is no currency bias, but explore how transactional and intrinsic features of the monies compete and affect each other's sphere of circulation and prices. To do so we apply the generalized search framework provided by Rupert & alii[2001] to dual monetary economics.

3. The basic structure of the model

The environment is much the same as the one in Trejos and Wright[1995] and Rupert, Schindler and Wright[2001]. The economy is populated with a $[0, 1]$ continuum of infinitevely-lived agents. Time is continuous and unbounded. There are N distinct, perfectly divisible but nonstorable goods and N types of agents with equal population. Each agent is specialized in consumption and production according to the following technology : a type j agent consumes good j and produces good $j+1$ (modulo N), for $j = 1, 2, .., N$. Agents meet each other bilaterally according to an anonymous random matching process with Poisson arrival rate β. This implies that one agent meets another agent of a particuliar type with probability $1/N$. In what follows we set $\beta/N = \alpha$ which measures the severity of the trading frictions. This specification makes double coincidence of wants impossible. Agents must consume in order to produce. This assumption makes it impossible to store more than one object in inventory. No agent consumes what he produces, and so they want to trade.

3.1. The competing monies

Trade is conducted through money. We assume that there are two candidate monies for the role of medium of exchange, money 1 and money 2. The proportion of agents holding money i is M_i $(i = 1, 2)$ with $M = M_1 + M_2 < 1/2$. Both are indivisible and yield a positive rate of return $\gamma_i > 0$ which can be interpreted as the instant utility of holding money i[3]. There are many ways to interpret γ_i. It can first be seen as money i's intrinsic value. For example in a metallist context money 1 and money 2 can both be gold coins but if $\gamma_2 > \gamma_1$ then money 2 is sounder or of better quality than money 1. γ_i can also account for the reverse probability of money i to be devaluaed in future. $\gamma_2 > \gamma_1$ then means that money 1 is more likely to be devalued than money 2. In short γ_i behaves like money i's rate of return[4]. Both monies entail bargaining upon their purchasing power, i.e. the quantities that will be exchanged against them. If one unit of money i is exchanged against q_i then we say that q_i is the price of money i. q_i is determined by bilateral bargaining in every single coincidence of wants meeting between a producer and a money i holder who desires the production good of his contracting party. Since agents of the same type are identical, endogeneous prices are the same for everybody. Each bargaining between a money i holder and a producer is directed by the parameter θ_i which represents money i's bargaining power[5]. The bargaining power of one money in exchange comes from its reputation. It is an exogeneous parameter. For instance, using euros for trade in Eastern Europe can give you an advantage over people using the home currency. This advantage stems from people's confidence in what they think is a stronger currency. Finally each money can be defined by a couple (γ_i, θ_i). Someway γ_i refers to the store of value's role of money and θ_i to its function of medium of exchange.

Subsequently we will say that one money gets weaker as soon as γ_i or θ_i becomes smaller. That is, we say that one money may loose its strength through a decline in its rate of return or a decline in its bargaining power. Conversly one money gets

[3] Money is a special good in this model since it cannot be destroyed by consumption and yields a positive rate of return whoever holds it.

[4] γ was first introduced in search monetary theory by Li and Wright (1998). It is said to be the instantaneous utility from holding money. The authors interpret a negative γ as a "storgae cost" like inflation. Li (2001) tells a different story : this instantaneous utility "*comes from possession of metal or selling the specie in world market.[...] Money holders buy goods from the world market in which coins are accepted by weight. There is a probability of meeting foreign traders with whom money holders can trade and derive utility from consuming the goods*". Besides the difficulty to give content to γ_i, introducing an instantaneous utility of holding money has a teleological interest since it amounts to introducing a new constraint for the circulation of each money. The utility brought by money in exchange always have to overcome this direct utility of holding money..

[5] Then $1 - \theta_i$ represents the bargaining power of the producer.

stronger when γ_i or θ_i increases. The major difference we conjecture between money 1 and money 2 is as follows: for our purpose we assume that money 2 holders always extract the entire surplus from trade whereas money 1 holders must share the surplus. That is to say we set $\theta_2 = 1$ and let θ_1 vary between 0 and 1. By doing this we want to capture the idea that money 2 behaves like a strong currency which suffers no doubt upon its purchasing power. Conversely money 1's bargaining power is subject to fluctuations and may be less than 1.

3.2. The exchange process

In accordance with the above specification each agent holds either one unit of money - money 1 or money 2 - or a production opportunity. Whenever there is a single coincidence of wants meeting between a producer a money i holder both agents must decide whether or not to trade. They bargain over q_i maximizing the surplus from trade according to the bargaining rule. When a producer meets a money 1 holder and if they want to trade then q_1 is produced and exchanged. q_1 depends heavily on θ_1 and if $\theta_1 < 1$ then both trading partners benefit from trade. It is also assumed that buyers trade the whole unit of money since money is indivisible. For money 2 holders the bargaining process takes the form of a take-it-or-leave-it offer made by the buyer. The seller accepts the offer if made no worse off by accepting. Thus the buyer gets the whole surplus from trade and leaves the seller indifferent between accepting or rejecting the offer. When agent j consumes q_i units of his consumption good he enjoys utility $u(q_i)$. The utility function is strictly increasing and twice differentiable on $[0, \infty[$, $u(0) = 0$, $u'(0) = \infty$ and $u''(q_i) < 0$. When a producers produces q_i for the money i holder he suffers a disutility cost equal to $c(q_i)$ with $c(0) = c'(0) = 0$. $c'(q_i) > 0$ and $c''(q_i) > 0$. We assume that there is a unique $\hat{q}_i$ such that $u(\hat{q}_i) = c(\hat{q}_i)$. Note that $u(q_i) > c(q_i)$ if and only if $q_i < \hat{q}_i$. All costs are expressed in terms of disutility. See figure 8.1 in section 8.

3.3. Miscellaneous

The Nash bargaining environment that we adopted here is such that all offers are made so that they are accepted in equilibrium. As Δ the delay between rounds of negociation goes to 0 the Nash solution to our bargaining problem takes the form of a maximand. Thus in this model the frictions come only from the decentralized exchange process and the threat of not finding a trading partner. Noting r the rate at which agents discount time, γ_i, θ_i, M_i and r are the unique parameters that drive the economy to potential steady states equilibria.

There are two trading situations in our economy. In the first one a money holder meets a producer without anything in hand. Whether the money holder is a type 1 or 2 will influence the trade process. In the second trading situation two money holders meet each other. Then there may be currency exchange. We rule out this last kind of trading situation by simply assuming that two distinct money holders part without trading and at no cost whenever they meet. This rules out currency trading.

4. The value of the monies

Let V_i be the expected lifetime utility of an agent holding money i and V_0 the equivalent for a producer. We can now write Bellman's equations for our three kinds of economic agents, producers and both kinds of money holders. We focus on steady states where all endogeneous variables are constant over time. If $q_i = Q_i$ is taken as given then Bellman's equations are:

$$rV_0 = \alpha M_1 \max\left[V_1 - V_0 - c(Q_1), 0\right] + \alpha M_2 \max\left[V_2 - V_0 - c(Q_2), 0\right] \quad (1)$$
$$rV_1 = \gamma_1 + \alpha(1-M)\max\left[u(Q_1) + V_0 - V_1, 0\right] \quad (2)$$
$$rV_2 = \gamma_2 + \alpha(1-M)\max\left[u(Q_2) + V_0 - V_2, 0\right] \quad (3)$$

The threat point of the bargaining process is equal to V_i in our model since we allow for the possibility of meeting other trading partners during the period of delay Δ after a rejected offer.

The participation constraints imply that:

$$V_1 - V_0 - c(q_1) \geq 0 \quad (4)$$
$$V_2 - V_0 - c(q_2) \geq 0 \quad (5)$$
$$u(q_1) + V_0 - V_1 \geq 0 \quad (6)$$
$$u(q_2) + V_0 - V_2 \geq 0 \quad (7)$$
$$V_i \geq 0 \quad (8)$$

Definition 1. *A symetric steady state dual monetary equilibrium is a vector of value functions $\vec{V} = (V_0, V_1, V_2)$ and quantities $\vec{q} = (q_1, q_2)$ such that:*

(i) $\vec{V}$ *satisfies* (1) *to* (8)
(ii) $\vec{q}$ *satisfies* (4), (5), (6) *and* (7)
(iii) $q_i \geq 0$

If (6) does not hold then we have a single currency equilibrium with only money 2 being used as a medium of exchange. If (7) does not hold then we have a single currency equilibrium only money 1 being used as a medium of exchange. If both (6) and (7) do not hold then both monies are hoarded.

Note that money cannot be dropped for it yields at least a postive rate of return γ_i. If money i does not circulate it means that agents prefer to hoard their cash and enjoy lifetime utility γ_i/r rather than spend it.

Let us consider the following trading scenario. We assume that, in every single coincidence of wants meeting, money 2 is always regarded as a foreign strong currency. The easiest way in our framework to model this trading pattern is to set $\theta_2 = 1 \geq \theta_1$. That is to say money 2 is stronger than money 1 in the sense of higher bargaining power. The implication is that in every single coincidence of wants meeting between a money 2 holder and a producer, money 2 holders always extract the entire surplus from trade leaving the producer indifferent between trading or not. Things are different for money 1. Let us assume that money 1 is the home currency. The crux of this paper is that we make its bargaining power (θ_1) and its rate of return (γ_1) vary, and study how such variations affect the main outcomes of the model: prices, the circulating pattern and the interactions between money 1 and money 2.

The rest of the section will be divided in two parts. In the first one we solve the model for q_1. In the second subsection we solve the model for q_2[6]. The following lemma and propositions should be regarded as transitonal results. The main results are discussed in section 5 and section 6.

4.1. The value of money 1

This model features divisible goods and people choose the quantity they will exchange through bargaining using the generalized Nash product. Then we have:

$$q_1 = \arg\max \left[u(q_1) + V_0 - V_1\right]^{\theta_1} \left[V_1 - V_0 - c(q_1)\right]^{1-\theta_1}$$

and

$$q_2 = \arg\max \left[u(q_2) + V_0 - V_2\right]^{1} \left[V_2 - V_0 - c(q_2)\right]^{0}$$

subject to the constraints $q_i \geq 0, \Delta_1^i = u(q_i) + V_0 - V_i \geq 0$ and $\Delta_0^i = V_i - V_0 - c(q_i) \geq 0$. The buyer-take-all hypothesis we have made for money 2 holders simplifies the determination of q_2 since now $c(q_2) = V_2 - V_0$. Then the model becomes:

$$\begin{aligned} rV_0 &= \alpha M_1 \max\left[V_1 - V_0 - c(Q_1), 0\right] \\ rV_1 &= \gamma_1 + \alpha(1-M)\max\left[u(Q_1) + V_0 - V_1, 0\right] \\ rV_2 &= \gamma_2 + \alpha(1-M)\max\left[u(Q_2) + V_0 - V_2, 0\right] \end{aligned}$$

Taking as given the Q_i that other agents are trading, any two agents bargain over the q_i they will exchange for money. In equilibrium $Q_i = q_i$. The first order condition for the Nash product on q_1 can be written :

$$e(q_1) = \theta_1 \left[V_1 - V_0 - c(q_1)\right] u'(q_1) - (1-\theta_1)\left[u(q_1) + V_0 - V_1\right] c'(q_1) = 0$$

A solution q_1 to $e(q_1)$ would be an unconstrained equilibrium if no other constraint Δ_1^1 or Δ_0^1 held with equality. But another type of monetary equilibrium can occur: suppose one constraint binds at $q_i = \bar{Q}_i > q_i^*$ the unconstrained equilibrium. If the unconstrained solution to the Nash bargaining problem taking $\bar{Q}_i$ as given is greater than $\bar{Q}_i$ then $\bar{q}_i = \bar{Q}_i$ is a constrained equilibrium.

Lemma 1. *If any monetary equilibrium q_1 exists, $q_1 \in [\bar{q}_1^g, \bar{q}_1^f]$ if $\gamma_1 \leq r.c(\hat{q}_1)$ and $q_1 \in [\bar{q}_1^g, \bar{q}_2^g]$ if $\gamma_1 \geq r.c(\hat{q}_1)$. If r is small enough and γ_1 is big enough then money 1 will not circulate and will be hoarded.*

[6] Note that this is the peculiar bargaining structure of money 2 that permits to solve for the value of each money sequentially.

Proof: Let us write $f(q_1) = (r + \alpha(1 - M) + \alpha M_1)\,\Delta_0^1 = (1-M)\,u(q_1) - (r+1-M)\,c(q_1) + \gamma_1$ and $g(q_1) = (r + \alpha(1 - M) + \alpha M_1)\,\Delta_1^1 = M_1\,[u(q_1) - c(q_1)] + r.u(q_1) - \gamma_1$. f is first increasing then decreasing. $f(0) = \gamma_1$ and $f(q_1) \geq 0$ if $q_1 < \bar{q}_1^f$. Note that $\bar{q}_1^f < \hat{q}_1$ if $\gamma_1 \leq r.c(\hat{q}_1)$ since $f(\hat{q}_1) = \gamma_1 - rc(\hat{q}_1)$. Now consider $g(q_1)$. $g(0) = -\gamma_1$. g is first increasing then decreasing. $g(q_1) \geq 0$ if $q_1 \in [\bar{q}_1^g, \bar{q}_2^g]$ and if r is not too small and γ_1 not too big. Note that $\bar{q}_1^g < \hat{q}_1$ since $g(\hat{q}_1) > 0$ when $f(\hat{q}_1) < 0$, i.e. when $\bar{q}_1^f < \hat{q}_1$. $\bar{q}_2^g < \hat{q}_1$ if $r.u(\hat{q}_1) = r.c(\hat{q}_1) < \gamma_1$ since $g(\hat{q}_1) = u(\hat{q}_1) - \gamma_1$. Note that g shifts down with an increase in γ_1 or when r decreases. Therefore if $\gamma_1 \leq r.c(\hat{q}_1)$, then any $q_1 > 0$ such that $e(q_1) = 0$ have to belong to $[\bar{q}_1^g, \bar{q}_1^f]$. Otherwise, if $\gamma_1 > r.c(\hat{q}_1)$ then $\bar{q}_1^f > \hat{q}_1$ but $\bar{q}_2^g < \hat{q}_1$, so any $q_1 > 0$ such that $e(q_1) = 0$ have to belong to $[\bar{q}_1^g, \bar{q}_2^g]$. ■ f and g are drawn on figures 8.2 and 8.3[7].

Lemma 1 says that, as γ_1 grows and r decreases the interval of values for money 1 narrows. There are also some values of γ_1 and r for which money 1 does not circulate. People are sufficiently patient and holding money 1 is so lucrative that buyers stay home. Both cases will now be considered.

4.1.1. First case: $\gamma_1 \leq r.c(\hat{q}_1)$ and $q_1 \in [\bar{q}_1^g, \bar{q}_1^f]$

In this configuration, money 1 does not have too much intrinsic value or does not yield too much instantaneous utility.

Proposition 1. *If $\gamma_1 \leq r.c(\hat{q}_1)$ there are always two positive values in exchange for money 1, one of them being constrained and equal to $\bar{q}_1^g$. As θ_1 departs from 0 the unconstrained solution grows from $\bar{q}_1^g$ to reach $\bar{q}_1^f$ when $\theta_1 = 1$. See figure 4.1 and figure 8.4.*

Proof: $e(q_1) = \theta_1\,[V_1 - V_0 - c(q_1)]\,u'(q_1) - (1 - \theta_1)\,[u(q_1) + V_0 - V_1]\,c'(q_1)$. First notice that $f(q_1) = -g(q_1) + [u(q_1) - c(q_1)]$. Let us begin with the unconstrained equilibrium. $e(\bar{q}_1^g) = \theta_1 f(\bar{q}_1^g) u'(\bar{q}_1^g) \geq 0$, holding with equality when $\theta_1 = 0$. $e(\bar{q}_1^f) = -(1 - \theta_1) g(\bar{q}_1^f) c'(\bar{q}_1^f) \leq 0$ holding with equality when $\theta_1 = 1$. Since $e(0) > 0$ and e is continuous on $[0, \infty)$, there exists at least one unconstrained monetary equilibrium $q_1 \in [\bar{q}_1^g, \bar{q}_1^f]$ such that $e(q_1) = 0$. Since $\frac{de}{d\theta_1} > 0$ for any $q_1 \in [\bar{q}_1^g, \bar{q}_1^f]$ we can conclude that e shifts up with an increase in θ_1. Now consider the values of q_1 such that the constraints bind. Let us note $e(\bar{Q}_1^g, q_1)$ the first order condition to the Nash bargaining problem when $\bar{Q}_1^g$ is taken as given. Since $e(\bar{Q}_1^g, 0) < 0$ and $e(\bar{Q}_1^g, \bar{q}_1^g) > 0$ we can conclude that the unconstrained solution to the Nash bargaining problem when $\bar{Q}_1^g$ is taken as given is greater than $\bar{Q}_1^g$. Therefore $\bar{q}_1^g = \bar{Q}_1^g$ is a constrained equilibrium. Since $e(\bar{q}_1^g) = \theta_1 f(\bar{q}_1^g) u'(\bar{q}_1^g) \geq 0$, holding with equality when $\theta_1 = 0$, this implies that there there exists a constrained equilibrium in $q_1 = \bar{q}_1^g$ as soon as $\theta_1 > 0$. Also note that $e(\bar{Q}_1^f, 0) > 0$ and that $e(\bar{Q}_1^f, \bar{q}_1^f) = -(1-\theta_1) g(\bar{q}_1^f) c'(\bar{q}_1^f) \leq 0$ holding with equality when $\theta_1 = 1$. We can conclude from above that $\bar{q}_1^f$ is not a constrained equilibrium. ■

[7] The suffix f or g over the bound $\bar{q}_1$ refers to the constraint that give birth to this bound.

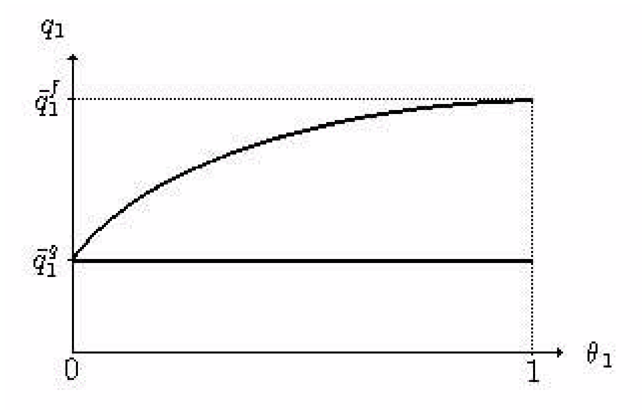

Figure 4.1: Value of money 1 when $\gamma_1 \leq r.c(\hat{q}_1)$

Proposition 1 shows that money 1's value in exchange increases with its associated bargaining power[8].

4.1.2. Second case: $\gamma_1 \geq r.c(\hat{q}_1)$ and $q_1 \in [\bar{q}_1^g, \bar{q}_2^g]$

Now money 1 yields sufficient utility for γ_1 to be at least equal to $rc(\hat{q}_1)$.

Proposition 2. *There exist $\bar{\theta} \in [0,1]$ which depends on r and γ_1 such that the following is true : for $\theta_1 < \bar{\theta}$, there is one constrained equilibrium equal to $\bar{q}_1^g$ and two unconstrained monetary equilibria. For $\theta_1 > \bar{\theta}$ only the constrained monetary equilibrium subsists. See figure 4.2 and figure 8.5.*

Proof: $e(\bar{q}_1^g) = \theta_1 f(\bar{q}_1^g)u'(\bar{q}_1^g) \geq 0$, with equality when $\theta_1 = 0$. $\bar{q}_1^g$ is still a constrained equilibrium. $e(\bar{Q}_2^g, 0) < 0$ and $e(\bar{Q}_2^g, \bar{q}_2^g) > 0$. That is, the unconstrained solution when $\bar{Q}_2^g$ is taken as given is less than $\bar{q}_2^g$. Then $\bar{q}_2^g$ is not a constrained equilibrium. $e(\bar{q}_2^g) = \theta_1 f(\bar{q}_2^g)u'(\bar{q}_2^g) \geq 0$, with equality when $\theta_1 = 0$. Also note that $e(q_1)_{|\theta_1=0} = -g(q_1)c'(q_1) \leq 0$, with equality when $q_1 = \bar{q}_1^g$ or $\bar{q}_2^g$, and that $e(q_1)_{|\theta_1=1} = f(q_1)u'(q_1) > 0 \, \forall \, q_1 \in [\bar{q}_1^g, \bar{q}_2^g]$ since $f(q_1) > g(q_1)$ on $[\bar{q}_1^g, \bar{q}_2^g]$. Since $\frac{de}{d\theta_1} = f(q_1)u'(q_1) + g(q_1)c'(q_1) > 0$ for any $q_1 \in [\bar{q}_1^g, \bar{q}_2^g]$, then we can conclude that there exists a $\bar{\theta} \in [0,1]$ such that for any $\theta_1 < \bar{\theta}$ there exists one unconstrained monetary equilibrium whereas for $\theta_1 > \bar{\theta}$ the unconstrained equilibrium vanishes. ■

4.1.3. Discussion

The above propositions show that the value of money 1 is linked to its bargaining power θ_1 in a way that depends on its intrinsic value γ_1 and the rate of time preference

[8] Note that contrary to the model K in Rupert & alii (2001) with $y > 0$ and $T_m = V_m$, the unconstrained solution is unique and increasing. This comes from the fact that money holders enjoy an *ex ante* advantage over the producers because of γ_1.

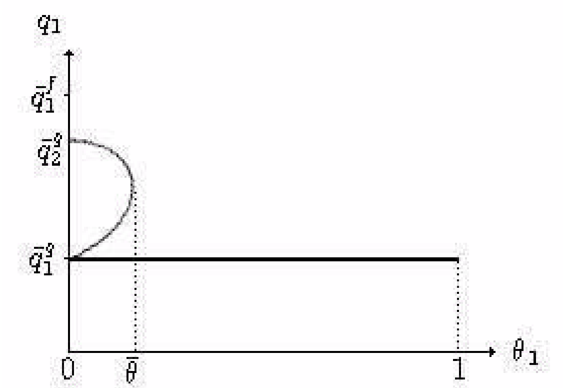

Figure 4.2: Value of money 1 when $\gamma_1 \geq r.c(\hat{q}_1)$

r. In an economy where money 1 does not have too much intrinsic value and where people value the present at least at $r = \gamma_1/c(\hat{q}_1)$, then money 1 increases its value in exchange when its bargaining power grows. Conversely, when the intrinsic value of money 1 is too high, money 1 will circulate only for some values of θ_1 below $\bar{\theta}$. See figure 4.2. Beyond $\bar{\theta}$ the holders of money 1 ask for too much production so that producers will not make the deal. Note that the value of money 1 does not rely on what happens for money 2. This is due to the peculiar bargaining strucuture we adopted for money 2.

We will see that things are very different for money 2 since money 2 holders must take into account the utility already provided by money 1 in exchange. The assumption that θ_1 is neither 1 (buyer-take-all) nor 0 (producer-take-all) implies that q_2 depends on q_1.

4.2. The value of money 2

We conjecture that money 2 holders always extract the entire surplus from trade when matched with producers. Then $c(q_2) = V_2 - V_0$ and money 2 will circulate if one can find a q_2 such that $rc(q_2) + rV_0 = \gamma_2 + \alpha(1 - M)\left[u(q_2) - c(q_2)\right].$

Proposition 3. *If $rV_0 < \gamma_2 - rc(\hat{q}_2)$ then money 2 does not circulate. If $\gamma_2 > rV_0 > \gamma_2 - rc(\hat{q}_2)$ then money 2 has a unique value in exchange. If $rV_0 > \gamma_2$ and if rV_0 is not too big then money 2 may have two values in exchange. See figure 4.3.* '

Proof: First note that money 2 will circulate if $u(q_2) + V_0 - V_2 \geq 0$ which is equivalent to $u(q_2) - c(q_2) \geq 0$, i.e. $q_2 < \hat{q}_2$. Note that $q_2 < \hat{q}_2$ requires that $\gamma_2 < rc(\hat{q}_2) + rV_0$, or $rV_0 > \gamma_2 - rc(\hat{q}_2)$ which could be either positive or negative. Now let us write $L(q_2) = rc(q_2) + rV_0 - \gamma_2 - \alpha(1 - M)\left[u(q_2) - c(q_2)\right]$. $L(0) = rV_0 - \gamma_2$ and $L(\hat{q}_2) = rc(\hat{q}_2) + rV_0 - \gamma_2$. Then if $rV_0 < \gamma_2$, $L(0) < 0$ and $L(\hat{q}_2) > 0$ if $rc(\hat{q}_2) > \gamma_2 - rV_0$. Since L is continuous on $[0, \hat{q}_2]$, the intermediate value theorem tells us that

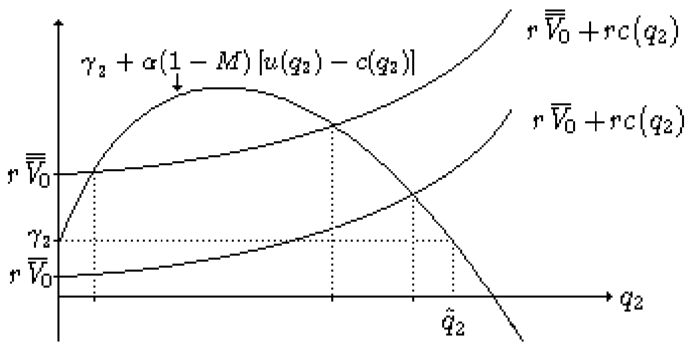

Figure 4.3: Illustration of proposition 3.

there exists at least on $q_2^* \in [0, \hat{q}_2]$ such that $L(q_2^*) = 0$. If $rV_0 > \gamma_2$ then $L(0) > 0$ and $L(\hat{q}_2) > 0$. But $L'(0) \to -\infty$ and since there is a unique q_2 such that $L'(q_2) = 0$, we can conclude that for values of rV_0 bigger than γ_2 but not too big, there exist two values q_2^* and q_2^{**} such that $L(q_2^*) = 0$ and $L(q_2^{**}) = 0$.■

4.2.1. Discussion

Proposition 3 says that when the rate of return on money 2 is too high, i.e. when $\gamma_2 > rV_0 + rc(\hat{q}_2)$, money 2 do not change hands. When money 2's rate of return has "good values", between rV_0 and $rV_0 + rc(\hat{q}_2)$, then there is one equilibrium value for money 2 in exchange. At last, when γ_2 is smaller than the flow return to a producer, then money 2 has two possible exchange values.

5. Currency competition, dollarization and Gresham's law

In this section we present the main results of the model. We study how modifications in the parametric values of money 1 affect the circulation of money 2 and its price. We will use proposition 3 which has several interesting implications for our purpose.

Proposition 4. *Dollarization*. *The weakening of money 1 through a decline in its bargaining power favors the circulation of money 2: bad money attracts good money.*

Proof: Let us start from a situation where money 1 is the only money in circulation and where $\theta_1 = 1$. In this configuration money 1 behaves like a strong currency for buyers extract the entire surplus from trade. Producers do not gain anything from trade, $rV_0 = 0$, and they are indifferent between accepting or rejecting money 1. A necessary condition for money 1 to circulate when $\theta_1 = 1$ is $\gamma_1 < rc(\hat{q}_1)$. This inequality means two things. First the nominal rate of return on money 1 should not be too high if we want money 1 to circulate rather than being stored. Second people should not be too patient, i.e. people cannot wait too much time for a trading opportunity. Symetrically, a sufficient condition for money 2 not to circulate is that $\gamma_2 > rc(\hat{q}_2)$ which means that storing money 2 yields too much utility to be used in exchange and that people are not impatient enough to use it. Let us now consider that the bargaining power of money 1 begins to fade. In a metallist system we would say that people suspect the sovereign of being about to debase money 1[9]. As soon as the bargaining power of money 1 declines producers start benefiting from trade with money 1 holders since the surplus from trade $u(q_1) - c(q_1)$ is now split between the two trading partners. A direct consequence is that rV_0 starts growing. The condition for money 2 *not* to circulate now becomes $\gamma_2 > rc(\hat{q}_2) + rV_0$ which is less easy to match than when money 1 was circulating with a bargaining power equal to 1. Money 2's rate of return now have to be greater than the flow return of a money 2 holder in exchange, $rc(\hat{q}_2)$, plus the flow return of a producer, rV_0. In other words when money 1 weakens through a decline in its bargaining power then it becomes more difficult for money 2 not to circulate. See figure 4.3.

Discussion: The interpretation is the following. As soon as the bargaining power of money 1 departs from 1 and starts decreasing, money 1 provides utility to both buyers and sellers. Then producers do not bargain with money 2 holders as they used to. They can now put forward the utility already brought by the circulation of money 1, rV_0. rV_0 behaves like the price money 2 holders have to pay if they want to use their money in exchange. In other words the weakening of money 1's bargaining power forces money 2 holders to lower the price they demand for money 2. Producers are now more eager to accept it and money 2 can circulate.

This result accounts for what is happening in most dual currency economies. Take a country where one domestic currency circulates without any doubt about its purchasing power. A foreign strong currency may circulate, but it is often confined to the role of store of value. As soon as the domestic currency looses part of its credibility the foreign currency can no longer rely on its high intrinsic content since it is now more remunerative to use it in exchange[10]. The decline of a national money contributes to monetize a currency that was used mainly as a store of value up to then[11].

[9] A decrease of θ_1 can be interpreted in a modern economy as well. Suppose Argentinian people expect the government to float the peso. We can expect peso's bargaining power to fall in daily transactions. Note that an *effective* debasement of money 1 would appear through a decline in γ_1.

[10] If we say that rV_0 represents the opportunity cost for a producer to meet a money 2 holder then we can rewrite the condition for money 2 not to circulate:$\gamma_2 - rV_0 > rc(\hat{q}_2)$ which means that the weakening of money 1's bargaining power lowers the rate of return of money 2 by rV_0. This lower rate of return will induce money 2 to circulate since now it is less rewarding to keep one's strong money home.

[11] In an economy where the use of one money does not hold any fixed cost there may be two equilibrium values for money, 0 and $q^* < \hat{q}$. As soon as there is such a fixed cost, like the opportunity cost of a double coincidence of wants meeting or the risk of being caught if it is an illegal currency, then money may have two values in exchange.One low value corresponds to the minimum value that money should buy in order to compensate this fixed cost. The high value is now less than q^* since the fixed cost is incorporated in the dynamic calculation. See for example Curtis and Waller (2000). This minimum value takes into account the fixed cost of using money. In our model we substitute the producer's opportunity cost of meeting a money 1 holder, which is a potential double coincidence of wants meeting, for the opportunity cost of meeting a money 2 holder with whom you will not gain anything. This lowers the value of money 2 in exchange as well as any likely double coincidence of

Note that when q_1 is too high (for some values of θ_1) then the opportunity cost of using money 2 in exchange becomes so high that money 2 can no longer play its role of medium of exchange. Producers will systematically refuse it in exchange since the use of money 1 is far more interesting. Money 2 holders do not exchange and enjoy lifetime utility γ_2/r.

Proposition 5. ***Gresham's law***. *The weakening of money 1 through a decline in its rate of return takes money 2 out of circulation: bad money drives out good money.*

Proof:Let us start from a situation with two circulating currencies. Both bargaining powers are fixed, $\theta_2 = 1$ and $\theta_1$can take one value between 0 and 1. Remember that $rV_0 = K.[(1-M)u(q_1) - (r+1-M)c(q_1) + \gamma_1]$ with $K = \frac{\alpha M_1}{r+\alpha M_1+\alpha(1-M)}$. So when γ_1 decreases so does rV_0. The lowering of the flow return of the producers put them in a inferior position when negociating with money 2 holders. The price that is then proposed is likely to be too high for producers to accept. Then money 2 stops circulating.

Discussion: The weakening of money 1's rate of return -or money 1's intrinsic value- makes it more difficult for money 2 to circulate. People prefer to store it rather to spend it. Money 1 prooves to be more useful in exchange than money 2. This result accounts for the mechanism known as Gresham's law. The explanation, however, is different that the ones that have been adopted in previous works. It does not rely on any fixed exchange rate between the two monies, nor does it depend on incomplete information about them. Gresham's law activates in this model because people compare the transactional utility and the store of value's role of the competing monies. Both monies can circulate but for some parametric values, in particular when the intrinsic value of one money is getting smaller, it is more useful to use one money in exchange and to spare the other one.

6. The relationship between q_2 and q_1

We can see from proposition 3 that, for fixed γ_1 and γ_2, the circulation of money 2 and its price depend on the value of money 1 since q_2 depends on rV_0 which is equal to $\frac{\alpha M_1}{r+\alpha M_1+\alpha(1-M)} * f(q_1)$. Therefore we are able to build a relationship between q_2 and q_1.

Let us write $\tilde{q}_1$ the value of money 1 that makes rV_0 maximum and $rV_0^1 = rV_0(\tilde{q}_1)$. See figure 8.2. Also note rV_0^2 the value of rV_0 above which money 2 does note circulate any longer. See figure 4.3. rV_0^1 depends on r and γ_1, decreasing with r and increasing with γ_1. rV_0^2 depends on r and γ_2, decreasing with r and increasing with γ_2. If we make the tractable assumption that $\gamma_2 = r.c(\hat{q}_2)$ then we can put forward the following proposition, altrough the proof is mainly graphical. See figure 8.7 and 8.8 for the construction of both relationships.

Proposition 6. *For $\gamma_2 = r.c(\hat{q}_2)$, if $\gamma_1 < r.c(\hat{q}_1)$ and if $rV_0^2 > rV_0^1$ then the relationship between the value of money 2 and the value of money 1 is depicted in figure 6.1.*

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wants meeting lowers the value of any circulating money.

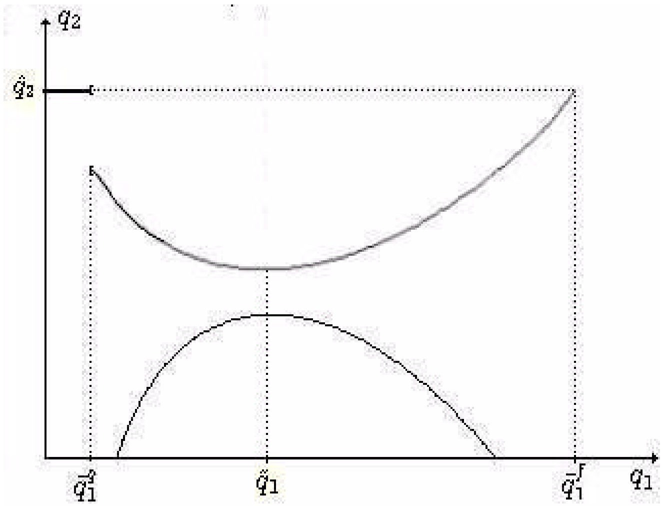


Figure 6.1: q_2 as a function of q_1 when $rV_0^2 > rV_0^1$

If $rV_0^2 < rV_0^1$ then the relationship between the value of money 2 and the value of money 1 is depicted in figure 6.2.

Commentary: Setting $\gamma_2 = r.c(\hat{q}_2)$ implies that when $rV_0 = 0$, $q_2^* = \hat{q}_2$. The fact that $rV_0^2 > rV_0^1$ or $rV_0^2 < rV_0^1$ will influence the kind of relationship between q_2 and q_1. In the first situation, that is to say when it is easy for money 2 to circulate since rV_0^2 is quite big, we can see that money 2 has value whatever q_1 may be. But in the second situation, when it is more difficult for money 2 to circulate since rV_0^2 is small, only some values of q_1, hence some values of θ_1, are compatible with the circulation of money 2.

Discussion: Let us interpret the figure 6.1. To do so we make q_1 vary from $\bar{q}_1^f$ to 0. For $\tilde{q}_1 \leq q_1 \leq \bar{q}_1^f$, the value of money 2 starts diminishing as soon as q_1 begins to depart from $\bar{q}_1^f$, i.e. when rV_0 begins to increase. If rV_0 goes on increasing then a second value for money 2 appears which increases along the growth in rV_0. But for $\bar{q}_1^g \leq q_1 \leq \tilde{q}_1$ a decrease of q_1 is now obtained through a decline in rV_0 since the present value of being a producer is an increasing function of q_1 when q_1 is low. Then the higher value of money 2 increases while the lower one decreases. At last when $q_1 < \bar{q}_1^g$, money 1 does not circulate. This enables money 2 holders to claim for the highest price then can get, that is $\hat{q}_2$. The reasoning is the same for the second relationship. See figures 8.3 and 8.4.

It is worth noting that, when $rV_0^2 > rV_0^1$, that is to say when it is easy for money 2 to circulate, money 2 circulates whatever the value of money 1. But when $rV_0^2 < rV_0^1$ there are some values of money 1 for which there is no value for money 2 in exchange. The explanation is that there are some intermediate values of money 1, around $\tilde{q}_1$, for which the surplus of the producer becomes so high that he will not make any deal

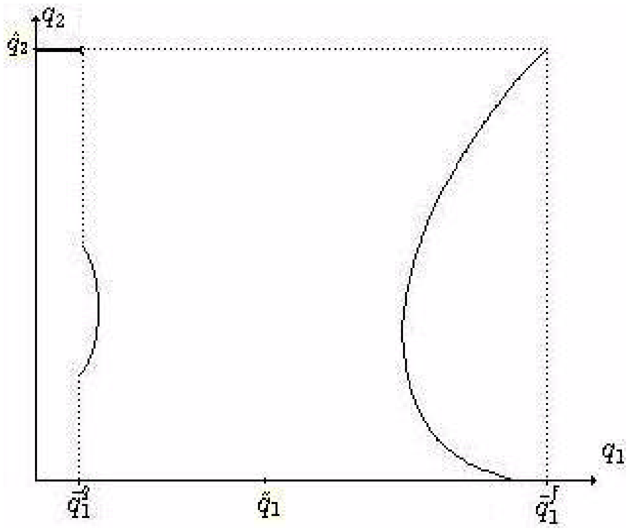


Figure 6.2: q_2 as a function of q_1 when $rV_0^2 < rV_0^1$

when meeting a holder of money 2.

7. Conclusion

We have tried to study how two distinct monies defined by their rate of return and bargaining power can circulate together. We found that the conditions under which the first money circulates may affect the circulation and the price of the other money. The explanation relies on the comparison between the transactional utility provided by the use of money in exchange and its utility as a store of value. Whether money 1 has low or high bargaining power changes the surplus of those of the producers matched with money 1 holders. Then money 2 holders have to take into account the utility brought by the circulation of money 1 to the producers. If money 2 holders were the only kind of agents holding money they could impose whatever price they wished. But now that one money is already circulating and allowing producers to make profitable exchanges money 2 holders cannot claim for too high a price, even though they get the entire surplus from trade because of the bargaining rule adopted here. Using this reasoning we were able to put forward propositions that fit either to the dollarization process or to Gresham's law. Thus we have a model that captures many features of dual currency economies where one foreign strong currency circulates alongside the domestic money whose purchasing power and confidence may be of great variability.

This paper may be impoved by going deeper into the building of the realtionship between q_2 and q_1. The results that are presented in section 6 are still under construction. Another way to precise the link between the two monies' value would be, among other things, to introduce the possibility for currency trading between money holders. It would then give rise to belief equilibria that could contradict the fundamentals of the competing monies.

8. Diagrams

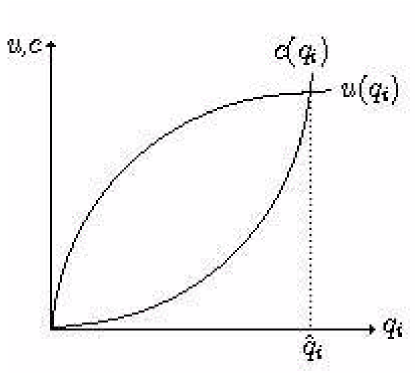


Figure 8.1: Utility and cost functions

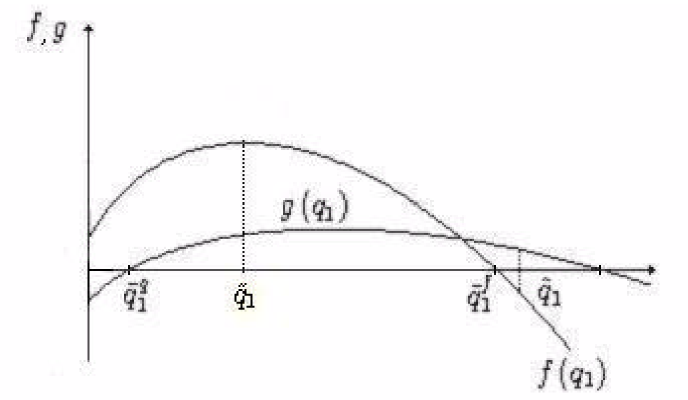
1. 

Figure 8.2: f and g when $\gamma_1 \leq r.c(\hat{q}_1)$

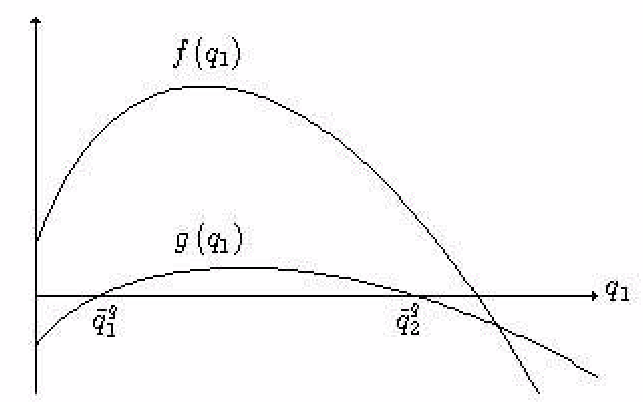


Figure 8.3: f and g when $\gamma_1 \geq r.c(\hat{q}_1)$

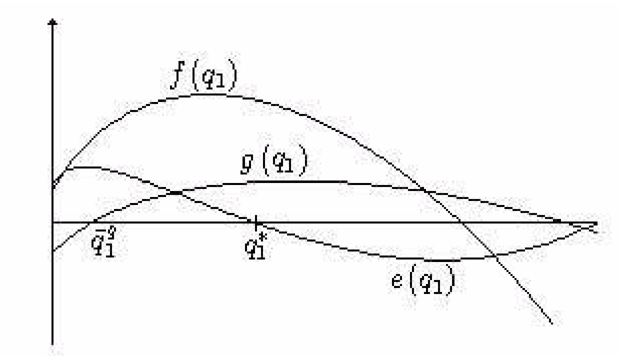


Figure 8.4: Value of money 1 when $\gamma_1 \leq r.c(\hat{q}_1)$

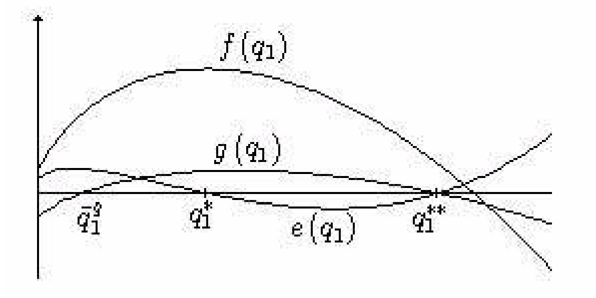


Figure 8.5: Value of money 1 when $\gamma_1 \geq r.c(\hat{q}_1)$

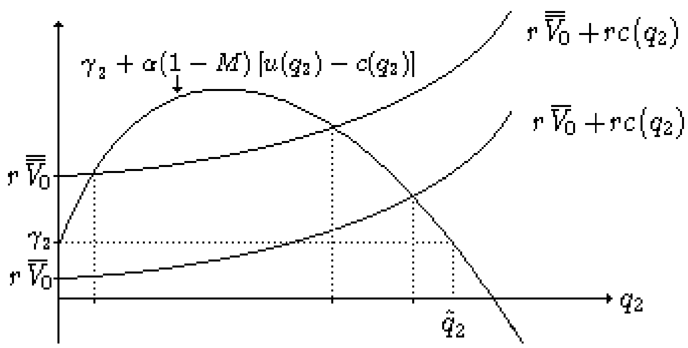


Figure 8.6: Illustration of proposition 3.

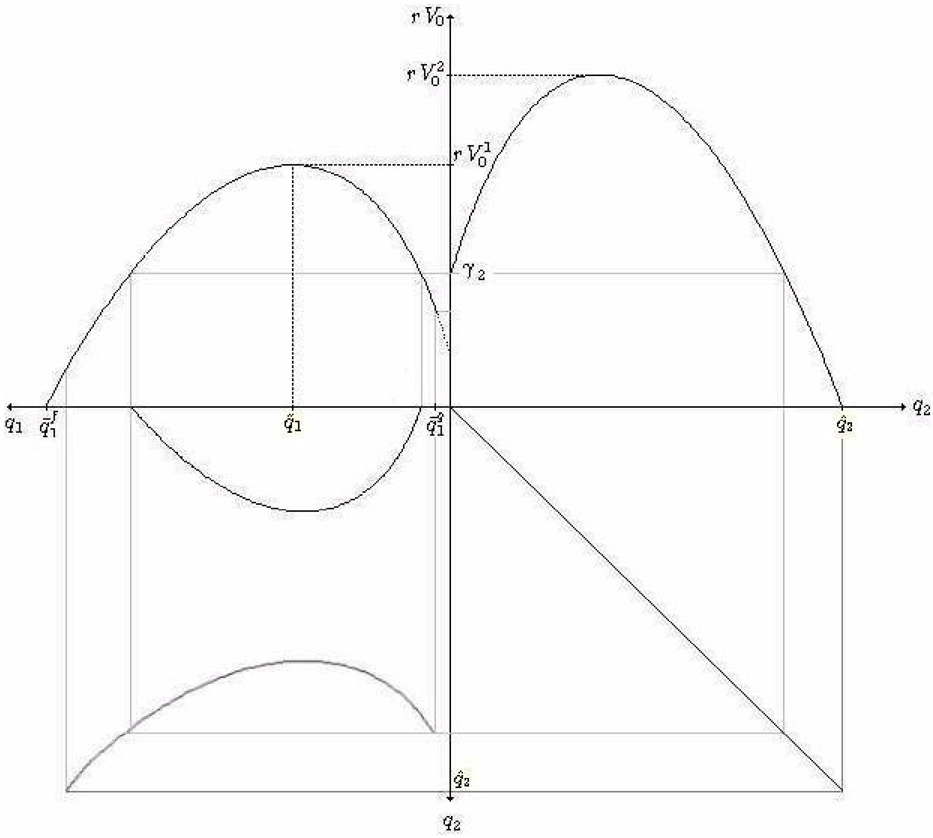


Figure 8.7: The construction of configuration 1.

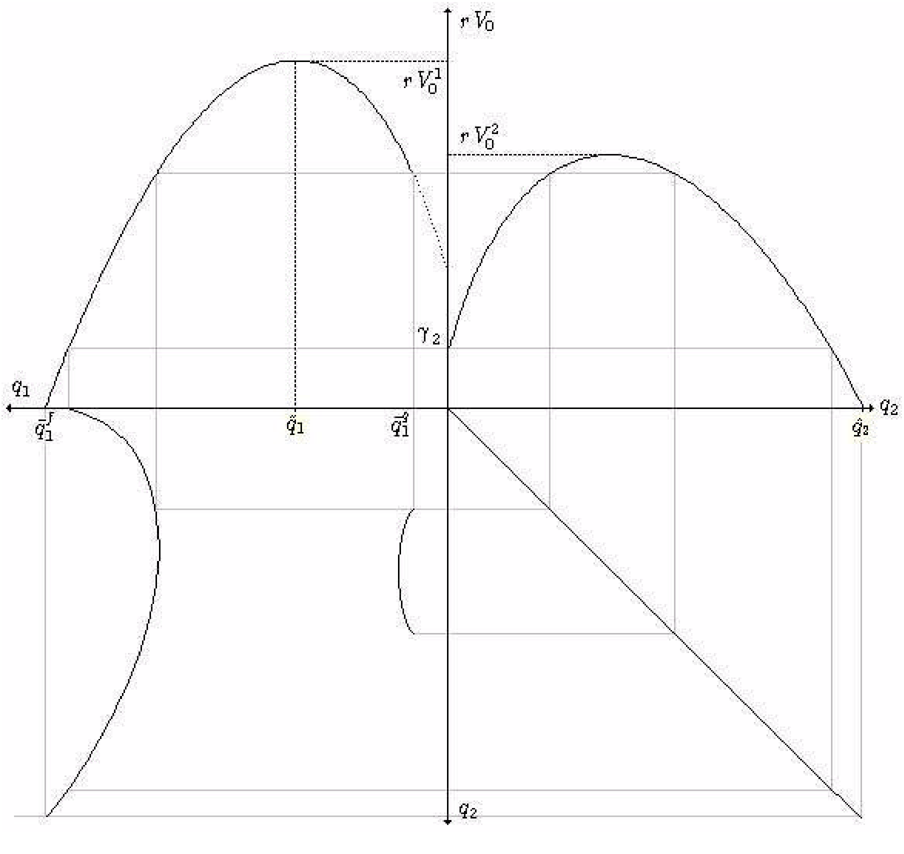


Figure 8.8: The construction of configuration 2.

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