Interbank Debt Networks, Systemic Bank Runs and the Domino Effect

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Abstract

In a Diamond & Dybvig's frame, banks embed in a network of interbank debts to face randomly located liquidity shocks. Banks endogenously choose a number of partners so as to be able to decentralize Pareto Optimal outcome. They are thus able to increase social welfare by a rise in investment in the produtive long term asset. The network structure, endogenously chosen by banks, tackles the issue of the unkown liquidity shock distribution, and avoids liquidity crises. As a consequence the rate of interest is driven to the risk free level. However, because of competion for funds, the structure chosen by banks does not maximize available liquidity in the network. As a consequence, the interbank market may turn unstable. A small and isolated unexpected liquidity shock may turn into a major systemic event. Formalizing the domino effect, crisis may then propagate along the financial network which can be forced to a generalised bank run. There is thus a trade-off between a risky interbank debt market and a safe payment mechanism which foregoes investment opportunities.

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Keywords: Network, Contagion, Bank

1 Introduction.

Financial distress and a propagation channel are both needed for contagious bank failures to occur. The first aspect has been highly scrutinized by economic theory, but the second one lacks of a convenient framework to analyze the consequences of interbank architecture on contagious bank runs. The way interbank architecture is organized is indeed essential to understand systemic risk to which banks are exposed.

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What we mean by interbank architecture refers at least to two different aspects: first the interbank debt market, and second the private interbank payment and clearing system. On the one side, interbank debt exchange plays a crucial role by channeling liquidity from institutions with a surplus of funds to those in need. Central Bank is not indeed the single liquidity provider to the financial system. Besides Central Bank interventions, financial institutions exchange funds through a completely private system of interbank short term loans. On the other side, a significant payment and clearing system has emerged outside the control of banking regulators and outside any Central Bank intervention trough which banks daily exchange huge amounts of liquidity. For e.g. the CHIPS network in the US functions as a private self-regulating interbank clearinghouse among financial institutions. SWIFT system is some kind similar in Europe to enable banks to directly compensate payments and exchange short term loans.

Consequently, banks are embedded within a network of financial relations which expands at the rhythm of financial globalization and liberalization[1]. This network of interbank relations expands with the development of financial flows between banks such as interbank loans, payment flows or all financial contracts between two financial institutions implying fund exchanges.

Being embedded in networks of interbank relations has unclear effect on the stability of financial systems. Indeed, thanks to interbank debts, financial institutions may be able to face their short term liquidity needs by borrowing from their partners. They may thus reduce the amount kept in short term reserves to face short term liquidity needs, and increase investment in long term productive assets. As a consequence interbank debt networks increase financial stability, and welfare by liquisity risk sharing. It's a standard diversification effect which drives idiosyncratic liquidity risk to the level of non-diversifiable market liquidity risk. However, in the mean time, banks are involved in a complex frame of interbank debt relations which expose them to systemic risk.

The questions we shall address are the following:

-first, what are the gains for a bank to embed in a network;

-second can bank endogenously choose the topology of the network they are embedded in so as to maximize depositors' welfare;

-third, how interbank network architecture induces systemic risk and may lead to contagious bank runs because of domino effect.

[1] Quantitatively, further to financial liberalization, the number of financial institutions carrying out interbank credit operations and the volumes of these operations have boomed. Those features have been particularly well documented by Furfine (2001 & 1999), or Bernard & Bisignano (2000). Rochet & Tirole (1996b) for example note "Over the past twenty years, the growing integration of financial markets, the development of new financial instruments, and the advance of computer technology have all contributed to a remarkable growth of financial activity in the main industrialized countries. One of the most significant consequence of this growth has been the unprecedented increase in the volume of trade on the large value interbank payment systems which itself has resulted in a massive increase in intra-day overdrafts on those systems". Documenting this fact, Rochet and Tirole cite figures extracted from Goodheart & Schoenmaker (1993). In the US, the daily volume of interbank flows amounts to USD bn 800. Consequently, banks are embedded within a continuously expanding network of financial relations.

To answer these questions we shall consider a standard stylized frame derived from Diamond Dybvig, were banks are operating within a network of debt contracts. Our main findings are the following.

In a framework where liquidity shock distribution is unknown, banks endogenously choose a number of partners so as to be able to decentralize Pareto Optimal outcome. Banks are thus able to increase social welfare by a rise in investment in the productive long term asset. The network structure, endogenously chosen by banks, tackles the risk stemming from the unknown liquidity shock distribution. Liquidity crises thus become a zero probability event. As a consequence the rate of interest is driven to the risk free level. However, because of competition for funds, the structure chosen by banks does not maximize available liquidity in the network. The chosen network is not the complete network. As a consequence, the interbank market may turn unstable in the following sense. A small and isolated unexpected liquidity shock may indeed turn into a major systemic event. Crisis may then propagate along the financial network which can be forced to a generalized bank run. Contagion can be defined here as the propagation of an isolated financial event -such as an isolated run- through a network of links. There is thus a trade-off between a risky interbank debt market and a safe payment mechanism which foregoes investment opportunities.

Despite their undisputable rise as autonomous financial players, very little attention has been devoted to interbank networks even in the considerable literature that developed on banking and banking risk management following Diamond and Dybvig's seminal work[2].

In fact, interbank market exposure appears only with Rochet & Tirole (1996a) where the authors raise the problem of moral hazard induced by lending / borrowing operations on the interbank market[3].

[2] The first models to be developed are indeed centered on single bank fragility. The canonical setup established by Diamond and Dybvig (1983) highlights an intrinsic banking fragility stemming from the role played by the bank as a maturity transformer. Any bank is indeed financed by short term deposits, while it finances long term investments while lending to the economy. This unmatched asset / liability maturity structure may expose banks to self-fulfilling bank-runs.

The sunspot run equilibrium hypothesis, made by Diamond and Dybvig, has been the first one to be dropped and replaced by the assumption that runs are consecutive to the release of new information about the viability of bank's investments. Main models under this hypothesis are Gorton (1985), Jacklin and Battacharya (1988), or Chari and Jaghanathan (1988). Based on an information those runs are "efficient" while the one highlighted by Diamond and Dybvig are not. Extensions of the bank run literature to multiple bank systems begins with Garber and Grilli (1989). In their model contagion among banks is channeled through income effect experienced by depositors. De Bandt (1985) and Temzelides (1997) rely both on a revision of depositors expectations on the expected return on bank's asset. The expectation revision follows either an aggregate and idiosyncratic shock (De Bandt) or reveals consumers learning on past bankruptcies of the banking system in a repeated version of Diamond and Dybvig's model (Temzelides). Chen's contribution (1999) is a step further. He introduces contagion in a multiple banking system by the use of rational herding behavior.

[3] Their paper show that peer monitoring among banks is an efficient way of tackling this issue. The model is however very sensitive to parameters value, and the banking system can be driven to bankruptcy for a small increase in the liquidity shock occurring in the first period.

Even multiple bank models do not study the way the banking system is structured and how it influences financial stability.

The contributions of Allen & Gale (2000), Freixas, Parigi & Rochet (2000) and Vivier-Lirimont (2004) are the first ones to introduce different shapes for the banking structure. Taking into account the way banking systems are structured to study banking issues thus appears to be a recent improvement in models of banking fragility. However, the previous contributions remain incomplete.

First, they only study some of the possible topologies and not all of them.

Second, all topologies are exogenously imposed, and no contribution has ever established a model in which networks are endogenously chosen by financial intermediaries as a control variable to increase social welfare and reduce liquidity risk.

Third, they do not explore the link between network density - defined as the number of edges adjacent to each node in the network - and financial stability.

Network theory concepts are of interest to enrich financial literature to determine the topology of these networks and the way this topology emerges.

Indeed, a recent and rapidly growing literature on network formation has developed since a few years. Based on strong theoretic tools[4], its main field of applications are information about job opportunities[5], trade and exchange of goods in non-centralized markets[6], R&D and collusive alliance among firms[7] or international alliances and trading agreements[8].

The plan is as follows. Section 2 exposes the allocation obtained when depositors have access to an autarkic bank and compares it to Central planner's allocation. Section 3 studies how first best outcome can be decentralized through the endogenous choice of an interbank network structure. Section 4 considers the influence of network architectures on the propagation of a located small rise in liquidity demand, and shows how interbank networks can lead to contagious bank runs. Section 5 concludes.

2 Model frame: stochastic preference for liquidity and risk sharing.

2.1 The model.

This section describes a simple model adapted from Allen and Gale (2000) and Vivier-Lirimont (2004). Within this frame, the interbank network enables an optimal risk sharing while liquidity shocks are stochastic.

[4] See in particluar Bala & Goyal (2000), Currarini (2002), Currarini & Morelli (2002), Dutta & Jackson (2000 & 2003), Dutta & Mutuswami (1997), Jackson(2003a, 2003b), Jackson & Watts (2001), or Jackson & Wolinsky (1996)

[5] Granovetter (1973), Calvo-Armengol (2000) or Ioannides & Datcher Loury (2002) for example

[6] Corominas-Bosch (1999, 2004), Weisbuch, Kirman & Herreiner (2000), or Wang & Watts (2002).

[7] Bloch (2001), Belleflamme & Bloch (2002), Goyal & Joshi (2000)

[8] Goyal & Joshi (2001) or Furusawa & Konishi (2002)

Time is discrete and divided into 3 periods $t = 0, 1, 2$. There is a single available and storable good (numeraire) which can be either consumed or invested. At $t = 0$, investment can be made in two different assets only. The first one is a liquid short term storage technology. One unit of good invested in this technology at date t produces one unit at date $t + 1$. The second one is a long term illiquid risky asset. Investment in this asset can only be made at the first date. One unit invested at $t = 0$ produces $R > 1$ units of good at period 2. Illiquidity is not complete. Should the long term asset be sold at the intermediate date, thus before it matures, it produces only r units of consumption good, with $0 < r < 1$. Let us assume that the early liquidation is a physical depreciation, the liquidation value being treated as the constant "scrap value" of investment.

Economy is constituted with n ex-ante identical regions. n is a strictly positive natural integer. We note $N_n = \{i_1, i_2...i_n\}$ the set of regions constituting the economy. In each one operates a single monopolistic bank. Topologically, each bank is represented by a network vertex. This regional structure can be interpreted in different ways. It can be considered as geographical areas for instance, or specialized branches in the banking industry for instance.

Each region is populated with a continuum of ex-ante identical depositors. Each depositor is endowed with one unit of consumption at $t = 0$ and receives no further endowment. Consumers have standard Diamond Dybvig preferences. With probability λ they are early consumers who value only date 1 consumption. With probability $1 - \lambda$ they are late consumers who value only date 2 consumption. Consumers preferences are thus given by

$$U(C_1, C_2) = \begin{cases} U(C_1) \text{ with probability } \lambda \\ U(C_2) \text{ with probability } 1 - \lambda \end{cases}$$

C_t being consumption at date $t = 0, 1, 2$. U is twice continuously differentiable, increasing and strictly concave.

Probability λ varies across regions. λ^i is the probability to be an early consumer in region i. There are two possible values for λ^i, a high value, noted λ^i_h, and a low value noted λ^i_l, with $0 < \lambda^i_l < \lambda^i_h < 1$. We will generally omit region's index when it is unnecessary. The value taken by λ depends on the state of nature, S_k. In each state of nature, a proportion p of the regions are hit by a low liquidity shock and a proportion $(1 - p)$ by a high liquidity shock. In a 4 region case with $p = \frac{1}{2}$, for example, the realization of the liquidity shock is given in the following table:

	Region 1	Region 2	Region 3	Region 4
State S_1	λ^1_h	λ^2_h	λ^3_l	λ^4_l
State S_2	λ^1_l	λ^2_l	λ^3_h	λ^4_h
State S_3	λ^1_h	λ^2_l	λ^3_h	λ^4_l
State S_4	λ^1_l	λ^2_h	λ^3_l	λ^4_h
State S_5	λ^1_l	λ^2_h	λ^3_h	λ^4_l
State S_6	λ^1_h	λ^2_l	λ^3_l	λ^4_h

Each region has the same ex-ante probability to face a high demand for liquidity which equals a $(1 - p)$. Aggregate demand for liquidity is known with probability 1 ex-ante, the distribution of those shocks is the single unknown variable.

All uncertainty is resolved at $t = 1$, when the state of nature is revealed. At time $t = 1$ each consumer knows his type, this is a private unobservable information. This characteristic is essential to the realization of bank runs which happen only when patient consumers do not reveal their true type and act as if they where short term consumers. As it is standard in such a model frame à la Diamond Dybvig, it is optimal for consumers to act cooperatively and to create a mutual bank that enables them to self insure against the risk of being type 1 (early consumers).

We shall study two benchmarks: the autarkic and the Central planner solutions.

2.2 The Autarkic Solution.

The bank then maximizes the consumer expected utility, subject to liquidity constraints. The amount redeemed by agents at date $t = 1$ is the realization of the liquidity risk of each agent. We assume the liquidity risk is highly volatile[9]. We shall assume thus the low demand for liquidity to be small, and the high demand to be very large such that we get $\lambda_l << \lambda_h$. Let us note $\gamma = (1 - p)\lambda_h + p\lambda_l$, the average liquidity shock. A bank in autarky located in region i maximizes the ex-ante expected utility of the representative consumer

$$\underset{b,k}{Max}\ p\left[\lambda_l^i U(C_1^i) + (1-\lambda_l^i)U(C_2^i)\right] + (1-p)\left[\lambda_h^i U(C_1^i) + (1-\lambda_h^i)U(C_2^i)\right] \quad (1)$$

Subject to the following constraints

$$b^i + k^i \leq 1 \quad (2)$$

$$\lambda_h^i C_1^i \leq b \quad (3)$$

$$\begin{cases} (1-\lambda_h^i)C_2^i \leq Rk^i \text{ with probability } (1-p) \\ (1-\lambda_l^i)C_2^i \leq Rk^i \text{ with probability } p \end{cases} \quad (4)$$

$$C_1^i \leq C_2^i \quad (5)$$

At $t = 0$, bank chooses a portfolio $(k^i, b^i) \geq 0$ where k is the per depositor amount invested in the long term asset (capital), while b is the per depositor amount invested in the short term asset (bonds), (Inequality 2). Inequality (3) is date 1 feasibility constraint, while date 2 feasibility constraint is given by inequality (4). The later depends on the realized shock at $t = 1$. Each bank has to abide by high demand for liquidity constraint at $t = 1$ because of the assumptions of the model in this case which make it impossible to borrow from any other institution (autarky) and because early liquidation is so costly that it is useless to get liquidity. However, at $t = 2$, the bank's resources have to cover

[9] The reason for this volatility are however not modeled here. This is just a formal assumption to prevent that bank with low liquidity reserves are not able to cope with a high demand for liquidity.

the payment towards late depositors which are in proportion λ_l^i with probability p or λ_h^i with probability $(1-p)$ depending on $t=1$ realized shock. Inequality (5) is the incentive constraint.

First order conditions give us the following solutions

$$\lambda_h^i \hat{C}_1^i = \hat{b}^i \tag{6}$$

$$(1-\lambda_h^i)\hat{C}_2^i = R\hat{k}^i \tag{7}$$

$$U'(\hat{C}_1^i) = R\frac{\lambda_h^i}{\gamma}U'(\hat{C}_2^i) \tag{8}$$

With

$$\frac{\lambda_h^i}{\gamma} > 1$$

As standard in models a la Diamond Dybvig, short term depositors benefit from the mutual bank. However, in autarky, a bank has to keep a high amount of liquid reserves to prevent itself from the risk of facing a high demand for liquidity, and consequently being driven to bankruptcy. The amount invested in the long term asset is relatively small.

2.3 Optimal risk sharing.

Central planner has the ability to transfer liquidity across regions at $t=1$, once the liquidity shock distribution is known but before depositors' withdrawals. There is no aggregate uncertainty. Cenral Planner program has thus only to abide by average liquidity constraints. Consequently, the allocation does not depend on the state of nature. All depositors are treated alike (they are all ex-ante identical). Each early consumer, receives C_1 and each late consumer C_2 whatever the bank in which he deposited his endowment, and whichever the state of nature may be. All banks face symmetric ex-ante situations with a $(1-p)$ probability to be hit by a high shock. Moreover they have the same technical abilities and they have access to the very same assets. As the program is solved for a representative region, we shall omit region's index.

$t=0$ constraint remains unchanged, i.e.

$$k + b \leq 1 \tag{9}$$

The payoffs make it optimal for the bank to pay for short term withdrawals by bond selling with γ being the average $t=1$ liquidity need ($\gamma = (1-p)\lambda_h + p\lambda_l$)
.

$t=1$ feasibility constraint is the following

$$\gamma C_1 \leq b \tag{10}$$

and $t=2$ feasibility constraint is the following

$$(1-\gamma)C_2 \leq Rk \tag{11}$$

Central planner maximizes (1) under constraints (9), (10), and (11), and under the following incentive constraint

$$C_1 \leq C_2 \tag{12}$$

FOC give us

$$\begin{aligned} (1-\gamma)C_2^* &= Rk^* \\ \gamma C_1^* &= b^* \\ U'(C_1^*) &= RU'(C_2^*) \end{aligned}$$

Central Planner's allocation is a first best allocation. Each bank keeps γC_1^* consumption units invested in the short term risk free asset to face early consumer demand for liquidity. This amount is the minimum level of reserves each bank has to keep in order to preserve the system from a global shortage of liquidity at $t = 1$. To reach optimum, at $t = 1$ short term funds are transferred to banks facing an excess demand for liquidity from its depositors from banks with available liquidity. At $t = 2$, borrowing banks refunds the lending ones by providing them long term liquidity units. Central Planner outcome exhibits an implicit rate of interest lying along the maturity transformation curve which equals $\frac{C_2^*}{C_1^*} - 1$. Optimum can be reached even if Central Planner does not know agent's type.

2.4 Welfare comparison.

When funds are transferable among regions, as there is no aggregate uncertainty about $t = 1$ liquidity demands, banks can store the average liquidity needs in short term asset. The remaining amount to be invested in the long term asset is superior when compared to autarky. Expected aggregate welfare can be written as follows.

In the autarkic case

$$\hat{W} = \gamma U\left(\frac{1-\hat{k}}{\lambda_h}\right) + (1-\gamma)U\left(R\frac{\hat{k}}{1-\gamma}\right) \tag{13}$$

In the Central Planner case

$$W^* = \gamma U\left(\frac{1-k^*}{\gamma}\right) + (1-\gamma)U\left(R\frac{k^*}{1-\gamma}\right) \tag{14}$$

When fund transfers are possible, aggregate welfare is strictly higher when compared to the autarkic case because long term asset payoff is greater than one. There is thus an incentive for banks to be able to transfer funds among regions.

Early liquidation of long term assets is a way to find liquidity at $t = 1$. However this solution is costly and exposes the bank to the risk of undergoing a bank run. Any bank can liquidate a maximum amount of $\beta(\lambda_h)$ units of long term asset without causing a run, (See appendix 1) with

$$\beta(\lambda_h) = r\left(k^* - \frac{(1-\lambda_h)C_1^*}{R}\right)$$

Should an autarkic bank i have b^* short term asset units, and face a high shock, λ_h if

$$(\lambda_h - \gamma)\, C_1^* > \beta(\lambda_h) \tag{15}$$

it is unable to even pay C_1 to late consumers at $t = 2$. There is an incentive for the latter to cheat and redeem by anticipation at $t = 1$. The bank is then driven to bankruptcy. This situation occurs either in case of a high shock for liquidity at $t = 1$, or if r, the early liquidation value of capital is small enough. Conversely, should it face a low demand for liquidity, λ_l it would have excess short term assets when compared with the amount redeemed by its early consumers.

To tackle this liquidity misallocation, banks have an incentive in opening an interbank market so as to enable liquid institutions to lend liquidity to illiquid ones, provided that the network structure enables efficient transfers and provided that the costs of such a network (rate of interest and access cost) are not too high. The issue to solve is to know whether it is possible or not to decentralize Central Planner's allocation through a network of interbank contacts.

3 Endogenous formation of the interbank network architecture.

As in Bhattacharya & Gale (1987) and Vivier-Lirimont (2004), a network of interbank relations enables to eliminate the liquidity shock distribution risk and decentralizes first best allocation. However, we are a step further to the previous contributions, in so far as we remove the exogenously imposed network architecture assumption to address the question of the endogenous network design choice.

By network we shall refer to undirected graphs. The reason for the choice of undirected graph lies in the very definition of the links. An ij link is a credit convention signed between bank i and bank j by which at $t = 0$ the two banks commit themselves into lending any available liquidity to one another (see *definition 2*). Through an ij link between bank i and bank j funds can be transferred in both direction (from i to j or from j to i). Directed graphs would enable fund transfers in a single direction.

The financial network can be considered as a mean to transfer funds among regions. Allen & Gale's (2000) way to transfer funds among banks consist in a network of interbank deposits. At $t = 0$, banks have the opportunity to deposit part of their available funds into one or several other financial institutions. Allen & Gale's article is based on a simple idea and studies the consequences of interbank deposits on banking stability. In their framework and in our's, liquidity shocks are randomly chosen between a high and a low value, while the amount of aggregate liquidity demand addressed to the banking system as a whole is perfectly known. To cope with illiquidity issues, banks hold cross deposits, which enable them to implement first best allocation. Their model takes into account 4 regions, in each region operates a single monopolistic bank. They study the impact on contagion of several networks architectures when the system has to face an unexpected rise in liquidity demand. Their paper concentrates on three network structures: a complete network called complete market structure, a circular network (incomplete market structure), and a two component network (disconnected incomplete market structure). They show the stability of the networks - which they call robustness - strongly depends on the completeness of the structure of interbank claims. Complete networks are shown in their paper to be more robust to contagion than incomplete structures. Closer to or work, Freixas, Parigi & Rochet (2000) follow scheme in which they study different scenarios of interbank exposures through open credit lines should a bank be bankrupt, or when the banking system has to face an unexpected rise in liquidity demand. They concentrate on two different topologies which are the circular and the complete networks.

The closest contribution to this paper is Vivier-Lirimont (2004) in which the author tackles the issue to know which exogenously imposed architectures both implement first best allocation and are characterized by pairwise stability[10]. He shows that strict conditions are to be fulfilled to have theses properties. He highlights two essential topological properties. If the network is asymmetric, banks have to be linked through a very short chain of intermediaries. If the network is symmetric, banks must have a minimum number of neighbors to avoid liquidity crises. Finally, stability and efficiency are not compatible as the single network architecture that maximizes welfare, is not pair-wise stable.

All previous contribution share the common feature of imposing an exogenous network architecture. Conversely, we shall concentrate here on the very issue of the formation of the network so as to avoid bankruptcy. We thus try to concentrate on the way networks are formed. Contrary to previous work, we do not impose any exogenous networks architecture.

We first determine under which conditions on neighborhood size, the first best outcome can be decentralized through a network of interbank relations. Second, we study the part played by the network in the contagion of an unexpected small and localized rise in liquidity demand.

3.1 Timing of events and definitions.

We define interbank networks as follows:

Definition 1 *A bank with less liquidity than needed is said to be illiquid. A*

[10]The concept of pairwise stability has firts been exposed by Jackson (1996). It simply indicates that a network is stable if no player wants to add a new link or if no players intends to break an existing link.

bank with more liquidity than needed is said to be liquid or over-liquid[11]*. We shall speak of over-liquidity when a former illiquid bank receives more liquidity than needed by several of its neighbors.*

A bank is said to be equilibrated when the amount of its available liquidity exactly matches the amount redeemable by its depositors at $t = 1$*.*

Definition 2 *A link* $g_{i,j} \in \{0, 1\}$ *where* $g_{i,j} = 1$ *means that bank* i *signed a bilateral credit convention with bank* j*.* $g_{i,j} = 0$ *means that no contract links* i *and* j*. A link between banks* i *and* j *is formed if and only if* $g_{i,j} = g_{j,i} = 1$*. The resulting link is noted* ij*. These links are open credit convention.*

Through these conventions, liquid banks commit to lending their excess liquidity to the requiring illiquid ones.

A network $g = \{(g_{i,j})\}$ *is a formal description of bilateral links existing among banks. Let* G *be the set of all non directed networks with* n *vertices.*

Definition 3 *The neighborhood* $N_i(g)$ *of a bank* i *is composed of the banks with which* i *is directly linked (not including* i *itself)*[12] $N_i(g) = \{j \in N_n : j \neq i, g_{ij} = 1\}$ *is the thus set of banks with which bank* i *signed open credit conventions in network* $g(s)$*, and* $\eta_i(g) = |N_i(g)|$ *is the cardinal of this set.*

Fund lending can only operate between 2 banks i *and* j *with a direct link, i.e. if* $g_{i,j} = g_{j,i} = 1$*.*

The timing of events is as follows.

$t = 0$. As a first step, banks cash deposits which are identical, normalized to one unit per depositor. As a second step, each bank chooses the network in which it is embedded by signing bilateral credit conventions. As a third step, banks allocate their funds.

$t = 1$. As a first step, liquidity shock distribution is revealed. As a second step, the interbank lending operations are carried out, liquid banks lend up to their lending limit to the requiring illiquid banks which they are linked with on a random basis. Funds can only be transferred along direct links. As a third step, early consumers redeem their deposits.

$t = 2$ First, interbank debt is refunded, and as a second step late depositors withdraw.

Each bank is embedded in a network of interbank contracts through which part or all of the set $N_n = \{i_1, i_2, ...i_n\}$ of banks are linked.

The cost for creating a link is to be considered to equal zero. As in Aghion & alii (2000) all loans however bear the same rate of interest within the network, no difference is made among debtors. This interest equals the safe interest rate r_1 plus an endogenous risk premium ρ. The latter compensates for the risk associated with the banking network system. Banks are risk neutral, if we

[11] An over-liquidity bank can be considered to be in a similar situation to an over-capitalised bank. But as in Diamond Dybvig's model banks have no capital, using this more common terminology would be incorrect.

[12] In technical terms we shall define the neighborhood $\Gamma(i)$ of a vertex i as the subgraph that consists of the vertices adjacent to i (not including i itself).

denote q as the probability for a bank in a given network to be bankrupt, we have

$$\rho = \frac{q}{1-q}(1+r_1) \tag{16}$$

3.2 Probability to be funded through the network for an illiquid bank.

The network in which the bank is embedded works as a liquidity provider. The single unknown variable in the model is the liquidity shock distribution within the network. Having access to a network of interbank relations is in fact a way for banks to have access to an external liquidity source. We have thus to determine the probability for an embedded bank to get funds from the interbank network when needed. Let η_i be the number of credit conventions signed by bank i, η_j the number of credit conventions signed by bank j, the probability, $P(\eta_i, \eta_j)$, that bank i gets one contact with a liquid bank through the interbank market in which it is embedded can easily be calculated as

$$P(\eta_i, \eta_j) = 1 - \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i}$$

(See appendix for the proof).

Lemma 1 *$P(\eta_i, \eta_j)$ is increasing in η_i decreasing in η_j, and increasing in p.(See appendix for the proof).*

We can note a first direct effect which can be labelled "competition for funds". Links are formed at $t = 0$, before liquidity shock distribution is known. Once the liquidity shock distribution has been revealed, funds are transferred to illiquid banks from their liquid partners on a random basis and along direct links only. Two illiquid banks linked to a single liquid bank are thus competing for liquidity. Consequently, bank i has an incentive to have as many liquid banks in its direct neighborhood as possible. That's the intuition behind the fact that $P(\eta_i, \eta_j)$ is increasing in η_i. But should the neighbors of its neighbors be illiquid too and i and them are competing for the same liquidity. That's he intuition behind the fact that $P(\eta_i, \eta_j)$ is decreasing in η_j. There is a positive direct effect of being linked with as many banks as possible. But there is a negative indirect effect from the neighbors of my neighbors which compete for the same potential liquidity should they be illiquid too. This negative externality will prevent the network from being complete.

As in Allen and Gale (2000), we shall focus on a symmetric version of the model. We consider the financial structure of each bank and available investment projects are all identical. We have $\eta_i = \eta_j = \mu$.

$$P(\mu) = 1 - \left[1 - \frac{1-(1-p)^{\mu}}{\mu}\right]^{\mu}$$

Proposition 1 *To maximize the probability of being funded through a direct contact banks have an interest in limiting the number of their partners. Indeed, $P(\mu)$ is non monotonic in μ, and reaches a unique global maximum in $\overline{\mu}$.*

(See appendix for the proof)

The non monotonicity of $P(\mu)$ is a direct consequence of the competition for fund effect we just highlighted. As illiquid neighbors of a single liquid bank compete for the same limited amount of liquidity to borrow, banks have an interest in limiting the number of nodes they are adjacent to. If banks want to maximize the probability of being able to borrow from a direct contact, they have to limit their number of partners. Consequently, banks do not embed in the complete network.

3.3 Value of the network.

The value of the network can be defined as an opportunity gain. The network enables indeed each bank to limit the amount invested in the short term asset to the average liquidity need, while (depending on network density) it may provide each network participant enough liquidity to face all withdrawals (even in the case of a high liquidity shock). It thus avoid to liquidate the long term asset when a bank with average reserves faces a high demand for liquidity. This opportunity gain can be defined as the value of the network for the banks involved. This gain depends on the probability of obtaining needed liquidity through the network, which itself depends on the neighborhood size of each bank.

To determine the value of the network, we shall consider a bank which invested b^* in the short term asset . If isolated and faced with a high demand for liquidity, it has to proceed to the force selling of $p(\lambda_h - \lambda_l)\frac{C_1^*}{r}$ units of long term asset to face $t = 1$ liquidity needs. The value of the network can thus be computed as the units of long term assets a bank with average bond reserves does not have to sell to meet the liquidity demand. Let θ be the amount of liquidity lent by liquid banks on the network. $P(\mu)$ is the probability for bank i to be linked with a liquid bank. An illiquid bank can thus expect $\mu\theta P(\mu)$ units of consumption from the interbank network. The value can of the network to bank i can be written as

$$v_i(g(\mu)) = \mu\theta P(\mu) - p(\lambda_h - \lambda_l)\frac{C_1^*}{r}$$

Definition 4 *Let π be a one to one mapping from N_n to N_n. Let $g^\pi = \{ij \mid \pi(i)\pi(j) \in g\}$ be the network obtained through this mapping. v is anonymous if $v(g^\pi) = v(g)$ for any g, π.*

Proposition 2 *The value produced by the network to $i, v_i(g(\mu))$ is anonymous and only depends on the network.*

The maximum value of the network is obtained for a degree μ_m of each vertex such that $\mu_m > \bar{\mu}$

(See Proof in appendix.)

The value produced from the network can be expressed in terms of the number of links of each player and on the number of links of potential partner. However, it does not depend on the identity of the potential partner but rather on the number of its partners itself. This property stems directly from the above expression of the value of the network which crucially relies on μ. However, should the network be asymmetric, the identity of the potential partner would play a role as the value of a partner comes both from its liquidity status and the liquidity status of its own partners. This property will be clearly be highlighted by the following subsection.

3.4 First best outcome decentralization and endogenous choice of a financial architecture.

The question being asked here aims at determining whether being embedded in an interbank network enables to decentralize first best outcome. To do so we have to consider a bank, which is adjacent to μ banks, and which satisfies planner's reserve constraint $\gamma C_1^* = b^*$.

If hit by a low liquidity shock, this bank has to pay C_1^* to each of its early depositors, the proportion of which is λ_l. The bank has thus enough liquid assets to cope with liquidity redeemed by its early consumers with its own reserves. In fact, it remains with excess liquidity which may be lent on the interbank market. This amounts to

$$(1-p)(\lambda_h - \lambda_l)C_1^* = \theta$$

Conversely, if hit by a high demand for liquidity shock, the bank has to pay C_1^* to a proportion λ_h of early depositors. But it only has b^* units of short term asset to face withdrawals. It faces thus an liquidity shortage amounting to $p(\lambda_h - \lambda_l)C_1^*$.

As the bank is embedded within a network of μ financial contacts, it has access to θ units of liquidity from each contact with probability $P(\mu)$. When illiquid, a bank with μ financial contacts can thus expect $\mu\theta P(\mu)$ units of consumption to borrow from the interbank market. The bank can face $t = 1$ withdrawals and is not bankrupt iff:

$$\begin{cases} \lambda_h C_1^* = b^* + \mu\theta P(\mu) \\ (1-\lambda_h)C_2^* = Rk^* + (1 + r_1 + \rho)\mu\theta P(\mu) \end{cases}$$

which is equivalent to

$$\begin{cases} \mu P(\mu) = \frac{p}{1-p} \\ r_1 + \rho = \frac{C_2^*}{C_1^*} \end{cases}$$

If this condition holds, it is then possible to decentralize first best no-bankruptcy outcome thanks to the interbank network.

Proposition 3 *Any $\tilde{\mu}$ regular network*[13] *decentralizes first best outcome. There is no bankruptcy in such a network. The rate of interest is thus driven to the risk free rate r_1 which lies along the maturity transformation curve.*

Banks endogenously choose a number of partners so as to be able to face any liquidity needs. The size of subset of neighbors which is endogenously chosen by each bank $\tilde{\mu}$ depends on the probability of a being hit by a low shock. $\tilde{\mu}$ is smaller than the degree of the complete network.

(See proof in appendix)

Both the amount of available liquidity and the structure of the interbank market enable any embedded bank to face any $t = 1$ (high or low) liquidity need while keeping only the average amount of bonds as reserves. The network thus prevents from a costly short selling of part of the long term asset. By signing $\tilde{\mu}$ credit convention, banks can increase the amount invested in long term asset, and keep reserves to the average level. As the long term asset is the single one with positive yield, the network increases social welfare and makes it possible to reach first best allocation. However, the degree of eacj vertex in the network is strictly higher than the degree that maximizes the probability to be funded through a direct contact. To implement first best no bankruptcy outcome, banks have to experience the negative externality created by competition for funds. Even without any cost of links, resulting network is not complete. Banks limit the number of links they create. Adding a link has indeed no linear effect on the amount of available liquidity from the subset of one's neighbors. This is a direct effect from what one could call "competition for funds" among illiquid institutions.

In such a $\tilde{\mu}$ regular network, the rate of interest is driven to the risk free rate r_1, as there is no risk for bankruptcy.

Network density, measured as the number of edges linked to each node within the network, depends on the size of the subgraph of liquid banks relative to the size of the subgraph of illiquid banks.

4 Systemic liquidity crisis and interbank network architecture.

As shown in the previous section, an interbank debt network enables to decentralize first best allocation. However, in the frame until then considered, period $t = 1$ total liquidity demand is known ex-ante. The point at stake is just an issue of liquidity distribution, which can be tackled by the interbank network. To study contagion, following Allen & Gale (2000) let us introduce state of nature S' in which the aggregate demand for liquidity overpass the system ability

[13] Network g with vertex set $V(g)$ is said to be $k-$regular if any vertex u within the network is linked to exactly k other vertex. Its degree is said to equal k.

$$\forall u \subset V(g), \deg(u) = k$$

to provide liquidity. We are to see this may drives the system to a widespread contagious crisis.

The market structure here considered is the one of subsection 3.4. Asset allocation requires each bank to have investment portfolio (b^*, k^*) and to offer (C_1^*, C_2^*) as a deposit contract where (b^*, k^*, C_1^*, C_2^*) stands for the first best outcome. To make the deposit contract effective, each bank signs $\tilde{\mu}$ open interbank credit conventions.

We shall take this allocation as given and we shall perturb the model. By perturbation we mean the realization of a state of nature $S^{'}$ to which the ex-ante assigned probability is 0 at $t = 0$. In this state demand for liquidity equals the average demand in each regions except for one region facing a slightly higher demand, say by ε, when compared to previous state. As the ex-ante probability assigned to $S^{'}$ is zero, it does not change allocations chosen at $t = 0$. At $t = 1$, the continuation equilibrium is however affected.

At continuation equilibrium , depositors have to choose whether they redeem or stay invested. Early depositors always withdraw their funds at $t = 1$, late depositors make a "now or later" withdrawal arbitrage decision. They choose to redeem according to the amount of consumption obtained doing so at each date. We shall assume, that late depositors always run when it is weakly optimal for them to do so (i.e. if the amount obtained by early withdrawal at $t = 1$ equals the amount obtained by late withdrawals at $t = 2$). Banks are constraint to pay C_1^* units of consumption to any depositors whishing to withdraw at $t = 1$. When banks are not able to face their liabilities they sell all their assets. The product of liquidation is equally shared among depositors. When the bank is able to face $t = 1$ liquidity withdrawals, remaining assets are sold at $t = 2$ and shared among patient depositors only. The issue of the game depends on the relative cost of liquidation of assets.

4.1 Asset Liquidation order.

At $t = 1$ the bank can be in three different situations: it is solvent if it can face liquidity demand by the exclusive use of its short term assets; it is insolvent if it has to sell part of its long term assets to face short term withdrawals; and it is bankrupt if the liquidation of all of its assets are not high enough to meet liquidity demand.

Obtaining resources is a costly process, the cost of which depends on the type of resources used by the bank. First, the bank sells its liquid assets on the bond markets, then it borrows liquidity on the interbank market, and then it sells its long term assets before they mature. To assure that long term assets are liquidated last we shall assume

$$\frac{R}{r} > \frac{C_2^*}{C_1^*}$$

Since first best allocation is independent of r, we can always ensure that this condition is satisfied by choosing r sufficiently small.

Liquidity is obtained at a relative cost that equals the cost of obtaining current consumption in term of future consumption. The smaller cost is incurred by the liquidation of bonds. One unit of consumption at $t = 1$ if reinvested in the short term asset produces one unit of consumption at the following date. The cost of obtaining liquidity by short term asset equals thus 1.

By borrowing on the interbank market to pay short term depositors withdrawals, the indebted bank proceeds to an inter-temporal transfer. It gets C_1^* unit of short term consumption, and will have to pay C_2^* units to the lending bank at $t = 2$ because of the rate of interest over loans. So it gives up part of its long term asset payoff to the lending bank. Consequently, the cost of liquidity by this mean is for a borrowing bank $\frac{C_2^*}{C_1^*}$ which is greater than one (see central planner FOC). By long term asset early liquidation, the bank gives up R units of future consumption to get r units of current consumption. The cost incurred by the bank to get liquidity by this mean is thus $\frac{R}{r}$.

Ranking relative cost of obtaining liquidity is thus

$$1 < \frac{C_2^*}{C_1^*} < \frac{R}{r}$$

To maximize depositor's welfare, the bank has thus to sell first its short term assets, then to borrow on the interbank market, and last to proceed to the early liquidation of its long term assets. This situation takes as given the fact that the interbank market works well and is not the carrier of financial distress.

4.2 Liquidation values.

The value of date 1 deposits is C_1^* if the bank is not bankrupt. If all depositors redeem early ($t = 1$), the bank is bankrupt but it is bound to pay C_1^* to each early depositor. Consequently, C_1^* is the value of the bank's asset when driven to bankruptcy. Let ω^i be the value of bank i's deposits at $t = 1$. Depositors are treated equally and receive ω^i. If $\omega^i < C_1^*$ depositors will try to redeem all of their deposits. To deal with liquidity demand bank i will try to borrow on the interbank market. Let m be the expected value of liquidity borrowed through the interbank market. When all depositors redeem, the aggregate demand for liquidity equals 1. Liabilities of the bank can thus be valued at ω^i which equals by definition bank's asset. Available liquidity is composed of the b^* units of short term asset, k^* units of long term assets plus what the bank may borrow on the interbank market m. The latter depends on the number of interbank credit conventions signed by i, $\tilde{\mu}$, on the unexpected liquidity shock, ε, and on the amount of liquidity liquid banks make available on the interbank market θ. ω^i equilibrium value is thus

$$\omega^i = b^* + rk^* + m(\tilde{\mu}, \varepsilon, \theta)$$

Let us note that the liquidation value of a bank is an increasing function of the amount that a bank may borrow on the interbank network. The interbank market plays a short term stabilizing role, in so far as the expected value of borrowings enables to move the value of asset closer to C_1^*.

4.3 Contagious bank runs.

Proposition 4 *The interbank network can ease the propagation of a liquidity crisis : first any liquid bank can turns illiquid because of its financial links, second the interbank market can lead to the chain bankruptcy of, or, to a chain bank run on all banks embedded in the network. The interbank network may turn a small and isolated liquidity crisis into a widespread systemic event with large effects on the financial system as a whole.*

What happens in the model can be summarized as follows.

Let us consider bank i coping with an unexpected $(\lambda_h + \varepsilon)$ demand for liquidity. All other bank face either the low or the high liquidity shock. To face withdrawals, bank i will borrow as much as possible on the interbank market. The choice of $\tilde{\mu}$ make it possible for bank i to borrow up to $p(\lambda_h - \lambda_l)C_1^*$ units of liquidity from the network. But it is unable to face the missing εC_1^* units. It thus has to proceed to force selling of part of its long term assets.

By assumption,

$$\varepsilon C_1^* > \beta(\lambda_h + \varepsilon)$$

This costly early liquidation drives depositors to a bank run. Long term consumers will not get at $t = 2$ a consumption amount superior to the one obtained should they redeem their deposits at $t = 1$. Bank i is thus bankrupt.

The interbank network then acts in a way that propagates illiquidity towards other financial institutions. Let us consider the $\frac{p}{1-p}$ banks which lend to i. They will not get refunded. They thus lose θ consumption units each. Missing liquidity in the system has been driven upwards from ε for one bank to θ for $\frac{p}{1-p}$ banks. Consequently, they will be no longer capable of facing their own financial $t = 2$ obligations. As depositors' information on banks asset is perfect in such a frame a la Diamond Dybvig, all of the $\frac{p}{1-p}$ banks' late depositors will behave unaccordingly to their type and redeem their deposits at $t = 1$. Those banks turn illiquid. Their market position from long turns short.

The reason for which those banks want to borrow on the interbank market is not known by interbank market participants. This hypothesis might seem to be strong but is not considering the fact that interbank lending is realized without monitoring in the model, as it is the case on interbank market such as Fed Funds (c.f. Furfine 2001; Bernard & Bisignano 2000). Liquid banks linked with the latter banks lend them the needed amount, despite the fact that those lons are not backed by any assets. From one bank to the next, the whole financial architecture established at date $t = 0$ may disappear.

This contagion does not necessarily takes time as in the above description. The general framework being as it is, bankruptcy of a bank may act as a signal of a systemic crisis on the interbank market. Banks would then refrain from lending, and institutions hit by a high liquidity shock are in a situation similar to the one occurring when they are isolated. The risk has however been transfered along the network from banks with a shortage of liquidity to banks with excess liquidity, which would have not been the case should banks have remained isolated.

The network has thus unclear effect on welfare, as it enables to increase investment in the long term asset, but, it may transfer liquidity shortages from fragile banks to liquid bnks. There is thus a trade-off between a risky interbank debt market and a safe payment mechanism which foregoes investment opportunities.

5 Conclusion

The aim of this article was to underline the ambivalent effect of the interbank market on financial stability. The interbank market can be modeled as a financial network through which liquidity is transferred among market participants as standard debt contracts. On the one side, we were able to show bank are able to endogenously choose a network architecture so as to decentralize first best outcome. Interbank networks increase the welfare of representative depositors. Banks have the opportunity to reduce their liquidity provisions as they are assured to be able to borrow enough from the interbank network to face a high liquidity shock. In that sense, the interbank market plays a stabilizing role. This result shows that bank program may be modified by including the choice of a network structure as a control variable in order to limit liquidity reserves. This result is in line with a standard diversifying effect.

On the other side, the interbank market may turn unstable. It can turn a small and isolated liquidity shock into a major systemic event. Located crises propagate along the financial network which can be forced to a generalized bank run. In a sense, we find the very same characteristics as the ones underlined by Diamond and Dybvig for a single bank, but on a systemic scale. There is thus a trade-off between a risky interbank debt market and a safe payment mechanism which foregoes investment opportunities.

Free banking markets theories have claimed that the interbank market can ensure by itself the liquidity required by any market participants. Central bank is thus not only unnecessary but also can lead to moral hazard if it acts as a lender of last resort. Free banking markets would then tend to act as banking market discipline device.

On the contrary, this articles tends to prove that the very architecture of interbank links makes interbank markets prone to global distress. The speed of interbank transactions makes it impossible for banks to carry peering operations before lending to a counterpart. It is not possible to distinguish illiquid but solvent borrowers from insolvent ones. Central bank appears thus as an essential liquidity provider to the market to reduce contagion. LTCM bankruptcy in 1998 documents this issue. It has been the latest systemic event that endangered the interbank market. Global crisis has been avoided by Fed's intervention as a liquidity provider.

Several questions remain however unsolved by this article, such as the change in interest rates with the number of participants to network, or the influence of topology itself (rather than the number of participants) on the stability of the system. These topics are to be considered for further research.

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6 Appendix

6.1 Amount of long term asset a bank can liquidate without causing a run

First consider all the networks with at least one isolated bank. Consider the isolated bank, its liquid reserve is limited to γC_1^*. With probability a $(1-p)$ the bank is hit by the high shock. Its liquidity shortage (amounting to $(\lambda_h - \gamma)\, C_1^*$) cannot be covered by any bank loan as the bank is isolated. The single solution it has is to sell by anticipation part of its long term asset. The bank must give the late consumers at least C_1^* at date 2 otherwise the would be better off withdrawing at date 1. So a bank with a fraction λ_h of early consumers can at most liquidate by anticipation a part α of its long term asset such as

$$R(k^* - \alpha) = (1 - \lambda_h)C_1^*$$

The amount of long term asset that can be liquidated by anticipation without causing a run is thus

$$\alpha = k^* - \frac{(1 - \lambda_h)C_1^*}{R}$$

This early liquidation produces r units of liquidity at $t = 1$. Any isolated bank can thus get by this way a maximum of $\beta(\lambda_h)$ without causing a run, with

$$\beta(\lambda_h) = r \left(k^* - \frac{(1 - \lambda_h)C_1^*}{R} \right)$$

If

$$(\lambda_h - \gamma)\, C_1^* > \beta(\lambda_h) \tag{17}$$

then the bank experiences a run and is driven to bankruptcy. We shall consider that the latter condition is satisfied. As a consequence, any isolated bank cannot implement first best outcome, without bearing the risk of being bankrupt with positive probability. Any isolated bank has to abide by autarkic outcome.

6.2 Probability to obtain liquidity through the interbank network. $P(\eta_i, \eta_j)$

Let us consider bank i facing liquidity needs at $t = 1$ and bank j is such as $g_{ij} = 1$. Among bank j's η_j contacts, there are k illiquid banks, plus i with probability

$$\binom{\eta_j - 1}{k} (1-p)^{\eta j - k - 1} p^k$$

Bank j 's liquidity is lent to bank i with probability $\frac{1}{k+1}$.

$$\binom{\eta_j - 1}{k} \frac{1}{k+1} (1-p)^{\eta j - k - 1} p^k$$

is thus the probability that the liquid bank j selects i among its illiquid contacts to lend its liquidity. By summation over k, we obtain the probability that bank i gets liquidity through j:

$$\frac{1-(1-p)^{\eta j}}{p\eta_j}$$

As j is liquid with a probability p

$$\left[1 - p\frac{1-(1-p)^{\eta j}}{p\eta_j}\right]^{\eta_i}$$

is the probability that no single contact of i among $\eta_i(g)$ is able to lend it liquidity. Hence

$$P(\eta_i, \eta_j) = 1 - \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i}$$

is the probability that bank i is able to find one unit of available liquidity through the interbank network in which it is embedded.

6.3 Proof of lemma 1.

Let us study the variations of $P(\eta_i, \eta_j) = 1 - \left[1 - \frac{1-(1-p)^{\eta j}}{p\eta_j}\right]^{\eta_i}$

6.3.1 Change in $P(\eta_i, \eta_j)$ relatively to η_i.

$\frac{\partial P(\eta_i,\eta_j)}{\partial \eta_i} = -\ln\left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right] * \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i}$

As $0 < 1 - \frac{1-(1-p)^{\eta j}}{\eta_j} < 1$

$\frac{\partial P(\eta_i,\eta_j)}{\partial \eta_i} > 0$

$P(\eta_i, \eta_j)$ is increasing in η_i. The probability for bank i to find available liquidity within the network is an increasing function of the number of edges adjacent to i.

6.3.2 Change in $P(\eta_i, \eta_j)$ relatively to η_j.

$$\frac{\partial P(\eta_i,\eta_j)}{\partial \eta_j} = -\eta_i * \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i - 1} * \left[-\frac{-\eta_j (1-p)^{\eta j} \ln(1-p) + (1-p)^{\eta j} - 1}{(\eta_j)^2}\right]$$

$$= \eta_i(g) * \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i - 1} * \left[\frac{(1-p)^{\eta j}\big(1-\eta_j \ln(1-p)\big) - 1}{(\eta_j)^2}\right]$$

$\frac{\partial P(\eta_i,\eta_j)}{\partial \eta_j}$ has the same sign as $(1-p)^{\eta j}\left(1-\eta_j \ln(1-p)\right) - 1$

Let us note $\Phi(\eta_j) = (1-p)^{\eta j}\left(1-\eta_j \ln(1-p)\right)$

$$\frac{\partial \Phi(\eta_j)}{\partial \eta_j} = \ln(1-p) * (1-p)^{\eta j)} \left(1-\eta_j \ln(1-p)\right) - (1-p)^{\eta j} \ln(1-p)$$

$$= \ln(1-p) * (1-p)^{\eta j}\left(-\eta_j \ln(1-p)\right)$$

$$= -\eta_j \left(\ln(1-p)\right)^2 (1-p)^{\eta j}$$

Hence $\frac{\partial \Phi(\eta_j)}{\partial \eta_j} < 0$

Let us study $\Phi(\eta_j)$ extrema

$\lim_{\eta_j(g)\to 0} \Phi(\eta_j) = 1$

$\lim_{\eta_j(g)\to +\infty} \Phi(\eta_j) = 0$

Hence $\left(\Phi(\eta_j) - 1\right) \in]-1;0[$

and consequently $\frac{\partial P(\eta_i,\eta_j)}{\partial \eta_j} < 0.$

The probability to find available funds through the interbank network is decreasing with the number of interbank open credit convention signed by bank i 's partners.

6.3.3 Change in $P(\eta_i, \eta_j)$ relatively to p

$$P(\eta_i, \eta_j) = 1 - \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j(g)}\right]^{\eta_i}$$

$$\frac{\partial P(\eta_i,\eta_j)}{\partial p} = \eta_i * \left[1 - \frac{1-(1-p)^{\eta j}}{\eta_j}\right]^{\eta_i - 1} * (1-p)^{\eta j - 1}$$

$$\frac{\partial P(\eta_i,\eta_j)}{\partial p} > 0$$

6.4 Proof of proposition 1.

The proof of proposition 2 lies in three steps. .

$$P(\mu) = 1 - \left[1 - \frac{1-(1-p)^{\mu}}{\mu}\right]^{\mu}$$

We denote $Q(\mu) = \left[1 - \frac{1-(1-p)^{\mu}}{\mu}\right]^{\mu}$ and $\phi(\mu) = \left[1 - \frac{1-(1-p)^{\mu}}{\mu}\right]$

$$\frac{\partial Q(\mu)}{\partial \mu} = \phi^{\mu}(\mu)\ln(\phi(\mu)) + \mu\phi^{'}(\mu)\phi^{\mu-1}(\mu)$$

$$= \phi^{\mu}(\mu)\left[\ln(\phi(\mu)) + \mu\frac{\phi^{'}(\mu)}{\phi(\mu)}\right]$$

$$= \Psi(\mu)\ Q(\mu)$$

With $\Psi(\mu) = \left[\ln(\phi(\mu)) + \mu\frac{\phi^{'}(\mu)}{\phi(\mu)}\right]$

First step: let us prove that $\frac{\partial Q(\mu)}{\partial \mu} \mid_{\mu\to 2} < 0$

The derivative of $\phi(\mu)$ relative to μ is $\phi^{'}(\mu) = \frac{(1-p)^{\mu}(\mu\ln(1-p)-1)+1}{\mu^2}$

We can thus get an expression for $\Psi(\mu)$.

$\Psi(\mu) = \ln\left(1 - \frac{1-(1-p)^{\mu}}{\mu}\right) + \frac{(1-p)^{\mu}(\mu\ln(1-p)-1)+1}{\mu-1+(1-p)^{\mu}}$

Hence

$\Psi(2) = \ln\left(1 - \frac{1-(1-p)^{2}}{2}\right) + \frac{(1-p)^{2}(2\ln(1-p)-1)+1}{1+(1-p)^{2}}$

$$\Psi(2) * \left(1 + (1-p)^2\right) = \ln\left(1 - \frac{1-(1-p)^{2}}{2}\right) * \left(1 + (1-p)^2\right) + (1-p)^2\left(2\ln(1-p) - 1\right) + 1$$

Let $\chi_2(p) = \ln\left(1 - \frac{1-(1-p)^{2}}{2}\right) * \left(1 + (1-p)^2\right) + (1-p)^2\left(2\ln(1-p) - 1\right) + 1$

We the study the sign of $\chi_2(p)$

$\frac{\partial\chi_2(p)}{p} = -2(1-p)\ln\left[\frac{1+(1-p)^2}{2}\right] - 2(1-p) - 2(1-p)\left(2\ln(1-p)\right)$

$= -2(1-p)\left\{\ln\left[\frac{1+(1-p)^2}{2}\right] + 1 + 2\ln(1-p)\right\}$

$\chi_2(p)$ and $\ln\left[\frac{1+(1-p)^2}{2}\right] + 1 + 2\ln(1-p)$ have opposite signs.

We define $z(p) = \ln\left[\frac{1+(1-p)^2}{2}\right] + 1 + 2\ln(1-p)$

We have $\frac{\partial z(p)}{\partial p} = -\frac{2(1-p)}{1+(1-p)^2} - \frac{2}{1-p} < 0$

$z(p)$ is strictly decreasing with p

With $\lim_{p\to 0} z(p) = 1$ and $\lim_{p\to 1} z(p) = -\infty$

If we choose $\underline{p}$ / $z(\underline{p}) = 0$

then we have

$\forall p < \underline{p}\ z(p) > 0$ and hence $\frac{\partial\chi_2(p)}{p} < 0$

$\forall p > \underline{p}\ z(p) < 0$ and hence $\frac{\partial\chi_2(p)}{p} > 0$

$\chi_2(p)$ decreasing on $(0, \underline{p})$ and increasing on $(\underline{p}, 1)$. It reaches a maximum at $\max\left(\chi_2(0); \chi_2(1)\right)$

But we have $\chi_2(0) = 0$ and $\chi_2(1) = 2\ln(1/2) + 1 < 0$

which implies $\chi_2(p) < 0$

and $\Psi(2) < 0$

which finally gives us $\frac{\partial Q(\mu)}{\partial \mu} \mid_{\mu\to 2} < 0$

Q.E.D.■

Second step: Let us show that $Q(\mu)$ increases towards its asymptotic values with the high values of μ.

One can easily check that $\Psi(\mu) \sim \left(\frac{1}{\mu}\right)^2$ when $\mu \to \infty$, implying that $\frac{\partial Q(\mu)}{\partial \mu} > 0$ for high values of μ. Therefore, $Q(\mu)$ increases towards its limit e^{-1} when $\mu \to \infty$. Q.E.D. ■

Third step: Let us show that $\frac{\partial Q(\mu)}{\partial \mu} < 0$ implies $\frac{\partial^2 Q(\mu)}{\partial \mu^2} > 0$

We have $\frac{\partial Q(\mu)}{\partial \mu} = \Psi(\mu)Q(\mu)$

Hence $\frac{\partial^2 Q(\mu)}{\partial \mu^2} = \Psi'(\mu)Q(\mu) + \Psi(\mu)Q'(\mu) = Q(\mu)\left(\Psi'(\mu) + \Psi^2(\mu)\right)$

As a consequence $\Psi'(\mu) > 0 \Rightarrow \frac{\partial^2 Q(\mu)}{\partial \mu^2} > 0$

We already know that $\lim_{\mu \to \infty} \phi(\mu) = 1$ and $\lim_{\mu \to \infty} \mu\phi'(\mu) = 0$

we thus have $\lim_{\mu \to \infty} \Psi(\mu) = 0$

Let us suppose that $\Psi'(\mu)$ is negative. Ψ is then decreasing towards its limit 0 when μ tends towards infinite. Ψ stays in the positive quarter section of the plan and we have $\Psi(\mu) > 0$.

Reciprocally if $\Psi(\mu) < 0$ then $\Psi'(\mu) > 0$ which implies $\frac{\partial^2 Q(\mu)}{\partial \mu^2} > 0$.

However we know that $\Psi(\mu) \leq 0$ is equivalent to $\frac{\partial Q(\mu)}{\partial \mu} < 0$. We thus have that $\frac{\partial Q(\mu)}{\partial \mu} \leq 0$ implies $\frac{\partial^2 Q(\mu)}{\partial \mu^2} > 0$.

Q.E.D. ■

We can conclude as a fourth step that

$\frac{\partial Q(\mu)}{\partial \mu} = 0$ for a given $\overline{\mu}$ (step 1 and 2)

and consecutively $\frac{\partial^2 Q(\mu)}{\partial \mu^2} > 0$.

Hence $Q(\mu)$ as no any local maximum, and there exists a unique $\overline{\mu}$ such that $Q(.)$ reaches an absolute minimum.

Moreover by continuity of $\frac{\partial^2 Q(\mu)}{\partial \mu^2}$ there exist $K > \overline{\mu}$ such as $Q(\mu)$ is strictly convex on $[1, K[$

This proves that $P(\mu)$ reaches a maximum in $\overline{\mu}$. This maximum is not the complete network.

6.5 Proof of proposition 2.

The proof of anonymity is a direct consequence of definition 4. The value produced from the network can be expressed in terms of the number of links of each player and on the number of links of potential partner. However, it does not depend on the identity of the potential partner but rather on the number of its partners itself. This property stems directly from the expression of the value of the network which crucially relies on μ which is identical to any vertex.

Second part of the demonstration relies on a simple study of $v_i(g(\mu))$.To maximise the value of the network bank i chooses μ so as to maximize $v_i(g(\mu))$

$$\underset{\mu}{Max}\ v_i(g(\mu)) = \mu\theta P(\mu) - p(\lambda_h - \lambda_l)\frac{C_1^*}{r}$$

To compare the value of μ maximizing $v_i(g(\mu))$ (i.e. μ_m), with the value of μ maximizing $P(\mu)$, we have to study function $H(\mu) = \frac{\partial v_i(g(\mu))}{\partial \mu}$

$$H(\mu) = \theta P(\mu) + \mu\theta \frac{\partial P(\mu)}{\partial \mu}$$

$v_i(g(\mu))$ reach a maximum when $H(\mu) = 0$, which implies that $\mu_m \in]\bar{\mu}, \infty[$. Indeed,

$$H(\bar{\mu}) = \theta P(\bar{\mu}) > 0$$

Q.E.D. ■

6.6 Proof of proposition 3.

Proposition 3 states that banks can decentralize first best no bankruptcy outcome, which drives the rate of interest to the risk free level. It lies in three steps.

First step.

We know that $lim_{\mu \to \infty} \mu P(\mu) = lim_{\mu \to \infty} \mu(1 - Q(\mu))$

But $lim_{\mu \to \infty} Q(\mu) = e^{-1}$

Moreover $lim_{\mu \to 1} \mu P(\mu) = p$

Then,for some $\tilde{\mu}$, and by continuity with have $\tilde{\mu} P(\tilde{\mu}) = \frac{p}{1-p}$.

Second step.

At $t = 1$ any illiquid bank is thus able to face short term withdrawals by borrowing on the interbank market the missings liquidity units. At $t = 2$, on the liability side, the borrowing bank has to pay C_2^* to each of its long term consummers (in proportion $(1 - \lambda_h)$), and to refund $p(1 + r_1 + \rho)(\lambda_h - \lambda_l)C_1^*$ to the lending bank. On the asset side, the lending bank has $Rk^* = (1 - \gamma)C_2^*$ units of long term asset.

$$(1 + r_1 + \rho) = (1 + \frac{C_2^*}{C_1^*})$$

is a sufficient condition for the asset to pay for all libilities. Symmetrically, wa could prove that the same consition ensures any lending bank to face both $t = 1$ and $t = 2$ withdrawals. To abide by $t = 2$ constraints both for lending and borrowing banks, the risk free rate lies along the maturity transformation curve, and is equal to $\frac{C_2^*}{C_1^*}$.

As a consequence, any $\tilde{\mu}$ regular network enables to decentralize first best non bankruptcy outcome. Bankruptcy risk, stemming from liquidity shock misallocation is then driven to zero, so the rate of interest on interbank loan is the risk free rate r_1.

Third step.

Let us prove that the degree of the network chosen by banks to decentralize first best no bankruptcy outcome $\tilde{\mu}$ is smaller than the degree of the complete network. To maximise the value of the network bank i chooses μ so as to maximize $v_i(g(\mu))$

$$\underset{\mu}{Max}\ v_i(g(\mu)) = \mu\theta P(\mu) - p(\lambda_h - \lambda_l)\frac{C_1^*}{r}$$

As for high values of μ, $\mu P(\mu)$ tends towards its asymptotic values (i.e. μ_m), $v_i(g(\mu))$ is asymptitically inreasing in μ with the high values of μ.

We indeed know that $lim_{\mu \to \infty} \mu P(\mu) = lim_{\mu \to \infty} \mu(1 - Q(\mu))$

and $lim_{\mu \to \infty} Q(\mu) = e^{-1}$.

As a consequence $v_i(g(\mu))$ reaches a maximum when the network is complete.

6.7 Proof of proposition 4.

Proof of proposition 4 requires several steps.

Step 1. To face liquidity shock λ, any bank can liquidate $\beta(\lambda)$ units of long term asset without causing depositors' run.

Let us consider a bank hit by a shock $\lambda_h + \varepsilon$. The $t = 0$ signed open credit conventions enables it to get an expected value of $p(\lambda_h - \lambda_l)C_1^*$ units of consumption on the interbank market from partner banks. It is thus not possible for the bank to find the missing εC_1 liquidity units from the interbank network. If $\varepsilon C_1 > r\left[k - \frac{1-\lambda_h-\varepsilon}{R}C_1^*\right]$ bank i is forced to bankruptcy.

Step 2. Let us then consider one of the $\frac{p}{1-p}$ lender bank of i , bank j. It has lent i θ units. With bank's i bankruptcy, bank j will not get refunded at $t = 2$. Bank j will thus be unable to pay its depositors at at $t = 2$ if $\theta > r\left[k - \frac{1-\lambda}{R}C_1\right]$. To fill in this financial gap and preserve its depositors' welfare, j has to become a borrower on the market. Because of interbank links, bank j's status on the market has turned from liquid to insolvent: j wants to borrow up to θ from its financial network partners. The excess demand of liquidity addressed to the market rises from ε for a bank to θ for $\frac{p}{1-p}$ banks. The interbank market structure should enable j to expect to borrow up to this limit on the market (we recall each bank is able to expect θ from the market). However, the system then highlights an unmatched liquidity structure, total liquidity needs exceeding total available liquidity. As a consequence, in expectation, the amount of available liquidity for each bank is strictly less than θ. The expected value of liquidity obtained through the interbank market, noted m is decreasing, so is the value of bank j's assets. We face a domino effect. The value of each of assets of bank j is thus

$$\omega^j = b^* + rk^* + m(\tilde{\mu}, \theta, \theta) < b^* + rk^* + m(\tilde{\mu}, \varepsilon, \theta)$$

Liquidity crisis propagation depends on the available information as well as the refunding priority order. As it is standard in a model a la Diamond Dybvig, we assume depositors' information about their bank balance sheet is perfect.

If refunding priority is given to interbank debt, bank j late depositors know they will not be paid the $t = 0$ expected amount. By hypothesis we have $\theta > \varepsilon C_1^*$, bank's j depositors have thus an interest in running to the bank, which in turn is forced to bankruptcy. Bank k, which is j's creditor, faces then the very same situation j had to deal with after i's bankruptcy. Available liquidity on the interbank network shrinks, while liquidity needs are on the rise. At least a proportion $\frac{p}{1-p}$ of the network is bankrupt. Depending on the timing of loans, from one bank to another, the value of bank's assets ω decreases and liquidity crisis propagates along the network. In the end, the whole banking system may be driven to bankruptcy.

The way this propagation takes place is not necessarily sequential. Available liquidity reduces with the crisis propagation along the financial network nodes. Consumers are able to expect what is to happen. As a consequence, the first bankruptcy can act as a signal of the forthcoming systemic crisis. This signal lead to systemic bank runs in every regions embedded in a particular banking network at $t = 1$.

If priority is given to depositors, the contagion scheme is different but the result is the same. Bank j 's late depositors are not incited to redeeming early. Region j does not undergo any bank run. However, bank k, j's creditor, will be the one unable to face its financial obligations against its late depositors at $t = 2$ (it has lost the θ units lent to j). Because of the information structure on the one side, and of the amount lent on the other side, (we still have $\theta > \varepsilon C_1^*$), $k's$ depositors run to the bank. To avoid bankruptcy, bank k in the interest of its depositors should borrows on the interbank market. The available liquidity shrinks on the interbank market. Expected available liquidity decreases for each bank, and so does the value of assets. The initial liquidity problem can thus propagate along the network.

In this case, the excess demand for liquidity cannot be satisfied by interbank loans.