Balance of payments crisis in an external debt-led growth regime

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Abstract

We study the long-run effect of capital account liberalization using the model of a small open economy with two sectors. In the case of a low technological level, the long-run evolution of the productive structure that follows the liberalization leads to *financial fragility* by creating the conditions under which balance of payments crises are possible. Higher capital inflows both have a stabilizing and a destabilizing effect, their overall effect depending on the relative technological level of each sector.

1 Introduction

What are the effects of capital account liberalization on developing countries? The last twenty years have witnessed numerous balance of payments (BoP) crises, often coupled with banking crises, in developing countries that had previously opened their economy to foreign capital flows. The south cone crises at the beginning of the eighties, the Mexican crisis of 1994, the Asian

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crises of 1997 and the Argentine crisis of 2001 were all preceded by massive capital inflows. Among the huge literature dedicated to the empirical analysis of those events, several works have shown that capital inflows were likely to initiate a boom-bust cycle that could dramatically end in a severe twin crisis.[1] Even if the opening of the capital account may be at the origin of the cycle, some papers insist on the amplifying role played by the domestic financial system, particularly the banking system (Hernández & Landerretche 1999). Boom-bust cycles are thus associated with a cyclical evolution of bank credit and the credit/GDP ratio strongly increases in the few years preceding the crisis.

All these papers show a growing consensus as to the possibly destabilizing short-run effect of capital inflows to developing countries, but the question of their long-run effect is still subject to debate. Among the factors that are at the origin of the cycles, pro-cyclical risk-taking of entrepreneurs and external creditors could eventually become less pronounced as agents learn from their errors, and the imperfect prudential regulation of the banking system might be improved after some time by the monetary authorities. Kaminsky & Schmukler (2003) give reasons for optimism as they observe that financial liberalization increases the amplitude of boom-bust cycles in the stock market in the short-run but decreases it in the long-run. They suggest financial liberalization might create the necessary incentives to implement institutional reforms that would stabilize the economy in the end. On the contrary, four hundred years of financial crises lead Rodrik (1998) to consider that boom-bust cycles "are the main story". Distortions and anomalies are the usual lot of financial markets so that the debate should be led in a world of second-best policy, where capital account convertibility might not be an adequate policy for developing countries. The argument, as it goes,

[1]For the positive link between capital inflows and lending booms in developing countries, see Hernández & Landerretche (1999) and Gourinchas, Valdés & Landerretche (2001). Lending booms are good predictors of twin crises (Kaminsky & Reinhart 1999, Tornell & Westermann 2002) although the typical lending boom usually ends up with a soft landing.

is that there is no evidence developing countries without capital controls outperform others,[2] whereas the dangers of capital account liberalization are clearly stated. According to Wyplosz (2001), capital account liberalization should ultimately be a first-best policy, although its benefits might be of second order, but it should only be implemented once a proper economic infrastructure is in place (that is, once the developing country has become a developed one), an evolution which might take decades.

This paper is a theoretical contribution to the debate. We study the long-run effect of capital account liberalization in a small open developing economy. To do this, we consider the financial opening as inducing a transition from a stationary state without movements of capital and with a moderate growth rate to another stationary state with continuous capital inflows, a higher growth rate and non zero external debt ratios (as long as no crisis occur). We dub the latter *external debt-led growth regime*. We show that balance of payments crises may occur in this new stationary state, depending on the structural characteristics of the economy. Our main finding is therefore that capital account liberalization has different effects on different economies: very productive ones will benefit without danger from external savings fuelling a steadier capital accumulation, while less productive economies will also experience higher growth rates during tranquil times, but at the cost of possible BoP crises. We also show that a deeper integration into international finance (in the sense of larger capital inflows) can, as the case may be, strengthen this fragility or, on the contrary, reduce it.

We build the model of a small open economy, with two sectors, producing standardized commodities for the world markets and a non-traded good. Because there is no intra-sectorial capital mobility in the short run, foreign capital inflows (corporate external debt) lead to a real appreciation which manifests itself by a rise in the relative price of non-traded goods with re-

[2]There is now a wide empirical literature on the correlation between capital controls and long-term growth. See Arteta, Eichengreen & Wyplosz (2001) for a recent and critical work on the subject.

gard to traded goods. In the long run, the relative size of the non-tradable sector increases, so that the real exchange rate is brought back to its long-run equilibrium value.[3] Thus, there are two different time-scales: the short time-scale of price movements, capital inflows and outflows and investment decisions, contrasting with the longer time-scale of capital accumulation and changes in its sectorial composition. As the adjustments of the productive structure are slow, sudden variations of prices and investment are likely to occur in the short run, thereby making crises possible.

The precise mechanism of the crisis involves multiple equilibria. On the one hand, a stop in the investment of the non-tradable sector reverses the capital flows to that sector and provokes a real depreciation. On the other hand, a real depreciation diminishes the profitability of the non-tradable sector and leads to a stop in the investment of this sector[4] if it is sufficiently large. During a BoP crisis, the stop in the investment of the non-tradable sector and the real depreciation occur simultaneously and validate each other.[5] The moment at which the change of equilibrium will occur is of course impossible to predict; but we can nevertheless examine the condition under which multiple equilibria arise. This condition depends on structural features of the economy, mainly the sectorial composition of capital which evolves in the long run. Thus, the model allows us to determine whether the endogenous long-run evolution of the economy eventually creates, or not, the conditions under which a crisis becomes possible.

As our aim is to discuss the possibility of crises in the new growth regime led by external debt, independently of short-run cyclical fluctuations, the model excludes factors that could originate a credit cycle.[6] For instance, to

[3]This mechanism is similar to the one studied by the so-called *Dutch disease* literature. See for instance Bruno & Sachs (1982).

[4]This mechanism requires the investment decisions to be taken with a short time horizon.

[5]The paper does not deal with sudden stops in capital inflows, although the framework developed here could easily be used to assess the vulnerability of a developing economy to such an exogenous shock.

[6]We do not deny the existence and relevance of these cycles, but our point is to construct

avoid over-indebtedness and over-investment, the external debt is entirely collateralized and risk-less, and a credit constraint limits the investment.

The stylized facts reproduced by the model are qualitatively consistent with the empirical evidence that has been documented in several papers. The real exchange rate appreciation which occurs in the phase of rapid growth preceding the crises is a well-established fact (Kaminsky & Reinhart 1999, Gourinchas et al. 2001, Tornell & Westermann 2002). This real appreciation is very likely to be partly the result from the massive capital inflows, although this might be truer of Latin America than of Asian countries (Calvo, Leidermann & Reinhart 1996, Athukorala & Rajapatirana 2003). The asymmetrical evolution of the two sectors was empirically observed by Tornell & Westermann (2002). In an event study of twin crises, they observe that the relative size of the non-tradable sector with regard to the tradable sector increases before the crises and decreases after. This sectorial asymmetry may be the consequence of differences in profitability. Ros & Lustig (2000) provide data on a relative profit squeeze in the Mexican tradable sector with regard to the non-tradable sector between 1987 and 1994. The model also emphasizes the central role played by investment in the crisis mechanism. This is consistent with the fact that investment is the component of GDP displaying the largest variability in the crises episodes, growing rapidly before and falling abruptly during the crisis, contrary to consumption which remains rather smooth (Gourinchas et al. 2001, Tornell & Westermann 2002). At last, the crises are often unpredicted and surprising, a fact accounted for by our multiple equilibria story.

This paper is related to the theoretical literature that tries to model the instability of emerging markets and the BoP crises following periods of

a theoretical framework from which they are *a priori* excluded. The crises we describe do not end a phase of over-enthusiastic expansion. Therefore, they are not likely to disappear thanks to some learning process. Empirically, the example of the Argentine crisis of the late nineties which was not preceded by a credit boom (contrary to the Mexican crisis of 1994), largely justifies our perspective.

rapid growth and large capital inflows. Contrary to our model, most of these works suggest an interpretation in terms of boom-bust cycles, applying the theories of the financial cycle to an open economy. Therefore, though they provide convincing explanations of the recent crises in emerging economies, they do not provide a suitable framework to assess the long-run effect of capital account liberalization.

Taylor (1998), for instance, describes a cycle that owes much to the work of Minsky (1986): during the upward phase, risky positions spread in the entire domestic financial system which becomes exposed to a devaluation.

Other works get their inspiration in the theory of the financial accelerator and transpose it to the case of a small open two-sector economy. In the model presented by Aghion, Bacchetta & Banerjee (1999), the growth of investment leads to an increase in the price of non-tradable inputs, deteriorating the profitability of firms and triggering thereby the turnaround of the cycle. These authors have the same perspective as the one we follow in this paper, in that they seek to determine the structural conditions of this cyclical instability. They show that an insufficiently developed domestic financial system is to blame.

In Schneider & Tornell (2000), the investment boom leads to a real appreciation which loosens the credit constraint of the firms producing non-traded goods because of a balance-sheet effect. This, in turn, fuels a still more rapid growth of the investment in this sector. This mechanism gives rise to multiple equilibria: if the economy finds itself in a risky temporary equilibrium with over-investment and real appreciation, a crisis can bring it back to an equilibrium with widespread bankruptcies and a depreciated real exchange rate.[7] The whole cycle is modeled in a dynamical way and risk-taking is rationalized by implicit bail-out guarantees.

[7]One could nevertheless object to the authors that the equilibrium with a depreciated real exchange rate is the only stable one. Indeed, the equilibrium with an appreciated real exchange rate is at the intersection of a constant supply curve with a upward-sloping demand curve.

The paper is organized as follows. Section 2 introduces the theoretical framework and describes the hypotheses of the model which is solved in section 3. In section 4, we study the effects of a capital account liberalization. The assumption made on the entrepreneurs' time horizon are discussed in section 5. Section 6 concludes.

2 The model

The model is in continuous time. $\dot{X}$ is the time derivative of X and $\partial_Y X$ is its partial derivative with regards to Y.

Firms

We consider a small open economy producing a traded good (t) and a non-traded good (n). Each good is produced by a continuum of firms according to a Cobb-Douglas technology of parameter $\alpha \in]0,1[$ with increasing returns. In the sector i, a representative firm j uses an amount L_i^j of labor and capital consisting in a quantity K_i^j of traded goods to produce an output $Y_i^j = A_i(K_i^j)^\alpha(\sigma_i L_i^j)^{1-\alpha}$. The efficiency of labor σ_i depends linearly of the sectorial capital-labor ratio ($\sigma_i = \sigma_0 K_i/L_i$) so that the aggregate sectorial production function is:

$$Y_i = a_i K_i \qquad i = n \text{ or } t \tag{1}$$

with $a_i = A_i \sigma_0^{1-\alpha}$.[8]

p_i is the price of the good i in some foreign currency used as account unit in international trade (say US dollars). We suppose that the traded goods consist of standardized commodities traded on organized world markets and

[8] We would get the same aggregate result from a Leontieff technology with an output-capital ratio a_i and a share of profit ω endogenously determined by a negotiation between managers and workers, the negotiation power of managers being α ($\omega = \text{argmax}\left[(\omega Y_i)^\alpha\big((1-\omega)Y_i\big)^{1-\alpha}\right] = \alpha$). As we are only interested in the aggregate result, we are indifferent to the accurate micro-foundation.

that their domestic producers are price-takers and face an infinitely elastic demand. Thus, p_t is exogenous; we make the further assumption that it stays constant. The relative price $\pi = p_n/p_t$ is the inverse of the real exchange rate. The exchange rate appreciates when π increases.

Consumers

Workers get wages representing a share $1-\alpha$ of the total output of their sector. To make things simple, we suppose it is entirely spent on consumption. Consumers have a utility function with a unitary elasticity of substitution between traded and non-traded goods, so that they spend a constant share $\mu \in]0,1[$ of their income in the purchase of non-traded goods.

External finance

There is no domestic financial system. There is, however, an international financial market, where risk-less bonds denominated in foreign currency and with an infinite maturity can be issued and traded. The holder of a 1$-bond receives an interest r per time unit over an infinite time horizon. As the economy is small, r is an exogenous variable.

Entrepreneurs have a certain knowledge necessary to produce and specific to their sector. They can always threaten their creditors to stop producing so as to renegotiate the interest rate down to r. This possibility prevents them to issue risky bonds and makes their creditors require fully collateralized bonds.

The fixed capital (consisting of the traded good) is used as collateral. We suppose that it does not depreciate. In the case of a bankruptcy, a fraction $h \in [0,1]$ of the fixed capital can be seized upon by external creditors. h is a measure of the degree of integration into international finance. For an economy fully integrated into the international financial market, $h = 1$ and the total amount of fixed capital can be pledged against foreign loans. The opposite case is that of $h = 0$: the entrepreneurs cannot issue debt on the international market and there are no capital inflows.[9] The variable h is a useful device that enables us to exogenously control the size of capital inflows. We will use it later to assess their impact on the economy thanks to comparative statics exercises.

Suppose the representative entrepreneur of the sector i wants to finance an increase in productive capacity $\dot{K}_i$. Each one of its creditor will lend him a limited amount of funds collateralized by some fraction of the firm's fixed capital. We suppose the creditors do not know the total debt of the firm. As a result, a creditor will collateralize its loan by the addition to physical capital this loan helps to finance (taking into account the internal funds used by the entrepreneurs in conjunction with it). Since all creditors do the same, they are sure that the total seizable quantity of fixed capital is enough to pay them all back in case of insolvency. The total increase of external debt $\dot{\mathcal{D}}_i$ financing an increase in fixed capital $\dot{K}_i$ is thus constrained by the following equation:

$$\dot{\mathcal{D}}_i \leq hp_t\dot{K}_i \tag{2}$$

The investment decision

At each moment, the representative entrepreneur of the sector i earns a revenue

$$\mathcal{F}_i = \alpha p_i Y_i - r\mathcal{D}_i = \alpha p_i a_i K_i - r\mathcal{D}_i \tag{3}$$

where $\mathcal{D}_i$ is the total amount of its external debt. If the cash-flow $\mathcal{F}_i$ is negative, the entrepreneur goes bankrupt and its fixed capital is liquidated. The following condition is a solvency condition.

Condition 1 (solvency). *The representative firm of the sector i is solvent and does not go bankrupt as long as*

$$\mathcal{D}_i \leq \frac{\alpha p_i a_i K_i}{r}$$

[9] As long as the economy remains financially opened, capital outflows from the entrepreneurs remain however possible. As we will see later, this is precisely what happens during a BoP crisis.

As the knowledge required to organize production is sector specific, the entrepreneurs cannot invest their revenue in the other sector. They are left with two possibilities: investing it in their own sector by building up new productive capacity, or buying bonds in the international financial market.[10]

If the entrepreneur chooses the former and wants to increase its physical capital by a quantity $\dot{K}_i$, he has to buy a quantity $\dot{K}_i$ of tradable goods. We suppose *he also has to incur some installation cost in the form of non-tradable goods.* This cost corresponds to all the services necessary to install the physical capital (including construction, consulting, etc.) that cannot be imported and have to be produced locally by the firms of the sector n. Because of this assumption, capital inflows financing domestic investment will increase the demand for non-traded goods and lead to a real appreciation. We choose a linear specification for this cost: to install 1 unit of capital, the entrepreneur has to consume η units of non-traded goods. The total cost of investment is then:

$$\mathcal{J}_i = (p_t + \eta p_n)\dot{K}_i \tag{4}$$

Having to choose between buying bonds and investing, the entrepreneur chooses the option that yields the largest expected rate of return. We make the simplifying assumption that the entrepreneurs have a short time horizon so that they only consider the instantaneous rate of return. We shall discuss in section 5 what becomes of those decisions when the entrepreneurs have a larger time horizon.

If the entrepreneur of the sector i invests its internal funds $\mathcal{F}_i$ and some additional amount of external finance $\dot{\mathcal{D}}_i \geq 0$ in the extension of its productive capacity ($\mathcal{J}_i = \mathcal{F}_i + \dot{\mathcal{D}}_i$), the instantaneous rate of return on its internal funds is:

$$\rho_i = \frac{1}{\mathcal{F}_i}\left(\alpha p_i a_i \frac{\mathcal{J}_i}{p_t + \eta p_n} - r\,\dot{\mathcal{D}}_i\right) = \frac{\alpha p_i a_i}{p_t + \eta p_n} + \left(\frac{\alpha p_i a_i}{p_t + \eta p_n} - r\right)\frac{\dot{\mathcal{D}}_i}{\mathcal{F}_i}$$

[10] Note that an entrepreneur from one sector may buy bonds emitted by a firm from the other sector on the international financial market. It is as if financial intermediation took place *outside* the country.

The entrepreneur chooses to increase its physical capital if and only if $\rho_i > r$. In this case, he invests all its revenue and contracts as much new debt as possible (the maximization program has a unique corner solution). On the contrary, if $\rho_i \leq r$, he uses all its revenue to buy bonds remunerated at the rate r (so that his net debt diminishes). A sufficient and necessary condition so that the entrepreneur contracts new debt and invests in physical capital is then:[11]

Condition 2 (investment). *The entrepreneur of the sector i contracts as much debt as possible and invests all its revenue in the extension of its productive capacity if and only if*

$$\frac{\alpha p_i a_i}{p_t + \eta p_n} > r$$

Using equation (2) where $\dot{\mathcal{D}}_i$ takes its largest possible value, one can express the variations of fixed capital and external debt and the total cost of investment as a function of the entrepreneur's cash-flow, when condition 2 is satisfied.

$$p_t \dot{K}_i = \frac{1}{1 - h + \eta\pi} \mathcal{F}_i \quad (5)$$

$$\dot{\mathcal{D}}_i = \frac{h}{1 - h + \eta\pi} \mathcal{F}_i \quad (6)$$

$$\mathcal{J}_i = \frac{1 + \eta\pi}{1 - h + \eta\pi} \mathcal{F}_i \quad (7)$$

Replacing $\mathcal{F}_i$ by its expression (equation (3)), we can now write the investment functions of each sector. To do this, let us first remark that the condition 2 takes the form $\pi > \pi_I$ for the sector n and $\pi < \pi_{II}$ for the sector t, with

$$\pi_I = \frac{r}{\alpha a_n - \eta r} \quad (8)$$

$$\pi_{II} = \frac{\alpha a_t - r}{\eta r} \quad (9)$$

[11] If condition 2 is satisfied, we directly have $\rho_i \geq \frac{\alpha p_i a_i}{p_t + \eta p_n} > r$. On the contrary, if $\rho_i > r$, then $\rho_i - r = \left(\frac{\alpha p_i a_i}{p_t + \eta p_n} - r\right)\left(1 + \frac{\dot{\mathcal{D}}_i}{\mathcal{F}_i}\right) > 0$. The second factor being positive, the first one is then strictly positive, which yields the desired inequality.

$\pi_{II} > \pi_I$ when $\alpha a_t > r(1 + \eta a_t/a_n)$.

We denote $d_i = \frac{\mathcal{D}_i}{p_t K_i}$ the debt-to-capital ratio. Of course, one has $d_i \leq h$. Note that, in the case of a bankruptcy (when condition 1 is not satisfied), there is no investment. This can only happen in a situation where the external debt is strictly positive.[12] According to condition 1, the sector t is insolvent if $rd_t > \alpha a_t$, and the sector n, if $\pi < \pi_B$ with

$$\pi_B = \frac{rd_n}{\alpha a_n} \tag{10}$$

One can easily check that $\pi_B < \pi_I$.

We can now write the sectorial investment functions. For the tradable sector,

$$g_t = \frac{\dot{K}_t}{K_t} = \begin{cases} 0 & \text{if } rd_t > \alpha a_t \\ 0 & \text{if } \alpha a_t > rd_t \text{ and if } \pi \geq \pi_{II} \\ \dfrac{\alpha a_t - rd_t}{1 - h + \eta\pi} & \text{if } \alpha a_t > rd_t \text{ and if } \pi < \pi_{II} \end{cases} \tag{11}$$

The investment function of the non-tradable sector is:

$$g_n = \frac{\dot{K}_n}{K_n} = \begin{cases} \dfrac{\alpha\pi a_n - rd_n}{1 - h + \eta\pi} & \text{if } \pi > \pi_I \\ 0 & \text{if } \pi \leq \pi_I \end{cases} \tag{12}$$

Market equilibrium for non-traded goods

We choose to study a "walrasian" equilibrium, where the price p_n is endogenously determined to equalize the supply of non-traded goods, given by the capital stock K_n, and the demand, an essential component of which stems from the investment. Note that the movements of p_n can stem from both variations of the price level in domestic currency and of the nominal exchange rate. Our model is compatible with all kinds of exchange rate regimes, though a hard peg requires flexible prices in domestic currency. We could as well study a "keynesian" equilibrium by supposing for example that p_n is exogenous. In this case, a utilization rate of productive capacity in the sector n

[12] If there is no debt, condition 1 is always satisfied.

would adjust the supply and the demand. The whole analysis of the model could easily be translated into this alternative framework.

The market clearing condition yields:

$$\underbrace{p_n Y_n}_{\text{supply}} = \underbrace{\mu(1-\alpha)(p_n Y_n + p_t Y_t)}_{\text{consumption}} + \underbrace{\eta p_n(\dot{K}_n + \dot{K}_t)}_{\substack{\text{installation cost}\\ \text{of investment}}}$$

Injecting (1) and (4) into this equation, one gets :

$$\big[1 - \mu(1-\alpha)\big]\pi a_n K_n = \mu(1-\alpha) a_t K_t + \eta\pi(\dot{K}_n + \dot{K}_t)$$

Let us define $\kappa = \dfrac{K_n}{K_t}$. κ is a measure of the sectorial composition of the economy. One can write the previous equilibrium in the following way:

$$\big[1 - \mu(1-\alpha)\big]\pi a_n \kappa = \mu(1-\alpha) a_t + \eta\pi\left(g_n\kappa + g_t\right) \tag{13}$$

3 Solving the model: short-run equilibrium and long-run dynamics

3.1 The short-run equilibrium

The model presented in the previous section can be decomposed in instantaneous (or short-run) equilibria and long-run dynamics. The short-run equilibrium is defined by the triplet (π, g_n, g_t) when it simultaneously satisfies the market equilibrium for non-traded goods (equation (13)) and the investment functions of both sectors (equations (11) and (12)). This equilibrium depends on the state variables κ, d_n et d_t whose evolution is governed by the long-run dynamics $\big(\kappa(\tau), d_n(\tau), d_t(\tau)\big)$. We analyze the short-run equilibrium in this sub-section, leaving it for the next one to study the long-run dynamics. In this sub-section, we consider a given instant τ with a fixed triplet $(\kappa, d_n, d_t) = \big(\kappa(\tau), d_n(\tau), d_t(\tau)\big)$.

We begin by making the following assumption which we assume to be valid henceforth:

Assumption 1.

$$\alpha a_t > r\left(1 + \eta\frac{a_t}{a_n}\right)$$

When this assumption is true, one has $\pi_{II} > \pi_I > 0$, and condition 1 (the solvency condition) is satisfied for the sector t.

One can graphically determine the short run equilibrium as the intersection of an II schedule (corresponding to equation (12)) and a NN schedule (corresponding to the system of equations (13) and (11)). The two schedules are represented on figure 1. This figure shows how three equilibria $\underline{\pi}$, π_I et $\bar{\pi}$ may arise because of the discontinuity of g_n at π_I along the II schedule. Among those equilibria, $\underline{\pi}$ et $\bar{\pi}$ are stable while π_I is unstable. Those results are rigorously derived in appendix A.1. Note that $\bar{\pi}$ can be greater than π_{II}.

The "good" equilibrium Let us examine the "good" equilibrium $\bar{\pi}$ first. In this equilibrium, the entrepreneurs from sector n invest all their internal funds in the extension of their productive capacity. They also use the newly bought fixed capital as a collateral to issue bonds on the international financial market. This leverage enables them to increase the amount of invested funds. If $\bar{\pi} < \pi_{II}$, the entrepreneurs from sector t also invest and the capital inflows that pour into the country amount to $\dot{\mathcal{D}}_t + \dot{\mathcal{D}}_n$. Those capital inflows finance a current account deficit of the same amount, partly due to a trade deficit stemming from the larger imports of capital goods allowed by the external indebtedness.

We show in appendix A.2 that $\partial_h\bar{\pi} > 0$, $\partial_\kappa\bar{\pi} < 0$ and $\partial_{d_i}\bar{\pi} \leq 0$. With a given composition of production, the real exchange rate is all the more appreciated as h is nearer to 1, i.e. as the economy is financially integrated into the international market. It is an essential mechanism of the model: in the short run, capital inflows that partly finance the purchase of (tradable) capital goods, also entail an increase in the demand of non-traded goods, because of the installation cost. This results in an increase of the relative price π, i.e., a real appreciation. Furthermore, a change in the productive structure towards the sector n (an increase of κ), leads to a real depreciation because it increases the supply of non-traded goods. An increase in the debt stock diminishes the payoff to entrepreneurs and the level of their investment, having then a negative impact on the relative price π.

The "bad" equilibrium In the "bad" equilibrium $\underline{\pi}$, condition 2 is only satisfied for the tradable sector. Its entrepreneurs invest all their internal funds in physical capital, with the leverage of external debt. But the real exchange rate is so depreciated that the non-tradable sector is less profitable than the international bonds. Instead of investing their internal funds in the extension of their productive capacity, the entrepreneurs of the sector n send them abroad to get the risk-less rate r. There is no investment in this sector.

Two cases are possible. If $\underline{\pi} > \pi_B$ (the case represented on figure 1), the sales of the n-firms, though insufficient to justify new investments, are large enough to cover the interest payments. The firms are solvent and production takes place in the usual way, generating a revenue that the entrepreneurs put on the financial market. Net capital inflows amount to $\dot{\mathcal{D}}_t - \mathcal{F}_n$, a magnitude much smaller than in the "good" equilibrium.

If, on the contrary, $\underline{\pi} < \pi_B$, the n-firms are insolvent and go bankrupt. The physical capital is liquidated: a fraction h is paid back to the creditors while what remains is put by the entrepreneurs on the international financial market. We will see in section 4 that this cannot be the case along the long-run trajectories we are interested in.

The same results of comparative statics apply to the "bad" equilibrium (see appendix A.2): $\partial_h \underline{\pi} > 0$, $\partial_\kappa \underline{\pi} < 0$ and $\partial_{d_i} \underline{\pi} \leq 0$.

Financial fragility and crises Let us define $\bar{\kappa}$ and $\underline{\kappa}$ by the following equations:

$$\bar{\kappa} = \frac{\mu(1-\alpha)a_t + \dfrac{\eta\pi_I(\alpha a_t - rd_t)}{1-h+\eta\pi_I}}{(1-\alpha)(1-\mu)\pi_I a_n + \dfrac{\alpha(1-h)\pi_I a_n + \eta\pi_I r d_n}{1-h+\eta\pi_I}} \tag{14}$$

$$\underline{\kappa} = \frac{\mu(1-\alpha)a_t + \dfrac{\eta\pi_I(\alpha a_t - rd_t)}{1-h+\eta\pi_I}}{\big[1-\mu(1-\alpha)\big]\pi_I a_n} \tag{15}$$

There is a "bad" equilibrium $\underline{\pi}$ if and only if $\kappa > \underline{\kappa}$, while the "good" equilibrium $\bar{\pi}$ exists if and only if $\kappa < \bar{\kappa}$ (appendix A.1). There are three possible configurations.

$\kappa \leq \underline{\kappa}$ When the productive structure is mainly oriented toward the production of traded goods, there exists a unique equilibrium $\bar{\pi}$, with an appreciated real exchange rate and capital inflows toward both sectors.

$\kappa \geq \bar{\kappa}$ When the productive structure is mainly oriented toward the production of non-traded goods, there exists a unique equilibrium $\underline{\pi}$, with a depreciated real exchange rate and capital inflows limited to sector t. There is no investment in the sector n and its firms might be insolvent.

$\underline{\kappa} < \kappa < \bar{\kappa}$ In the intermediate situation, there exists two stable equilibria $\underline{\pi}$ and $\bar{\pi}$. The state of the economy is *a priori* undetermined.

Suppose that the economy is initially in the "good" equilibrium $\bar{\pi}$. In the third configuration of dual equilibria, the economy is liable to jump down to the "bad" equilibrium at each and every moment. We interpret this sudden and unpredictable transition from $\bar{\pi}$ to $\underline{\pi}$ as a balance of payments crisis. We dub the situation in which two equilibria coexist *financial fragility.*

What does such a crisis look like ? The main sign is a sudden real depreciation. If one were to look for it in a real economy, one would of course have to expect a *nominal* depreciation, as the prices in domestic currency of the non-traded goods are not likely to be sufficiently flexible to fall down so fast. The real depreciation is a consequence of sudden capital outflows from the sector n which are triggered by the stop in the investment of this sector. The stop in investment is in turn provoked by the real depreciation. This stop will not last forever, of course, as the long-run dynamics will move κ downward, thus leading to an increase in $\underline{\pi}$ (see comparative statics in appendix A.2) that will eventually make it profitable to resume investing. But it might nonetheless last for a long period of time: contrary to the boom-bust cycle story, with its short-lived recessions, our model is able to account for severe and long-lasting recessions. Capital outflows play a crucial role in the story. Indeed, the crisis might even happen when $h = 0$, that is, without capital inflows: the very possibility that the internal funds, usually financing domestic investment, flee out of the country and provoke a real depreciation, is the central mechanism of the crisis.

This BoP crisis is of the "self-fulfilling prophecy" kind. A situation of financial fragility is indeed defined by the fact that the stop in the investment of sector n entails a drop of demand for n-goods that leads to a sufficiently large real depreciation and to such a profit-squeeze to justify the initial stop. A first way to interpret it is to consider that the initiative of this change is taken by the entrepreneurs of the sector n who modify their expectation of the short-run rate of return of the investment projects. In this case, the crisis comes from a *coordination failure* and is the direct consequence of pessimistic *animal spirits*, as in Weil (1989). Another way to interpret it is to see the origin of the crisis in the market mechanism (the expectations of the market participants as represented by the choices of the commissaire priseur who announces the real exchange rate). In a real economy, the foreign exchange market is the obvious place where the change of expectation might take place.[13]

[13] One could imagine a crisis of a second kind, triggered by a stop in capital inflows into the sector n. This would happen if this stop entailed a sufficiently large fall in the demand of n-goods to make the economy jump to the configuration where there exists a unique "bad" equilibrium $\underline{\pi}$. We can show, taking into account the long-run dynamics, that this

Our main point is that the economy becomes financially fragile when κ exceeds $\underline{\kappa}$: a large non-tradable sector, when compared to the tradable sector, destabilizes the economy. We can give an intuitive rationale for this by writing the market equilibrium for non-traded goods (equation (13)) in the following way:

$$\left[1-\mu(1-\alpha)\right]\pi a_n = \frac{\mu(1-\alpha)a_t}{\kappa} + \eta\pi\left(g_n + \frac{g_t}{\kappa}\right)$$

When the investment g_n comes down to zero, one of the components of demand vanishes and a fall of the price π is necessary to achieve market clearing. The necessary fall is all the larger as the residual fraction of demand (stemming from sector t) is low. The demand stemming from sector t thus stabilizes the movements of the real exchange rate. The way we just rewrote equation (13) clearly shows that a rise in κ tends to diminish the importance of this stabilizing component of demand.

Let us notice that $\partial_h \underline{\kappa} > 0$. A deeper integration into international financial markets diminishes the size of the financial fragility zone. This is due to the fact that a deeper integration (a higher h) fuels a higher investment g_t in sector t. This increases the stabilizing component of demand and smoothes the fall of the relative price in the case of a stop in the investment of sector n. On the contrary, $\partial_{d_t} \underline{\kappa} < 0$: an increase in the corporate debt of sector t increases the size of the financial fragility zone.

3.2 The long-run dynamics

Whereas π is a jump variable that instantaneously adjusts the short-run equilibrium and is therefore likely to have a discontinuous evolution, $\kappa = \frac{K_n}{K_t}$, d_n and d_t, the state variables of the model, have a smooth evolution with time τ. We dub *long-run dynamics* the trajectory $(\kappa, d_n, d_t)(\tau)$, and *long-run equilibrium* the stationary state of these dynamics.

Using equation (2) when the credit constraint holds, one gets $\dot{d}_i = g_i(h - d_i)$. As long as the short-run equilibrium is $\bar{\pi}$ and the conditions 1 and 2 are

can never happen.

satisfied in both sectors, the dynamical system $(\kappa, d_n, d_t)(\tau)$ is:

$$\left\{\begin{array}{lllll} \dot{\kappa} & = & \kappa(g_n - g_t) & = & \dfrac{\kappa\big[\alpha(\bar{\pi}a_n - a_t) - r(d_n - d_t)\big]}{1 - h + \eta\bar{\pi}} \\ \dot{d}_n & = & g_n(h - d_n) & = & \dfrac{\alpha\bar{\pi}a_n - rd_n}{1 - h + \eta\bar{\pi}}(h - d_n) \\ \dot{d}_t & = & g_t(h - d_t) & = & \dfrac{\alpha a_t - rd_t}{1 - h + \eta\bar{\pi}}(h - d_t) \end{array}\right. \tag{16}$$

Under the same assumptions, there exists a unique stationary state, locally stable (see appendix A.4), denoted (κ^*, d_n^*, d_t^*). One has $d_n^* = d_t^* = h$. We define

$$\bar{\pi}^* = \frac{a_t}{a_n}$$

the relative price in this stationary state. It is the *long-run* equilibrium value of $\bar{\pi}$ which equalizes the profitability of both sectors when they have the same stock of external debt. As there is no capital mobility between sectors, their profitability can be different in the short run. If this is the case, the entrepreneurs of the more profitable sector will have higher internal funds and will invest more than those of the other sector. This indirect way reproduces the effects of a slow capital mobility and ensures the convergence toward the long-run equilibrium.

Let us notice that the assumption 1 can be simply written as $\pi_I < \bar{\pi}^* < \pi_{II}$. It has a very simple interpretation: it just means that investment takes place in the two sectors in the long-run equilibrium.

The value of κ^* is computed by solving the short-run equilibrium (equations (13), (11) and (12)) for $\pi = \bar{\pi}^*$ and $d_n^* = d_t^* = h$. Under assumption 1, $\pi_I < \bar{\pi}^* < \pi_{II}$, and one gets:

$$\kappa^* = \frac{\mu(1-\alpha)a_t + \dfrac{\eta\bar{\pi}^*(\alpha a_t - rh)}{1 - h + \eta\bar{\pi}^*}}{\big[1 - \mu(1-\alpha)\big]a_t - \dfrac{\eta\bar{\pi}^*(\alpha a_t - rh)}{1 - h + \eta\bar{\pi}^*}} \tag{17}$$

This long-run value κ^* is an increasing function of h. This is easily checked by computing the following partial derivative:

$$\partial_h \frac{\eta\bar{\pi}^*(\alpha a_t - rh)}{1 - h + \eta\bar{\pi}^*} = \frac{\eta\bar{\pi}^*\big[\alpha a_t - r(1 + \eta\bar{\pi}^*)\big]}{(1 - h + \eta\bar{\pi}^*)^2}$$

which is positive under assumption 1. In the long run, a deeper financial integration moves the productive structure toward the production of non-tradable goods.

4 The effects of capital account liberalization

4.1 A scenario of capital account liberalization

In this section, we study a scenario of capital account liberalization in a small open economy. We make the assumption 1. We suppose that the economy is not financially opened for $\tau < 0$. The entrepreneurs can neither issue bonds on the international financial market ($h = 0$), nor use their internal funds to buy bonds in that market. Investment is always equal to domestic savings and there is no crisis. We also suppose that the economy is in the long-run equilibrium corresponding to $h = 0$ for $\tau < 0$.

When $\tau = 0$, the capital account is liberalized and h takes a value between 0 and 1 (for the needs of the description, we suppose $h > 0$).

At this moment, h jumps from 0 to a strictly positive value while the triplet $(\kappa, d_n, d_t)_{(\tau=0)} = (\kappa^*_{|h=0}, 0, 0)$ remains constant. At the same time, $\bar{\pi}$ jumps from its long-run value $\bar{\pi}^*$ to a higher short-run value corresponding to the new h. $\dot{\kappa}$, $\dot{d}_n$ and $\dot{d}_t$ therefore become positive (see equation (16)) so that κ and the two debt stocks will increase in the subsequent moments (when $\tau > 0$).

The capital inflows impulsed by the liberalization thus immediately lead to a real appreciation. This makes the sector n more profitable than the sector t and accelerates capital accumulation in the former more than in the latter. The initial appreciation might be so large that $\bar{\pi}(0) > \pi_{II}$. In this case, investment only takes place in the sector n in the aftermath of the opening but the growth of this sector eventually brings $\bar{\pi}(\tau)$ under the threshold π_{II}.

At first, $\kappa(\tau)$ is an increasing function of time τ: the productive structure evolves toward the production of non-traded goods. Meanwhile, the debt-to-capital ratio rises in each sector to reach its long-run value h. If no crisis occurs, the evolution of the three state variables leads to a continuous decrease in $\pi = \bar{\pi}$ (see the comparative statics in appendix A.2), followed by a return to the long-run equilibrium.

Yet, as d_t increases, the threshold $\underline{\kappa}(\tau)$ of the financial fragility zone decreases; and as $\kappa(\tau)$ increases, the economy moves toward that dangerous zone. If, at some point, $\kappa(\tau)$ goes beyond $\underline{\kappa}(\tau)$, the economy becomes financially fragile, making it possible for a crisis to occur.

As a matter of fact, $\kappa(\tau)$ may rise up to some maximal value greater than κ^* before decreasing back and converging towards κ^* (possibly with small oscillations).[14] Indeed, when $\kappa(\tau)$ reaches its long-run value κ^* for the first time, the relative price $\bar{\pi}(\tau)$ is still greater than $\bar{\pi}^*$, since the debt ratios $d_n(\tau)$ and $d_t(\tau)$ are still smaller than h,[15] so that $\kappa(\tau)$ goes on rising. It is maximal when the decrease in $\bar{\pi}(\tau)$ and the increase in $d_n(\tau) - d_t(\tau)$ bring about equal profits in both sectors. Therefore, the economy may become financially fragile (and remain so) after some time, but it may also be immune to BoP crises in the long-run after being financially fragile during some transitory phase.

These different situations are displayed in figure 2 which shows an example of trajectory for $\kappa(\tau)$ and $\underline{\kappa}(\tau)$. In this example, when the degree of integration into the international financial market h is low, the economy never becomes financially fragile (figure 2a); for intermediate levels of h, it is only financially fragile during a transitory phase (figure 2b); when h is high, it becomes financially fragile after some time and remains so in the long-run equilibrium (figure 2c).

[14] This was the case in all our numerical simulations.

[15] From the comparative statics results derived in appendix A.2, one has $\bar{\pi}(\kappa^*, d_n, d_t) > \bar{\pi}^* = \bar{\pi}(\kappa^*, h, h)$ as long as $d_n < h$ and $d_t < h$.

4.2 Financial fragility in the long run

We now investigate whether the long-run equilibrium is financially fragile. First, we are able to derive analytical conditions for the financial fragility in this stationary state. Second, beyond the financial turbulence an economy might sail through on its route from financial autarky to an external debt-led growth regime, our main objective is to assess the structural fragility of the new regime. To do this, we have to compare κ^* with $\underline{\kappa}^*$ ($\underline{\kappa}^*$ is the value taken by the threshold $\underline{\kappa}$ in the long-run equilibrium, i.e. for $d_t = h$).[16]

- If $\kappa^* \leq \underline{\kappa}^*$, the economy is not financially fragile in the long run. Depending on the largest value attained by $\kappa(\tau)$ during the transitory dynamics, it might have gone through a period of financial fragility. If it has safely escaped from it, it can reach the new stationary state without any further danger. The effect of capital account liberalization is then to set up a new growth regime led by external debt in which capital accumulation is faster and the relative size of the non-tradable sector is larger. This regime, once fully in place, cannot be destabilized by an endogenous BoP crisis.[17]

- If, on the contrary, $\kappa^* > \underline{\kappa}^*$, the economy becomes financially fragile, and remains so, after a lapse of time (possibly zero) following the liberalization. A BoP crisis of the kind described in section 3.1 can occur at each moment. As the stationary state lasts forever, the crisis is very likely to occur in the end. We can show that this crisis does not trigger any bankruptcy (see appendix A.3).

To know which of these two cases apply to any given economy, one has to determine the sign of $\kappa^* - \underline{\kappa}^*$ as a function of the different model's parameters.

[16] Notice that the inequality $\kappa^* < \bar{\kappa}$ is always valid in the long-run equilibrium. Indeed, if $\kappa^* \geq \bar{\kappa}$, the only possible short-run equilibrium is $\underline{\pi}$ which cannot be a long-run equilibrium because of assumption 1.

[17] It may of course be destabilized by an exogenous shock leading to a stop in external finance for both sectors, a danger from which an external debt-led economy is seldom entirely immune.

The two following propositions provide some answers.

Proposition 1. *For all* (μ, α, r, η)*, there exists a unique function* $A_t(a_n, h) > \frac{a_n r}{\alpha a_n - \eta r}$ [18] *such that*

- *if* $\frac{a_n r}{\alpha a_n - \eta r} < a_t < A_t(a_n, h)$*, then* $\kappa^* > \underline{\kappa}^*$ *and the economy is financially fragile in the long run,*
- *if* $a_t \geq A_t(a_n, h)$*, then* $\kappa^* \leq \underline{\kappa}^*$ *and the economy is not financially fragile in the long run.*

Additionally, $A_t(a_n, 1)$ *is a decreasing function of* a_n*.*

Proof. See appendix A.5. □

Proposition 2. *For all* (μ, α, r, η)*, there exists two functions* $A_n(a_t, h)$ *and* $A'_n(a_t, h)$ [19] *such that* $A_n(a_t, h) \geq A'_n(a_t, h) > \frac{\eta a_t r}{\alpha a_t - r}$ [20] *and*

- *if* $\frac{\eta a_t r}{\alpha a_t - r} < a_n < A'_n(a_t, h)$*, then* $\kappa^* > \underline{\kappa}^*$ *and the economy is financially fragile in the long run,*
- *if* $a_n \geq A_n(a_t, h)$*, then* $\kappa^* \leq \underline{\kappa}^*$ *and the economy is not financially fragile in the long run.*

Proof. See appendix A.6 □

The two propositions show the existence of a range of parameters in which the economy is financially fragile in the long-run. This confirms analytically what we had already observed in the numerical simulations presented in figure 2. They also provide an economic interpretation of this range of parameters: the conditions that make BoP crises possible are created by a relatively weak level of productivity in any sector.[21] Figure 3 sums up in a unique graph the effect of sectorial productivity on long-run financial fragility.

[18] $\frac{a_n r}{\alpha a_n - \eta r}$ corresponds to the lower bound of a_t when assumption 1 is satisfied.

[19] Although we could not prove it, all our numerical simulations suggest that $A_n = A'_n$.

[20] $\frac{\eta a_t r}{\alpha a_t - r}$ corresponds to the lower bound of a_n when assumption 1 is satisfied.

[21] Note however that the productivities are always sufficiently large to justify foreign capital inflows in the long-run: this is the meaning of our assumption 1.

There is an alternative interpretation of these two propositions. A capital account liberalization is seldom an isolated reform, but most likely takes place in a reform package including, among others, external trade liberalization, in line with the *Washington consensus* (Williamson 1990). It could then be argued that the external trade liberalization might have some positive impact on total factor productivity,[22] thus creating conditions for stability that the capital account liberalization alone might have not achieved. Proposition 1 and 2 can therefore be understood as indicating the minimum sectorial productivity gains that have to be achieved so that the financial opening does not lead to crises in the long run.

4.3 The effect of capital inflows

So far, we have shown that a financially opened economy may experience crises that were not possible in an economy closed to capital flows, but we know nothing of the effect a variation in the integration degree h might have. The origin of financial fragility, as we defined it, is to be found in the possibility of capital *outflows* made by domestic entrepreneurs, but we still do not know whether the capital *inflows* impulsed by the liberalization, and whose magnitude is indicated by h, strengthen or not this phenomenon. We therefore have to study what effect a variation of h has on the sign of $\kappa^* - \underline{\kappa}^*$.

In many cases, of course, it has no effect. For all (a_n, a_t) such that $a_t \geq \max_{h \in [0,1]} A_t(a_n, h)$ (north-east of the diagram (a_n, a_t) in figure 3), the economy is never financially fragile in the long run, whatever the degree of integration into international finance. This is the case of economies with a high "aggregate" technological level. For all (a_n, a_t) such that $a_t < \min_{h \in [0,1]} A_t(a_n, h)$ (south and west of the diagram (a_n, a_t) in figure 3), the economy is always financially fragile in the long run, whatever the magnitude of capital inflows. This second configuration can be understood as economies with a low "aggre-

[22]As there is no consensus on this point, neither in the empirical research, nor in the theoretical works, it is not our intention to discuss it here. We only consider the *possibility* that the productivities a_n et a_t might increase during the reforms.

gate" technological level. It is only in an intermediary range of parameters, in the vicinity of the decreasing curve $a_t = A_t(a_n, 1)$, that the magnitude of capital inflows has an effect on the possibility of crises.

This effect is ambiguous. On the one hand, as we noticed in section 3.2, an increase in h moves the long-run sectorial composition toward the financial fragility zone (κ^* is an increasing function of h), a mechanism that originates the dynamics triggered by the opening of the capital account and described in section 4.1. But on the other hand, an increase in h has also a stabilizing effect on the economy, since it moves the threshold $\underline{\kappa}^*$ upward. The reader may recall that $\partial_h \underline{\kappa} > 0$ whereas $\partial_{d_t} \underline{\kappa} < 0$ (section 3.1). Between those two contradictory effects, the former is the stronger. The sign of $\partial_h \underline{\kappa}^*$ is the same as that of $\partial_h \left[\frac{1}{1-h+\pi_I} \left(1 - \frac{h}{\bar{\pi}^* \left(1 + \frac{1}{\pi_I}\right)} \right) \right] = \frac{\bar{\pi}^* - \pi_I}{\bar{\pi}^*(1-h+\pi_I)^2}$ which is positive under assumption 1.

The overall effect of h depends of the relative technological levels in the two sectors. Let us examine the variations of the function $h \mapsto A_t(a_n, h)$. Numerical simulations led us to distinguish three different configurations, represented on figure 4 and characterized by different values of a_n. (1) When a_n is large enough (figure 4a), $h \mapsto A_t(a_n, h)$ is an increasing function: the economy becomes financially fragile for higher values of h and is not subject to crises for lower values of h. In this case, the deeper the integration into international finance (or the larger the capital inflows), the more likely the economy is to be financially fragile in the long run. (2) When a_n is low (figure 4c), on the contrary, this function is decreasing: the cause of long-run financial fragility is then to be found in too weak a financial integration. (3) At last, in a small range of intermediate values of a_n (figure 4b), this function may have an "inverted U-shape": the stationary state is then immune to BoP crises when the economy is either deeply or almost not integrated into international finance.[23] Note that in the intermediary range of parameters

[23] Analytically, the shape of the function $h \mapsto A_t(a_n, h)$ is constrained by the fact that the equation $\kappa^* - \bar{\kappa}^* = 0$ can be reduced to the roots of a polynomial of order 2 in h: for

we are interested in, a high level of a_n means a low level a_t and vice versa. [24]

By comparing the extreme points $h = 0$ et $h = 1$, we can analytically confirm the role of the relative technological levels a_n and a_t in determining the effect of h on the long-run financial fragility. This is the aim of the following proposition.

Proposition 3. *For all (μ, α, r, η), there exists a unique $B_n > \frac{r}{\alpha}$ such that*

- $A_t(a_n, 1) - A_t(a_n, 0) > 0$ *(< 0) when $a_n > B_n$ ($< B_n$)*
- *for all (a_n, a_t) such that $a_n > B_n$ and $A_t(a_n, 0) \leq a_t < A_t(a_n, 1)$, there exists a unique $h_0 \in [0, 1]$ with*

$$\begin{aligned} \forall h \quad & 0 \leq h \leq h_0 \Rightarrow \kappa^* \leq \underline{\kappa}^* \quad \textit{(no financial fragility)} \\ & h_0 < h \leq 1 \Rightarrow \kappa^* > \underline{\kappa}^* \quad \textit{(financial fragility)} \end{aligned}$$

- *for all (a_n, a_t) such that $a_n < B_n$ and $A_t(a_n, 1) \leq a_t < A_t(a_n, 0)$, there exists a unique $h_0 \in [0, 1]$ with*

$$\begin{aligned} \forall h \quad & 0 \leq h < h_0 \Rightarrow \kappa^* > \underline{\kappa}^* \quad \textit{(financial fragility)} \\ & h_0 \leq h \leq 1 \Rightarrow \kappa^* \leq \underline{\kappa}^* \quad \textit{(no financial fragility)} \end{aligned}$$

Proof. See appendix A.7. □

Thus, when the productivity of the non-tradable sector is high ($a_n > B_n$) and the productivity of the tradable sector is low ($a_t < A_t(B_n, 1)$), capital inflows tend to make the economy more fragile. In a certain range of t-sector productivities, there exists a maximal degree of integration into international finance (or a maximal magnitude of capital inflows) beyond which the economy becomes financially fragile in the long run. On the contrary, when the sector n is less productive ($a_n < B_n$) and the sector t has a high productivity ($a_t > A_t(B_n, 1)$), capital inflows tend to stabilize the economy in the long run and we get the inverse result. This situation is illustrated by the frames (a) et (b) of figure 3.

each a_n, there is at most two values of h separating different regimes.

[24] Proposition 1 establishes that $A_t(a_n, 1)$ is decreasing in a_n.

5 Notes on the entrepreneurs' time horizon

We derived the previous results with the assumption that the time horizon of the entrepreneurs did not exceed the present moment. This assumption played an important role in our multiple equilibria story since the discontinuity of the demand function at π_I required the investment to be a function of the current prices. We now turn to the case of longer time horizons. We first briefly justify why rational entrepreneurs in strategic interaction with each other would take inter-temporal investment decisions by considering the current prices (the argument draws on Keynes (1937)). Then, we examine under what conditions the different trajectories of the model are rational expectation equilibria.

With infinitely-lived investment projects, entrepreneurs should take into account the whole trajectory of future prices before deciding to invest or to buy foreign bonds. Denoting γ their discount rate and supposing they have an infinite time horizon, the rate of return on a bond purchased at date τ is

$$\int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} r = r/\gamma$$

while the rate of return on investment in sector i is

$$\int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} \frac{\alpha a_i p_i(s)/p_t - rh}{1 - h + \eta\pi(\tau)}$$

In the case of the tradable sector, this rate of return is equal to $\frac{1}{\gamma}\frac{\alpha a_t - rh}{1-h+\eta\pi(\tau)}$ and does not depend on future values of the real exchange rate, so that the investment condition 2 remains valid. This is not the case of the non-tradable sector, whose investment condition has to be modified:

Condition 3. *At time τ, an entrepreneur of sector n invests if and only if $f(\tau) > 0$, with*

$$f(\tau) = \int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} \pi(s) - \frac{1}{\gamma}\frac{r}{\alpha a_n}[1 + \eta\pi(\tau)]$$

This condition is of course equivalent to condition 2 in the limit $\gamma \to +\infty$.

Radical uncertainty As the future relative prices $\left(\pi(s)\right)_{s>\tau}$ depend on the future decisions that will be taken by other entrepreneurs, a given entrepreneur is in a situation of radical uncertainty. If rationality, in the sense of rational expectation equilibrium, is not common knowledge, he simply does not know what the future prices will be. This situation was briefly analyzed by Keynes (1937), who suggest three possible methods used by the entrepreneurs to handle that uncertainty, all of which could justify our simplifying assumption. (1) The entrepreneurs choose to ignore the possible changes of future prices. If the future prices are expected to remain equal to the current price, condition 3 is equivalent to condition 2 and we are back to our former assumption. (2) The entrepreneurs consider that the current price already expresses a correct calculation of expected future prices. A high relative price (greater than π_I) results from a high aggregate investment of sector n which indicates that the rate of return is expected to justify investment, and vice-versa. (3) Each entrepreneur decides to imitate his fellows. The decisions of the entrepreneurs are then coordinated with the help of a *convention* and our model studies the particular convention stating that a current price greater than π_I is a sign of prosperous business justifying investment.

Are expectations necessarily missed? If we accept the hypothesis that reasonable and practical entrepreneurs may take their decisions on the basis of current prices, we enter a world of almost systematically missed expectations. But we still have to address the following question: are those missed expectations necessary to the results of the model? could it not be that perfectly well expected future prices might also be compatible with them? In other words, we have to determine whether the several trajectories of future relative prices $\left(\pi(s)\right)_{s>\tau}$ described in the model are rational expectation equilibria.

A trajectory without a crisis can be a rational expectation equilibrium.

From condition 3, this is the case when

$$\forall \tau \geq 0 \quad \int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} \bar{\pi}(s) > \frac{1}{\gamma} \frac{r}{\alpha a_n} [1 + \eta \bar{\pi}(\tau)]$$

This will be the case some time after the financial opening.[25]

On the contrary, with an infinite time horizon, a trajectory with a crisis and with a symmetric behavior of the entrepreneurs is not a rational expectation equilibrium. To prove it, let us consider the case of a crisis happening at time τ so that $\pi(\tau) = \underline{\pi}(\tau) < \pi_I$ and examine the decision taken by an entrepreneur with an infinite time horizon.

- If no entrepreneur of the sector n ever invest again, $g_n = 0$ and κ decreases since the tradable sector goes on growing. This entails a continuous rise in $\underline{\pi}$. As $\kappa \to 0$, $\underline{\pi} \to +\infty$. If the entrepreneur foresees this evolution, he will eventually find it profitable to invest, at least when $\underline{\pi}$ goes above π_I. A trajectory where the investment of sector n would stop forever is therefore impossible.

- If the entrepreneurs of the sector n stop investing until a certain date $T > \tau$ characterized by $f(T) > 0$ and start investing again after that, prices will continuously increase until T, discontinuously rise at T to attain some value strictly greater than $\bar{\pi}^*$ and decrease to converge to the long-run value $\bar{\pi}^*$. In this case, the function $f(\tau)$ of condition 3 can be written:

$$\begin{aligned} f(\tau) &= \int_{\tau}^{T} ds\, e^{-\gamma(s-\tau)} \underline{\pi}(s) + \int_{T}^{+\infty} ds\, e^{-\gamma(s-\tau)} \bar{\pi}(s) - \frac{1}{\gamma} \frac{r}{\alpha a_n} [1 + \eta \underline{\pi}(\tau)] \\ &\geq \int_{\tau}^{T} ds\, e^{-\gamma(s-\tau)} \underline{\pi}(s) + \frac{\bar{\pi}^*}{\gamma} e^{-\gamma(T-\tau)} - \frac{1}{\gamma} \frac{r}{\alpha a_n} (1 + \eta \bar{\pi}^*) \end{aligned}$$

The last term goes to $\frac{1}{\gamma} \left[\bar{\pi}^* - \frac{r}{\alpha a_n} (1 + \eta \bar{\pi}^*) \right] > 0$ when $\tau \to T$. As a

[25] Indeed, as the economy converges to the new stationary state, $\int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} \bar{\pi}(s)$ gets close to $\int_{\tau}^{+\infty} ds\, e^{-\gamma(s-\tau)} \bar{\pi}^* = \bar{\pi}^*/\gamma > \frac{1}{\gamma} \frac{r}{\alpha a_n} [1 + \eta \bar{\pi}(\tau)]$. The last inequality is equivalent to $\bar{\pi}(\tau) < \pi_{II}$, which is satisfied after some time, since $\bar{\pi}(\tau)$ converges to $\bar{\pi}^*$ and $\bar{\pi}^*$ is strictly lower than π_{II} from assumption 1.

consequence, the entrepreneur will find it profitable to start investing again at a date strictly lower than T.

- The two previous points enable us to conclude that a trajectory with a crisis can never be a rational expectation equilibrium with an infinite time horizon.

This result can be extended to a case where the entrepreneurs start investing again depending on a sunspot variable with some subjective probability distribution.

If the entrepreneurs' time horizon is infinite, a crisis can only happen if future relative prices are not perfectly foreseen. In particular, if all entrepreneurs take the same decision, the expected length of the crisis will always be greater than its actual length. Note however that the initial decision to stop investing might be fully justified by the following evolution of relative prices, although the end of the crisis is expected to happen later than it actually will. We now show that crises may occur without missed expectations if the time horizon is bounded.

Suppose the investment decisions are taken by project managers who are evaluated on a regular basis by the entrepreneurs, each evaluation period having the same length $\delta < \infty$, and that these project managers compute the expected rate of return with a time horizon bounded by the next evaluation date.[26] Then, a crisis may happen at date τ if it lasts until the end of the current evaluation period. Let us denote $T(\tau)$ the maximal date beyond which the non-tradable sector will necessarily start investing again, and τ^- the date of the last evaluation. The length of the crisis is bounded by $p\,\delta - (\tau - \tau^-)$, where $p = \max\{n \in \mathbb{N} |\, \tau^- + n\,\delta < T(\tau)\}$. If the project managers start investing again at the beginning of the $p+1^{\text{st}}$ evaluation period, the whole trajectory is a rational expectation equilibrium.

[26]This assumption can be thought of as a short-cut for a more detailed model of industrial organization which would wholly specify the strategic interaction between entrepreneurs and project managers as well as the incentives of the latter.

Therefore, the model does not require, for a crisis to occur, either irrational and excessively pessimistic entrepreneurs, or entrepreneurs with a zero time horizon. The crisis is possible if the frequency with which project managers are evaluated is high enough.

6 Conclusion

We have built a model describing the long-run dynamics of a small open two-sector developing country after a capital account liberalization. Because changes in the sectorial composition of supply take time, demand shocks are absorbed by fluctuations of the relative price of both sectors, the real exchange rate. When the economy is open to capital flows, a sudden real depreciation and a fall in the investment of the non-tradable sector can simultaneously occur and validate each other, thereby provoking a balance of payments crisis. The conditions which make these crises possible depend on structural characteristics of the economy, in particular the sectorial composition of fixed capital: a crisis may only occur when the size of the sector n is sufficiently large compared with that of the sector t. These crises do not require any excessively optimistic behavior of entrepreneurs or creditors, leading to over-investment or over-indebtedness. They do not require systematically missed expectations either, as long as the investment decisions have a sufficiently short time horizon, which might be the case if project managers are evaluated at short regular intervals. Thus, this formal framework seems well suited to discuss the long-run effects of capital account liberalization, once the possible learning process is over.

We have shown how the capital account liberalization leads to a growth regime financially fragile in the long run —meaning by this that balance of payments crises may occur in it— in economies where any sector has a low productivity (though still large enough to justify capital inflows in the long-run). On the contrary, high sectorial productivities keep the economy away from the danger of balance of payments crises.

In an intermediate range of productivities, the magnitude of foreign capital inflows determines whether the economy is financially fragile in the long run. The foreign capital inflows have a double effect. On the one hand, they provoke a real appreciation in the short run and accelerate the capital accumulation in the non-tradable sector, which tends to make the economy more fragile. On the other hand, they finance the investment of the tradable sector which is a stabilizing component of demand in the market for non-traded goods. When the productivity of the non-tradable sector is low whereas the productivity of the tradable sector is high, the second effect dominates the first one and larger capital inflows tend to diminish the intrinsic danger of the financial opening. On the contrary, when the productivity of the non-tradable sector is higher and the tradable sector has a low productivity, the first effect dominates the second and capital inflows strengthen the financial fragility.

What do these results imply as regards the effect of a reform policy that includes capital account liberalization in a developing country? The general lesson is that the opening of the capital account leads to a higher growth rate in the long run but also creates new risks of crises. The exact effects differ with the structure of the economy. This work is therefore a case for taking into account the particular characteristics of the economy when trying to assess the possible success of such a reform in terms of macroeconomic performance.

An important point is the complementarity of the implemented reforms. Indeed, sufficiently strong productivity gains always allow to open the economy to capital flows without making balance of payments crises possible. Will they be fostered by other measures? It is particularly crucial to study with great care the productivity gains possibly induced by an external trade reform preceding the financial opening: will they be sufficient to make it safe?

The intensity of technical progress matters a great deal; so does its direction. For a low productivity of the tradable sector and a higher productivity of the non-tradable sector, an increase in the size of capital inflows may be destabilizing. In this case, a policy of capital controls aimed at reducing their intensity might allow to reap the benefits of higher growth rates fueled by external savings while preventing crises to occur, at least until future productivity gains take place in the tradable sector.

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A Appendix

To simplify notations, we choose, without loss of generality, the unit of the non-traded good so that $\eta = 1$. Besides, we denote $\beta = \mu(1-\alpha)$.

A.1 Short-run equilibria

For a given g_n, the system of equations (13) and (11) has a unique solution (π, g_t), which defines a first strictly increasing relation between π and g_n (represented by the NN schedule on figure 1) :

$$g_n = \mathfrak{S}(\pi) = \begin{cases} (1-\beta)a_n - \dfrac{\beta a_t}{\kappa\pi} - \dfrac{\alpha a_t - r d_t}{\kappa(1-h+\pi)} & \text{si} \quad \pi < \pi_{II} \\ (1-\beta)a_n - \dfrac{\beta a_t}{\kappa\pi} & \text{si} \quad \pi \geq \pi_{II} \end{cases}$$

Denote $g_n = \mathfrak{D}(\pi)$ the second relation given by equation (12) (represented by the II schedule on figure 1). A market equilibrium π^{e} must satisfies $(\mathfrak{S} - \mathfrak{D})(\pi^{\mathrm{e}}) = 0$.

When $\pi < \pi_I$, there exists a unique equilibrium, characterized by $g_n = 0$, if and only if the solution of $\mathfrak{S}(\pi) = 0$ is strictly lower than π_I. This is the case when $\kappa > \underline{\kappa}$ with

$$\underline{\kappa} = \frac{\beta a_t + \dfrac{\pi_I(\alpha a_t - r d_t)}{1-h+\pi_I}}{(1-\beta)\pi_I a_n}$$

We denote $\underline{\pi}$ this equilibrium. We say an equilibrium π^{e} is stable when $\dfrac{\partial}{\partial\pi}(\mathfrak{S}-\mathfrak{D})(\pi^{\mathrm{e}}) > 0$ (the slope of the NN schedule must be greater than the slope of the II schedule at the equilibrium). With this definition, $\underline{\pi}$ is a stable equilibrium.

When $\pi > \pi_I$, the equilibrium must satisfies the equation $\mathfrak{S}(\pi) = \dfrac{\alpha\pi a_n - r d_n}{1-h+\pi}$. This equation is of the second degree in π and it is easy to check that the product of its roots is strictly negative. Therefore, it has a unique strictly positive solution. This solution is an equilibrium if and only if it is strictly greater than π_I. This is the case when $\kappa < \bar{\kappa}$, with

$$\bar{\kappa} = \frac{\dfrac{\beta a_t}{\pi_I} + \dfrac{\alpha a_t - r d_t}{1-h+\pi_I}}{(1-\beta)a_n - \dfrac{\alpha\pi_I a_n - r d_n}{1-h+\pi_I}}$$

We also check that this solution is stable.

When both $\underline{\pi}$ and $\bar{\pi}$ are equilibria, and only in that case, π_I is an unstable equilibrium.

To sum it up, there are three possible configurations:

1. a unique stable "good" equilibrium $\bar{\pi}$, greater than π_I.
2. a unique stable "bad" equilibrium $\underline{\pi}$, lower than π_I.
3. three equilibria, $\underline{\pi} < \pi_I < \bar{\pi}$, $\underline{\pi}$ and $\bar{\pi}$ being stable, whereas π_I is unstable.

A.2 Comparative statics in the short-run equilibrium

By differentiating $\big(\mathfrak{S} - \mathfrak{D}\big)\big(\bar{\pi}, h, \kappa, d_i\big) = 0$ with regards to h, κ and d_i, we find the partial derivatives of $\bar{\pi}$: $\partial_h \bar{\pi} = -\dfrac{\partial_h(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}{\partial_\pi(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}$, $\partial_\kappa \bar{\pi} = -\dfrac{\partial_\kappa(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}{\partial_\pi(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}$ and $\partial_{d_i} \bar{\pi} = -\dfrac{\partial_{d_i}(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}{\partial_\pi(\mathfrak{S} - \mathfrak{D})(\bar{\pi})}$.

From appendix A.1, the equilibrium $\bar{\pi}$ is stable and we have $\partial_\pi(\mathfrak{S} - \mathfrak{D})(\bar{\pi}) > 0$. Furthermore, it is very easy to check that $\partial_h(\mathfrak{S} - \mathfrak{D})(\bar{\pi}) < 0$, $\partial_\kappa(\mathfrak{S} - \mathfrak{D})(\bar{\pi}) > 0$ and $\partial_{d_i}(\mathfrak{S} - \mathfrak{D})(\bar{\pi}) \geq 0$.

Therefore, we conclude that $\partial_h \bar{\pi} > 0$, $\partial_\kappa \bar{\pi} < 0$ and $\partial_{d_i} \bar{\pi} \leq 0$.

The calculation and the results are analogous in the case of the equilibrium $\underline{\pi}$.

A.3 No bankruptcy during the crises in the long-run equilibrium

The equilibrium $\underline{\pi}$ leads to bankruptcies if $\underline{\pi} < \pi_B$, i.e. when $(\mathfrak{S} - \mathfrak{D}_1)(\pi_B) > 0$. We can write this condition in the form $\kappa < \kappa_B$ with

$$\kappa_B = \frac{\mu(1-\alpha)a_t + \frac{\eta \pi_B}{1-h+\eta\pi_B}(\alpha a_t - r d_t)}{[1-\mu(1-\alpha)]\pi_B a_n}$$

Let us study the long-run equilibria. We first consider the case $h = 1$. We have:

$$\kappa^* - \kappa_B^* = \frac{\mu(1-\alpha)a_t + \alpha a_t - r}{[1-\mu(1-\alpha)]a_t - (\alpha a_t - r)} - \frac{\mu(1-\alpha)a_t + \alpha a_t - r}{[1-\mu(1-\alpha)]\pi_B a_n}$$

The sign of this expression is the same as the sign of

$$[1-\mu(1-\alpha)]\pi_B a_n - \big[[1-\mu(1-\alpha)]a_t - (\alpha a_t - r)\big] = -(a_t - \frac{r}{\alpha})(1-\alpha)(1-\mu) < 0$$

from assumption 1.

As κ^* is an increasing function of h and κ_B^* decreases with h, $\kappa^* - \kappa_B^*$ is strictly negative for all $h \in [0, 1]$. There is no bankruptcy in the "bad" equilibrium when the crisis occurs in the long-run equilibrium.

A.4 Stability of the long-run equilibrium

As long as no crisis occurs, the dynamical system (16) has a unique stationary point $(\kappa, d_n, d_t) = (\kappa^*, h, h)$, characterized by $\bar{\pi} = \bar{\pi}^* = a_t/a_n$. We show here that this equilibrium is locally stable.

The Jacobian matrix of the dynamical system at this point is:

$$J(\kappa^*, h, h) = \begin{pmatrix} \partial_\kappa \dot{\kappa} & \partial_{d_n} \dot{\kappa} & \partial_{d_t} \dot{\kappa} \\ \partial_\kappa \dot{d}_n & \partial_{d_n} \dot{d}_n & \partial_{d_t} \dot{d}_n \\ \partial_\kappa \dot{d}_t & \partial_{d_n} \dot{d}_t & \partial_{d_t} \dot{d}_t \end{pmatrix} = \begin{pmatrix} \partial_\kappa \dot{\kappa} & \partial_{d_n} \dot{\kappa} & \partial_{d_t} \dot{\kappa} \\ 0 & -g_n & 0 \\ 0 & 0 & -g_t \end{pmatrix}$$

The three eigenvalues of this matrix are $\partial_\kappa \dot{\kappa}$, $-g_n$ et $-g_t$. They are all negative. Indeed,

$$(\partial_\kappa \dot{\kappa})(\kappa^*, h, h) = \frac{\alpha \kappa^* \big[(1-h)a_n + \eta a_t\big]}{(1-h+\eta\bar{\pi}^*)^2} \, \partial_\kappa \bar{\pi} < 0$$

because $\partial_\kappa \bar{\pi} < 0$ (see appendix A.2). The long-run dynamics are therefore locally stable around the long-run equilibrium.

A.5 Proof of proposition 1

When $a_t = \frac{a_n r}{\alpha a_n - \eta r}$, $\kappa^* - \underline{\kappa}^* > 0$. Furthermore, $\kappa^* - \underline{\kappa}^* \longrightarrow -\infty$ when $a_t \to -\infty$. Because of the continuity of $\kappa^* - \underline{\kappa}^*$, this proves the existence of A_t. Its uniqueness comes from the fact that equation $\kappa^* - \underline{\kappa}^* = 0$ can be reduced to a polynomial equation of degree 2 in a_t and cannot have more than two solutions.

By explicitly writing A_t for $h = 1$, one easily checks that its partial derivative with regards to a_n is strictly negative. Thus, A_t is a decreasing function of a_n when $h = 1$.

A.6 Proof of proposition 2

When $a_n = \frac{\eta a_t r}{\alpha a_t - r}$, $\kappa^* - \underline{\kappa}^* > 0$. Furthermore, $\kappa^* - \underline{\kappa}^* \longrightarrow -\infty$ when $a_n \to -\infty$. Because of the continuity of $\kappa^* - \underline{\kappa}^*$, the function $a_n \mapsto \kappa^* - \underline{\kappa}^*$ has an odd number of roots on $]\frac{\eta a_t r}{\alpha a_t - r}, +\infty[$. The equation $\kappa^* - \underline{\kappa}^* = 0$ can be reduced to a polynomial equation of order 3 in a_n. Therefore, it has either 1 or 3 roots in this interval. We denote A_n the greater and A'_n smaller root.

A.7 Proof of proposition 3

To simplify notations, we choose, without loss of generality, the unit of the non-traded good so that $\eta = 1$. Besides, we denote $\beta = \mu(1 - \alpha)$.

The proposition 1 enables us to directly compute $A_t(a_n, h)$ as a function of h by using the formula of the roots of a second order polynomial. We want to solve the inequality $A_t(a_n, 1) - A_t(a_n, 0) > 0$ as a function of a_n. This will prove the first part of the proposition. The second part follows immediately since $\kappa^* - \underline{\kappa}^*$ can be reduced to a polynomial of order 2 in h. When $\min(A_t(a_n, 0), A_t(a_n, 1)) \leq a_t < \max(A_t(a_n, 0), A_t(a_n, 1))$, this polynomial takes different values with opposite signs in 0 and 1, and has therefore a unique root h_0 in $[0, 1]$.

By writing explicitly $A_t(a_n, 1) - A_t(a_n, 0)$, and reducing it to a common denominator, we show that the inequality $A_t(a_n, 1) - A_t(a_n, 0) > 0$ is equivalent to $x - y > 0$ with

$$
\begin{aligned}
x &= \alpha^2\beta(1-\beta)a_n^3 + r^2\alpha(1-\alpha+\beta)a_n + 2r^3\alpha - r\alpha^2(2\beta a_n + r)a_n \\
y &= a_n\sqrt{\alpha(1-\beta)\left[4a_n r\alpha\beta(1-\alpha-\beta)(\alpha a_n - r)(r + a_n\beta) + \alpha(1-\beta)\left(r^2 + 2a_n r\beta - a_n^2\alpha\beta\right)^2\right]}
\end{aligned}
$$

We have $y > a_n\alpha(1-\beta)(r^2 + 2a_n r\beta)$ and $x > -r\alpha^2(2\beta a_n + r)a_n$. Therefore,

$$x + y > a_n r\alpha(r + 2a_n\beta)(1 - \alpha - \beta) > 0$$

Multiplying by the conjugate, we can write our inequality $(x - y)(x + y) > 0$, which yields,

$$(\alpha a_n - r)(\alpha\beta a_n^2 - r(1-\alpha)a_n - r^2) > 0$$

The left member, as a function of a_n, has at most two roots in the interval $]r/\alpha, +\infty[$. Since $(\alpha\beta a_n^2 - r(1-\alpha)a_n - r^2)$ goes to $+\infty$ when a_n goes to $+\infty$ and is equal to $-r^2(1-\beta)/\alpha < 0$ when $a_n = r/\alpha$, it has a unique root B_n. The inequality is then equivalent to $a_n > B_n$.

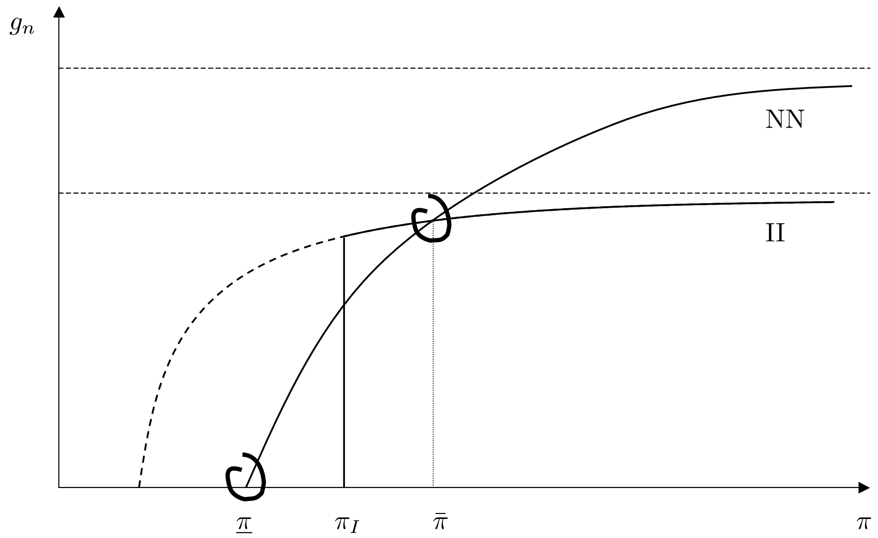
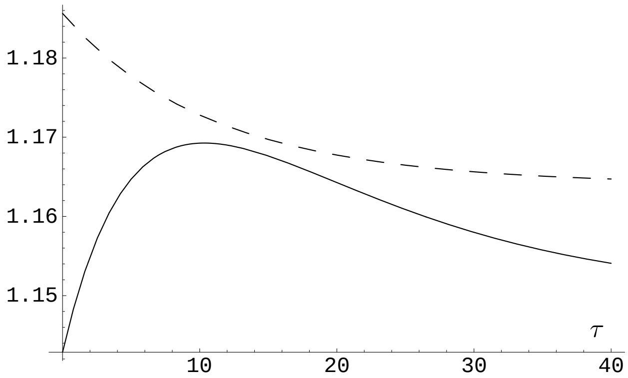
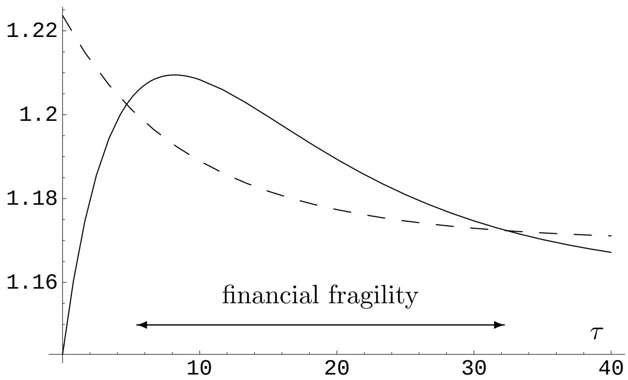


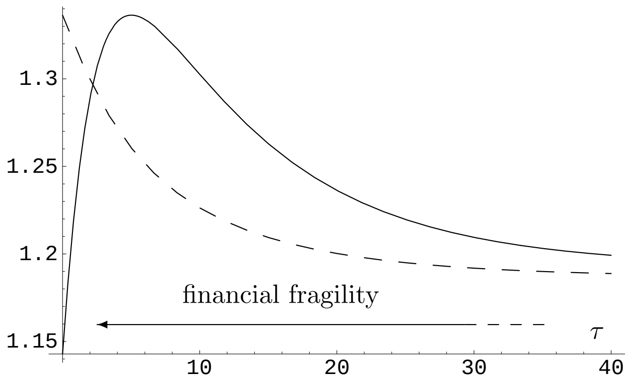
Figure 1: Short run equilibrium of the market for non-traded goods



(a) $h = 0.2$



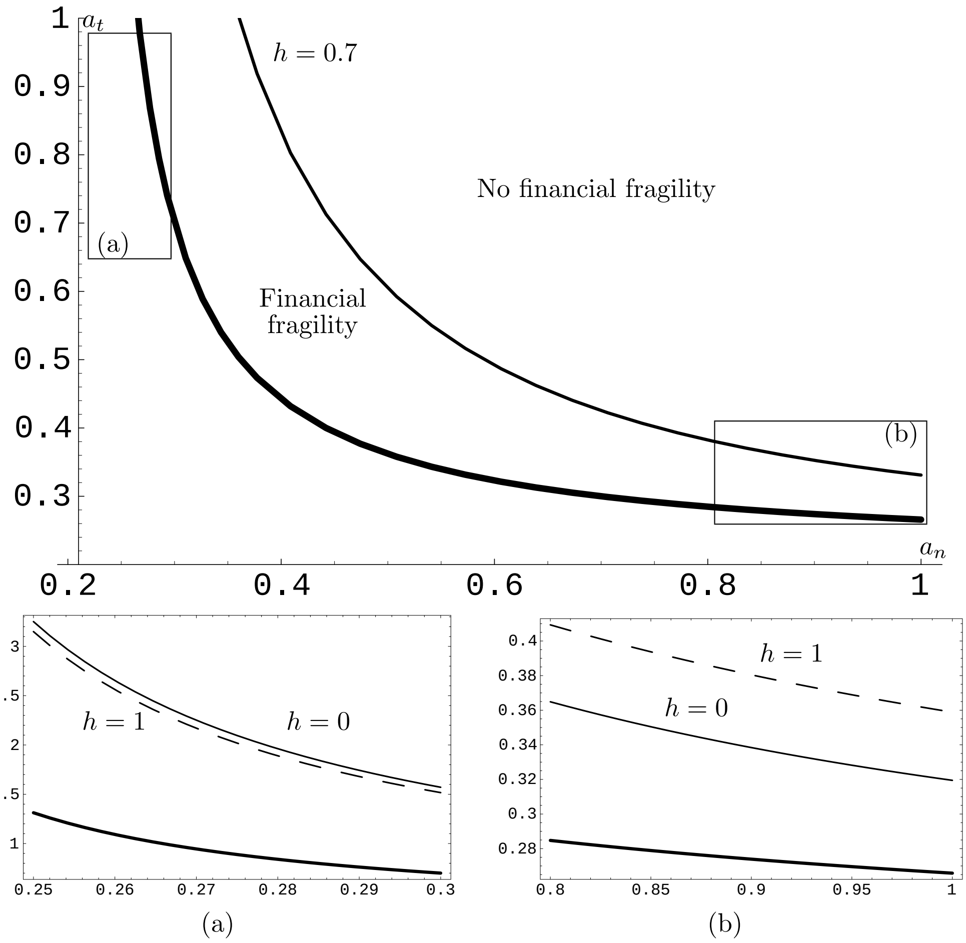
(b) $h = 0.4$



(c) $h = 0.7$

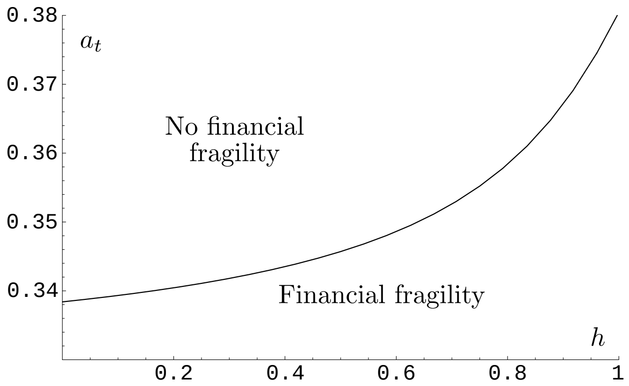
Note: This graph shows the evolution of $\kappa(\tau)$ (solid line) and of the threshold $\underline{\kappa}(\tau)$ (dotted line), for different values of h and for $\alpha = 1/3$, $\mu = 0.7$, $a_t = 0.3$, $a_n = 1.2$, $r = 7\%$ and $\eta = 1$.

Figure 2: Long-run dynamics of the productive structure $\underline{\kappa}$

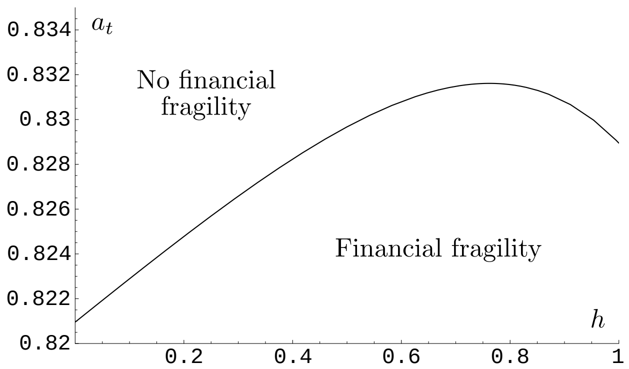


Note: This graph shows the partition of the plane (a_n, a_t) in financially fragile or non-financially fragile zones, for $\alpha = 1/3$, $\mu = 0.7$, $r = 7\%$, $\eta = 1$ and $h = 0.7$. The bold line represents the lower bound of assumption 1. The thin line (solid or dotted) represents the upper bound of the financial fragility zone and corresponds to the function $a_n \mapsto A_t(a_n, h)$. Two close-up shots (a) and (b) display the effect of a variation of h from 0 to 1.

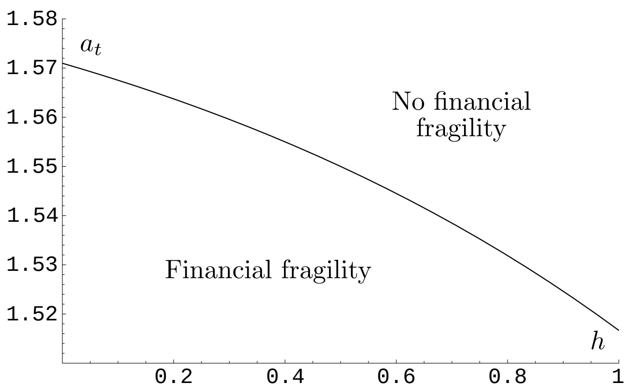
Figure 3: The effect of sectorial productivities



(a) $a_n = 0.9$



(b) $a_n = 0.4$



(c) $a_n = 0.3$

Note: This graph shows the partition of the plane (h, a_t) in financially fragile or non-financially fragile zones as a function of a_n, for $\alpha = 1/3$, $\mu = 0.7$, $r = 7\%$ and $\eta = 1$. The curve dividing the plane corresponds to the function $h \mapsto A_t(a_n, h)$.

Figure 4: The effect of the degree of integration into international finance h