A Search Model of Electronic Cash Cards[¤]

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Abstract

The aim of this article is to take into account the various costs related to the use of two types of currencies: …at money and electronic cash cards. A search theoretic model of monetary exchange is used to study the conditions for a type of currency to be spent when there are shocks to production, i.e. when the quantities of goods o¤ered can be small or big. First, depending on the transaction costs and the utility derived from consuming a good in big quantity, it is shown that there exist equilibria where each type of currency is, is not, or is partially spent on small goods. Second, the e¤ects on the equilibrium solutions of a change in the fraction of transactions that can be done with electronic cash cards are analyzed. Finally, the supply of electronic money is endogenized in order to determine when …at currency holders switch to e-money.

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1 Introduction

Although much of the money used by individuals in their everyday transactions is in the form of banknotes and coins, if experiments with various forms of electronic cash (or purse) cards succeed then perhaps notes and coins will become as obsolete as cowrie shells or manillas.

Currently, France is leaping toward a cashless future with a nationwide launch of "smart cards" to dispense people with pocket change, and speed small transactions. Called "Moneo", these electronic cash cards were introduced two years ago in some French regions. In November 2002, the service expanded to include Paris. According to the technology company that launched the project, some 850 000 consumers now regularly use Moneo cards at 80 000 grocery shops, parking lots or vending machines. Because this card is anonymous, there are no privacy or identity theft concerns.[1] But if an owner loses his or her smart card, the cash that is stored on the card can be used by whoever …nds it (in the storage limit of 100 euros) as no pin number is required to dispense cash. In Japan, 650 000 electronic purses known as "Edy" cards are in circulation and can be used at 2 100 stores, mainly in the Tokyo area. But unlike in France, where users can upload money from their bank accounts onto cash cards in banks, post o¢ces, and any participating shop or supermarket, in Japan, the cards can only be re…lled at special machines.

The question that can be raised is whether or not electronic cash cards will be used, and if so, how long will it take to be widely accepted ? Indeed, contary to …at money, Banks generally charge between 0.4 and 0.9 percent per transaction, and consumers have to pay an annual fee between 6 and 13 euros for using electronic money.

The aim of this article is to take into account the various costs related to the use of …at money and electronic cash card to determine to conditions for each type of currency to be accepted in all transactions. More precisely, a search theoretic model of monetary exchange is used to study the conditions for a type of currency to be spent when the quantities of goods o¤ered can be small or big. First, it is shown that depending on the transaction costs and the utility derived from consuming a good in big quantity, there exist equilibria where each type of currency is, is not, or is partially spent on small goods. Second, the question of legal tender laws, and the e¤ects on the equilibrium solutions of a change in the fraction of transactions that can be done with electronic cash cards, are analyzed. Finally, the supply of electronic money is endogenized in order to determine when …at currency holders switch to electronic money.

I adopt the search-theoretic framework of Kiyotaki and Wright (1991, 1993). Trades are decentralized, and agents meet in pairs and at random. Barter is

[1] There exists two distinct types of electronic money: identi…ed and anonymous electronic money (also known as digital cash). Identi…ed e-money contains information revealing the identity of the person who originally withdrew the money from the bank. Also, in much the same manner as credit cards, identi…ed e-money enables the bank to track the money as it moves through the economy. Anonymous e-money works just like real paper cash. Once anonymous e-money is withdrawn from an account, it can be spent or given away without leaving a transaction trail.

ruled out by restrictions on preferences and technology. As a consequence, people need money to carry out trades. However, there exist two different types of currencies in this economy: fiat money and electronic money or cash cards. The conditions for a type to be spent when there are shocks to production, i.e. when the quantites of goods offered can be small or big, are studied. Apart from the search models of Lotz and Rocheteau (2002) and Lotz (2002), a few theoretical papers are devoted to the introduction of a new type of currency. Ritter (1995) and Norwood (2001) study the transition from a barter to a monetary equilibrium. Moreover, most of the search models with two currencies focus on the possibility of counterfeiting (Kultti, 1996; Green and Weber, 1996) and on the emergence of an international currency (Matsuyama et al., 1993; Zhou, 1997), or the use of a foreign currency for internal trade (Soller Curtis and Waller, 2000). Shy and Tarkka (2002) study the market for electronic cash cards. However, contary to this model, I offer micro-foundations to the use of electronic money in trades.

This paper is organized as follows: in the second and third section, the model with only one fiat currency in the economy (with and without transaction costs) is described as a benchmark. In the fourth section, a dual-currency search model, where fiat money and electronic cash cards may simultaneously circulate, is analysed. In section five, the effects on the equilibrium solutions of a change in the fraction of transactions that can be done with electronic cash cards are studied. In the last section, the supply of electronic money is endogenized in order to determine when fiat currency holders switch to e-money.

2 Assumptions

The aim of this article is to take into account the costs related to the use of two types of currencies: fiat money and electronic cash cards. We could add the use of charge cards (credit cards) but for simplification we start with two types of currencies. We concentrate on the conditions for a type of currency to be spent when there are shocks to production, i.e. when the quantities of goods offered are small or big.

The assumptions are the following: We use an indivisible search model of monetary exchange. Time is continuous. People meet at random according to a Poisson arrival rate α. Barter is ruled out and there is no double coincidence of wants. There is a proportion M of money holders. Some people hold fiat currency (in proportion M^F) and some others hold electronic cash cards (in proportion M^E). By definition, $M = M^F + M^E$. There is a fraction $1 - M$ of producers (or sellers).

When a producer meets a consumer, he can either produce a big or a small quantity of goods. These shocks to production are i.i.d. and the same for all agents. The probability that a seller can produce a big quantity is β. We concentrate on equilibria where sellers always accept money if offered, so the probability ¦ that a producer takes money is equal to 1. However, the probability that a consumer spends his money on small goods is γ. This variable is endogenous since the opportunity cost of spending money on a small good is you could save it and get a big good later. While consuming the good in big quantity (Q) gives utility $U = u(Q)$, the good in small quantity (q) only gives utility $u = u(q)$, where $U > u > 0$. Finally, production is instantaneous and costless, with no loss in generality.

In this model, because double coincidence of wants is impossible, money is essential. However, different types of currencies can be used as a medium of exchange. It is assumed that the disutility cost of accepting currency is η, with $0 \leq \eta \leq u$. It is also assumed that the desutility cost of accepting an Electronic Cash Card (ECC) is $\rho < \eta$ (gain of time, less conterfeiting than for coins and bills), and that sellers and ECC holders agree to respectively pay a fee C^S and C^E per transaction to the Cash Card Issuer for using this ECC in trade. Sellers also pays a fee $\overline{C}$ per period of time to the cash card issuer. To complicate a little bit the model, we could also assume that currency holders may lose their unit of money with some probability ω^F (in this case, they become sellers but at the same time a seller becomes a currency holder with probability λ^F, i.e. finds the unit of money). In the same way, an ECC holder may lose his cash card, with some probability ω^E, and a seller becomes an ECC holder with probability λ^E.

3 One currency model without transaction costs

First, let us start the model described above with only one fiat currency in the economy and no transaction costs as a benchmark. We concentrate on equilibria where money is always accepted: ¦ = 1. It can be shown that whatever the parameters of the model, this equilibrium always exist. We concentrate on steady-state equilibria.

The probability of simple coincidence is x and the Poisson arrival rate is α.

The probability that a seller can produce a big (resp. small) quantity is β (resp. $1 - \beta$) and the probability that a consumer spends his money on small goods is γ. Each individual is in one of the two following states: seller (S) or fiat money holder (F). The agent's rate of time preference is $r > 0$.

In steady-state, the Bellman equations can be written in the following way:

$$rV_S = \alpha x M \{\beta (V_F - V_S) + (1 - \beta)\gamma (V_F - V_S)\} \quad (1)$$

$$rV_F = \alpha x (1 - M) \quad \beta(U + V_S - V_F) + (1 - \beta)\max_{\gamma} \gamma (u + V_S - V_F) \quad (2)$$

where V_S (resp. V_F) is the expected lifetime utility of an agent in the state S (resp. F).

Notice that the maximisation problem in (2) implies that the buyer chooses whether or not to spend his money on small goods when he meets a seller who produces a small quantity (which happens with probability $1 - \beta$). Sellers always accept money but can only trade a small quantity depending on the probability that a buyer spends his money or not on small goods. Let γ be the probability that a money holder is willing to spend his money on small goods in a trade with a producer. The best response condition is described as:

$$\gamma = \begin{cases} 1 & > 0 \\ [0,1] \quad \text{if } u + V_S - V_F & = 0 \\ 0 & < 0 \end{cases} \tag{3}$$

Rearranging the Bellman equations, the sign of $u + V_S - V_F$ depends on the sign of $T(\gamma)$ where:

$$T(\gamma) = -\alpha x \beta (1 - M) U + (r + \alpha x \beta) u + \alpha x (1 - \beta) M u \gamma$$

Therefore, the best response condition (3) can be rewritten as:

$$\gamma = \begin{cases} 1 & T(1) > 0 \\ [0,1] \quad \text{if} & T(\gamma) = 0 \\ 0 & T(0) < 0 \end{cases}$$

$T(\gamma)$ is a linear function in U, u and γ. Accorging to Figure 1, it can be seen that when U is big enough, the unique equilibrium is $\gamma = 0$. U big means that $T(1) < 0$, i.e.

$$U > \frac{r + \alpha x \beta (1 - M) + \alpha x M}{\alpha x \beta (1 - M)} u \equiv \overline{U}$$

When U is small, the unique equilibrium is $\gamma = 1$. U small means that $T(0) > 0$, i.e.

$$U < \frac{r + \alpha x \beta}{\alpha x \beta (1 - M)} u \equiv \underline{U}$$

where $\overline{U} > \underline{U} > u$.

Finally, when U is neither big nor small, there are three possible equilibria: $\gamma = 0$, $\gamma = 1$ and $\gamma = \gamma^*$ (satisfying $T(\gamma^*) = 0$) where

$$\gamma^* = \frac{\alpha x \beta (1 - M) U - (r + \alpha x \beta) u}{\alpha x (1 - \beta) M u}$$

The conditions for $0 < \gamma^* < 1$ are the following:

$$\begin{aligned} \gamma^* &> 0 \ , \ U > \underline{U} \\ \gamma^* &< 1 \ , \ U < \overline{U} \end{aligned}$$

The best response function is represented in Figure 2. According to it, when U is big enough ($U > \overline{U}$), buyers will never spend their money on small goods. Indeed, the opportunity cost of spending money on a small good is too big as buyers can save their unit of money and get a big good later. On the contrary, when U is small enough ($U < \underline{U}$), buyers will always spend their money on small goods. Finally, when U is neither big nor small ($\underline{U} < U < \overline{U}$), there are three possible equilibria: the two previous pure equilibria and a mixed strategy which can be interpreted either as one in which buyers spend their money on small goods with probability γ^* ($0 < \gamma^* < 1$), or as one in which the fraction γ^* of buyers always accept to spend their money on small goods while the fraction $1 - \gamma^*$ never accept.

Figure 1: The ¡ (γ) function

Figure 2: The best response function

4 One currency model with transaction costs

Now, let us describe the model when the desutility cost of accepting currency (or the transaction cost) is η, with $0 \le \eta \le u$. We still concentrate on equilibria where money is always accepted: $\Pi = 1$. Because of η, we must verify the condition for this type of equilibrium to exist.

The previous model becomes:

$$\begin{aligned} rV_S &= \alpha x M \left\{ \beta (V_F - V_S - \eta) + (1-\beta)\gamma (V_F - V_S - \eta) \right\} \\ rV_F &= \alpha x (1-M) \left\{ \beta (U + V_S - V_F) + (1-\beta) \max_{\gamma} \gamma (u + V_S - V_F) \right\} \end{aligned}$$

For the fiat currency to be accepted, it must be verified that $V_F - V_S - \eta \geq 0$. This is true iff:

$$\eta \leqslant \frac{\alpha x (1-M)\left[\beta U + (1-\beta)\gamma u\right]}{r + \alpha x (1-M)\left[\beta + (1-\beta)\gamma\right]} \equiv \overline{\eta}$$

The best response condition remains:

$$\gamma = \begin{cases} 1 & > 0 \\ [0,1] & \text{if } u + V_S - V_F \; = 0 \\ 0 & < 0 \end{cases}$$

and $T(\gamma)$ becomes:

$$T(\gamma) = -\alpha x \beta \left[(1-M) U + M\eta\right] + (r + \alpha x \beta) u + \alpha x (1-\beta) M (u - \eta) \gamma$$

The best response condition can still be rewritten as:

$$\gamma = \begin{cases} 1 & T(1) > 0 \\ [0,1] & \text{if } \; T(\gamma) = 0 \\ 0 & T(0) < 0 \end{cases}$$

When U is big enough, the unique equilibrium is $\gamma = 0$. U big means that $T(1) < 0$, i.e.

$$U > \frac{\left[r + \alpha x \beta (1-M)\right] u + \alpha x M (u - \eta)}{\alpha x \beta (1-M)} \equiv \overline{\overline{U}}$$

When U is small, the unique equilibrium is $\gamma = 1$. U small means that $T(0) > 0$, i.e.

$$U < \frac{(r + \alpha x \beta) u - \alpha x \beta M \eta}{\alpha x \beta (1-M)} \equiv \underline{U}$$

where $\overline{\overline{U}} > \underline{U} > u$.

Finally, when U is neither big nor small, there are three possible equilibria: $\gamma = 0$, $\gamma = 1$ and $\gamma = \gamma^{**}$ where

$$\gamma^{**} = \frac{\alpha x \beta \left[(1-M) U + M\eta\right] - (r + \alpha x \beta) u}{\alpha x (1-\beta) M (u - \eta)}$$

The conditions for $0 < \gamma^{**} < 1$ are the following:

$$\begin{aligned} \gamma^{**} &> 0 \ , \ U > \underline{\underline{U}} \\ \gamma^{**} &< 1 \ , \ U < \overline{\overline{U}} \end{aligned}$$

where $\gamma^{**} > \gamma^{*}$.

5 Fiat money and electronic cash cards

Now, let us consider a dual-currency search model where fiat money and electronic cash cards may simultaneously circulate. It is assumed that the desutility cost of accepting an Electronic Cash Card (ECC) is $\rho < \eta$ (gain of time, less conterfeiting than for coins and bills), and that sellers and ECC holders agree to respectively pay a fee C^S and C^E per transaction to the Cash Card Issuer for using this ECC in trade. Sellers also pay a fee $\overline{C}$ per period of time to the cash card issuer.

In steady-state, the model with two different types of currencies can be written in the following way:

$$\begin{aligned} rV_S &= \alpha x M^F \left\{\beta(V_F - V_S - \eta) + (1-\beta)\gamma^F(V_F - V_S - \eta)\right\} \\ &\quad + \alpha x M^E \left\{\beta\left(V_E - V_S - \rho - C^S\right) + (1-\beta)\gamma^E\left(V_E - V_S - \rho - C^S\right)\right\} - \overline{C} \\ rV_F &= \alpha x(1-M)\left\{\beta(U + V_S - V_F) + (1-\beta)\gamma^F(u + V_S - V_F)\right\} \\ rV_E &= \alpha x(1-M)\left\{\beta\left(U + V_S - V_E - C^E\right) + (1-\beta)\gamma^E\left(u + V_S - V_E - C^E\right)\right\} \end{aligned}$$

where V_S, V_F and V_E are the expected lifetime utility of a seller (S), a fiat currency holder (F) and an electronic cash card holder (E). For simplification, we assume with no loss in generality that $\alpha x = 1$.

Let γ^F be the probability that a money holder is willing to spend his fiat currency on small goods in a trade with a producer, and γ^E the probability that a money holder is willing to spend his electronic money on small goods. The best response conditions are described as:

$$\gamma^F = \begin{cases} 1 & \\ [0,1] & \text{if } u + V_S - V_F \\ 0 & \end{cases} \begin{matrix} > 0 \\ = 0 \\ < 0 \end{matrix} \quad \text{and} \quad \gamma^E = \begin{cases} 1 & \\ [0,1] & \text{if } u + V_S - V_E - C^E \\ 0 & \end{cases} \begin{matrix} > 0 \\ = 0 \\ < 0 \end{matrix}$$

Moreover, for currency F and E to be accepted, we must verify that $V_F - V_S - \eta > 0$ and $V_E - V_S - \rho - C^S > 0$.

Rearranging the Bellman equations, the sign of $u + V_S - V_F$ depends on the sign of $T^F\left(\gamma^F, \gamma^E\right)$ where:

$$\begin{aligned} T^F\left(\gamma^F, \gamma^E\right) = &\left[r + \beta + (1-\beta)\left(\gamma^F M^F + \gamma^E M^E\right)\right] u - (1-M)\beta U - \overline{C} \\ &- M^F\left[\beta + (1-\beta)\gamma^F\right]\eta \\ &- M^E\left[\beta + (1-\beta)\gamma^E\right]\left(\rho + C^S + \frac{(1-M)C^E\left[\beta + (1-\beta)\gamma^E\right]}{r + (1-M)[\beta + (1-\beta)\gamma^E]}\right) \end{aligned}$$

Rearranging the Bellman equations, the sign of $u + V_S - V_E - C^E$ depends on the sign of $T^E\left(\gamma^F, \gamma^E\right)$ where:

$$\begin{aligned} T^E\left(\gamma^F,\gamma^E\right) = & \Bigg\{ r + \beta + (1-\beta)\left(\gamma^F M^F + \gamma^E M^E\right) + \\ & M^F \left(\frac{(1-M)(1-\beta)\left(\gamma^E - \gamma^F\right)}{r + (1-M)\left[\beta + (1-\beta)\gamma^F\right]} \right) \left[\beta + (1-\beta)\gamma^F\right] \Bigg\} \left(u - C^E\right) \\ & - (1-M)\beta\left(U - C^E\right) - \overline{C} \\ & - M^F\left[\beta + (1-\beta)\gamma^F\right](\eta - A) \\ & - M^E\left[\beta + (1-\beta)\gamma^E\right]\left(\rho + C^S\right) \end{aligned}$$

and

$$A = \frac{(1-M)C^E\left[\beta + (1-\beta)\gamma^E\right] + (1-M)(1-\beta)\left(\gamma^F - \gamma^E\right)u}{r + (1-M)\left[\beta + (1-\beta)\gamma^F\right]}$$

Definition 1 For given γ^F and γ^E compute in $T^F\left(\gamma^F,\gamma^E\right)$ and $T^E\left(\gamma^F,\gamma^E\right)$. An equilibrium is a couple $\left(\gamma^F,\gamma^E\right)$ such that:

$$\begin{aligned} \text{if } \gamma^j &= 1 \;\; (j = F, E) \;\; \text{then } T^j \geq 0 \\ \text{if } \gamma^j &= 0 \;\; (j = F, E) \;\; \text{then } T^j \leq 0 \\ \text{if } \gamma^j &\in (0,1) \;\; \text{then } T^j = 0 \\ \text{and } V_F - V_S &\geq \eta \;;\; V_E - V_S \geq \rho + C^S \end{aligned}$$

In this dual-currency search model, there exist nine equilibrium solutions.

It can be verified that $V_F - V_S \geq \eta$ iff:

$$\eta \leq \frac{M^E\left[\beta + (1-\beta)\gamma^E\right]\left(\rho + C^S + B\right) + (1-M)\left[\beta U + (1-\beta)\gamma^F u\right] + \overline{C}}{r + M^E\left[\beta + (1-\beta)\gamma^E\right]\left(1 + \frac{(1-M)(1-\beta)\left(\gamma^F - \gamma^E\right)}{r + (1-M)\left[\beta + (1-\beta)\gamma^E\right]}\right) + (1-M)\left[\beta + (1-\beta)\gamma^F\right]}$$

where:

$$B = \frac{(1-M)C^E\left[\beta + (1-\beta)\gamma^E\right] + (1-M)(1-\beta)\left(\gamma^F - \gamma^E\right)u}{r + (1-M)\left[\beta + (1-\beta)\gamma^E\right]}$$

and $V_E - V_S \geq \rho + C^S$ iff:

$$\rho + C^S \leq \frac{M^F\left[\beta + (1-\beta)\gamma^F\right](\eta - A) + (1-M)\left[\beta\left(U - C^E\right) + (1-\beta)\gamma^E\left(u - C^E\right)\right] + \overline{C}}{r + M^F\left[\beta + (1-\beta)\gamma^F\right]\left(1 + \frac{(1-M)(1-\beta)\left(\gamma^E - \gamma^F\right)}{r + (1-M)\left[\beta + (1-\beta)\gamma^F\right]}\right) + (1-M)\left[\beta + (1-\beta)\gamma^E\right]}$$

5.1 Case 1 : $\gamma^F = \gamma^E = 0$

Let us consider the first case: $\gamma^F = \gamma^E = 0$. The value functions become:

$$\begin{aligned} rV_S &= M^F\beta(V_F - V_S - \eta) + M^E\beta\left(V_E - V_S - \rho - C^S\right) - \overline{C} \\ rV_F &= (1-M)\beta(U + V_S - V_F) \\ rV_E &= (1-M)\beta\left(U + V_S - V_E - C^E\right) \end{aligned}$$

We must verify for this type of equilibrium to exist that:

$$\begin{aligned} V_F - V_S - \eta &\geqslant 0 \; ; V_E - V_S - \rho - C^S \geqslant 0 \\ T^F(0,0) &< 0 \; ; T^E(0,0) < 0 \end{aligned}$$

It can be verified that $V_F - V_S \geqslant \eta$ iff:

$$\eta \leqslant \frac{M^E\beta\left[\rho + C^S + \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] + (1-M)\beta U + \overline{C}}{r + \beta(1-M^F)}$$

and $V_E - V_S \geqslant \rho + C^S$ iff:

$$\rho + C^S \leqslant \frac{M^F\beta\left[\eta - \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] + (1-M)\beta\left(U - C^E\right) + \overline{C}}{r + \beta(1-M^E)}$$

Moreover, $T^F(0,0) < 0$ iff:

$$U > \frac{(r+\beta)u - M^F\beta\eta - M^E\beta\left[\rho + C^S + \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] - \overline{C}}{\beta(1-M)} \equiv \widehat{U}_{(0,0)}$$

and $T^E(0,0) < 0$ iff:

$$U > \frac{(r+\beta)\left(u - C^E\right) - M^F\beta\left[\eta - \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] - M^E\beta\left(\rho + C^S\right) + (1-M)\beta C^E - \overline{C}}{\beta(1-M)} \equiv \widetilde{U}_{(0,0)}$$

It is easy to verify that $\widehat{U}_{(0,0)} > \widetilde{U}_{(0,0)}$.

5.2 Case 2 : $\gamma^F = \gamma^E = 1$

Let us consider the second case: $\gamma^F = \gamma^E = 1$. The value functions become:

$$\begin{aligned} rV_S &= M^F(V_F - V_S - \eta) + M^E\left(V_E - V_S - \rho - C^S\right) - \overline{C} \\ rV_F &= (1-M)\left\{\beta(U + V_S - V_F) + (1-\beta)(u + V_S - V_F)\right\} \\ rV_E &= (1-M)\left[\beta\left(U + V_S - V_E - C^E\right) + (1-\beta)\left(u + V_S - V_E - C^E\right)\right] \end{aligned}$$

We must verify for this type of equilibrium to exist that:

$$\begin{aligned} V_F - V_S - \eta &\geqslant 0 \; ; V_E - V_S - \rho - C^S \geqslant 0 \\ T^F(1,1) &> 0 \; ; T^E(1,1) > 0 \end{aligned}$$

It can be verified that $V_F - V_S \geq \eta$ iff:

$$\eta \leq \frac{M^E\left[\rho + C^S + \frac{(1-M)C^E}{r+(1-M)}\right] + (1-M)[\beta U + (1-\beta)u] + \overline{C}}{r+(1-M^F)}$$

and $V_E - V_S \geq \rho + C^S$ iff:

$$\rho + C^S \leq \frac{M^F\left[\eta - \frac{(1-M)C^E}{r+(1-M)}\right] + (1-M)\left[\beta\left(U - C^E\right) + (1-\beta)\left(u - C^E\right)\right] + \overline{C}}{r+(1-M^E)}$$

Moreover, $T^F(1,1) > 0$ iff:

$$U < \frac{[r+\beta+(1-\beta)M]u - M^F\eta - M^E\left[\rho + C^S + \frac{(1-M)C^E}{r+(1-M)}\right] - \overline{C}}{\beta(1-M)} \equiv \widehat{U}_{(1,1)}$$

and $T^E(1,1) > 0$ iff:

$$U < \frac{[r+\beta+(1-\beta)M]\left(u - C^E\right) - M^F\left[\eta - \frac{(1-M)C^E}{r+(1-M)}\right] - M^E\left(\rho + C^S\right) + (1-M)\beta C^E - \overline{C}}{\beta(1-M)} \equiv \widetilde{U}_{(1,1)}$$

It is easy to verify that $\widehat{U}_{(1,1)} > \widetilde{U}_{(1,1)}$.

5.3 Case 3 : $\gamma^F = 1$, $\gamma^E = 0$

Let us consider the third case: $\gamma^F = 1$ and $\gamma^E = 0$. The value functions become:

$$\begin{aligned}
rV_S &= M^F(V_F - V_S - \eta) + \beta M^E\left(V_E - V_S - \rho - C^S\right) - \overline{C} \\
rV_F &= (1-M)\left\{\beta(U + V_S - V_F) + (1-\beta)(u + V_S - V_F)\right\} \\
rV_E &= (1-M)\beta\left(U + V_S - V_E - C^E\right)
\end{aligned}$$

We must verify for this type of equilibrium to exist that:

$$\begin{aligned}
V_F - V_S - \eta &\geq 0 \,;\, V_E - V_S - \rho - C^S \geq 0 \\
T^F(1,0) &> 0 \,;\, T^E(1,0) < 0
\end{aligned}$$

It can be verified that $V_F - V_S \geq \eta$ iff:

$$\eta \leq \frac{\beta M^E\left[\rho + C^S + \frac{(1-M)\beta C^E + (1-M)(1-\beta)u}{r+(1-M)\beta}\right] + (1-M)[\beta U + (1-\beta)u] + \overline{C}}{r + \beta M^E\left(1 + \frac{(1-M)(1-\beta)}{r+(1-M)\beta}\right) + (1-M)}$$

and $V_E - V_S \geq \rho + C^S$ iff:

$$\rho + C^S \leq \frac{M^F\left[\eta - \frac{(1-M)\beta C^E + (1-M)(1-\beta)u}{r+(1-M)}\right] + (1-M)\beta\left(U - C^E\right) + \overline{C}}{r + M^F\left(1 - \frac{(1-M)(1-\beta)}{r+(1-M)}\right) + (1-M)\beta}$$

Moreover, $T^F(1,0) > 0$ iff:

$$U < \frac{\left[r+\beta+(1-\beta)M^F\right]u - M^F\eta - M^E\beta\left[\rho + C^S + \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] - \overline{C}}{\beta(1-M)} \equiv \widehat{U}_{(1,0)}$$

and $T^E(1,0) < 0$ iff:

$$U > \frac{\left[r+\beta+(1-\beta)M^F\right]\left(u - C^E\right) - M^F\left[\eta - \frac{(1-M)C^E}{r+(1-M)}\right] - M^E\beta\left(\rho+C^S\right) + (1-M)\beta C^E - \overline{C}}{\beta(1-M)} \equiv \widetilde{U}$$

It can be verified that $\widehat{U}_{(1,0)} > \widetilde{U}_{(1,0)}$.

5.4 Case 4 : $\gamma^F = 0$, $\gamma^E = 1$

Let us consider the fourth case: $\gamma^F = 0$ and $\gamma^E = 1$. The value functions become:

$$\begin{aligned} rV_S &= \beta M^F(V_F - V_S - \eta) + M^E\left(V_E - V_S - \rho - C^S\right) - \overline{C} \\ rV_F &= (1-M)\beta(U + V_S - V_F) \\ rV_E &= (1-M)\left\{\beta\left(U + V_S - V_E - C^E\right) + (1-\beta)\left(u + V_S - V_E - C^E\right)\right\} \end{aligned}$$

We must verify for this type of equilibrium to exist that:

$$\begin{aligned} V_F - V_S - \eta &\geq 0 \, ; V_E - V_S - \rho - C^S \geq 0 \\ T^F(0,1) &< 0 \, ; T^E(0,1) > 0 \end{aligned}$$

It can be verified that $V_F - V_S \geq \eta$ iff:

$$\eta \leq \frac{M^E\left[\rho + C^S + \frac{(1-M)\left(C^E-(1-\beta)u\right)}{r+(1-M)}\right] + (1-M)\beta U + \overline{C}}{r + M^E\left(1 - \frac{(1-M)(1-\beta)}{r+(1-M)}\right) + (1-M)\beta}$$

and $V_E - V_S \geq \rho + C^S$ iff:

$$\rho + C^S \leq \frac{\beta M^F\left[\eta - \frac{(1-M)\left(C^E-(1-\beta)u\right)}{r+(1-M)\beta}\right] + (1-M)\left[\beta\left(U - C^E\right) + (1-\beta)\left(u - C^E\right)\right] + \overline{C}}{r + \beta M^F\left(1 + \frac{(1-M)(1-\beta)}{r+(1-M)\beta}\right) + (1-M)}$$

Moreover, $T^F(0,1) < 0$ iff:

$$U > \frac{\left[r+\beta+(1-\beta)M^E\right]u - M^F\beta\eta - M^E\left[\rho + C^S + \frac{(1-M)C^E}{r+(1-M)}\right] - \overline{C}}{\beta(1-M)} \equiv \widehat{U}_{(0,1)}$$

and $T^E(0,1) > 0$ iff:

$$U < \frac{\left[r + \beta + (1-\beta) M^E\right]\left(u - C^E\right) - \beta M^F \left[\eta - \frac{(1-M)\beta C^E}{r+(1-M)\beta}\right] - M^E\left(\rho + C^S\right) + (1-M)\beta C^E - \overline{C}}{\beta(1-M)},$$

It can be verified that $\widetilde{U}_{(0,1)} < \widehat{U}_{(0,1)}$. Therefore, it is immediate that there does not exist an equilibrium where electronic cash card is used for small transactions whereas fiat money is not.

5.5 Comments

First, let us analyse the results we would have obtained if the fee per transaction paid by the electronic cash card holder (C^E) were equal to zero. We can see that in case 1 ($\gamma^F = \gamma^E = 0$), $\widehat{U}_{(0,0)}$ would equal $\widetilde{U}_{(0,0)}$. Therefore, the equilibrium where both currencies (fiat and electronic money) are not spend on small goods would exist for values of U big enough: $U > \widehat{U}_{(0,0)}$. In case 2 ($\gamma^F = \gamma^E = 1$), $\widehat{U}_{(1,1)}$ would equal $\widetilde{U}_{(1,1)}$. Therefore, the equilibrium where both types of currencies are spend on small goods would exist for values of U small enough: $U < \widehat{U}_{(1,1)}$. Other equilibria, where only one type of currency is spent on small goods, would be impossible. We will show below that the crucial and interesting variable in this model is the fee per transaction paid by the cash card holder (C^E).

Second, the smaller the desutility cost of accepting fiat money (η) or the quantity of electronic money in the economy (M^E), the smaller the desutility cost of accepting electronic money ($\rho + C^S$) has to be for this type of currency to be accepted in exchange. Similarly, the smaller $\rho + C^S$ or the quantity of fiat money in the economy (M^F), the smaller the desutility cost of accepting fiat money (η) has to be for this currency to be accepted in exchange.

Third, let us determine the equilibrium solutions of the model. It is necessary to compare the values of $\widehat{U}_{(\gamma^F,\gamma^E)}$ and $\widetilde{U}_{(\gamma^F,\gamma^E)}$ in order to discuss the solutions obtained. Case 4 described above is not an equilibrium solution so we concentrate on the three first cases. We know that:

$$\widehat{U}_{(0,0)} > \widetilde{U}_{(0,0)} \;;\; \widehat{U}_{(1,1)} > \widetilde{U}_{(1,1)} \;;\; \widehat{U}_{(1,0)} > \widetilde{U}_{(1,0)}$$

Moreover, it can be verified that:

$$\begin{aligned} \widehat{U}_{(1,0)} &> \widehat{U}_{(0,0)} \\ \widehat{U}_{(1,1)} &> \widehat{U}_{(1,0)} \text{ if } u - \left(\rho + C^S\right) > \theta C^E, \text{ where } \theta < 1 \end{aligned}$$

Therefore, when $\rho + C^S$ and C^E are not big (which is always the case in this model), we can conclude that:

$$\widehat{U}_{(1,1)} > \widehat{U}_{(1,0)} > \widehat{U}_{(0,0)} > \widetilde{U}_{(0,0)}$$

We can also verify that:

$$\widetilde{U}_{(1,1)} > \widetilde{U}_{(1,0)} \text{ if } u - \left(\rho + C^S\right) > C^E$$

and:

$$\tilde{\Pi}_{(1,0)} > \tilde{\Pi}_{(0,0)} \text{ if } u - \eta > (1 - \theta) C^E, \text{ where } \theta < 1$$

Therefore, when $\rho + C^S$, η and C^E are not big, which we assume, we can conclude that:

$$\tilde{\Pi}_{(1,1)} > \tilde{\Pi}_{(1,0)} > \tilde{\Pi}_{(0,0)}$$

6 Conclusion

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