

Current Account and Real Interest Rate An Intertemporal Model for G7 Countries

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Abstract

I develop an extended version of the intertemporal model of the current account. First, introducing real interest rate variations in the optimizing program, I stress the importance of this variable in explaining the current account and investment dynamic. Second, considering imperfect substitutability among traded goods, I study the relation between terms of trade variations and current account. Third, I provide a new explanation to the Glick-Rogoff (1995) paradox, based on measurement errors.

Empirical study spanning the period 1970-1997 is carried on G7 countries, individually and pooled. Results strongly support the impact of terms of trade on current account, whereas they are unconvincing about the impact of real interest rate.

Key words: Current Account - Investment - Optimizing Behavior - Linearization - Shock : Anticipated, not Anticipated, Permanent, Transitory - Productivity - Terms of Trade - Real Interest Rate - Seemingly Unrelated Regressions - Pooled Estimations.

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1 Introduction

Over the last twenty years, the traditional approach to the current account, based on export supply and import demand functions, has been challenged by the intertemporal approach (cf., Obstfeld and Rogoff, 1995). Since these approaches belong to different theoretical paradigms and have sometimes opposed policy recommendations, it is worth considering their empirical support. Here, I am interested in the most developed intertemporal approach that considers investment and consumption decisions to model the current account in a stochastic context. My work is based on the seminal paper of Glick and Rogoff (1995).

In that paper, authors study current account and investment dynamics in response to productivity shocks. They show that the relation depends on the nature of the shock: for one part, they oppose specific (to the country) and common (to all the countries) productivity shocks, and for the other part, they distinguish between transitory and permanent productivity shocks. In the case of a specific permanent productivity shock, the net permanent income increases more than actual current income (since the investment implies adjustment costs), so that saving decreases, and the current account, the difference between saving and investment, falls by more than investment increases. Hence, one of the most important conclusion of the paper is that a specific productivity shock, when it is permanent, has a higher impact on the current account than on the domestic investment. But estimations on G7 countries conclude to the opposite: instead of being less than one, the ratio is more than two. That is what is called the Glick and Rogoff's paradox.

The authors explain the paradox by evoking the near non-stationarity of productivity shock. Using numerical simulations, they show that a weak variation in the persistence of productivity shock (the autoregressive coefficient in the univariate decomposition), has a big impact on the magnitude of the specific productivity coefficient in the two equations. As the period length doesn't allow to discriminate between these weak differences, the permanent or transitory nature of productivity shocks remains a controversial point, so that this explanation is not convincing and casts some doubts on the validity of the approach. Other authors have challenged the paradox.

Iskan (2000) extends the analytical framework to a two-sectors economy, producing tradable and nontradable goods. Since for nontradable goods, production equals consumption, specific nontradable productivity shocks have an identical impact on investment and current account. Hence, if a large part of productivity shocks occurs in the nontradable sector, the coefficient on productivity in the current account and investment equations are near the same. But this explanation is unsatisfactory for three reasons. Firstly, in the model, non tradable relative price is exogenous, albeit it is affected by productivity shocks (cf., Balassa, 1964, and more recently Rogoff,

1992). Secondly, at the extreme limit, when all the productivity shocks occur in the nontradable sectors, the model concludes to an equality between the two coefficients, whereas in the Glick and Rogoff (1995) estimations, they are opposed to their theoretical order. Thirdly, it is difficult to believe that the major source of productivity is in the nontradable sector, as often the inverse is supposed to be true. In fact, the author doesn't solve the paradox. Though proposing a complete consistent intertemporal approach, he stresses the difference in data sources as an explaining factor.

Kent (1997) provides another explanation. In its cross-country study of current account dynamic, the author considers another shock affecting income: terms of trade variation, along with productivity shocks. Hence, Glick and Rogoff estimations could be subject to an omitted variable bias. According to the intertemporal approach, the current account response to a term of trade shock depends on its persistence. If the shock is permanent, saving remains constant, whereas investment increases, so that the instantaneous effect is negative. If the shock is transitory, saving increases, as the consumer smooths consumption, so that the current account effect is undeterminate (cf., Persson and Svensson, 1985).

In that paper, I try to improve the Intertemporal model of an open economy, and to solve the paradox, by estimating structural equations on G7 countries. As it remains an open issue for a long time, I am not interested in the opposition between transitory and permanent productivity shocks, and consistently with a broad range of the literature, I consider only permanent productivity shocks. First, I am preoccupied by the impact of real interest rates. As the approach considers intertemporal decisions, it is surprising to neglect real interest rate variations. On the period, they have been important, and one can reasonably consider that they have affected saving and investment decisions. So I introduce exogenous interest rate variations due to sticky prices. Second, I consider that national productions are imperfectly substitutable. Hence, I oppose domestic productivity to foreign productivity instead of specific versus common productivity. In that case, relative price effects are important. As G7 economies are big ones, terms of trade could be endogenous, equilibrating the world market for domestic good. So, I study their exogeneity with productivity shocks. Third, I show that the Glick and Rogoff's paradox might simply reflect the bad measure of productivity when Total Factor Productivity is used.

The rest of the paper is organized as follow. In the second section, I present the model. I calculate investment dynamic, consumption dynamic, and current account responses to shocks. I obtain the two fundamental equations relating current account and investment variations to shocks in the real interest rate and domestic productivity. In the third section, extending the model to a two countries world economy, I introduce the effects of terms of trade and foreign productivity. In the fourth section, I estimate

investment and current account equation on G7 countries, individually and pooled. Firstly, I realize standard regressions, similar to those of Glick and Rogoff (1995). Secondly, I study productivity and growth, and modifying estimations, I present an explanation to the Glick Rogoff paradox. Section five presents some concluding remarks and extensions.

2 Theoretical Model

National account identity ensures that the current account, CA , equals the difference between production, Y , augmented by external repayment on external claims, $(R - 1)B$, and consumption augmented by investment, $C + I$

$$CA_t = Y_t - (C_t + I_t) + (R_t - 1)B_t \quad (1)$$

Investment and saving decisions being intertemporal, the current account reflects a calculus made by the agent concerning anticipated revenue. So, specifying Y , the shocks affecting production, and expectations made by the agent, one can study the current account dynamic.

As to simplify, I do not consider capital depreciation, investment equals the difference between successive capital stocks : $I_t = K_t - K_{t-1}$. I use a standard production function in which investment imply an adjustment cost, represented by a quadratic term, and capital enters production stage with one lag. So, the production depends positively on the productivity level, A_t , an exogenous variable, on the capital available during the passed period, K_{t-1} , and depends negatively on the investment of the current period:

$$Y_t = A_t K_{t-1}^\alpha \left[1 - \frac{\theta}{2} \left(\frac{I_t^2}{K_{t-1}} \right) \right] \quad (2)$$

I consider three sources of shocks. First, real interest rate shocks. Though the real interest rate, remains a stationary variable, it fluctuates around its mean. These variations reflect imperfect price adjustments that are stochastic instead of being determinist. As they affect the discount factor, they have non linear effects, and equations have to be linearized. I linearize only at the first order and I suppose that the real interest rate follows a first order autoregressive process around its mean¹:

$$(R_t - \bar{R}) = \rho (R_{t-1} - \bar{R}) + \varepsilon_t^R \quad 0 < \rho < 1 \quad \text{and} \quad \varepsilon_t^R \equiv iid(0; \sigma_R^2) \quad (3)$$

Second, I consider domestic and rest of the world productivity shocks. In this section, to simplify the presentation, I introduce only domestic productivity, A . Shocks are assumed to be permanent. This implies that all

¹Abusively R_t is called the real interest rate, though it is one plus it.

the productivity variations are unexpected. So, the shock, ε^A , is simply the difference between two productivity levels:

$$A_t = A_{t-1} + \varepsilon_t^A \quad \text{and} \quad \varepsilon_t^A \equiv iid(0; \sigma_A^2) \quad (4)$$

Third, I consider terms of trade shocks. Under some conditions, one can show that they impact current account and investment in the same way as domestic productivity. So, in that section, I avoid them from mathematical calculus. In the next section, I will introduce them without constraining their persistence.

The way in which these shocks affect investment and current account depends on their effect on the expected profitability of investment and so on revenue. So I begin to study the influences on capital stock.

2.1 Investment and Capital

Investor maximizes the actualized sum of expected profits, taking account of expected productivity, expected real interest rate, and investment cost²:

$$Max \sum_{p=0}^{\infty} E_t \left[\prod_{k=0}^p R_{t+k}^{-1} (Y_{t+p} - I_{t+p}) \right] \quad (5)$$

At the equilibrium, the investment cost of a one additional unit of capital in the present period equals to its expected actualized marginal productivity in $t + 1$ (since the capital invested enters production stage with one period lag), augmented by the diminishing cost of adjustment in the next period (due to the increase in the capital stock) and by the residual value of capital for periods after:

$$\frac{\partial Y_t}{\partial K_t} = E_t \left[\frac{1}{R_{t+1}} \frac{\partial Y_{t+1}}{\partial K_{t+1}} \right] \quad (6)$$

This problem is a stochastic optimization problem. Since it is a non linear second order problem, it is solvable, but involves linearization³. The solution is given by Abel Blanchard (1984) or Shapiro (1986). Adapting the authors's results to the model, one can show that the linearized dynamic for capital stock is:

$$K_t = \lambda_0 K_{t-1} + \sum_{i=0}^{\infty} \lambda_1^i [\gamma_R E_t(R_{t+i}) + \gamma_A E_t(A_{t+i})] \quad (7)$$

²Conventionally, $E_t(X_{t+p})$ denotes the expectation made by the agent in t concerning the value of X p periods further. Note that this in the case considered, the price of capital remains constant, by oppoition to other studies concerned uniquely with investment.

³See Sargent (1987), 333-346, for a general presentation and discussion of this technic.

Where λ_0 and λ_1^{-1} are the roots of the characteristic equation derived from the linearized Euler equation⁴: $0 < \lambda_0 < 1$ and $0 < \lambda_1 < \bar{R}^{-1}$. The capital stock in the present period equals the stock in the passed period increased by a positive function of the total effect of shocks affecting its present expected profitability. Without calculus, one can determinate the sign of the γ coefficients. An expected increase in real interest rate, the discount factor, diminishes the actualized marginal capital value, so γ_R is negative. By opposition, an expected increase in productivity increases the profitability, so γ_A is positive. Of course, the independence between real interest rate and productivity dynamics is a central assumption behind this reasoning.

Using the capital dynamic, the equation 7, at periods t and $t - 1$, and applying the rational expectations operator, one obtains the investment dynamic as a function of passed investment, real interest rate and productivity variation. Hence, investment's variation obtained by subtracting I_{t-1} , is given by the following relation⁵:

$$\Delta I_t = (\lambda_0 - 1) I_{t-1} + \omega_R \Delta R_t + \omega_A \Delta A_t \quad (8)$$

The coefficient on lagged investment is negative since $\lambda_0 < 1$. The coefficient on the real interest rate is negative too, and increase in absolute term with the persistence of the shock. Finally, the coefficient on productivity variation is positive.

2.2 Consumption Dynamic

On the demand side, the representative agent maximizes its expected intertemporal utility, U_t , which equals the actualized sum of instantaneous anticipated utilities, V_{t+p} :

$$U_t = \sum_{p=0}^{\infty} (1 + \beta)^{-p} E(V_{t+p}) \quad (9)$$

Where β is the subjective rate of time preference. To simplify calculus, I consider the following instantaneous utility function, which is the quadratic

⁴ $K_{t-1} + a_0 K_t + a_1 E_t K_{t+1} = -\gamma E_t X_{t+1}$, with γ and X_t are two vectors representing determinants of investment and their impact.

⁵ Where The result is obtained by considering the following relation, $I_t = \lambda_0 I_{t-1} + \sum_{i=0}^{\infty} \lambda_1^i [E_t(X_{t+i}) - E_{t-1}(X_{t+i-1})]$, derivated from 7. Hence, $\omega_R = \gamma_R / (1 - \lambda_1 \rho)$ and $\omega_A = \gamma_A / (1 - \lambda_1)$. As shown by Glick and Rogoff, in the case where productivity shocks are transitory, the only change in the investment dynamic concerns ω_A .

function⁶:

$$V_{t+p} = C_{t+p} - \frac{\sigma}{2} C_{t+p}^2 \quad (10)$$

Since the optimum implies a non negative marginal utility consumption, I assume that σ , the positive parameter, is little enough for its inverse to be superior at the stationary consumption level, \bar{C} .

In an open economy, the agent differs the impacts on its revenue by smoothing its consumption: he can borrow (lend) from the rest of the world to finance its consumption today (tomorrow). if B_t is the net stock of claims on the rest of the world, then noting that $\Delta B_{t+1} = CA_t$, one can express the net wealth dynamic using the current account equation, the equation 1:

$$B_{t+1} = R_t B_t + (Y_t - I_t) - C_t \quad (11)$$

At the optimum, the consumption path fills two condition. First, in each period, the marginal utility of current consumption equals its opportunity cost, the marginal actualized utility of saving in the current period and consuming at the next period⁷. Second, over the lifetime, the intertemporal constraint is respected: the expected present value of indebtedness equals the present expected claim on the rest of the world.

For this problem to have a solution, it can be shown that the equality between the discount factor and the mean real interest rate is a necessary condition⁸. So, by hypothesis, $\bar{R} = 1 + \beta$. Linearizing the Euler equation associated with the consumer problem and denoting by a minuscule the real interest rate centered around its mean⁹:

$$E_t(C_{t+1}) = C_t + \eta E_t(r_{t+1}) \quad \eta > 0 \quad (12)$$

Albeit the utility function is identical, the consumption doesn't follow the same dynamic as in Hall (1978) seminal paper (a random walk). It is comparable to that of Hall (1986). When real interest rate fluctuates, but remains stationary, the consumption follows a random walk with an damped

⁶Considering a CRRA function wouldn't affect qualitative results since linearization is made at the first order.

⁷

$$V'_{C_t} = E_t(R_{t+1} V'_{C_{t+1}}) / (1 + \beta)$$

⁸This explains why shocks affecting real interest rate need to be transitory, and so why I assume a stationary real interest rate, though, on the empirical side, regarding the last thirty years, this is all but sure.

⁹Where $\eta = [1/\sigma - \bar{C}] / \bar{R}$

drift. The expected consumption path is increasing when the expected real interest rate is above its mean. This intertemporal substitution effect can be explained by applying the rational expectation operator to the equation ???. If the expected real interest rate is above its mean, it is because a positive shock has occurred at the present period, a part of which will be damped tomorrow. Hence, consumption decreases today, when real interest rate is at its highest level, and increases tomorrow, when, though remaining above its mean, it is below its present level.

Equation 12 expresses the expected consumption variation, not the actual one. To obtain actual consumption variations, consider the intertemporal constraint, which equalizes the actualized net income flows (LHS), to the actualized consumption flows (RHS):

$$\sum_{p=0}^{\infty} E_t \left[(Y_{t+p} - I_{t+p}) \prod_{k=0}^p R_{t+k}^{-1} \right] + B_t = \sum_{p=0}^{\infty} E_t \left[C_{t+p} \prod_{k=0}^p R_{t+k}^{-1} \right] \quad (13)$$

In this equation, the LHS is the net present expected permanent income, Y^p augmented by the financial wealth. It is impacted in two ways by interest rate variations: the substitution effect coming from the revision of the revenue sequence following a productivity shock, Y^1 , and the wealth effect. Appendix 1.1 shows that using the constraint and generalizing the equation 12 p periods ahead, one obtains the actual variation in the consumption:

$$\Delta C_t = \frac{\bar{R} - 1}{\bar{R}} \left[\tilde{Y}_t^1 - \delta_0 \varepsilon_t^R \right] + \eta \rho r_{t-1} \quad \delta_0 > 0 \quad (14)$$

Where δ_0 is defined in the appendix 1.1, and a tilde denotes the expectation variation on the same variable between two consecutive periods¹⁰. At the stationary equilibrium, consumption path is flat: it simply equals to the flow associated with permanent income. Actual consumption variation is the sum of an expected change due to the real interest rate mean reverting, and a unexpected change due to the modification of the net permanent income. Hence, to calculate consumption variation, one must calculate the revision of anticipation on the net permanent income \tilde{Y}_t^1 . Linearizing production function, equation 2, yields :

$$Y_t = \alpha_A A_t + \alpha_K K_{t-1} + \alpha_I I_t \quad \text{by definition, } \alpha_A, \alpha_K > 0 \text{ and } \alpha_I < 0 \quad (15)$$

With α_K and α_A positive and α_I negative. So current account is affected by real interest rate shocks only through their indirect effect on investment, whereas productivity shocks impact it through this channel the direct one.

¹⁰ $\tilde{X}_{t+p} = E_t(X_{t+p}) - E_{t-1}(X_{t+p})$

Hence, if it is clear that an positive real interest rate shock increases current revenue (since it diminishes current investment), the total impact of productivity is undeterminate. On the other side, the effect on permanent income are well known. Using properties of the expectations's variation, appendix 1.2 shows that summing anticipating errors on investment and capital stock, one obtains :

$$\tilde{Y}_t^1 = \frac{\bar{R}\varphi_1\omega_R}{\bar{R}-\rho}\varepsilon_t^R + \frac{\bar{R}(\varphi_1\omega_A + \alpha_A)}{\bar{R}-1}\varepsilon_t^A \quad (16)$$

Since χ_1 is positive and ω_R is negative, the net permanent income expectation decreases with a real interest rate shock: the expectation in period t , after observation of the shock, is below the expectation in $t - 1$. In that case, though stationary capital stock remains the same, planned investment is slowed, and so, present revenue too. The more persistent is the real interest rate shock, the more important is the variation of the expected net permanent income due to it. By opposition, a positive productivity shock implies a favorable revision of the net permanent income, firstly by the direct effect (the permanent actualized exogenous increase in the revenue), and secondly, by the revision of investment planning (this effect is "symmetric" to the interest rate effect).

Using equation 16 into equation 14 yields the actual consumption variation in function of productivity shock and real interest rate shock:

$$\begin{aligned} \Delta C_t = & \left(\frac{\bar{R}\varphi_1\omega_R}{\bar{R}-\rho} - \delta_0 \right) \frac{\bar{R}-1}{\bar{R}}\varepsilon_t^R \\ & + (\varphi_1\omega_A + \alpha_A)\varepsilon_t^A + \eta\rho r_{t-1} \end{aligned} \quad (17)$$

Consumption instantaneously diminishes with a real interest rate shock, and increases with a productivity shock. Using the equation 3, one can show that its impact is stronger on the consumption than on current income. Hence, saving decreases with a productivity shock¹¹.

2.3 Current Account

Equations 8, 3 and 17 decompose investment, production and consumption dynamics in function of exogenous shocks. So, one can study the current account variation:

$$\Delta CA_t = (R_t - 1)CA_{t-1} + \Delta(Y - I - C)_t \quad (18)$$

¹¹ $\Delta Y'_{\varepsilon_t^A} < \alpha_A$ since the direct effect is diminished by the indirect effect on the investment. As φ_1 and ω_A are positive, $\Delta Y'_{\varepsilon_t^A} < \Delta C'_{\varepsilon_t^A}$.

Using these equations and substituting I_{t-1} for ΔK_{t-1} , one can find, in the appendix 1.3, the equation for the current account. Using the dynamics of exogenous variable, equations 3 and 4, one can express this equation in function of observable variables instead of structural shocks (the two being different in the case of real interest rate since it is mean-reverting):

$$\begin{aligned} \Delta CA_t = & \phi_0 \bar{R} + \varphi_2 I_{t-1} + (R_t - 1) CA_{t-1} \\ & - \frac{\varphi_2 \omega_A}{R - \lambda_0} \Delta A_t + \phi_1 \Delta R_t - \phi_0 R_{t-1} \end{aligned} \quad (19)$$

All the coefficient signs are known excepted ϕ_0 , so that the constant and the coefficient on lagged real interest rate are unknown, but of opposite sign. As $\varphi_2 > 0$, lagged investment has a positive impact on current account: first it has increase current capital and second it has reduced the need for further investment. Lagged current account too has a positive impact, due to the variation of the net claim and so the increase in financial revenues. A positive productivity shock, ΔA_t , decreases the current account. In absolute term, the coefficient is higher than in the investment equation since, as seen, consumption increases more than current revenue. A positive real interest rate variation increases it.

3 Terms of Trade introduction

Albeit Iscan (2000) found a negligible impact for relative prices on current account, on a theoretical ground, it is wise to model traded goods as imperfect substitute. Moreover, this result is opposed to that of Mendoza (1995), who, in a cross country study, estimates that terms of trade variations can account for a large part of variations in gross domestic product (nearly one-half). So, I consider a two goods world economy, an import and an export good. The agent consumes only the import good, and the export good is the only good produced by the national economy. So, terms of trade, P_t , are the relative price of the national production. Their variations affect the consumer wealth and so the current account. Hence, taking them into account may increase the explanatory power of the model.

Terms of trade shocks may affect investment too. Whereas few authors have considered the origin of the capital stock as a possible condition for these shocks to affect investment dynamic, the case where the capital is imported must be separated from the case where it is produced domestically. In the latter, an increase in terms of trade, has no effect on investment in the model, since capital and export good are the same goods and have the same price. Hence, an increase in terms of trade has the same effect on marginal productivity and on marginal cost. In the case where capital is imported, an increase in terms of trade, increasing marginal productivity

	Pooled	CD	FR	IT	JA	UK	US	WD
1960-1997								
ρ	-0.10	-0.12	-0.11	-0.13	-0.09	-0.26	-0.08	-0.18
t	4.21	1.08	1.30	1.52	2.67	1.68	1.12	1.59
1970-1997								
ρ	-0.18	-0.25	-0.19	-0.19	-0.16	-0.28	-0.18	-0.26
t	5.33	1.51	1.74	1.76	3.87	1.57	1.78	1.69

Table 1: Terms of trade decomposition (autoregressive coefficient)

value of capital, without modifying its cost, increases investment. It has the same impact as a productivity shock. As actually, capital is imported and produced domestically, terms of trade shocks must impact positively investment.

In the literature, the debate on the effect of terms of trade shocks has been conditioned on the temporal nature of the shock (cf., Kent, 1997). Above a certain level of persistence, the term of trade shock has a negative impact on the production diminished by consumption. So, if the shock is highly persistent, the current account will decrease by more that the investment will increase. When all the capital is produced domestically, a term of trade shock has no impact on investment, but only on consumption and so on the current account. Hence, countries with the least persistent terms of trade shocks may exhibit a positive relationship between current account and terms of trade shocks, and countries with the most persistent terms of trade shocks may share a negative relationship¹².

Table 1 reports the autoregressive coefficient obtained from OLS regression, ρ , and its t-ratio, t , for each G7 country. If the estimated persistence varies across countries, it is not clear wether series are stationary or not¹³. In most of the cases, the lagged level is not significant, even when standard significance level are used. Hence, terms of trade series may have an unit-root. But, on the period, terms of trade have been affected by breakdown (oil shocks), and it is well known that this can mislead conclusions obtained from standard ADF-test. So, I think these results are inconclusive, and I do not restrict shocks to be transitory or permanent.

In this two goods economy, there can't be any common productivity shocks. But the current account is affected by foreign productivity: when it increases, it pushes up the demand for domestic goods and so the current account. Hence, considering terms of trade leads to oppose domestic and

¹²It is theoretically possible to calculate the autoregressive coefficient above which the relation between terms of trade and current account becomes negative. The technic is near the same than that used by Glick and Rogoff (1995) to explain the paradox.

¹³In fact; persience was estimated using the same regression as in ADF tests, with one lag and a constant: $\Delta tot_t = c + \rho tot_{t-1} + d\Delta tot_{t-1}$. So t -ratio affected to ρ is sufficient to test stationarity.

	CD	FR	IT	JA	UK	US	WD
1960-1997							
$\Delta p_t, \Delta a_t$	-0.04	0.11	-0.24	0.08	-0.15	0.45	0.07
$\Delta p_t, \Delta a_{t-1}$	0.03	-0.07	-0.10	0.03	-0.45	-0.09	0.14
1970-1997							
$\Delta p_t, \Delta a_t$	0.01	0.08	-0.40	0.12	-0.12	0.53	0.07
$\Delta p_t, \Delta a_{t-1}$	0.06	-0.17	-0.21	-0.04	-0.42	-0.12	0.16

Table 2: Correlation between terms of trade and productivity

foreign productivity.

As it could be the argued that for the big economies considered here, terms of trade respond endogenously to productivity shocks, I calculate the extent to which these two variables are correlated. Results are shown in table 2, for two periods, and using contemporaneous or lagged productivity. The overall weak correlation suggests that terms of trade might be an additional source of exogenous shocks explaining investment and current account. Moreover, it allows to treat terms of trade as exogenous.

4 Empirical Results on G7 countries

I estimate the two fundamental equations for current account and investment, for all the G7 countries on a national basis and on a pooled basis¹⁴. From the model (equations 19 and 8), structural equations and theoretical signs on coefficients are the following:

$$\begin{aligned}
 \Delta CA_t &= b_0 + b_1 I_{t-1} + b_2 t + b_2 \Delta a_t + b_3 \Delta a_t^* & (20) \\
 &\quad + \quad \quad \quad - \quad \quad \quad - \quad \quad \quad + \\
 &\quad + b_4 \Delta r_t + b_5 r_{t-1} + b_6 \Delta p_t + b_7 p_{t-1} \\
 &\quad + \quad \quad \quad ? \quad \quad \quad ? \quad \quad \quad ? \\
 \Delta I_t &= c_0 + c_1 I_{t-1} + c_2 t + c_3 \Delta a_t + c_4 \Delta r_t + c_5 \Delta p_t & (21) \\
 &\quad - \quad \quad \quad + \quad \quad \quad + \quad \quad \quad - \quad \quad \quad +/0
 \end{aligned}$$

Data used come from the European Macroeconomic Database, EURO-STAT. It is well known that current account variability has greatly increased since the beginning of the exchange rate float (cf., Tesar, 1991). So, though data are available on the period 1960-1997, I choose to limit estimations to the subperiod 1970-1997¹⁵.

¹⁴CD is Canada, FR is France, IT is Italia, JA is Japan, UK is United Kingdom, US is United States, and WD is western Germany.

¹⁵Since Glick and Rogoff (1996) estimations were carried on the 1960-90 period, we observed if the paradox was not due to the period choice which covers differing contexts, in terms of capital mobility.

In all the cases, to obtain real data from nominal ones, I use value added deflator. Investment, is gross investment. It is expressed in national currency at constant prices, like the current account. Productivity is measured by total factor productivity, so shocks, are proxy by its logarithmic variation, Δa . Foreign productivity shock, Δa^* , is the weighted average of G7 foreign productivities, weights being equal to the mean of each country share in the total trade of G7 countries, including intra-trade. Terms of trade, p , refer to goods and services. It is the logarithm of the price of exports divided by the price of import. Real interest rates are long-term real interest rates on public bonds, r . After inspection of current account and investment data, I consider a dummy for each serie¹⁶.

Coefficients are made comparable across countries by multiplying them by the country GDP mean on the period (excepted lagged endogenous and lagged investment). For convenience, they are multiplied by 100, so that each of them represents the impact as a percentage of mean GDP. For the pooled regressions, the same technic is used, and to avoid heteroscedasticity, each equation in the pooled estimation is scaled by standard errors of the individual regression. Two cases are considered: the first in which lagged investment and exogenous economic variables are constrained to have the same impact across countries, and the second in which only exogenous economic variables are constrained to have the same impact¹⁷.

The equation 1 shows that by construction disturbances in current account and investment equations at a given time are likely to be negatively correlated, reflecting some common immeasurable or omitted factor. Coefficients in equations displaying contemporaneous correlation are more efficiently estimated using SURE technic (Seemingly Unrelated REgressions). So I use this technic, though in practice results differs marginally from these obtained by OLS¹⁸.

On the tables, the estimated coefficient figures on the first line, whereas its standard deviation figures between brackets on the second line. One (two, three) asterisk denotes that the variable is significative at the 10% (5%, 1%) level. Adjusted R^2 is the original R^2 corrected by the number of regressors and DW is the Durbin-Watson statistic for autocorrelation of residuals (at the first order).

¹⁶For the current account, dummies are put in 1982 (CD), 1991 (JA,UK,WD), 1992 (US), and 1993 (FR,IT). For the investment, dummies are put in 1974 (JA), 1975 (FR), 1982 (CD), 1991 (UK, US), 1993 (IT, WD), so that each has an economic interpretation (first oil shock, Gulf war, EMS crisis and German reunification).

¹⁷Hence, in all the tables, when there is no coefficient on the pooled estimation, it means that it is country specific, free to vary accross countries.

¹⁸Nason and Rogers (1999) use a different technic. The authors estimate a VAR on investment and current account and identify structural shocks by imposing conditions to determinate specific and common shocks.

4.1 A Standard Estimation on G7 Countries

In the first step, regressions were run on equations 20 and 21, and in most of the cases, residuals were autocorrelated (results are not reported here). So, in a second step, in table 3, I present the results for these equations, augmented by the lagged endogenous variable (see column *lagged*) as regressor.

Considering Durbin-Watson statistic (see column *DW*), there is no evidence of autocorrelation at the first order. Estimations explain between 42% and 90% of the actual variation in the investment and current account in G7 countries.

The coefficient on lagged investment is not of the right sign (positive in the current account equation and negative in the investment equation). In the pooled regressions, it is not significant in the current account equation for which it is of the wrong sign. In most of the cases, terms of trade variations have a positive impact on current account and on investment. This result is puzzling since the investment enters with a negative sign in the current account. Hence, the impact of terms of trade variations on actual income less consumption must be strongly positive, and so the shock might be transitory. It may be the case since lagged terms of trade are significant too. But, considering table 1, this is all but sure. Whatever, the magnitudes are weak, compared to the impact of productivity. In almost all the cases, expected France and Western Germany, real interest rates are not significant (in level or variation). Though correctly signed, they don't appear in the pooled regressions. The foreign productivity shock is significant and of the good sign. The fact that it doesn't appear in all country regressions is perhaps explained by our measure, common to all the countries, and not based on each country foreign trade. Note that the magnitude is near the same than that of the domestic productivity coefficient (a statistical test accepts the equality).

Finally, no country respects the correct order for the relation in the productivity magnitude. Results obtained without the introduction of real interest rate or terms of trade and without lagged endogenous display the same conclusion concerning the relative impact of specific productivity shocks on current account and investment. Hence, the paradox is not due to data source¹⁹. Moreover, correlation between residuals is very high (it ranges from -8% to -38%, with a mean of -26%). To conclude, estimations report paradoxical and inconclusive support for the theoretical model.

¹⁹Glick and Rogoff use BLS or OECD data.

sur	0	C	Dum.	Trend	Lagged	Inv(-1)	PIIT	ΔTOT	ΔRLT	PIIT*	TOT ₋₁	RLT ₋₁	R ²	adjR ²	DW	corr.	
CD	CA:	54.88** (16.66)	3.48** (0.62)	0.27** (0.1)	16.27 (13.93)	-17.87* (10.51)	2.07 (13.3)	2.29 (5.15)	8.81 (9.12)	9.33 (19.95)	-12.42*** (3.73)	-9.18* (5.05)	64.2	41.8	2.29	-33.8	
	I:	0.76 (0.46)	-3.72*** (0.62)	0.37*** (0.08)	60.55*** (8.07)	-39.31*** (9.05)	8.08 (9.08)	16.48*** (4.34)	16.1*** (6.23)	16.1*** (6.23)				83.4	77.3	2.12	
FR	CA:	-11.37** (5.39)	0.9** (0.39)	0.02 (0.04)	5.4 (12.88)	-4.63 (6.75)	-58.46*** (10.03)	4.16** (1.99)	13.23** (6.31)	47.98*** (11.84)	2.63** (1.24)	1.08 (5.35)	65.2	43.4	2.36	-35.3	
	I:	0.34 (1.19)	0.44 (0.57)	0.07** (0.03)	44.38*** (9.11)	-10.5 (7.05)	69.37*** (10.96)	-1.07 (2.43)	2.33 (6.53)	2.33 (6.53)				81.2	74.2	2.42	
IT	CA:	-10.55 (10.91)	6.07*** (0.76)	0.13** (0.06)	8.08 (11.75)	-37.43*** (12.97)	-16.44 (10.17)	4.97 (3.66)	2.47 (8.72)	-14.79 (20.2)	3.62 (2.5)	-3.44 (6)	82.6	71.7	2.26	-25.9	
	I:	1.87 (1.44)	-3.77*** (0.66)	0.08*** (0.03)	36.95*** (8.82)	-16.27* (8.41)	28.93*** (7.2)	-2.53 (2.82)	3.77 (5.11)	3.77 (5.11)				85.1	79.5	2.31	
JA	CA:	33.5*** (6.62)	1.92*** (0.66)	-0.2** (0.09)	5.13 (12.52)	8.46 (5.42)	-15.11 (9.91)	2.03 (1.73)	-7.1 (6.08)	23.6 (19.69)	-6.55*** (1.39)	-6.36 (5.9)	80.9	65.0	2.19	-7.7	
	I:	-2.96** (1.18)	0.97 (1.76)	0.1 (0.11)	33.39** (13.01)	0.7 (7.21)	100.55*** (26.03)	2.52 (2.23)	-2.13 (6.36)	-2.13 (6.36)				82.0	73.7	1.62	
UK	CA:	-15.57 (17.68)	2.41** (0.99)	-0.01 (0.05)	23.14 (16.16)	4.25 (11.76)	-7.04 (12.74)	5.89 (4.96)	-2.49 (6.56)	-3.15 (25.23)	3.29 (3.93)	-6.32 (6.89)	41.6	5.1	1.89	-34.7	
	I:	0.23 (1.22)	-1.92** (0.86)	0.05 (0.04)	43.46*** (13.39)	-6.81 (9.29)	24.42*** (8.41)	0.13 (3.91)	1.21 (4.2)	1.21 (4.2)				62.0	48.0	2.09	
US	CA:	12.02 (8.15)	2.67*** (0.5)	-0.18*** (0.06)	15.7 (12.25)	26.66*** (8.65)	-7.13 (11.2)	3.3 (2.93)	-9.97 (9.23)	10.81 (14.2)	-2.92* (1.75)	-1.83 (5.01)	71.2	53.2	1.73	-8.1	
	I:	0.79 (0.78)	-0.9** (0.46)	0.1** (0.04)	56*** (8.97)	-16.66*** (8.32)	76.11*** (8.44)	-4.72* (2.55)	7.62 (8.29)	7.62 (8.29)				89.2	85.2	2.02	
WD	CA:	-21.19 (17.89)	-3.35*** (0.69)	-0.01 (0.04)	-4.21 (11.08)	-6.07 (10.67)	-43.2*** (15.73)	0.69 (3.63)	37.61** (15.31)	77.24*** (25.68)	4.46 (3.94)	51.99** (24.15)	73.4	56.8	2.40	-37.7	
	I:	0.86 (1.19)	-2.19*** (0.57)	0.06*** (0.02)	41.82*** (7.99)	-10.46 (6.51)	40.7*** (7.49)	5.89** (2.82)	-4.05 (8.54)	-4.05 (8.54)				82.2	75.6	1.70	
Pooled	CA:					-0.56 (2.62)	-17.55*** (2.97)	2.65*** (0.93)	0.6 (1.86)	18.41*** (4.77)	-2.04*** (0.62)	0.1 (1.7)					
	I:					-14.56*** (2.44)	40.25*** (2.55)	3.16*** (0.79)	-1.44 (1.64)								
	CA:					-16.29*** (3.16)	-16.29*** (3.16)	4.08*** (0.93)	1.81 (1.85)	14.07*** (5.11)	-1.86*** (0.58)	0.11 (1.63)					
	I:					39.23*** (2.7)	39.23*** (2.7)	3.06*** (0.83)	-1.63 (1.66)								

TABLE 1 : Standard Complete Estimation using SURE (1970-1997)

	CD	FR	IT	JA	UK	US	WD
1960-1997							
Level	0.90	0.95	0.94	0.99	0.84	0.81	0.91
Variation	0.92	0.96	0.96	0.97	0.89	0.84	0.94
1970-1997							
Level	0.88	0.92	0.92	0.97	0.84	0.81	0.89
Variation	0.93	0.97	0.97	0.96	0.88	0.84	0.95

Table 3: Correlation between measured TFP variation and Growth

4.2 Another Explanation to the Paradox

We investigate the productivity measure as an explanation of bad estimation results. Total Factor Productivity Variation, Δa_t , is not observable, it is measured by the following equality:

$$\Delta a_t = \Delta y_t - \alpha \Delta k_t - (1 - \alpha) \Delta l_t \quad (22)$$

Where α , the share of capital in the national production, is proxy by its average on the period (between 0.5 and 0.7). The table 4 reports correlation between TFP variation and real growth, in level, or in variation for each country. The high correlation between these two variables can't be explained by the equation 22. So I suspect estimations to be incompatible with this productivity measure.

In table 2, I present new estimations, obtained by subtracting non-productivity growth from the LHS of equations 20 and 21. R^2 is comparable to that of prior regressions (it ranges from 42% to 88%). In some cases, there might be residual autocorrelation (as for the investment equation for the United States). Correlation between residuals, $corr(\varepsilon_{CA}, \varepsilon_I)$, is reduced (-4% on mean).

First of all, we can remark that the paradox is explained. New estimations display the right relative magnitude for domestic productivity coefficient: it has a bigger impact on the current account than on the investment, for five of the seven countries and the two pooled regressions. In that case, coefficients are highly significant, at the 1% level. Second, two puzzling results disappear. Terms of trade variations have a positive significant weak impact on the current account, but do not have a significant influence on the investment. This corresponds to the theoretical case where shocks are too much transitory to modify investment behavior, but remains significant to triggers consumption smoothing behavior. This is confirmed by the significance of lagged terms of trade. Real interest rates have a significant negative impact on investment. The fact that they don't affect the current account in the pooled regression remains puzzling. Moreover, at the country level, the sign is not conditioned by the debtor/creditor country position (US has

	CA :	C	Dum.	Trend	Lagged	Inv(-1)	PIIT	Δ TOT	Δ RLT	PII*	TOT ₋₁	RLT ₋₁	R ²	adjR ²	DW	corr.
CD	CA :	118.97*** (44.4)	6.55*** (1.47)	0.05 (0.25)	20.51 (37.1)	6.6 (25.88)	16.63 (33.68)	-1.59 (12.64)	43.49* (23.03)	-50.08 (53.1)	-27.22*** (9.93)	9.24 (13.23)	59.1	33.6	1.39	4.1
	I :	-1.93*** (0.53)	0.46 (0.71)	0.11 (0.09)	-4.57 (10.17)	-11.04 (10.44)	5.96 (10.38)	6.24 (4.96)	15.87** (7.14)				23.9	-4.1	1.79	
FR	CA :	4.14 (8.14)	2.71*** (0.59)	0.07 (0.05)	46.79** (19.2)	-21.74** (9.81)	-113.55*** (14.68)	2.37 (2.85)	21.43** (9.13)	68.36*** (17.73)	-0.26 (1.88)	4.08 (8.04)	81.9	70.6	2.70	-12.6
	I :	1.22 (0.95)	0.14 (0.48)	0.13*** (0.02)	13* (7.44)	-22.61*** (5.62)	30.22*** (8.88)	0.05 (1.95)	-0.63 (5.2)				74.5	65.1	1.78	
IT	CA :	-21.51 (15.65)	10.09*** (1.04)	0.16** (0.08)	39.09** (16.25)	-30.14* (17.9)	-33.59** (14.04)	9.04* (5.04)	-13.17 (12.21)	34.31 (28.94)	5.09 (3.59)	-10.08 (8.61)	87.6	79.9	2.32	-10.2
	I :	-0.58 (1.48)	-0.21 (0.59)	0.1*** (0.03)	-3.48 (9.12)	-13.41 (8.62)	29.79*** (7.37)	2.79 (2.88)	-14.56*** (5.23)				71.2	60.6	1.54	
JA	CA :	35.8*** (10.41)	1.57 (1.03)	-0.01 (0.15)	34.68* (19.65)	-8.03 (8.56)	-47.59*** (15.71)	-0.41 (2.73)	-15.35 (9.6)	27.76 (31)	-7.17*** (2.19)	-17.76* (9.3)	76.6	57.2	1.87	-18.9
	I :	-3.95*** (0.92)	2.44* (1.37)	0.15* (0.08)	-2.51 (10.17)	-6.57 (5.66)	99.01*** (20.39)	2.11 (1.75)	-3.76 (4.99)				82.1	73.7	1.63	
UK	CA :	-75.18*** (26.68)	5.36*** (1.36)	0 (0.07)	60.03** (23.55)	-7.89 (16.35)	11.55 (18.15)	13.07* (6.9)	-27.82*** (9.36)	-131.89*** (37.81)	16.93*** (5.93)	-30.33*** (10.38)	73.6	57.1	1.99	0.2
	I :	2.08** (0.84)	0.32 (0.6)	0.11*** (0.03)	-37.82*** (9.48)	-25.84*** (6.42)	15.64*** (5.78)	0.5 (2.69)	-2.58 (2.89)				76.3	67.6	1.70	
US	CA :	-5.95 (13.99)	3.87*** (0.87)	-0.3*** (0.1)	87.16*** (21.15)	39.43*** (15.12)	-35.98* (19.52)	22.66*** (5.12)	5.97 (16.12)	-87.38*** (24.39)	0.86 (3)	3.08 (8.59)	83.1	72.5	2.31	25.3
	I :	-1.56 (1.39)	0.24 (0.82)	0 (0.08)	-18.6 (15.9)	0.3 (14.86)	19.38 (15.12)	3.88 (4.56)	24.91* (14.84)				41.4	19.8	0.96	
WD	CA :	-53.03 (38.02)	-4.33*** (1.47)	-0.04 (0.08)	22.13 (23.21)	-12.83 (21.92)	-97.4*** (32.63)	-9.37 (7.23)	61.55* (31.56)	153.49*** (54.31)	10.98 (8.37)	114.87** (51.32)	64.7	42.7	1.60	-12.6
	I :	0.74 (1.24)	-0.39 (0.63)	0.05** (0.02)	-30.42*** (8.88)	-12.34* (6.82)	26.59*** (7.75)	3.35 (2.9)	-12.6 (8.8)				65.1	52.2	1.40	
Pooled	CA :															
	I :															
	CA :															
	I :															

TABLE 2 : Modified LHS, Complete Estimation using SURE (1970-1997)

a positive coefficient whereas it is the major debtor country on the period, an Japan, which is the major creditor country, has a negative coefficient).

5 Concluding Remarks

In that paper, I tried to solve the Glick and Rogoff's paradox. As shown by differing estimations, this paradox must be due to some measurement error concerning the productivity. In contrast to some empirical results, I showed that relative prices variations have an influence on the current account. Hence, it may be interesting to discriminate between common and specific terms of trade shocks among the G7 countries.

The puzzling result remains the absence of influence of real interest rate on the current account. On this point, empirical evidences supporting the conclusion of the model are weak. Maybe, estimations should consider the indebtedness instead of the current account that encompasses other financial flows.

References

- [1] Abel, A.B. and Blanchard, O. (1986) The present value of profits and the cyclical movements in investment. *Econometrica*, **54**, March, 249-73.
- [2] Balassa, B. (1964). The purchasing-power parity doctrine : a reappraisal. *Journal of Political Economy*, **72**, 584-96.
- [3] Glick, R. and Rogoff, K. (1995) Global versus country-specific productivity shocks and the current account. *Journal of Monetary Economics*, **35**, 159-192.
- [4] Hall, R.E. (1988) Intertemporal substitution in consumption. *Journal of Political Economy*, **96**, n° 2, 339-357.
- [5] Hall, R.E. (1978) Stochastic implications of the life-cycle permanent income hypothesis: theory and evidence. *Journal of Political Economy*, **86**, December, 971-87.
- [6] Hossain, F. (1999) Transitory and permanent disturbances and the current account : an empirical analysis in the intertemporal framework. *Applied Economics*, **31**, 965-974.
- [7] Iscan, T.B. (2000) The terms of trade, productivity shocks, and the current account. *Journal of Monetary Economics*, **45**, 587-611.

- [8] Kent, C. (1997) The response of the current account to terms of trade shocks: a panel-data study. *Research Discussion Paper*, 9705, Reserve Bank of Australia, October.
- [9] Mankiw, M.G. and Rotemberg, J.J. and Summers, L.H. (1985) Intertemporal substitution in macroeconomics. *Quarterly Journal of Economics*, 225-51.
- [10] Mendoza, E. (1995) The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, **36**, 1, 101-137, February.
- [11] Nason, J.M., and Rogers, J. (1999) Investment and the current account in the short run and in the long run. *International Finance Discussion Papers*, Board of Governors of Federal Reserve System, **647**, October.
- [12] Obstfeld, M.A. and Rogoff, K. (1995) The intertemporal approach to the current account. *Handbook of International Economics*, **III**, 1731–1797.
- [13] Persson, T. and Svensson, L.E.O. (1985) Current account dynamics and the terms of trade: Harberger-Laursen-Metzler two generations later. *Journal of Political Economy*, **93**, n° 1, 43-65.
- [14] Razin, A. (1995) The dynamic-optimizing approach to the current account: theory and evidence. *Understanding interdependence, the macroeconomics of the open economy*, P.B. Kennen editor, Princeton University Press, 169–198.
- [15] Rogoff, K. (1992) Traded goods consumption smoothing and the random walk behavior of the real exchange rate. *BOJ Monetary and Economic Studies*, **10**, n° 2, 1-29, November.
- [16] Sachs, J. (1982) The current account in the macroeconomic adjustment process. *Scandinavian Journal of Economics*, **84**, 147–59, June.
- [17] Sargent, T. (1987). *Macroeconomic Theory*. Princeton University Press.
- [18] Shapiro, M.D. (1986) Investment, output, and the cost of capital. *Brookings Papers on Economic Activity*, **17**, 1, 111-164.
- [19] Tesar, L. (1991) Savings, investment, and international capital flows. *Journal of International Economics*, **31**, 55-78.

Appendix I

Mathematical Calculus

1. From the equation 13 to the equation 14

Iterating the equation 12 p periods ahead and applying the rational expectation operator to the equation 3 yields:

$$E_t(C_{t+p}) = C_t + \frac{1 - \rho^p}{1 - \rho} \rho \eta r_t \quad (23)$$

Using this equation in the Linearized RHS of the intertemporal constraint, equation 13, one obtains:

$$\sum_{p=0}^{\infty} E_t \left[C_{t+p} \prod_{k=0}^p R_{t+k}^{-1} \right] = \frac{\bar{R}}{\bar{R} - 1} C_t + \delta_1 r_t \quad (24)$$

$$\text{where } \delta_1 = \frac{\eta \bar{R}^2 - \bar{C}(\bar{R} - 1)}{\bar{R}(\bar{R} - 1)(\bar{R} - \rho)} \rho > 0$$

Linearizing the LHS of the intertemporal constraint, the equation 13, and using the generalized version of the equation 3 yields :

$$E_t(Y_t^p) = E_t(Y_t^1) - \delta_2 r_t \quad (25)$$

where

$$E_t(Y_t^1) = \sum_{p=0}^{\infty} \bar{R}^{-p} E_t(Y_{t+p} - I_{t+p}) \quad \text{and} \quad \delta_2 = \frac{\bar{R} \bar{Y}}{\rho(\bar{R} - \rho)} > 0$$

Using equations 24 and 25 one obtains the simplified intertemporal constraint:

$$\frac{\bar{R}}{\bar{R} - 1} C_t = E_t(Y_t^p) - \delta_1 r_t + B_t \quad (26)$$

Subtract C_{t-1} in the LHS of the equation above:

$$\Delta C_t = \frac{\bar{R} - 1}{\bar{R}} [E_{t-1}(Y_t^1) - \delta_0 r_t + B_t] - C_{t-1} \quad (27)$$

Where $\delta_0 = \delta_1 + \delta_2 > 0$. Using the Euler Equation, equation 12, at the $t - 1$ period, one can express C_{t-1} as a function of $E_{t-1}(C_t)$ and $E_{t-1}(r_t)$. But $E_{t-1}(C_t)$ respects the intertemporal constraint anticipated in period $t - 1$, so that one can express it in function of passed expectations about present variables:

$$E_{t-1}(C_t) = \frac{\bar{R} - 1}{\bar{R}} [E_{t-1}(Y_t^1) - \delta_0 E_{t-1}(r_t) + E_{t-1}(B_t)] \quad (28)$$

Finally, using the relation 28 to substitute for C_{t-1} in the equation 27, one obtains:

$$\Delta C_t = \frac{\bar{R} - 1}{\bar{R}} [(E_t - E_{t-1})(Y_t^1 - \delta_0 r_t)] + \eta E_{t-1}(r_t) \quad (29)$$

The equality between lagged current account and variation of the net claim on the rest of the world implies that the net claim is predetermined, $E_{t-1}(B_t) = B_t$. Hence, it vanishes from the equation 27. Using the revision of expectations operator and the univariate decomposition of r_t , one finds the equation 14 in the text.

1. From the equation 3 to the equation 16

Using the linearized production function in the permanent income equation, one obtains:

$$\tilde{Y}_t^1 = \alpha_k \sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{K}_{t+p-1} + (\alpha_I - 1) \sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{I}_{t+p} + \frac{\alpha_A \bar{R}}{\bar{R} - 1} \varepsilon_t^A \quad (30)$$

First, calculate the expectation in t about investment in $t + p$. Using the equation 7:

$$E_t(I_{t+p}) = \lambda_0 E_t(I_{t+p-1}) + \omega E_t(\Delta X_{t+p}) = \dots = \lambda_0^{p+1} I_{t-1} + \omega \sum_{s=0}^p \lambda_0^s E_t(\Delta X_{t+p-s}) \quad (31)$$

So, using the definition of the expectation variation:

$$\tilde{I}_{t+p} = E_t(I_{t+p}) - E_{t-1}(I_{t+p}) = \omega \sum_{s=0}^p \lambda_0^s \widetilde{\Delta X}_{t+p-s} \quad (32)$$

Productivity shocks being permanent, $E_t(\Delta A_{t+p-s}) - E_{t-1}(\Delta A_{t+p-s}) = 0$ excepted for $s = p$, for which the difference equals the shock, ε_t^A . One can show that for real interest rate, $\widetilde{\Delta X}_{t+p-s} = \rho^{p-s-1}(\rho - 1)\varepsilon_t^R$ for $p > s$ and $\widetilde{\Delta X}_t = \varepsilon_t^R$. So, using these results in the equation above:

$$\sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{I}_{t+p} = \frac{\bar{R}}{\bar{R} - \lambda_0} \left[\frac{(\bar{R} - 1)}{(\bar{R} - \rho)} \omega_R \varepsilon_t^R + \omega_A \varepsilon_t^A \right] \quad (33)$$

Capital expectation error in period p equals the sum of all the investment expectation error between the current period and p :

$$\tilde{K}_{t+p} = \sum_{s=0}^p \tilde{I}_{t+s} = \frac{\rho^{p+1} - \lambda_0^{p+1}}{\rho - \lambda_0} \omega_R \varepsilon_t^R + \frac{1 - \lambda_0^{p+1}}{1 - \lambda_0} \omega_A \varepsilon_t^A \quad (34)$$

$$\begin{aligned} \sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{K}_{t+p-1} &= \bar{R}^{-1} \sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{K}_{t+p} \\ &= \frac{1}{\bar{R}} \left[\frac{\omega_R \varepsilon_t^R}{\rho - \lambda_0} \sum_{p=0}^{\infty} \bar{R}^{-p} (\rho^{p+1} - \lambda_0^{p+1}) + \frac{\omega_A \varepsilon_t^A}{1 - \lambda_0} \sum_{p=0}^{\infty} \bar{R}^{-p} (1 - \lambda_0^{p+1}) \right] \end{aligned} \quad (35)$$

Using formulas on the infinite sum of a geometric progression, the equation 35 becomes:

$$\sum_{p=0}^{\infty} \bar{R}^{-p} \tilde{K}_{t+p-1} = \frac{\bar{R}}{\bar{R} - \lambda_0} \left[\frac{\omega_R}{\bar{R} - \rho} \varepsilon_t^R + \frac{\omega_A}{\bar{R} - 1} \varepsilon_t^A \right] \quad (36)$$

Using these equalities in the equation 30, one obtains the equation 16 in the text, where:

$$\varphi_1 = \frac{[\alpha_K + (\alpha_I - 1)(\bar{R} - 1)]}{\bar{R} - \lambda_0} > 0 \quad (37)$$

The sign of φ_1 is derived from the intertemporal condition of profitability for the investment: the present value of output gains due to marginal investment, adjustment cost to marginal investment, $\alpha_K / (\bar{R} - 1)$ is superior to the adjustment to marginal investment, $\alpha_I - 1$.

1. From the equation 18 to the equation 19

Using the equation 18, we see that to calculate current account variation, we only need to calculate variation of the net income, $\Delta(Y - I)$, as consumption variation is given by the equation 17. Using the investment dynamic and the equality between lagged capital and lagged investment, one obtains:

$$\Delta(Y - I)_t = \varphi_2 I_{t-1} + (\alpha_I - 1) \omega_R \Delta R_t + [(\alpha_I - 1) \omega_A + \alpha_A] \Delta A_t \quad (38)$$

$$\text{where } \varphi_2 = (\alpha_K + (\alpha_I - 1)(\lambda_0 - 1)) > 0$$

Considering the equation 37, as $\lambda_0 < 1$ and $\alpha_I < 0$, $\varphi_2 > 0$. Using the last equation and the equation 17 in the equation 18, one obtains the equation 19 where the following notations are used:

$$\phi_0 = \eta\rho + \frac{(1 - \rho)(\bar{R} - 1)}{\bar{R}} \left(\frac{\bar{R}\varphi_1\omega_R}{\bar{R} - \rho} - \delta_0 \right) \quad (39)$$

$$\phi_1 = (\alpha_I - 1) \omega_R - \left(\frac{\bar{R}\varphi_1\omega_R}{\bar{R} - \rho} - \delta_0 \right) \frac{\bar{R} - 1}{\bar{R}} > 0 \quad (40)$$