Sticky Wages in a Stochastic DGE Model of the Business Cycle*

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Abstract

In this paper a stochastic dynamic general equilibrium (DGE) model with capital accumulation is augmented by sticky wages. Wages are set in a staggered way as in Taylor (1980) implying that the optimal wage will be set for two periods. Prices are also sticky since there are adjustments cost of prices as in Rotemberg (1982). It is confirmed that wage staggering has a higher potential to generate persistent output responses to a money growth shock. Interestingly, adjustment costs of capital contribute strongly to output persistence. If it is not costly to adjust capital there is no output persistence at all. Price adjustment costs can strengthen the effects of money growth shocks on output in the presence of costly capital adjustment.

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1 Introduction

The question whether money growth shocks can contribute significantly to the explanation of observed business cycle fluctuations has been explored intensively in the last years. Starting with the seminal paper of Chari, Kehoe and McGrattan (2000)[1] many authors have investigated various nominal and real rigidities that could strengthen the transmission of a monetary policy shock. Most of these papers conclude that a model with sticky prices is not able to generate enough output persistence even when augmented by several other real rigidities like capital adjustment costs or variable capital utilization, among others. Hence, there has been an attempt to incorporate other nominal rigidities, especially sticky wages.

Recently Ascari (2003) has provided a unifying framework for the analysis of price and wage staggering in dynamic stochastic general equilibrium models. He simplifies the models so that an exact analytical solution can be obtained. He is therefore able to identify the influence of several specific model parameters on the persistence of a money growth shock. Bénassy also explores the implications of staggered prices and wages analytically (see Bénassy (2000a) and Bénassy (2000b). Most other papers in the literature examine simulation results of calibrated versions of the models under investigation and provide some intuition deduced from simplified equilibrium conditions.[2]

Ascari (2003) concludes that, first, labor immobility across sectors plays a key role in enabling both wage and price staggering models to exhibit persistence. Second this channel is the more important the higher intertemporal elasticities of substitution are. Ascari considers both rigidities separately. In his wage models labor can be immobile because there are industry specific households organized as unions which have monopoly power since labor cannot move across industries (workers organized by skills). This kind of labor immobility is also analyzed in Ascari (2000a). Another way to model immobile labor is to assume that households organized as unions supply differentiated labor inputs to the firms (workers organized by industries). Huang and Liu (2002), Erceg (1997) and Gerke (2003) are examples for this research branch. The model in this paper also belongs to this class. The approach of Bénassy is some kind of combination of both of Ascari's labor

[1] Note that this paper was already issued as a working paper in 1996.

[2] Another exception is Andersen (2004).

immobilities and is unique in the literature.

Unfortunately - and despite of Ascari's unifying paper - the models differ substantially in the way price and wage stickiness is rationalized and implemented. There are mainly two ways to incorporate sticky wages:[3] The first is the well known Calvo pricing scheme (see Calvo (1983)) where household unions face a fixed probability of being able to change their wage rate. The second are Taylor type wage contracts (see Taylor (1980)) where the unions set the wage for a specified period of time, e.g. 2 or 4 periods. The Calvo approach is used extensively in Bénassy's work. Woodford (2003) also assumes Calvo pricing while Chari, Kehoe and McGrattan (2000), Huang and Liu (2002), Erceg (1997) and Gerke (2003) use Taylor contracts. In addition the approaches differ with respect to their specific assumptions about production functions, capital accumulation, implied or assumed money demand functions, utility functions, capital adjustment costs etc. Some include sticky prices, others do not. It is therefore only natural that results are likely to differ substantially. Some peculiarities are summarized below not to explore in detail the reasons but just to demonstrate the diversity and to point to the main differences in the assumptions.

On the one hand Huang and Liu (2002) find that wage staggering has a much higher potential to create empirically observed reactions of output to a money growth shock than price staggering while on the other hand Ascari (2000a) concludes that high persistence is an unlikely outcome. The reasons for these different conclusions can be due to the steady state inflation rate. Ascari shows that in general the degree of persistence is lower the higher the steady state growth rate of money and thus the inflation rate.[4] Huang and Liu study a model with a zero steady state rate of money growth. So possibly their results break down once the model is generalized along these lines. Bénassy (2000a) can show that both output and employment can display a hump-shaped response. The most important parameters in his model are the probability of wage adjustment and the autocorrelation coefficient of the money growth process. He concludes that the reason for the failure Chari, Kehoe and McGrattan (2000) to produce persistence in output is caused by a too short duration of the price contracts of only a quarter. But Chari et al. consider Taylor contracts. Bénassy conjectures that Ascari's model fails

[3] I will only refer to wage stickiness since this is the main focus of the paper.

[4] In a related paper Ascari (2000b) examines the influence of a positive inflation rate in a model with Calvo *price* staggering in a similar model as in Chari, Kehoe and McGrattan (2000). He can show that higher inflation now causes a *higher* persistence in output.

to produce a hump because he uses Taylor contracts of only two quarters in conjunction with a random walk for money. Again this difference can be due to the inflation rate which is zero at the steady state in Bénassy's approach.

Gerke (2003) studies several different versions of Taylor type wage staggering in a model with price adjustment costs as in Rotemberg (1982). Most of his versions fail to produce persistence. But he can create a hump-shaped output response in a variant with a utility function that allows for different values of the elasticity of substitution between consumption and real money balances. His model is one of the few exceptions which consider a positive steady state inflation rate. The paper of Erceg (1997) can be interpreted as an extension of the Chari, Kehoe and McGrattan (2000) model. He uses both Taylor type wage and price staggering combined with different assumptions about the way capital is used: it can be either fixed or factor specific at the firm level or mobile in the aggregate. His model incorporates firm specific adjustment costs of capital and can generate considerable persistence. In the language of Ascari (2003) it belongs to the type of model with immobile workers organized by industries and is thus successful in creating a persistent output response. But Erceg's setup differs from Ascari's with regard to capital accumulation. It is virtually absent in Ascari's analysis, so it is again not obvious which mechanism is responsible for Erceg's success.

This paper tries to combine some aspects of the work in Gerke (2003) as well as in Erceg (1997). I study only two period Taylor type wage contracts - not four period contracts as in Gerke's work. But I use the price adjustment cost version of Rotemberg (1982) also incorporated in Gerke while Erceg investigates also Taylor type price contracts which also last for four periods. I depart from the assumption of four period contracts since I belief that the longer these contracts last the higher is the possibility of a cyclical reaction of the variables since the order of the difference equations grows. I consider adjustment costs of capital but assume that households accumulate capital and rent or sell it to the firms while in Erceg each firm decides on its capital stock itself. There is a zero inflation steady state. The main result of the paper confirms the finding of Ascari (2003) that immobile labor leads to more persistence in output. But it turns out that capital adjustment costs contribute significantly to output persistence. Without them money growth shocks cannot stimulate a persistent reaction of output.

The paper is organized as follows: Section 2 presents the model along with the main assumptions on household and firm behavior. In Section 3 the results are presented using impulse response functions and are related to other results in the literature. Section 4 concludes.

2 The Model

2.1 The Labor Market Intermediary

The labor market intermediary buys in every period $n_{i,t}$ units of labor at the nominal wage rate $W_{i,t}$ from the household $i \in [0,1]$ in order to bundle them to the aggregate labor input n_t. Then he offers this labor aggregate to the firms. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with $\epsilon_w > 1$.

$$n_t = \left(\int_0^1 n_{i,t}^{(\epsilon_w - 1)/\epsilon_w} di \right)^{\epsilon_w/(\epsilon_w - 1)} \tag{1}$$

The bundler maximizes his profits over $n_{i,t}$ given the above production function and given the aggregate nominal wage W_t. So the problem can be written as

$$\max_{n_{i,t}} \left[W_t n_t - \int_0^1 W_{i,t} n_{i,t} di \right] \text{ s.t. } \quad n_t = \left(\int_0^1 n_{i,t}^{(\epsilon_w - 1)/\epsilon_w} di \right)^{\epsilon_w/(\epsilon_w - 1)} \tag{2}$$

The first order conditions for each household i imply

$$n_{i,t}^d = \left(\frac{W_{i,t}}{W_t} \right)^{-\epsilon_w} n_t \tag{3}$$

where $-\epsilon_w$ measures the constant wage elasticity of labor demand from each household i. It is assumed that households offer exactly this amount of labor demanded so that demand always equals supply: $n_{i,t}^d = n_{i,t}^s =: n_{i,t}$. Since the labor market intermediary operates under perfect competition he does not make any profits. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only wage rate W_t that is consistent with this requirement is given by

$$W_t = \left(\int_0^1 W_{i,t}^{(1-\epsilon_w)} di \right)^{1/(1-\epsilon_w)} \tag{4}$$

When wages are set for just two periods as explored in the next section the wage equation simplifies. With wages set for two periods half of the households adjust their wage in period t and half do not. Moreover all adjusting households choose the same wage. Define $W_{i,s,t}$ as the nominal wage at time t of any household i who has set its wage $t-s$ periods ago. Then the wage index W_t is given by

$$W_t = \left(\frac{1}{2}W_{i,0,t}^{1-\epsilon_w} + \frac{1}{2}W_{i,1,t}^{1-\epsilon_w}\right)^{1/(1-\epsilon_w)} \tag{5}$$

2.2 The Household

I consider a MIU-setup where the household i is assumed to have preferences over consumption $c_{i,t}$, leisure $1-n_{i,t}$ and real money balances $M_{i,t}/P_t$. In this model the household sets its wage rate. The household cannot decide on its labor supply because it supplies exactly what the labor market intermediary demands. So the instantaneous utility function is given by

$$\begin{aligned} & u\left(c_{i,t}, \frac{M_{i,t}}{P_t}, \left(\frac{W_{i,s,t}}{W_t}\right)^{-\epsilon_w} n_t, a_t\right) \\ = & \frac{\left[a_t\left(\eta c_{i,t}^{\nu} + (1-\eta)\left(\frac{M_{i,t}}{P_t}\right)^{\nu}\right)^{\frac{1}{\nu}}\right]^{1-\sigma} - 1}{1-\sigma} - \frac{a_t\Theta\left[\left(\frac{W_{i,s,t}}{W_t}\right)^{-\epsilon_w} n_t\right]^{1+\gamma}}{1+\gamma} \end{aligned} \tag{6}$$

σ is the degree of risk aversion while η is a share parameter and ν determines the interest elasticity of the implied money demand function. In this function $c_{i,t}$ and $M_{i,t}/P_t$ are combined to a composite good via a CES aggregator. Labor is separable because households will differ according to their labor supply.[5] Θ is a weighting parameter and a_t is a taste shock which is the same for all households. s can take the values 0 and 1 since the household sets its wage for two periods. This implies that $W_{i,0,t} = W_{i,1,t+1}$. Note that the steady state inflation rate is zero in this model so that there is no indexation of wages.

[5]Note that labor must be additively separable because their is no longer a continuum of identical households each supplying the same continuum of differentiated labor. See Woodford (2003), p. 222 for more details on this point.

The household's budget constraint can be written as follows:

$$\begin{aligned} & c_{i,t} + i_{i,t} + \frac{M_{i,t}}{P_t} + \frac{B_{i,t}}{P_t} \\ = \; & \frac{W_{i,s,t}}{P_t} n_{i,t} + z_{i,t} k_{i,t-1} + \frac{M_{i,t-1}}{P_t} + (1 + R_{t-1}) \frac{B_{i,t-1}}{P_t} + \frac{M^s_{i,t}}{P_t} + \gamma_i \frac{\Xi_t}{P_t} \end{aligned} \tag{7}$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \tag{8}$$

are the nominal profits of the intermediate goods producing firms and γ_i is the share household i receives from these profits. The uses of wealth are real consumption $c_{i,t}$, real investment $i_{i,t}$, holdings of real money balances $M_{i,t}/P_t$ and real bonds $B_{i,t}/P_t$. There are several sources of the household's wealth. It earns money working in the market at the desired wage rate $W_{i,s,t}$ supplying $n_{i,t} = (W_{i,s,t}/W_t)^{-\epsilon_w} n_t$ units of labor. It can spend its money holdings carried over from the previous period $M_{i,t-1}/P_t$. It receives a capital income equal to $z_{i,t}k_{i,t-1}$ by selling capital to the firms where $z_{i,t}$ denotes the real return on capital $k_{i,t}$. There are also previous period bond holdings including the interest on them $(1 + R_{t-1}) (B_{i,t-1}/P_t)$. Finally the household receives a monetary transfer $M^s_{i,t}$ from the monetary authority and a share γ_i of profits form the intermediate goods firms, respectively. The transfer is equal to the change in money balances, i.e.

$$M^s_{i,t} = M_{i,t} - M_{i,t-1} \tag{9}$$

The capital stock increases according to the following law of motion:

$$k_{i,t} = (1 - \delta) k_{i,t-1} + \phi \left(\frac{i_{i,t}}{k_{i,t-1}} \right) k_{i,t-1} \tag{10}$$

There are costs of adjusting the capital stock which are captured by the ϕ function. δ is the rate of depreciation. The detailed properties of ϕ will be discussed in the calibration subsection. Because this equation cannot be explicitly solved for $i_{i,t}$ a second Lagrange multiplier $(\theta_{i,t})$ has to be introduced into the optimization problem of the household.

The Lagrangian is then given by:

$$
\begin{aligned}
L_i &= E_0 \Bigg[\sum_{t=0}^{\infty} \beta^t u \left(c_{i,t}, m_{i,t}, \left(\frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t, a_t \right) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{i,t} \Bigg(z_{i,t} k_{i,t-1} + \frac{W_{i,s,t}}{P_t} \left(\frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t + m_{i,t-1} \frac{P_{t-1}}{P_t} + m_{i,t}^s \\
&\quad + (1 + R_{t-1})\, b_{i,t-1} \frac{P_{t-1}}{P_t} + \gamma_i \frac{\Xi_t}{P_t} - c_{i,t} - i_{i,t} - m_{i,t} - b_{i,t} \Bigg) \qquad (11) \\
&+ \sum_{t=0}^{\infty} \beta^t \theta_{i,t} \left((1-\delta)\, k_{i,t-1} + \phi \left(\frac{i_{i,t}}{k_{i,t-1}} \right) k_{i,t-1} - k_{i,t} \right) \Bigg]
\end{aligned}
$$

Here small variables indicate real quantities, i.e. for example $m_{i,t} = M_{i,t}/P_t$. Households optimize over $c_{i,t}, W_{i,s,t}, i_{i,t}, k_{i,t}, m_{i,t}$ and $b_{i,t}$ taking prices and the initial values of the price level P_0 and the capital stock $k_{i,0}$ as well as the outstanding stocks of money $M_{i,0}$ and bonds $B_{i,0}$ as given. s can take the values 0 and 1 and the household sets the wage rate for two periods so that $W_{i,0,t} = W_{i,1,t+1}$ as explained above. The first order conditions then read

$$
\frac{\partial L_i}{\partial c_{i,t}} = \beta^t D_1 u(\cdot, t) - \beta^t \lambda_{i,t} = 0 \qquad (12)
$$

$$
\begin{aligned}
\frac{\partial L_i}{\partial W_{i,0,t}} &= -\epsilon_w \beta^t D_3 u(\cdot, t) \left(\frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w - 1} \frac{n_t}{W_t} + \beta^t \lambda_{i,t} \left(\frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w} \frac{n_t}{P_t} \\
&\quad -\epsilon_w \beta^t \lambda_{i,t} \frac{W_{i,0,t}}{P_t} \left(\frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w - 1} \frac{n_t}{W_t} \\
&\quad +\beta^{t+1} E_t \Bigg[-\epsilon_w D_3 u(\cdot, t+1) \left(\frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w - 1} \frac{n_{t+1}}{W_{t+1}} \qquad (13) \\
&\quad +\lambda_{i,t+1} \left(\frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w} \frac{n_{t+1}}{P_{t+1}} \\
&\quad -\epsilon_w \lambda_{i,t+1} \frac{W_{i,0,t}}{P_{t+1}} \left(\frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w - 1} \frac{n_{t+1}}{W_{t+1}} \Bigg] = 0
\end{aligned}
$$

$$
\frac{\partial L_i}{\partial i_{i,t}} = -\beta^t \lambda_{i,t} + \beta^t \theta_{i,t} \phi' \left(\frac{i_{i,t}}{k_{i,t-1}} \right) \left(\frac{1}{k_{i,t-1}} \right) k_{i,t-1} = 0 \qquad (14)
$$

$$\frac{\partial L_i}{\partial k_{i,t}} = E_t\beta^{t+1}\lambda_{i,t+1}z_{i,t+1} - \beta^t\theta_{i,t} + E_t\beta^{t+1}\theta_{i,t+1}\Bigg[(1-\delta) + \phi\left(\frac{i_{i,t+1}}{k_{i,t}}\right) + \phi'\left(\frac{i_{i,t+1}}{k_{i,t}}\right)\left(-\frac{i_{i,t+1}}{k_{i,t}^2}\right)k_{i,t}\Bigg] = 0 \quad (15)$$

$$\frac{\partial L_i}{\partial m_{i,t}} = \beta^t D_2 u(\cdot, t) - \beta^t\lambda_{i,t} + E_t\beta^{t+1}\lambda_{i,t+1}\frac{P_t}{P_{t+1}} = 0 \quad (16)$$

$$\frac{\partial L_i}{\partial b_{i,t}} = -\beta^t\lambda_{i,t} + E_t\beta^{t+1}\lambda_{i,t+1}(1+R_t)\frac{P_t}{P_{t+1}} = 0 \quad (17)$$

$D_x u(\cdot, t+y)$ denotes the first partial derivative of the u-function with respect to the x-th argument evaluated at period $t+y$. The derivatives with respect to $\lambda_{i,t}$ and $\theta_{i,t}$ are omitted since they are equal to the intertemporal budget constraint and the capital accumulation condition respectively. ϕ' denotes the derivative of the ϕ-function with respect to the investment to capital ratio which is regarded as one argument. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t\to\infty}\beta^t\lambda_{i,t}x_{i,t} = 0 \qquad \text{for } x = m, b, k \quad (18)$$

It is assumed that there exists a contingent claims market where all households can insure themselves against all idiosyncratic risks. This implies that the household's decisions for consumption, money, bonds, investment and capital are all identical. In addition the factor prices and the Lagrange multipliers will also be identical. All households who reset their wage in the same period face identical decision problems so that they choose the same wage rate. This means that the index i can be dropped (see Christiano, Eichenbaum and Evans (2001), p. 13, for a more thorough discussion).
Equation (13) determines the optimal wage rate of the household. Using (12) to replace λ_t by the marginal utility of consumption it can be rearranged yielding the following formula:

$$W_{0,t} = \frac{\epsilon_w}{1-\epsilon_w}\frac{D_3u(\cdot,t)\,n_t + \beta E_t D_3 u(\cdot,t+1)\left(\frac{W_{t+1}}{W_t}\right)^{\epsilon_w} n_{t+1}}{D_1u(\cdot,t)\,\frac{n_t}{P_t} + \beta E_t D_1 u(\cdot,t+1)\left(\frac{W_{t+1}}{W_t}\right)^{\epsilon_w}\frac{n_{t+1}}{P_{t+1}}} \quad (19)$$

The households set their optimal nominal contract wage as a constant markup $\epsilon_w/(\epsilon_w-1)$ over some kind of marginal rate of substitution between consumption and leisure which is given by the ratio of some weighted marginal disutilities of labor to some weighted marginal utilities of consumption. These

weights are given by n and n/P in the two periods for which the wage is set and the growth factor of the aggregate nominal wage rate, respectively.[6] The formula is similar in its structure to the one that results in a model with price staggering for the intermediate goods producing firms.
The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1+R_t) = E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta}\frac{P_{t+1}}{P_t}\right] \tag{20}$$

Supposed the Fisher equation is valid the real interest rate r_t is implicitly defined as

$$(1+r_t) = E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\beta}\right] \tag{21}$$

because P_{t+1}/P_t equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate. The derivative with respect to money determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money yields the following equation:

$$D_2 u(\cdot, t) = D_1 u(\cdot, t)\frac{R_t}{1+R_t} \tag{22}$$

This specification allows to estimate an empirical money demand function. A detailed description will be presented in the calibration section. The efficiency conditions for investment implies that λ_t equals θ_t times the change in adjustment costs.

$$\lambda_t = \theta_t \phi'\left(\frac{i_{i,t}}{k_{i,t-1}}\right) \tag{23}$$

2.3 The Finished Goods Producing Firm

The firm producing the final good y_t in the economy uses $y_{j,t}$ units of each intermediate good $j \in [0,1]$ purchased at price $P_{j,t}$ to produce y_t units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with $\epsilon_p > 1$.

$$y_t = \left(\int_0^1 y_{j,t}^{(\epsilon_p-1)/\epsilon_p} dj\right)^{\epsilon_p/(\epsilon_p-1)} \tag{24}$$

[6] Ascari (2003) derives the same formula in his 'craft unions' case.

The firm maximizes its profits over $y_{j,t}$ given the above production function and given the price P_t. So the problem can be written as

$$\max_{y_{j,t}} \left[P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj \right] \text{ s.t. } \quad y_t = \left(\int_0^1 y_{j,t}^{(\epsilon_p - 1)/\epsilon_p} dj \right)^{\epsilon_p/(\epsilon_p - 1)} \tag{25}$$

The first order conditions for each good j imply

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} y_t \tag{26}$$

where $-\epsilon_p$ measures the constant price elasticity of demand for each good j. Since the firm operates under perfect competition it does not make any profits. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price P_t that is consistent with this requirement is given by

$$P_t = \left(\int_0^1 P_{j,t}^{(1-\epsilon_p)} dj \right)^{1/(1-\epsilon_p)} \tag{27}$$

2.4 The Intermediate Goods Producing Firm

Intermediate good firms operate under a Cobb-Douglas-technology which is subject to an aggregate random productivity shock a_t.

$$y_{j,t} = a_t n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha} \tag{28}$$

Here $n_{j,t}$ is the labor input employed in period t by a firm j, similarly $k_{j,t-1}$ is the capital stock, and $0 < \alpha < 1$ is labor's share.

Each intermediate goods producing firm faces costs of adjusting its price $P_{j,t}$. The adjustment costs can be measured in units of the final good and are given by

$$\frac{\phi_p}{2} \left[\frac{P_{j,t}}{P_{j,t-1}} - 1 \right]^2 y_t \tag{29}$$

where $\phi_p > 0$. This equation captures both costs that stem from adjusting prices as well as costs that emerge through the misallocation of supply and demand, see Rotemberg (1982). These costs increase with greater price increases and also with the amount of the final good produced.[7]

Intermediate goods firms maximize their profits which are given by

$$\Xi_{j,t} = P_{j,t}y_{j,t} - P_t w_t n_{j,t} - P_t z_t k_{j,t-1} - P_t \frac{\phi_p}{2}\left[\frac{P_{j,t}}{P_{j,t-1}} - 1\right]^2 y_t \qquad (30)$$

where w_t is the aggregate real wage rate. Note that firms cannot supply more of the good j as is demanded by the final good firm. This demand is given in (26). Inserting this restriction in the profit function as well as in the production function (28) allows to write down the intermediate goods firms' optimization problem which is a dynamic one due to the adjustment costs.

$$\max_{n_{j,t},k_{j,t-1},P_{j,t}} E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \frac{\Xi_{j,t}}{P_t}$$

$$\text{s.t.} \quad \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p} y_t = a_t n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha} \qquad (31)$$

$\beta^t \lambda_t / P_t$ is the pricing kernel. It is equal to the marginal value of an additional unit of profits to the household.[8] The Lagrangian for this problem can be written as follows:[9]

$$\begin{aligned} L_j \;=\; & E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{P_t}\left[P_{j,t}\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p} y_t - P_t w_t n_{j,t} - P_t z_t k_{j,t-1}\right.\right. \\ & \left. -P_t \frac{\phi_p}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 y_t\right] \qquad (32) \\ + & \left.\sum_{t=0}^{\infty} \beta^t \xi_t \left(a_t n_{j,t}^{\alpha} k_{j,t-1}^{1-\alpha} - \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p} y_t\right)\right] \end{aligned}$$

The first order conditions are given below:

$$\frac{\partial L_j}{\partial k_{j,t-1}} = -\beta^t \lambda_t z_t + \beta^t \xi_t (1-\alpha) a_t n_{j,t}^{\alpha} k_{j,t-1}^{-\alpha} = 0 \qquad (33)$$

[7] This kind of modeling sticky prices is extensively used in the literature, see for example Dib and Phaneuf (2001), Gerke (2003) and Ireland (1997).

[8] Formally it is given by $\partial L_i / \partial \Xi_t$ in the household's optimization problem where $\gamma_i = 1$.

[9] $P_t w_t = W_t$ is the aggregate nominal wage rate which is the relevant wage for each intermediate goods firm. The same holds for z_t which is also not firm specific. In addition the Lagrange multiplier ξ_t is the same across all firms.

$$\frac{\partial L_j}{\partial n_{j,t}} = -\beta^t \lambda_t w_t + \beta^t \xi_t \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} = 0 \tag{34}$$

$$\begin{aligned}
\frac{\partial L_j}{\partial P_{j,t}} \quad = \quad & \beta^t \lambda_t \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p} \frac{y_t}{P_t} - \epsilon_p \beta^t \lambda_t \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p} \frac{y_t}{P_t} \\
& -\beta^t \lambda_t \phi_p \left[\frac{P_{j,t}}{P_{j,t-1}} - 1\right] \frac{y_t}{P_{j,t-1}} \\
& +E_t \beta^{t+1} \lambda_{t+1} \phi_p \left[\frac{P_{j,t+1}}{P_{j,t}} - 1\right] \frac{P_{j,t+1}}{P_{j,t}^2} y_{t+1} \\
& +\beta^t \epsilon_p \xi_t \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon_p - 1} \frac{y_t}{P_t} = 0
\end{aligned} \tag{35}$$

The first two conditions can be rearranged to yield the familiar microeconomic conditions for profit maximization generalized to markup pricing:

$$\lambda_t z_t \quad = \quad \xi_t \left(1-\alpha\right) a_t n_{j,t}^{\alpha} k_{j,t-1}^{-\alpha} \tag{36}$$

$$\lambda_t w_t \quad = \quad \xi_t \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \tag{37}$$

Dividing by ξ_t on both sides results in

$$\frac{\lambda_t}{\xi_t} z_t \quad = \quad \left(1-\alpha\right) a_t n_{j,t}^{\alpha} k_{j,t-1}^{-\alpha} \tag{38}$$

$$\frac{\lambda_t}{\xi_t} w_t \quad = \quad \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \tag{39}$$

where λ_t/ξ_t is the markup factor μ_t. This reflects the market power of the firms since factor prices w_t, z_t are not just equal to the marginal products of labor and capital respectively.

In a symmetric equilibrium every firm will make the same choices so that

$$P_{j,t} = P_t, n_{j,t} = n_t, k_{j,t-1} = k_{t-1} \text{ for all } t \tag{40}$$

So (38) and (39) hold with all j's eliminated. This means that the efficiency condition for the optimal price of the firms simplifies considerably because all ratios of $P_{j,t}/P_t$ are then equal to one.

$$\begin{aligned}
& \lambda_t \left(1-\epsilon_p\right) y_t - \lambda_t \phi_p \left[\frac{P_t}{P_{t-1}} - 1\right] \frac{P_t}{P_{t-1}} y_t \\
& +E_t \beta \lambda_{t+1} \phi_p \left[\frac{P_{t+1}}{P_t} - 1\right] \frac{P_{t+1}}{P_t} y_{t+1} + \epsilon_p \xi_t y_t = 0
\end{aligned} \tag{41}$$

In case that there are no adjustment costs of prices, i.e. $\phi_p = 0$, the markup is constant and equal to $\mu_t = \lambda_t/\xi_t = \mu = \lambda/\xi = \epsilon_p/(\epsilon_p - 1)$.[10]

2.5 Market Clearing Conditions and Other Equations

The aggregate resource constraint is derived using the resource constraint of households, firms, the government and the monetary authority. Due to the adjustment costs of prices some resources have to be used to finance them so that the condition deviates from the standard one and is given by

$$y_t = c_t + i_t + \frac{\phi_p}{2}\left[\frac{P_t}{P_{t-1}} - 1\right]^2 y_t \tag{42}$$

where the assumption of a symmetric equilibrium has already been taken into account. It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero: $b_t = 0$ for all t.[11]

Real marginal cost ψ_t is just the reciprocal of the markup so that

$$\psi_t = \frac{1}{\mu_t} \tag{43}$$

From the definition of the markup ψ_t is thus linked to the Lagrange multipliers in the following way:

$$\psi_t = \frac{\xi_t}{\lambda_t} \tag{44}$$

The aggregate real wage is just the nominal wage divided by the price level:

$$w_t = \frac{W_t}{P_t} \tag{45}$$

2.6 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent

[10] In the steady state with zero inflation μ is also equal to $\epsilon_p/(\epsilon_p - 1)$ but irrespective of the value of ϕ_p.

[11] See Flodén (2000), p. 1413. He argues that bonds are introduced to determine the nominal interest rate.

but non permanent effects the level of money follows an AR(2)-process. Assume that money grows at a factor g_t:

$$M_t = g_t M_{t-1} \tag{46}$$

If $\widehat{g}_t$ follows an AR(1)-process $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_{g_t}$ then money will follow an AR(2)-process.[12] Note that inflation is zero at the steady state so also money growth is zero there ($g = 1$).

There is another shock in the model, namely the productivity shock a_t. As mentioned above this shock can also act as a taste shock. So one can easily analyze the model's impulse responses to this productivity/ taste shock. Under these circumstances $\widehat{a}_t$ follows an AR(1)-process

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \epsilon_{a_t} \tag{47}$$

with ϵ_{a_t} white noise and $0 < \rho_a < 1$.

2.7 The Steady State

Imposing the condition of constancy of the price level in the steady state ($P_t = P_{t-1} = P$) on the nominal interest rate equation reveals the familiar condition from RBC models that $\beta = 1/(1 + R)$. In addition, as there is no steady state price inflation, $R = r$. The two period wage setting of the households implies $W_0 = W_1$. Using this in the wage index reveals that $W_0 = W_1 = W$. There is also no wage inflation. Nevertheless the nominal wage rates of the households differ since in every period only half of the households adjust their wage while the other half is passive and cannot reoptimize.[13] The optimal steady state real wage rate of the optimizing households can be derived from (13).

$$\frac{W_0}{P} = -\frac{\epsilon_w}{\epsilon_w - 1} \frac{\partial u/\partial n}{\partial u/\partial c} \tag{48}$$

It is given by a constant markup $\epsilon_w/(\epsilon_w - 1)$ over the marginal rate of substitution between consumption and labor $-(\partial u/\partial n)/(\partial u/\partial c) = dc/dn$. At the

[12] A hat ($\widehat{\ }$) represents the relative deviation of the respective variable from its steady state (see the Appendix). ρ_g lies between 0 and 1 and ϵ_{g_t} is white noise.

[13] Gerke (2003) considers a model with a positive steady state inflation rate which allows for further asymmetries. See the discussion later.

steady state each household's individual labor supply n_i is equal to aggregate labor supply n because $n_i = (W_0/W)^{\epsilon_w}\, n = n$ since $W_0 = W$ for all i.

The capital accumulation equation tells us that $\phi\,(i/k) = \delta$ at the steady state. It is assumed that $\phi' = 1$ in steady state to ensure that Tobin's q is equal to one ($q = 1/\phi'$). As a consequence of the requirement that the model with adjustment costs of capital should display the same steady state as the model without them i/k is equal to $\phi\,(i/k)$. Using this in the efficiency condition for capital it can be shown that the rental rate on capital is $z = r + \delta$ as in a standard RBC model. With the help of (39) and the steady state for z it is possible to pin down k/n which amounts to

$$\frac{k}{n} = \left(\frac{r+\delta}{a}\frac{\mu}{1-\alpha}\right)^{-1/\alpha} \tag{49}$$

Real marginal costs are determined by $\psi = 1/\mu$ while μ is given by the steady state of the efficiency condition for the optimal price (41). This results in $\mu = \epsilon_p/(\epsilon_p - 1)$. ψ can be used to calculate w using (39) as well:

$$w = \psi a \alpha \left(\frac{k}{n}\right)^{1-\alpha} \tag{50}$$

The calculation of the steady state value of consumption is tedious. From the production function one knows that labor productivity is given by

$$\frac{y}{n} = a\left(\frac{k}{n}\right)^{1-\alpha} \tag{51}$$

This productivity can be combined with the investment to capital ratio to calculate the investment share:

$$\frac{i}{y} = \left(\frac{i}{k}\frac{k}{n}\right) / \left(\frac{y}{n}\right) \tag{52}$$

Now one can derive the consumption share using the aggregate resource constraint.

$$\frac{c}{y} = -\frac{i}{y} + 1 \tag{53}$$

Note that $y = c + i$ at the steady state because $P/P - 1 = 0$ in (42) so that the presence of adjustment costs does not have any influence. To get the level of c the level of y and i have to be determined: $y = n \cdot y/n$, $i = y \cdot i/y$. Finally $c = y - i$ is the consumption steady state value.

(48) can be used to calculate the preference parameter Θ since n will be given exogenously. Using (22) the ratio of m over c depends only upon β, η and ν.

$$m = c \left[\frac{\eta}{1-\eta} (1-\beta) \right]^{\frac{1}{\nu-1}} \tag{54}$$

In turn Θ can be determined as a function of these parameters and c, m, w and n by solving (48).

$$\Theta = \frac{\epsilon_w - 1}{\epsilon_w} \left[a \left(\eta c^\nu + (1-\eta) m^\nu \right)^{\frac{1}{\nu}} \right]^{-\sigma} \left[\eta c^\nu + (1-\eta) m^\nu \right]^{\frac{1}{\nu}-1} \eta c^{\nu-1} w n^{-\gamma} \tag{55}$$

2.8 Calibration

In order to compute impulse responses the parameters of the model have to be calibrated. It is possible to either specify β or r exogenously. Here β will be set to 0.99 implying a value of r of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature. ψ and μ can be determined by fixing a value for the elasticity of the demand functions for the differentiated products, ϵ_p. This elasticity being equal to 4 causes the static markup $\mu = \epsilon_p/(\epsilon_p - 1)$ to be 1.33 which is in line with the study of Linnemann (1999) about average markups. The wage elasticity of the demand for the household's labor inputs ϵ_w is given by 10, a value that is also used in Erceg (1997) as well as in Gerke (2003). In order to determine the steady state real wage w the productivity shock a has to be specified, along with calculating k/n, see below. As there is no information available about that parameter it is arbitrarily set at 10.[14] n is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case, σ, the parameter governing the degree of risk aversion, is set to 2. γ, which is equal to the inverse of the intertemporal elasticity of labor supply, is chosen to be equal to 1, as in Gerke (2003). The parameters ν and η are calibrated by estimating an empirical money demand function the form of which is implied by the efficiency conditions of

[14]In contrast to the well known basic neoclassical model of King, Plosser and Rebelo (1988) there is no escape from specifying parameters such as a at the steady state. The system cannot be reduced until only deep parameters remain to be calibrated.

the household. This functional form is obtained by solving (22) for m_t and taking logarithms:

$$\ln m_t = \frac{1}{\nu - 1} \ln \frac{\eta}{1-\eta} + \frac{1}{\nu - 1} \ln \left(\frac{R_t}{1 + R_t} \right) + \ln c_t \tag{56}$$

Estimates of Chari, Kehoe and McGrattan (2000) reveal that $\eta = 0.94$ and $\nu = -1.56$. They use US data from Citibase covering 1960:1-1995:4 regressing the log of consumption velocity on the log of the interest rate variable $R_t/(1+R_t)$. Since the focus is on the qualitative results of the model the money demand function is not estimated for specific German data. The implied value of Θ is then equal to 0.0035 while m/c is given by 2.06.

As this model considers the role of capital accumulation several other technological parameters have to be calibrated. The most common one is the depreciation rate δ which is set to 0.025 implying 10% depreciation per year. Labor's share α is 0.64 whereas the elasticity of Tobin's q with respect to i/k is set to -0.5.[15] This value is also used in King and Wolman (1996). The presence of adjustment costs of capital dampens the volatility of investment and is a common feature in equilibrium business cycle models. Using r, δ, a, α and ψ the ratio k/n can be determined. The sensitivity parameter of the intermediate goods producing firms' adjustment cost function ϕ_p is equal to 3.95 in the benchmark case, the same value used in Gerke (2003). Ireland (1997) estimates a value of 4.05 for ϕ_p using US data and a maximum likelihood approach. The model studied here implies that the steady state costs of price adjustment are essentially zero because steady state inflation is zero.

For the exogenous money growth process $\rho_g = 0.5$ is used. As the focus of the paper is on persistency of money shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

3 Impulse Response Functions

The solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. The theoretical background of this algorithm

[15]It can be shown that this elasticity is given by $-[\phi''/\phi' \cdot (i/k)]$.

is developed in King and Watson (1999) whereas computational aspects and the implementation are discussed in King and Watson (2002).

How is a monetary policy shock transmitted in this model? An intuition could be the following. A positive money growth shock leads to higher resources of the household. Thus household's demand for goods rises and this in turn causes a rise in the labor demand of the firms to enable them to increase production. Higher consumption reduces the marginal utility of consumption and higher labor demand lowers the marginal disutility of labor. In turn - as the household set its optimal wage as a constant markup over the ratio of some weighted average of marginal utilities of labor to some weighted average of marginal utilities of consumption - it raises its wage rate.[16] At the same time the household takes into account that a higher wage would lower the demand for its specific labor since its relative wage would be higher than that of those households who will not change their wage. This substitution effect together with the income effect - due to the reduced labor income - dampens the rise in the optimal wage of the household so that the wage rate will not rise proportionally with aggregate demand. Since firms set prices as a markup over marginal costs and since marginal costs are determined by the aggregate wage rate which itself is influenced by the optimal wage rate the rise in prices will be dampened as well. In addition prices will react weaker because firms face price adjustment costs.

Figures 1-4 show the impulse responses of selected variables to a one percent shock to the money growth rate. They overall confirm the intuition above.

Figure 1 displays the reaction of output, consumption, investment and labor. The responses are strongest in the period of the shock and approach smoothly the steady state. They display considerable persistence compared to models with staggered prices. Using as a metric of persistence the ratio of the period $t + 1$ reaction of output to the period t reaction as proposed by Andersen (2004) for two period contracts - defined as the contract multiplier in Huang and Liu (2002) - reveals a value of 0.52 which can be considered as relatively high compared with Andersen's results.[17] All these aggregates converge to their steady states form above showing no cyclical reaction. In Figure 2 real marginal costs react moderately (0.32% deviation from steady state) and show a hump. Unfortunately the nominal interest rate rises so that the model

[16] See (48) and note that $\partial u/\partial n < 0$.

[17] His values range between 0.55 and 0.87.

does not display the liquidity effect. Figure 3 reveals that inflation peaks in the initial period but also that the price level does not overshoot here. Due to the small increase in real marginal costs firms raise their prices only by a small amount which gives rise to a persistent reaction of the price level. The real wage is hump-shaped and countercyclical in this model version. The household adjusts its optimal wage carefully causing a persistent wage response as well. As a consequence the wage index $\widehat{W}_t$ rises smoothly. Real money balances rise and show a small hump 11 quarters after the money growth shock.

Gerke (2003) considers a similar model with four-period wage staggering. The main differences between his model and the one studied here are - besides the length of the wage contracts - a positive steady state inflation rate and the absence of capital adjustment costs. In his benchmark model he uses a utility function that is additively separable in consumption and leisure as in Walsh (1998), p. 69, where consumption and money are aggregated by some kind of Cobb-Douglas function. The impulse responses in this model are cyclical for output, investment, labor and the optimal wage. In a sensitivity analysis he uses a different utility function that is very similar to the preference specification (6). In this case output displays a hump-shaped response in Gerke's model. This result does not hold here. In light of Ascari (2000a) it may be conjectured that the reason is possibly the positive inflation rate in Gerke's model. Ascari finds that the negative relationship between persistence in output and the inflation rate is also affected by the intertemporal elasticity of substitution of labor and the elasticity of substitution between the differentiated goods in a *non-linear* way. So it depends on the specific values of these parameters used whether the model generates plausible persistent output reactions.

Gerke also reports that results are not sensitive with regard to the price adjustment cost parameter ϕ_p in his benchmark model. In my model the opposite is true. Interestingly the model here can generate considerable persistence when ϕ_p is very high implying high costs of price adjustment for the firms. Figure 5 shows the responses for $\phi_p = 100$. The contract multiplier rises to 0.80 compared to 0.52. But when adjustment costs of capital are zero as in Gerke a higher ϕ_p does not increase but *decrease* persistence. This result is very interesting as it shows that there is a non-linear relationship between price and capital adjustment costs: Only for zero or low adjustment costs of capital a higher ϕ_p leads to a lower contract multiplier and less persistence in output. For moderate and high capital adjustment costs a higher ϕ_p causes a higher contract multiplier. Figure 6 represents the result for zero adjustment costs and $\phi_p = 100$ where output is even cyclical. These results are similar to those in Gerke's benchmark model, see his Figures 6.2 and 6.3, especially concerning the relative strength of the reactions and the smoothness of consumption. The cyclicality emerges only under high price adjustment costs here. In Figure 7 ϕ_p is set to zero along with zero adjustment costs. This leads to a quite persistent output response. Consumption is even hump-shaped now but investment reacts too strongly relative to output. The contract multiplier is equal to 0.31. In the benchmark model with very high adjustment costs of capital (with an elasticity of Tobin's q with respect to the investment to capital ratio equal to -500) persistence can only be increased by a small amount. Investment's reaction to a money growth shock is now extremely small, see the scale in Figure 8, while at the same time output does only have a slightly higher persistence with a contract multiplier of 0.57, compared to 0.52.

A higher intertemporal elasticity of substitution for consumption - a lower value for σ - also enhances the persistence of output, confirming Gerke's results. In this case output and consumption react stronger than investment and households raise their wage rate very strongly leading to an overshooting of $\widehat{W}_{0,t}$.

In an early paper on the role of wage staggering in a dynamic stochastic setting Erceg (1997) stresses the role of the wage elasticity of the demand for the households' differentiated labor inputs ϵ_w and the form of the money demand function to create persistence in output. In his model both prices and wages are set in a staggered way for four periods. He argues that in a model with capital accumulation a high value of ϵ_w is not sufficient to explain a persistent output reaction to a money growth shock. In addition the money demand function has to be income based with an income elasticity equal to one. But in a model like the one at hand the money demand function implied by (22) and given in (56) is consumption based and the implied income elasticity would be lower than one as consumption varies much less than output in response to money growth shock. Erceg proposes in turn a model with adjustment costs of capital at the firm level and claims that in this case output reacts with considerable persistence to a money growth shock. The difference to the model at hand is the assumption in Erceg that firms accumulate their own capital and not households. Moreover adjustment costs of capital are modeled differently: Firms operate using the effective stock of capital which is given by subtracting a quadratic term in new investment from $k_{j,t}$. The model at hand can generate a persistent output reaction *although* the money demand function is consumption based and for a moderate value of ϵ_w, as Figure 1 reveals. There is no need of a higher elasticity of money demand as in Erceg. Variations of the wage elasticity of the demand for households' labor ϵ_w change the benchmark results considerably. Using Erceg's value would correspond to $\epsilon_w = 33.\bar{3}$ and results in a contract multiplier of 0.65. Real money balances are hump-shaped in this case. Interestingly output persistence is sensitive with regard to the price elasticity of the demand for intermediate goods ϵ_p. With $\epsilon_p = 1.1$ which implies a very low elasticity and an unrealistic high markup factor of 11 the model creates a persistent output impulse response, see Figure 9, with a contract multiplier equal to 0.84.

Andersen (2004) stresses the role of capital accumulation as an important propagation mechanism. He develops a model that has an analytical solution and can show which technology and preference parameters are important for a persistent output reaction. His model does not assume that individual labor supply plays a decisive role for wage formation as in my model. Specifically he uses some kind of wage bargaining model where unions trade off wages and employment. He integrates the idea that involuntary unemployment plays an important role, especially in a European context, in an otherwise quite standard dynamic stochastic equilibrium model. His main result is that neither capital accumulation nor nominal contracts alone can generate plausible impulse responses but that the interaction of both mechanisms can strengthen persistence up to unit roots. The result that a higher capital accumulation parameter increases persistence does not hold here. Higher capital accumulation would imply a lower value of the depreciation rate δ. Using $\delta = 0.01$ generates an even less persistent output reaction with a contract multiplier of 0.48. It must be noted that Andersen's capital accumulation is very unusual. He employs a parameterized version of the adjustment cost function $\phi\left(I/K\right)$ where $\phi\left(I/K\right) = \left(I/K\right)^{\delta}$, K and I in levels. His equation for the evolution of the capital stock then reads

$$K_{t+1} = K_t \left(\frac{I_t}{K_t}\right)^{\delta} = K_t^{1-\delta} I_t^{\delta} \qquad 0 \leq \delta \leq 1 \tag{57}$$

Andersen argues that higher capital accumulation is then associated with a higher δ. But $\delta = 1$ implies $K_{t+1} = I_t$ which means full depreciation of the capital stock whereas $\delta = 0$ implies $K_{t+1} = K_t$ which is constancy of capital. So it is not clear why a *higher* δ leads to stronger capital accumulation.

Changing nevertheless δ to 0.1 indeed leads to a higher contract multiplier of 0.58 compared to 0.52 in the benchmark case. Variations of his marginal value of wage income to unions cannot be conducted in the model at hand so that implications cannot be compared.

Huang and Liu (2002) study a model with both wage and price staggering and conclude that also in a model augmented by capital accumulation wage staggering has a much higher potential to generate persistence in output than price staggering. Their setup deviates with regard to capital adjustment costs. These are modeled in a similar way as the price adjustment costs in (29) without dependence on y_t. Huang and Liu again stress the influence of ϵ_w on the model outcome. The higher the wage elasticity of household's labor supply the higher the contract multiplier. They yield a value as high as 0.56 for $\epsilon_w = 6$ while in my model the value would be 0.46 for that case. This confirms the intuition from above: The higher the demand elasticity for the differentiated labor inputs the higher would be the loss in the demand for labor of the specific household and the higher the stickiness of the optimal wage rate, the closer the reset wage to the existing one. In turn the higher will be the persistence of output.[18]

The model is sensitive to variations in γ, the inverse of the intertemporal elasticity of substitution of labor. For $\gamma = 0$ this elasticity tends to infinity and *lowers* the contract multiplier of output to 0.38, see Figure 10. Interestingly, for very high values of γ and thus low intertemporal elasticities of substitution of labor the multiplier increases slightly up to 0.59 for $\gamma = 100000000$. This contradicts results of Ascari (2003) who finds that the degree of persistence seems to be extremely insensitive to the value of the intertemporal elasticity of labor. Additionally, the standard direction of the influence of $1/\gamma$ is reversed.

Results of Bénassy (2000a) concerning the ability to produce hump-shaped responses do not carry over to this model. Bénassy can derive a condition for a hump in employment that depends on the relation between the probability that households can adjust their wage and the autocorrelation coefficient of the money growth process ρ_g. This would require setting $\rho_g > 0.5$.[19] Varying ρ_g appropriately does not help in creating humps.

[18]See also Ascari (2003) on this point. He can show analytically that for $\epsilon_w \to \infty$ output has a unit root.

[19]Otherwise the contract length would have to be changed complicating the model setup considerably.

4 Conclusions

A stochastic dynamic general equilibrium model has been proposed to explain persistent reactions of output and inflation to a money growth shock. Wages are set in a staggered way for two periods while there are also adjustment costs of prices at the firm level. The results confirm the finding from the literature that sticky wages have a higher potential in explaining persistence in output than sticky prices. The wage elasticity of the demand for differentiated labor inputs in conjunction with the assumption of immobile labor plays an important role for creating this result.

But the paper also demonstrates that sticky prices can contribute to persistence when prices are costly to adjust for the firms. In the presence of adjustment costs of capital price adjustment costs intensify the persistence of output. When there are no costs of adjusting the capital stock there is no persistence even in a model with wage staggering.

Future research should focus more intensively on the role of the diverse modeling procedures of sticky wages and prices. It is not yet clear whether Calvo pricing and Taylor staggering lead to the same results. This is of special interest as models with Taylor type staggering seem to inherit a higher possibility of cyclical reactions as Calvo pricing models. This is confirmed by Gerke (2003) in his chapters D and E. Moreover sticky prices that emerge through adjustment costs appear to have a higher potential to generate persistence in models with costly capital adjustment. Models with Taylor price staggering of two periods are not able to generate a persistent response of output to a money growth shock.

A Appendix

A.1 Household's Equations

The Taylor approximation for the consumption decision is given by

$$\begin{aligned} 0 = \quad & -mD_{12}u\left(c,m,n,a\right)\widehat{P}_t \\ & +cD_{11}u\left(c,m,n,a\right)\widehat{c}_t - D_1u\left(c,m,n,a\right)\widehat{\lambda}_t \\ & +mD_{12}u\left(c,m,n,a\right)\widehat{M}_t + aD_{14}u\left(c,m,n,a\right)\widehat{a}_t \end{aligned} \tag{58}$$

A hat ($\widehat{\ }$) represents the relative deviation of the respective variable from its steady state ($\widehat{a}_t = \left(a_t - a\right)/a$). $D_iu\left(\cdot\right)$ denotes the first partial derivative

of the u-function with respect to the i-th argument. Similarly $D_{ij}u(\cdot)$ denotes the partial derivative of $D_i u(\cdot)$ with respect to the j-th argument, all evaluated at the steady state. It should be noted that $D_{3j}u(\cdot) = D_{j3}u(\cdot)$ for $j = c, m$ will be equal to zero because of the separability assumption in the utility function.
The efficiency condition for the optimal nominal wage is determined by

$$\begin{aligned}
& -\beta c \frac{D_{11}u(c,m,n,a)}{D_1 u(c,m,n,a)} \widehat{c}_{t+1} - \beta m \frac{D_{12}u(c,m,n,a)}{D_1 u(c,m,n,a)} \widehat{M}_{t+1} \\
& +\beta n \frac{D_{33}u(c,m,n,a)}{D_3 u(c,m,n,a)} \widehat{n}_{t+1} + \beta \left(m \frac{D_{12}u(c,m,n,a)}{D_1 u(c,m,n,a)} + 1 \right) \widehat{P}_{t+1} \\
& +\epsilon_w n \frac{D_{33}u(c,m,n,a)}{D_3 u(c,m,n,a)} \widehat{W}_{t+1} \\
& +\beta a \left(\frac{D_{34}u(c,m,n,a)}{D_3 u(c,m,n,a)} - \frac{D_{14}u(c,m,n,a)}{D_1 u(c,m,n,a)} \right) \widehat{a}_{t+1} \\
= \; & (1+\beta) \left(1 + \epsilon_w n \frac{D_{33}u(c,m,n,a)}{D_3 u(c,m,n,a)} \right) \widehat{W}_{0,t} \\
& +c \frac{D_{11}u(c,m,n,a)}{D_1 u(c,m,n,a)} \widehat{c}_t + m \frac{D_{12}u(c,m,n,a)}{D_1 u(c,m,n,a)} \widehat{M}_t \\
& -n \frac{D_{33}u(c,m,n,a)}{D_3 u(c,m,n,a)} \widehat{n}_t - \left(m \frac{D_{12}u(c,m,n,a)}{D_1 u(c,m,n,a)} + 1 \right) \widehat{P}_t \\
& -\epsilon_w n \frac{D_{33}u(c,m,n,a)}{D_3 u(c,m,n,a)} \widehat{W}_t + a \left(\frac{D_{14}u(c,m,n,a)}{D_1 u(c,m,n,a)} - \frac{D_{34}u(c,m,n,a)}{D_3 u(c,m,n,a)} \right) \widehat{a}_t
\end{aligned} \tag{59}$$

The efficiency condition for money determines the respective demand function. So one gets

$$\begin{aligned}
& \beta D_1 u(c,m,n,a) \widehat{P}_{t+1} - \beta D_1 u(c,m,n,a) \widehat{\lambda}_{t+1} \\
= \; & c D_{21} u(c,m,n,a) \widehat{c}_t + m D_{22} u(c,m,n,a) \widehat{M}_t \\
& -D_1 u(c,m,n,a) \widehat{\lambda}_t \\
& + \left[\beta D_1 u(c,m,n,a) - m D_{22} u(c,m,n,a) \right] \widehat{P}_t \\
& +a D_{24} u(c,m,n,a) \widehat{a}_t
\end{aligned} \tag{60}$$

The nominal interest rate follows, according to (20),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R} \widehat{R}_t + \widehat{\lambda}_t \tag{61}$$

in the approximated form, with R (respective r for the real rate) as the steady state values. The real rate r_t was deduced via the Fisher equation (see (21)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \tag{62}$$

Optimal investment is determined from the efficiency condition for i_t:

$$0 = -\widehat{\lambda}_t + \widehat{\theta}_t + \frac{\phi''}{\phi'}\frac{i}{k}\widehat{i}_t - \frac{\phi''}{\phi'}\frac{i}{k}\widehat{k}_{t-1} \tag{63}$$

The first order condition for capital implies:

$$\beta z\widehat{\lambda}_{t+1} + \beta z\widehat{z}_{t+1} + \beta(1-\delta)\widehat{\theta}_{t+1} - \beta\frac{\phi''}{\phi'}\frac{i}{k}\frac{i}{k}\widehat{i}_{t+1} = -\beta\frac{\phi''}{\phi'}\frac{i}{k}\frac{i}{k}\widehat{k}_t + \widehat{\theta}_t \tag{64}$$

Capital evolves over time according to

$$\widehat{k}_t = (1-\delta)\widehat{k}_{t-1} + \delta\widehat{i}_t \tag{65}$$

A.2 The Labor Market Intermediary's Equation

Since the focus is on a symmetric equilibrium the only equation that remains for the labor market intermediary is the wage index.

$$0 = \frac{1}{2}\widehat{W}_{0,t} + \frac{1}{2}\widehat{W}_{0,t-1} - \widehat{W}_t \tag{66}$$

In order to avoid too many variables $\widehat{W}_{1,t}$ is dropped and replaced by $\widehat{W}_{0,t-1}$.

A.3 Intermediate Goods Firm's Equations

The optimum conditions of profit maximization problem determine the real wage and the rental rate of capital (see (38) and (39)).

$$0 = (\alpha-1)\widehat{n}_t + (1-\alpha)\widehat{k}_{t-1} + \widehat{\xi}_t - \widehat{\lambda}_t + \widehat{a}_t - \widehat{w}_t \tag{67}$$

$$0 = \alpha\widehat{n}_t - \alpha\widehat{k}_{t-1} + \widehat{\xi}_t - \widehat{\lambda}_t + \widehat{a}_t - \widehat{z}_t \tag{68}$$

The production function is given by the Cobb-Douglas-functions of the intermediate goods firms and valid in aggregate variables.

$$0 = -\widehat{y}_t + \alpha\widehat{n}_t + (1-\alpha)\widehat{k}_{t-1} + \widehat{a}_t \tag{69}$$

The Taylor approximation for optimal price setting (41) is given by

$$
\begin{aligned}
& -\beta\mu\phi_p \widehat{P}_{t+1} \\
= \; & \mu\left(1-\epsilon_p\right)\widehat{\lambda}_t - \left(\mu\phi_p + \beta\mu\phi_p\right)\widehat{P}_t + \left[\mu\left(1-\epsilon_p\right)+\epsilon_p\right]\widehat{y}_t + \epsilon_p\widehat{\xi}_t + \mu\phi_p\widehat{P}_{t-1}
\end{aligned} \tag{70}
$$

A.4 Market Clearing Conditions and Other Equations

The Taylor expansion of the aggregate market clearing condition is given by[20]

$$0 = -\widehat{y}_t + \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t \tag{71}$$

The markup μ_t is determined by the ratio of price over nominal marginal cost and as there is no steady state inflation it follows that $\mu_t = 1/\psi_t$. So the Taylor approximation can be written as

$$0 = \widehat{\mu}_t + \widehat{\psi}_t \tag{72}$$

Real marginal cost are linked to the Lagrange multipliers by

$$0 = \widehat{\psi}_t + \widehat{\xi}_t - \widehat{\lambda}_t \tag{73}$$

The real wage equation is represented by

$$0 = -\widehat{w}_t + \widehat{W}_t - \widehat{P}_t \tag{74}$$

A.5 The Monetary Authority and further Equations

To close the model one needs to assume some exogenous process for the money supply. Here it will be assumed that the growth rate of $\widehat{M}_t$ follows an AR(1)-process. This means that the level of money will follow an AR(2)-process (see the discussion in the main text). In order to model this properly one has to add the equation

$$0 = \widehat{M}_t - \widehat{g}_{M_t} \tag{75}$$

[20] The adjustment cost term does not appear in this equation because the steady state inflation rate is zero in this model. This is fundamentally different for a positive inflation rate, see Gerke (2003), p. 175. It also confirms the result of Ascari (2000b) that a positive inflation rate not only changes the steady state (long run properties of the model) but also the dynamics (short run properties).

where $\widehat{g}_{M_t}$ is the exogenous stochastic process that will have the same characteristics as $\widehat{M_t}$.
As it is interesting to study the implications for the inflation rate Π this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \tag{76}$$

There are now 22 variables

$$\widehat{c}_t, \widehat{i}_t, \widehat{y}_t, \widehat{\lambda}_t, \widehat{\theta}_t, \widehat{k}_t, \widehat{k}_{t-1}, \widehat{n}_t, \widehat{w}_t, \widehat{z}_t, \widehat{\mu}_t,$$
$$\widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{W}_{0,t}, \widehat{W}_{0,t-1}, \widehat{W}_t, \widehat{\Pi}_t, \widehat{M}_t, \widehat{\xi}_t$$

but only 19 equations so three tautologies must be added to the model. These are

$$\widehat{W}_{0,t} = \widehat{W}_{0,t} \tag{77}$$
$$\widehat{P}_t = \widehat{P}_t \tag{78}$$
$$\widehat{k}_t = \widehat{k}_t \tag{79}$$

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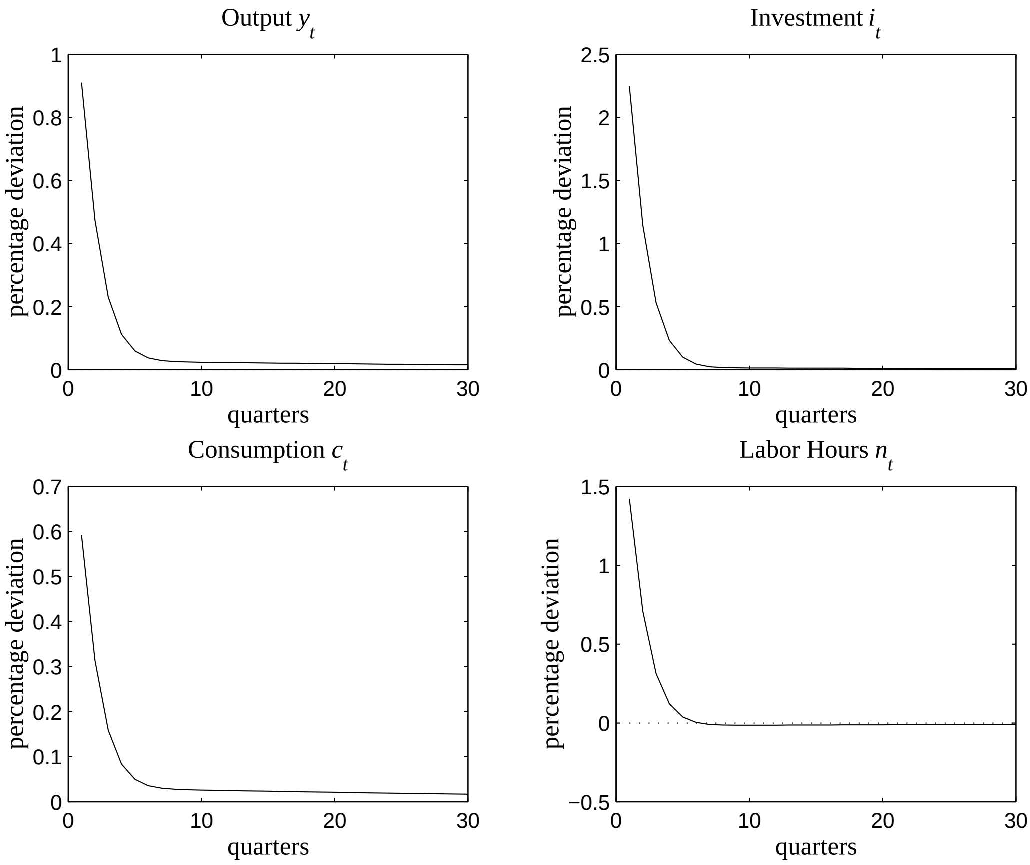

Figure 1: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$

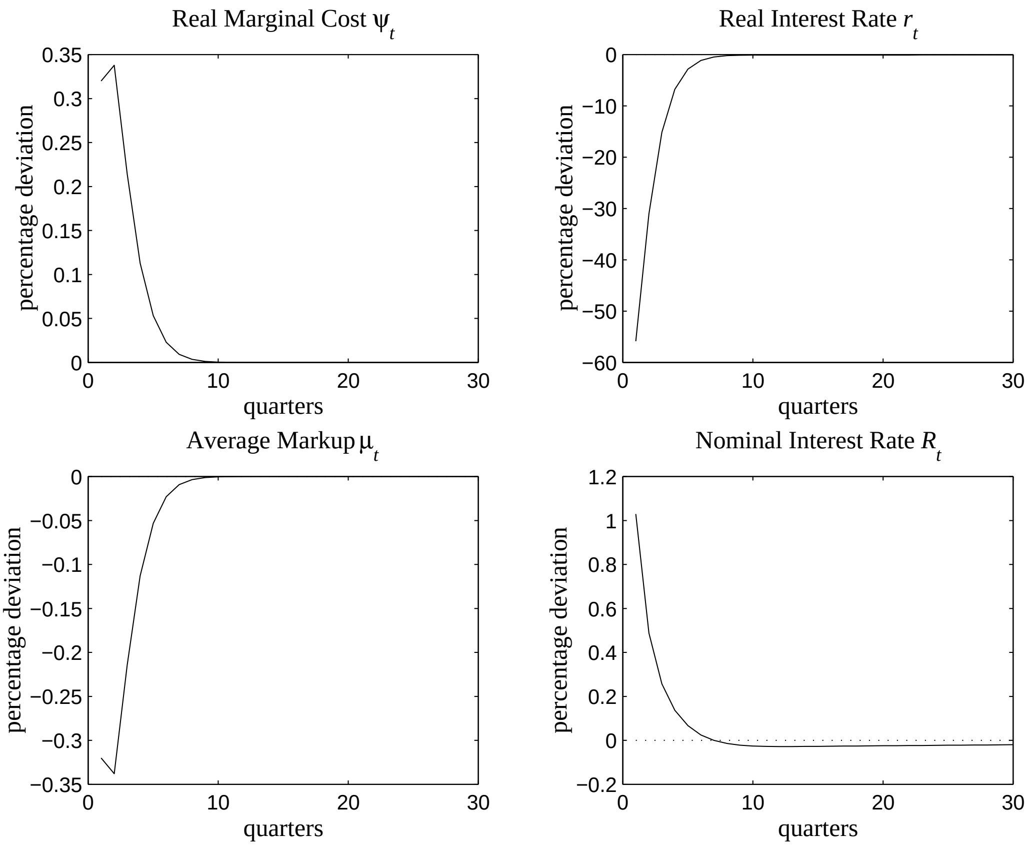

Figure 2: Impulse Response Functions for $\widehat{\psi}_t, \widehat{r}_t, \widehat{\mu}_t, \widehat{R}_t$

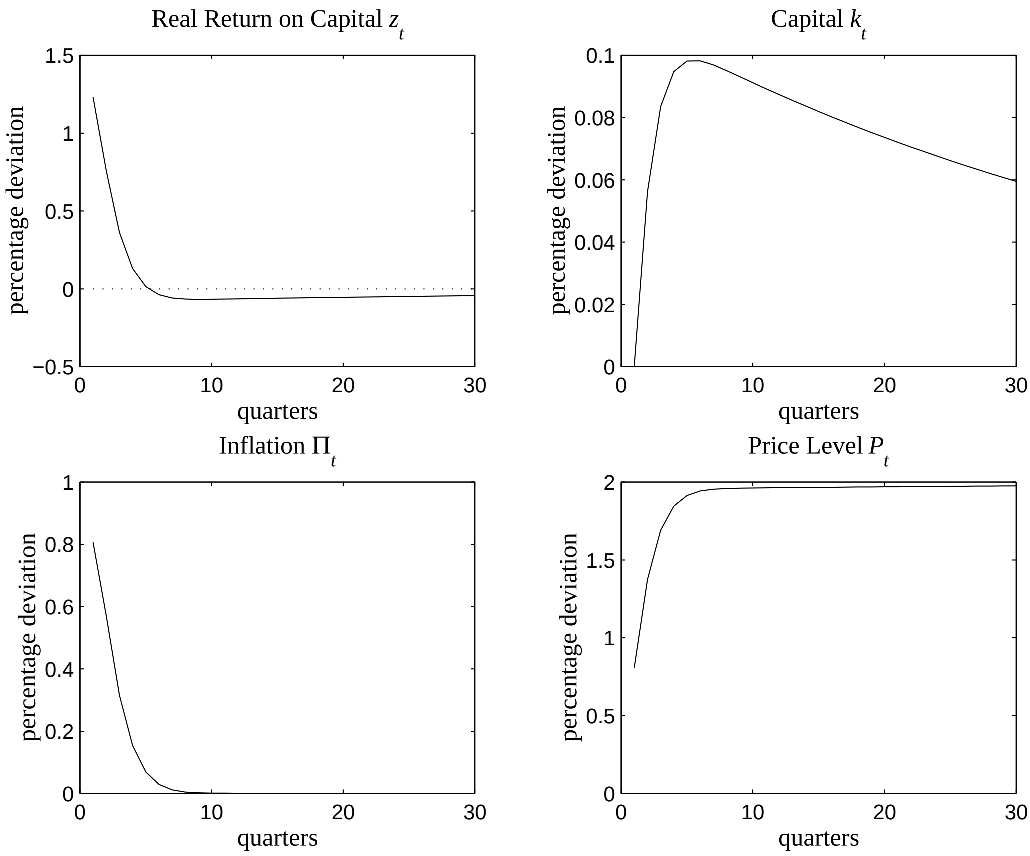

Figure 3: Impulse Response Functions for $\widehat{z}_t, \widehat{k}_t, \widehat{\Pi}, \widehat{P}_t$

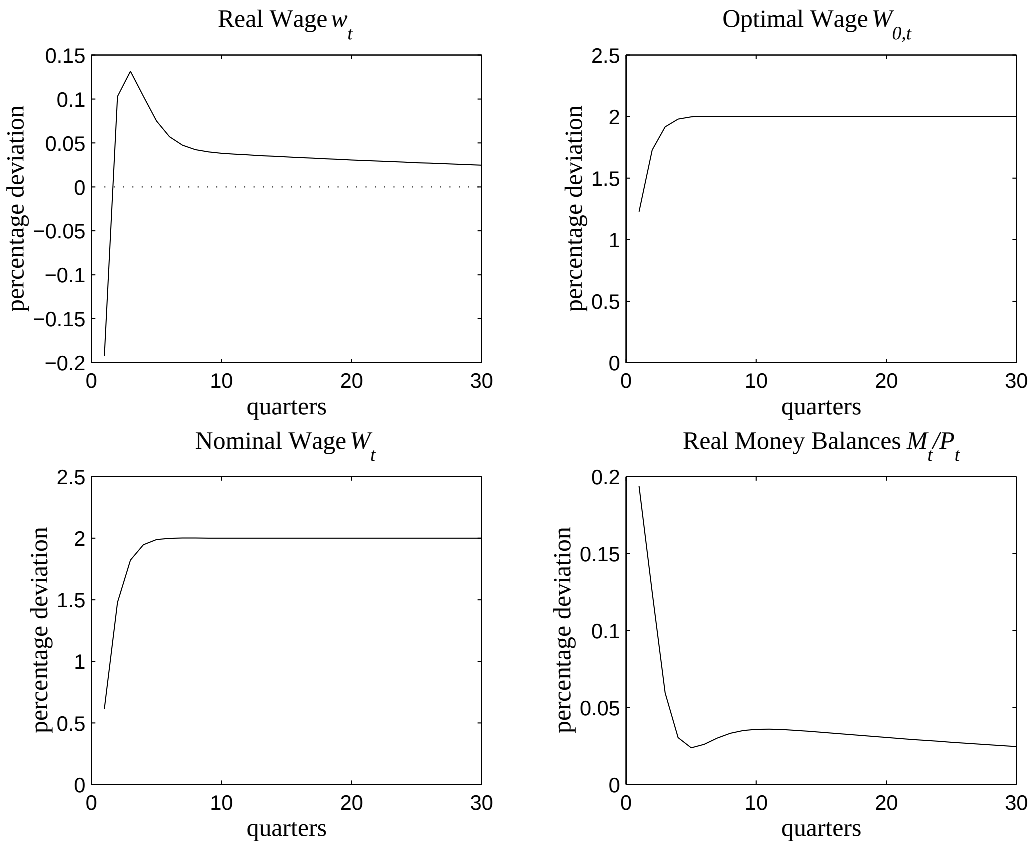

Figure 4: Impulse Response Functions for $\widehat{w}_t, \widehat{W}_{0,t}, \widehat{W}_t, \widehat{M}_t - \widehat{P}_t$

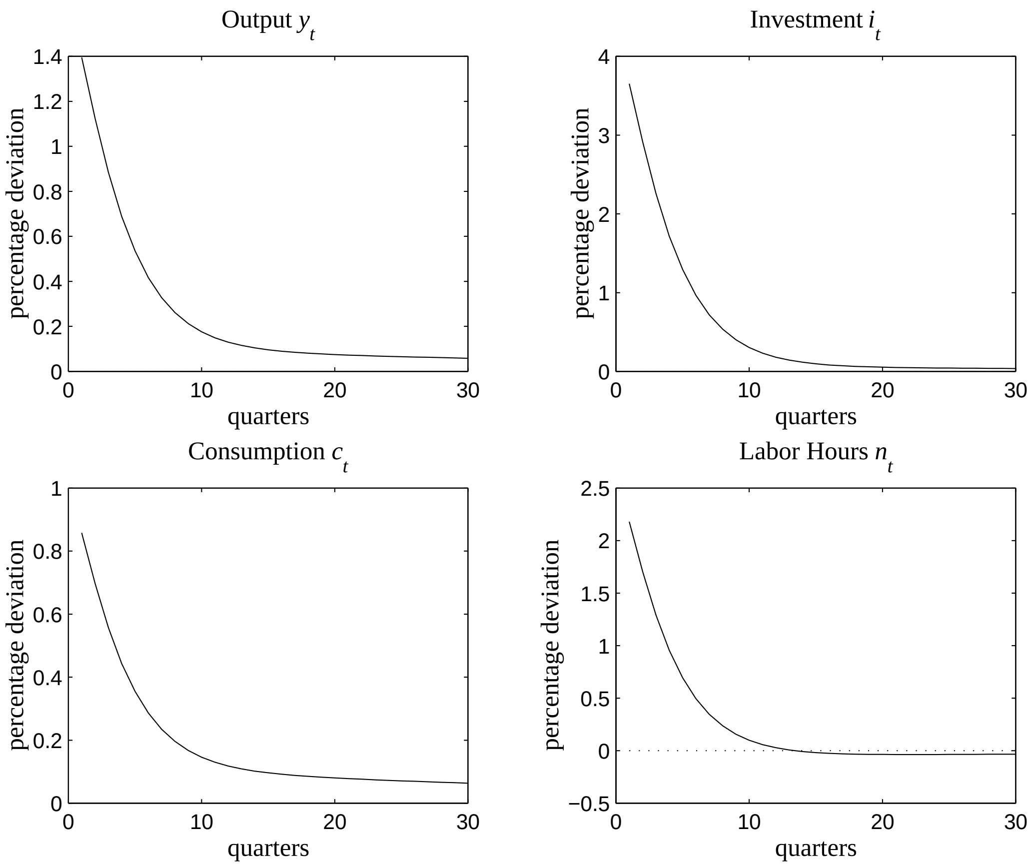

Figure 5: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, very high price adjustment costs ($\phi_p = 100$) and benchmark capital adjustment costs (Tobin's q elasticity of -0.5)

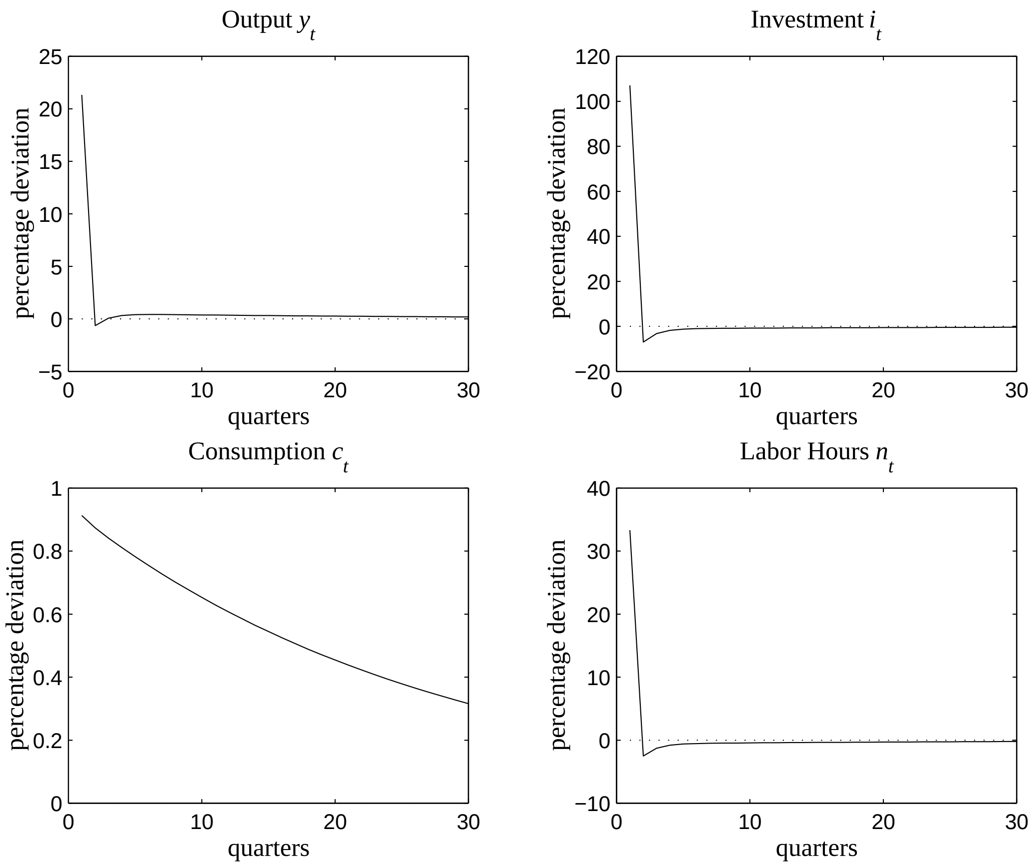

Figure 6: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, very high price adjustment costs ($\phi_p = 100$) and zero capital adjustment costs (Tobin's q elasticity of 0)

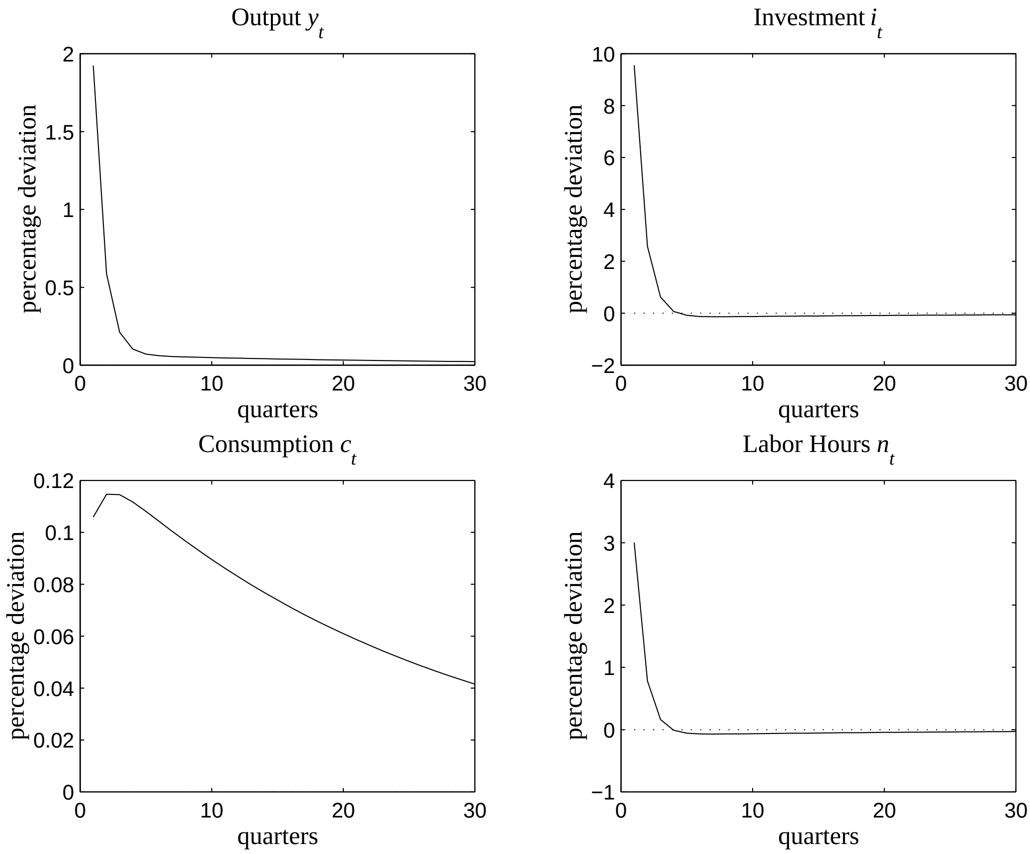

Figure 7: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, zero price adjustment costs ($\phi_p = 0$) and zero capital adjustment costs (Tobin's q elasticity of 0)

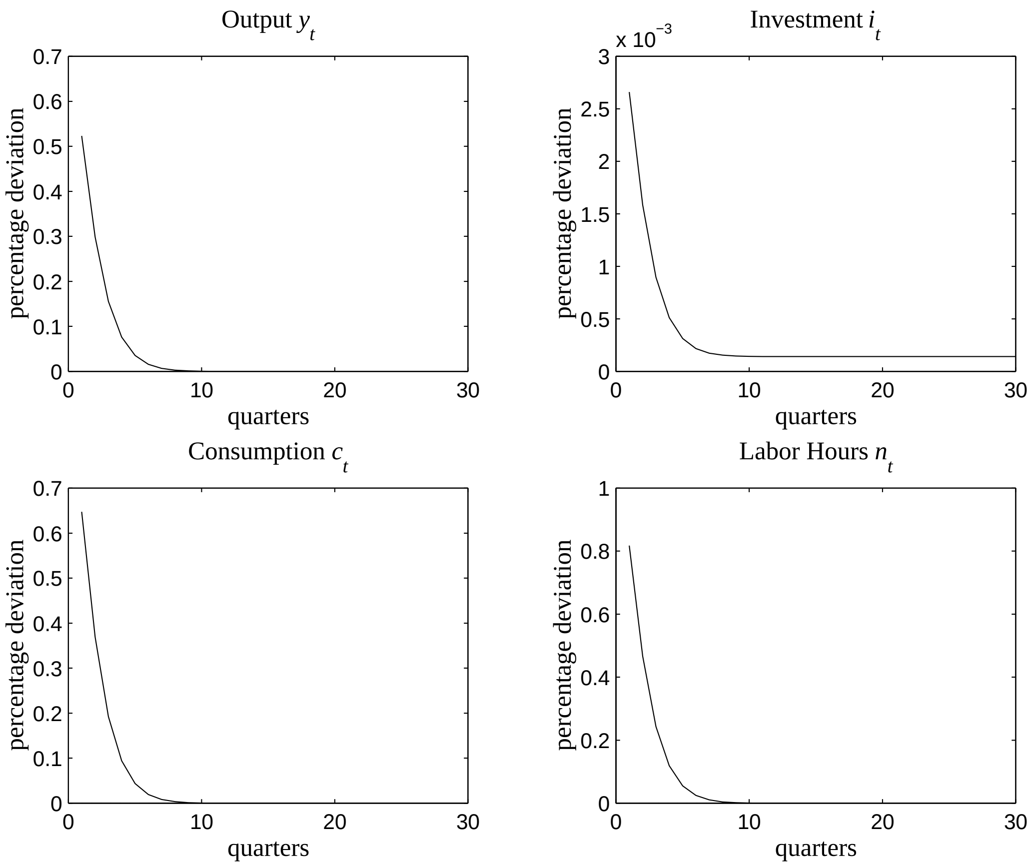

Figure 8: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, benchmark price adjustment costs ($\phi_p = 3.95$) and high capital adjustment costs (Tobin's q elasticity of -500)

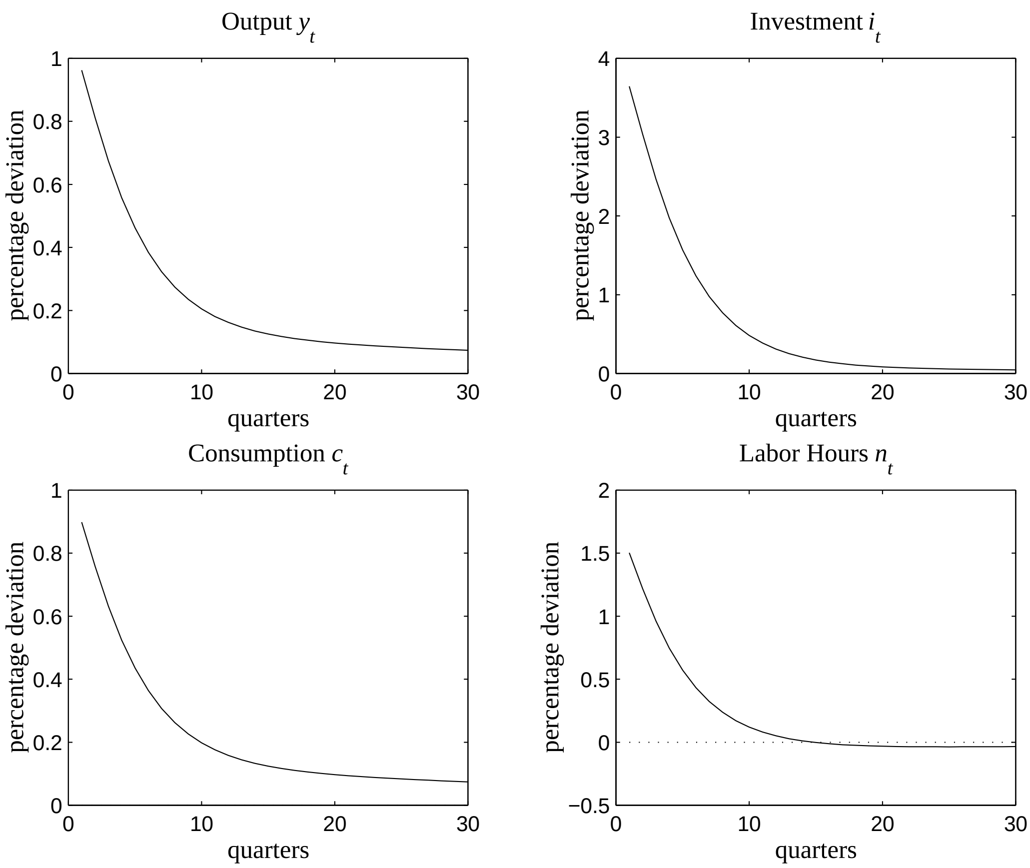

Figure 9: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, very low price elasticity ($\epsilon_p = 1.1$)

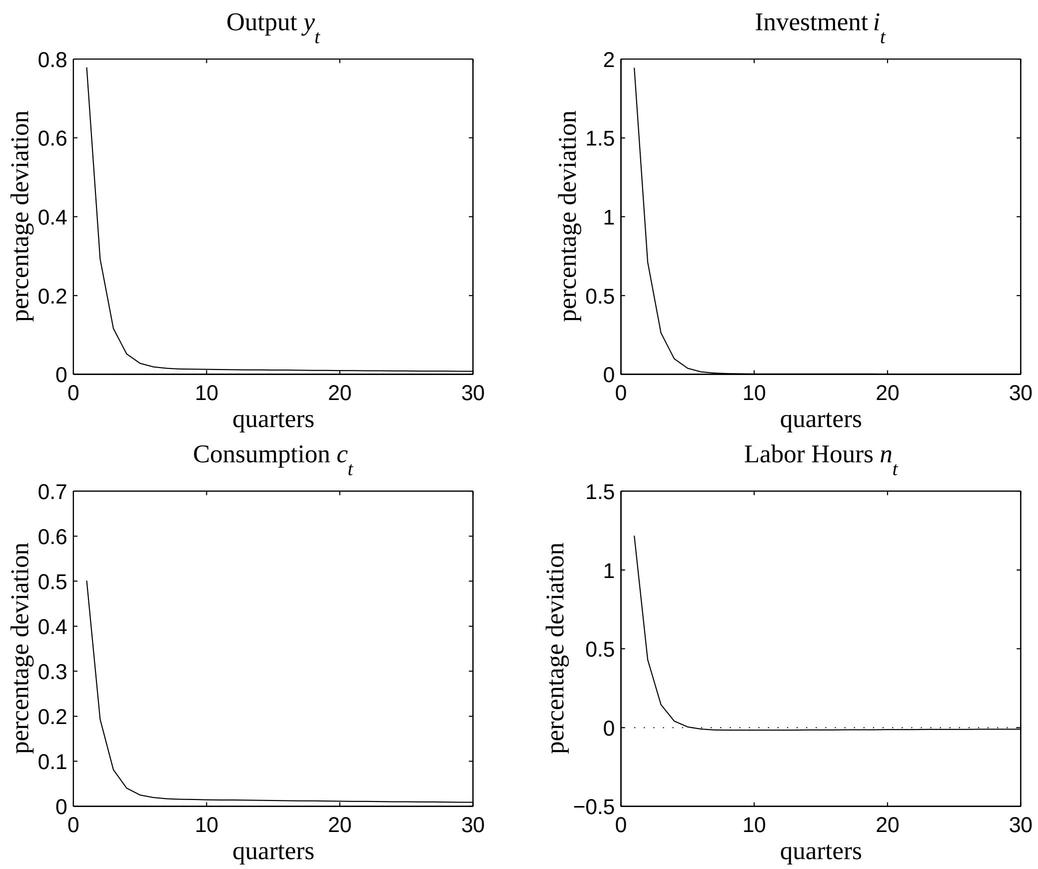

Figure 10: Impulse Response Functions for $\widehat{y}_t, \widehat{i}_t, \widehat{c}_t, \widehat{n}_t$, infinite labor supply elasticity ($\gamma = 0$)