When and How Much Does a Peg Increase Trade? The Role of Trade Costs and Import Demand Elasticity under Monetary Uncertainty

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Abstract

This paper extends recent research in stochastic new open-economy macroeconomics (NOEM) to study the effects of the exchange-rate regime on international trade in a more realistic, yet rigorous, analytical set-up. We essentially embed trade in similar and different output mixes within a common framework and focus on the implications of impediments to cross-border transactions under alternative invoicing, namely producer's currency pricing (PCP) versus consumer's currency pricing (CCP). Given separable utility and symmetry in structure and in the distributions of national money shocks, the only source of uncertainty in the model, our principal contribution is to show that with (some degree of) PCP – although not (full) CCP – a peg slightly reduces expected trade, measured in terms of GDP, relative to a float under elastic import demand. Inelastic import demand, possible under the same taste for diversity but dissimilar outputs arising from differences in endowments, reverses this conclusion.

JEL Classification: F10, F33, F41.

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1 Introduction

The literature that has directly or indirectly addressed the question whether the exchange-rate regime matters for trade has not arrived yet at a satisfactory answer. A fixed exchange rate has often been claimed to substantially increase trade, mostly on empirical grounds and notably in Rose (1999), as far as recent research is concerned. But in theoretical work focusing on monetary uncertainty under high substitutability of cross-country output and no trade costs, Bacchetta and van Wincoop (2000 a) have warned that this is not necessarily the case. In related analysis in Mihailov (2003), still under frictionless trade, we have furthermore shown that alternative modelling of the currency of price setting in open economies with nominal rigidity implies certain distinction among the trade effects of the exchange-rate regime. Our main point was that under (complete) consumer's currency pricing (CCP),[1] as assumed in the quoted paper by Bacchetta and van Wincoop, a peg versus float does not matter for trade prices and, hence, flows because the pass-through and expenditure-switching channel of the international transmission of money shocks is closed. However under (some degree of) producer's currency pricing (PCP),[2] when pass-through and expenditure switching are operating, a peg can stabilize trade-to-output *variability*. Nevertheless it cannot, neither can a float, increase the *expected* trade share in GDP, irrespective of the assumed currency of price stickiness.

The objective of the present paper is thus to examine further the effects of the exchange-rate regime on trade prices and flows in a more careful manner, by also looking into some of their key *non-monetary* determinants. Wishing to achieve analytical clarity in uncovering the mechanisms of such effects as well as direct comparability with earlier results, we build on a "baseline" stochastic new open-economy macroeconomics (NOEM)[3] set-up, such as that in Mihailov (2003). But a major import of the present paper is that it embeds trade in similar vs. different output mixes within a *common* theoretical framework, at the same time taking an explicit account of the implications of impediments to cross-border transactions under alternative invoicing,[4] namely CCP vs. PCP; whereas the previous literature, classic as well as NOEM, has usually modelled in separation either trade of differentiated brands belonging to the same homogeneous product[5] or trade arising from complete specialization in the production

[1] Also termed pricing-to-market (PTM) or local currency pricing (LCP) and possible under market segmentation.

[2] The standard assumption of open-economy models, both theoretical and empirical, in the Mundell-Fleming-Dornbusch tradition.

[3] NOEM is defined as a line of research and classified across differing assumptions in the recent survey by Lane (2001). A narrower and more technical summary of the basic NOEM methodology is also provided in Sarno (2001).

[4] Friberg (1998) points out to the fact that the currency of *price setting*, the currency of *invoicing* and the currency of *payment*, although theoretically corresponding to three distinct stages of a typical international trade transaction and hence potentially different, practically coincide "with few known exceptions". Therefore in what follows we use "invoicing" and "price setting" interchangeably (without talking at all about the "currency of payment").

[5] As in Obstfeld-Rogoff (1995, 1998, 2000, 2001) models, to quote just the earliest NOEM examples.

of just one national good-type.[6] Our unified approach becomes feasible, it is true, at the cost of a highly stylized environment, by essentially attributing the primary cause of the international exchange of goods to identical tastes for diversity and not to Ricardian comparative advantage in productivity. Nevertheless, our microfounded general-equilibrium parallel of consumer's to producer's currency pricing under monetary uncertainty and costs of cross-border transactions has provided valuable insights into trade determination, in particular about the role of nominal and real factors in it. In essence, it has permitted us to derive and interpret conditions when a peg would dominate a float in generating more expected trade-to-output and when a float would do that instead.

Our focus on trade, and not on welfare, is justified in the following two, complementary aspects. First, most NOEM comparisons of exchange-rate regimes have been centered around welfare issues, whereas international trade has been covered only marginally. Yet it is important to also look at trade for a purely *theoretical* reason, namely because the underlying relative prices and the subsequent flows of goods predetermine – through microfounded consumption and leisure choices – the ultimate equilibrium allocations of these welfare ingredients, themselves sensitive to the specification of utility. Moreover, and as a consequence of not undertaking welfare analysis, we are able to allow for a rather general utility function, although separable in consumption and leisure. Second, and now from a *policy-oriented* perspective, much has been debated on the trade implications of a monetary union, i.e. a fixed exchange-rate regime at its extreme, within the context of a united Europe. In an attempt to extend NOEM research in a direction that throws more light on the theory that should underpin such important but perhaps thus far somewhat misleading public discussion, we consider it worthwhile to address analytically the present topic.

It turns out from our study that the effects of the exchange-rate regime on both expected trade shares and their variability ultimately depend on whether import demand is elastic or inelastic, once an international trade friction and distinct cross-country substitutability have been explicitly incorporated, like we do here, into the baseline NOEM set-up. In a preview of our principal findings, we could say that, first, with production of *similar* brands national trade shares in GDP drastically fall relative to the case of costless exchange. The reason is that obstacles to trade such as distance or tariffs induce a home bias in consumption, much stronger under CCP than under PCP, in the optimal behavior of agents with identical tastes. A major contribution is to show that this home bias is, however, considerably mitigated to more empirically relevant levels by each of two additional features of our model: (i) allowing for production of *different* output mixes, i.e. for inelastic import demand, under (even full) CCP; (ii) allowing for (even partial) PCP, which introduces expenditure switching to the cheaper nationally-specific good, determined by the particular realization of the nominal exchange rate under float and to the extent this is feasible given cross-country output substitutability. But the most important

[6] As in Corsetti-Pesenti (1997, 2001 a, b, 2002) extensions, under unit substitutability across national good-types, of the original Obstfeld-Rogoff framework.

result the paper derives is that, unlike in the frictionless-trade NOEM research, the exchange-rate regime affects under (some) PCP *expected* trade-to-output, in a way depending on the *interaction* of trade costs with the degree of substitutability between the nationally-produced composites: under *elastic* demand of similar products a peg slightly reduces expected trade-to-output relative to a float, in both economies we study and, hence, for the world as a whole; under demand which is *inelastic* because of complete specialization in two different but equally-valued good-types, a peg slightly increases expected trade-to-output relative to a float. Another new point from our analysis is that non-monetary factors such as transport or tariff frictions and the substitutability of output mixes also determine, via the optimally arising consumption bias, *both* the expected level and the variability of trade-to-GDP. As to the trade stabilization a peg can achieve under (some degree of) PCP, a contribution of the present work is to clarify that its extent would be greater for countries, or currency blocs, which produce less substitutable good-types for meaningful costs of exchanging them and are located closer to one another or apply weaker restrictions in their bilateral trade.

The paper is further down organized as follows. Section 2 outlines our extended stochastic NOEM model of exchange rate and trade determination and highlights the differences in its initial assumptions under CCP vs. PCP. The third section studies under symmetry and float how *trade costs* and distinct *type* and *brand* consumption substitutabilities affect international relative prices and, consequently, agents' optimization and the resulting equilibrium relationships across our alternative invoicing. Section 4 then focuses on the effects of the *nominal* exchange-rate regime on both the expected level and the variability of trade-to-output ratios, whereas the fifth section clarifies the role played by their *real* determinants. Section 6 concludes and appendices A and B contain, respectively, a detailed derivation of our key equilibrium expressions and the proofs of propositions.

2 The Extended Model

In this section, we first briefly outline the set-up in Mihailov (2003). We then explain how our two extensions here, the "transport" cost friction and the distinct cross-country substitutability, have been analytically integrated within it.

2.1 Our Baseline

General Environment We study a stochastic economy which exists in a single period[7] and is made up of two countries, *H(ome)* and *F(oreign)*, assumed

[7] Extension to sequential dynamics is straightforward: it will only violate ex-ante symmetry right after the first period and thus require recursive simulation. However, since the relevant measure of variables under uncertainty is their *expected* level, with which we are concerned here, simulating and summing over a sufficiently large number of *periods* will essentially replicate the analytically derived results over multiple *states* of nature we provide further down.

of equal size. A continuum of differentiated *brands*, each produced and sold by a monopolistically competitive firm, is available for consumption. Brands as well as their producers are indexed by i in H and i^* in F. Firms in Home are uniformly distributed on $[0,1]$ and those in Foreign on $(1,2]$. To have a role for the nominal exchange-rate regime, we assume sticky prices motivated by menu costs. Monopolistic competition enables each firm to optimally choose the price(s) at which it sells its product. Prices are set in advance, i.e. in our *ex-ante* state 0 (*before* uncertainty has been resolved), and remain valid for the *ex-post* state $s \in S$ (*after* money shocks have been observed).

Governments and Shocks In each country, there is a government whose only role is to proportionally transfer cash denominated in national currency to all domestic households in a random way.[8] We interpret such a money supply behavior as a flexible exchange-rate system and model it in terms of stochastic money-stock growth rates. Moreover, we restrict it to be *jointly symmetric*, in the following sense. For $\forall s \in S$, μ_s and μ_s^* are, respectively, H-money stock and F-money stock *net* rates of growth, having the same mean and variance. For the sake of symmetry, ex-ante (state 0) national money holdings of the representative households in Home and Foreign are assumed *identical* in terms of units of each country's currency:[9] $M_0 = M_0^*$. The ex-post (state s) cash balances, i.e. the domestic-currency budgets with which Home and Foreign households dispose for transactions purposes in the realized state of nature $s \in S$, are then respectively given by $M_s \equiv M_0 + \mu_s M_0 = (1+\mu_s) M_0$ and $M_s^* \equiv M_0^* + \mu_s^* M_0^* = (1+\mu_s^*) M_0^*$.

The only difference between float vs. peg in terms of the joint distribution (up to second moments, inclusive) of national money growth *shocks* (μ_s, μ_s^*) and, hence, of the resulting ex-post money *stocks* (M_s, M_s^*) thus arises from their *covariance* terms. It is imposed by the definition itself of a fixed vs. flexible exchange-rate regime: under (pure) *float*, the correlation of national money stocks is 0; under (credible) *peg*, this correlation is 1. In essence, our fixed exchange-rate version is thus isomorphic to a model where a monetary union or a single-currency area is hit by just one, common money shock.

Households In H and F, there is a continuum of households assumed *identical*. The population in each of these economies is supposed constant and is normalized to 1. The representative household (in H as well as in F) likes diversity and consumes *all* brands on the interval $[0,2]$. It also supplies labor, earning the equilibrium wage, and owns an equal proportion of domestic firms, receiving their profits (in the form of dividends).

The representative household in Home[10] maximizes utility:

[8] Seigniorage is then repaid in a lump-sum fashion at the end of the period, to make agents willing to hold money, as is standard in the finite-horizon literature.

[9] At an initial *equilibrium* exchange rate of 1, as will be discussed later.

[10] The notation in which the model is further on set out generally refers to Home, but for Foreign symmetric relationships hold (unless otherwise stated).

$$\underset{c_s, l_s}{Max} \quad u(c_s, l_s), \quad \forall s \in S. \tag{1}$$

Our utility function is assumed to be well-behaved (i.e. to exist, be continuous, twice differentiable and concave) and separable in its two arguments. l_s is (hours of) leisure and c_s is a constant elasticity of substitution (CES) real consumption index defined by the following Dixit-Stiglitz (1977) aggregator (interpretation of its parameters will be given soon):

$$c_s \equiv \left\{ \left(\frac{1}{2}\right)^{\frac{1}{\nu}} \left[\left(\int_0^1 c_{i,s}^{\frac{\varphi-1}{\varphi}} di \right)^{\frac{\varphi}{\varphi-1}} \right]^{\frac{\nu-1}{\nu}} + \left(\frac{1}{2}\right)^{\frac{1}{\nu}} \left[\left(\int_1^2 c_{i^*,s}^{\frac{\varphi-1}{\varphi}} di^* \right)^{\frac{\varphi}{\varphi-1}} \right]^{\frac{\nu-1}{\nu}} \right\}^{\frac{\nu}{\nu-1}}. \tag{2}$$

In this representative agent economy, the aggregate constraints on (per-) household behavior coincide with those of the identical households. They are standard in NOEM but, for completeness, we briefly present them below.

Time Endowment Constraint The endowment of hours to the representative household (in Home) is normalized to 1 in each state,

$$l_s + n_s \equiv 1, \quad \forall s \in S. \tag{3}$$

so that $n_s \equiv 1 - l_s$ is the (Home) household's labor (supply).

Cash-in-Advance (CiA) Constraint Households need to carry cash before going to the goods market. Moreover, we restrict them to hold and receive from their monetary authority only *domestic* currency. Thus (for Home)

$$\underbrace{c_s P_s}_{H \text{ national } expenditure \text{ (in } H \text{ currency)}} \leq \underbrace{M_s}_{\text{available } cash \text{ in } H \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{4}$$

National Money Market Equilibrium Since with concave utility (and no dynamics) CiA constraints are *binding* and there is no investment and government spending in the model, the nominal value of national output sold (for consumption) is equal to the total stock of money in each of the countries:

$$Y_s = M_s, \quad \forall s \in S. \tag{5}$$

Aggregate Budget Constraint = National Income Identity With a nominal wage rate of W_s and total hours of work amounting to $1-l_s$, the nominal labor income of the (Home) representative household is given by $W_s(1 - l_s)$. Nominal dividends from firm profits earned by this household are denoted by Π_s.

In equilibrium, all income from the activity of firms is distributed to domestic households (but this happens only at the end of the single period we consider):

$$\underbrace{\underbrace{W_s(1-l_s)}_{\text{labor income}} + \underbrace{\Pi_s}_{\text{ownership income}}}_{H \text{ national (factor) } income \text{ (in } H \text{ currency)}} \equiv \underbrace{Y_s}_{H \text{ national } output \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{6}$$

First-Order Conditions The following "compact" first-order condition can be derived in a familiar way from the above-described constrained optimization problem for the H representative household:

$$W_s = \frac{u_{l,s}}{u_{c,s}} P_s, \quad \forall s \in S. \tag{7}$$

$u_{l,s}$ and $u_{c,s}$ in (7) are the marginal utilities of leisure and consumption, respectively, in the realized state s. The *real* wage rate is thus equal, in equilibrium, to the ratio of these marginal utilities.

Firms Production is effected by firms which are owned by *domestic* households. We abstract from an international stock market, as well as of risk-sharing issues in general. To simplify this initial NOEM analysis of costly trade in similar vs. different output mixes within a unified framework, we focus here on *identical* technology in terms of labor input for producing a unit of output (although national endowments may differ) common to all firms in Home and Foreign. In H it is:

$$y_s = n_s = 1 - l_s, \quad \forall s \in S. \tag{8}$$

As the production function is identical across countries, international trade does not arise in the model from comparative advantage but from the equal preference to consume each of the national good-types. Although Ricardian trade theory is, certainly, important for an analysis like ours, it will complicate matters here and is left for future work. For the same reason, productivity shocks are abstracted away in the present paper as well.[11]

We now turn to the analytical integration of the trade friction and cross-country substitutability parameters into our baseline studied in Mihailov (2003) and summarized above.

2.2 Incorporating Iceberg Costs

Although heavily exploited in many NOEM models, the key *pricing-to-market* (PTM) assumption – which changes crucially their equilibrium outcomes – has not yet received an explicit and solid grounding within this line of literature. To

[11] Within the (New-)Keynesian modelling perspective of which we make use here this is not so unusual since output is anyway demand-determined.

rationalize *market segmentation* and the ensuing *possibility* for PTM behavior by monopolistically competitive firms, we introduce *symmetric* costs of international trade in goods, $\tau\,(\equiv \tau^*)$, in the set-up under CCP vs. PCP analyzed in Mihailov (2003). Following Obstfeld and Rogoff's (2001) NOEM application of ideas in the traditional literature,[12] we model them as being of the "iceberg" type, i.e. real losses in transit expressed in per cent of the quantity shipped: $0 \leq \tau < 1$. Although we model our τ parameter in a quite literal, "melting iceberg" fashion, we would nevertheless wish to interpret it in a much more general context, essentially capturing all kinds of frictions or impediments to international trade (or transaction costs, in a still broader sense). These may normally range from obstacles of a subjective (policy) nature such as tariff and non-tariff barriers to considerations of an objective (physical) character such as transport costs that are themselves a function of distance and transportation technology.

In both our CCP and PCP versions, the iceberg cost parameter $\tau \in [0, 1)$ enters the model via firms' production cost structure. Under this assumption a fixed fraction τ of each good shipped abroad "melts" in transit. Therefore firms have to also produce the *additional output* that is eventually lost when crossing the "ocean", given that there is demand corresponding to the remaining (i.e. "surviving") part of the output produced for export. A wedge of τ is consequently driven between output *produced* and output *consumed* in *real* terms. For a *real* (*Foreign*) import demand of $c^*_{i,s}$, a *Home* firm $i \in [0, 1]$ must ensure (and hence, produce) a *real* (*Home*) export supply of $\frac{c^*_{i,s}}{1-\tau}$.[13] A simple calculation shows why: a real quantity of $\frac{c^*_{i,s}}{1-\tau}$ is produced and shipped abroad from which only $c^*_{i,s}$ arrives and is consumed. The difference,

$$\frac{c^*_{i,s}}{1-\tau} - c^*_{i,s} = \tau \frac{c^*_{i,s}}{1-\tau}, \tag{9}$$

"melts" in transit, so *real* losses due to such a trade friction are a constant fraction τ of the amount shipped by the exporting producer.

2.3 Distinguishing Brand from Type Substitutability

The original NOEM set-up, e.g. Obstfeld-Rogoff (1995, 2000) or Bacchetta-van Wincoop (1998, 2000 a, b), was one of frictionless trade and a unique consumption substitutability. Subsequent contributions, such as Corsetti-Pesenti (1997, 2001 a, b, 2002), Obstfeld-Rogoff (1998), Galí-Monacelli (2002) and, notably,

[12] Exogenous real *"iceberg" costs* of international trade originate in the modelling approach common to the Ricardian comparative advantage trade and payments theory: to mention just the most prominent classic studies, in Samuelson (1952) and Samuelson (1954). Transport costs of that type are assumed too in the seminal paper by Dornbusch, Fisher and Samuelson (1977) and its NOEM interpretations in Obstfeld and Rogoff (1996: Chapter 4, Section 5, pp. 235-257) and Kraay and Ventura (2002). Trading frictions, not necessarily modelled as iceberg costs, have also recently been employed outside NOEM, by Martin and Rey (2000), Sercu and Uppal (2000), Parsley and Wei (2000) and Betts and T. Kehoe (2001), among others.

[13] The logic for a *Foreign* firm $i^* \in (1, 2]$ is, certainly, symmetric.

Tille (2000, 2002), have extended it to include a second parameter, determining cross-country output substitutability, but have assigned to it a *unitary* value. To allow for a richer setting, in the present paper we relax the latter restriction, although under symmetry. Our model thus involves two distinct substitutability parameters: $\varphi\,(\equiv \varphi^*) > 1$, the elasticity of substitution between any two nationally-produced *differentiated brands*,[14] and ν, with $0 \leq \nu \leq \varphi > 1$, the elasticity of substitution between the *composite good-types* across countries. The good-type is, in effect, the nationally-specific output mix, itself an aggregation of all domestically produced brands.

Such a substitutability decomposition proves to be a useful analytical device. It allows us to distinguish trade between countries producing the same, but *diversified* across brands, output type (under $\nu \equiv \varphi > 1$) from trade between countries *specializing* in only one of two different output types, each diversified across *national* brands (under $\nu < \varphi > 1$). In a more general sense or as a metaphor, we could refer to these alternative extremes as complete *diversification* of (world) production and complete *specialization* of (national) production, respectively. Our model thus conveniently nests two conceptually different types of international trade, namely the exchange of *similar* vs. *different* "output mixes". To our knowledge, they have not been explicitly compared within a coherent framework in the existing literature, with Tille (2002) providing a very recent exception. Although retaining the usual NOEM restriction of unit substitutability, Tille's (2002) analysis allows for even greater generality than our present study by introducing two sectors in each of the two countries and by varying sectors' relative size. Yet he does not explore how transport costs and non-unitary substitutability of national output composites influence trade prices and flows, which we do here.

In both our CCP and PCP versions, the substitutability parameters ν and φ enter the model via the symmetric *preference* structure embodied in (2). In general, we further down assume that $0 < \nu < \varphi > 1$. Such an assumption seems the appropriate one in our stylized context. The reason is that $\nu < \varphi$ implies that there is less substitutability across the aggregate national outputs of the two countries than between any two differentiated brands produced in each of these countries, because of naturally (geographically) predetermined complete national *specialization* in production. Consumption substitutability is thus *lower across types than across brands* in the unified international trade framework we study.[15] Moreover unlike φ, ν is not restricted to the elastic region of its domain, a feature that is related to some lasting debates in the empirical trade and development literature[16] and that has important theoretical

[14] Following NOEM modelling tradition, φ is assumed to be *larger* than 1. The reason is that otherwise the marginal revenue of firms will be negative (see, for instance, Obstfeld and Rogoff (1996), Chapter 10, footnote 2, p. 661).

[15] In the special case of $\nu \equiv \varphi > 1$, our two elasticity parameters coincide so that the set-up reduces to world production of the same homogeneous good-type.

[16] A number of studies have argued that world demand for many products, in particular primary commodities, is income- and price-inelastic. This has also been advanced as a major explanation behind the secular decline in the terms of trade of such goods. Todaro and Smith (2002), p. 522, for instance, refer to World Bank (1994), Table 2.5, to claim that the elasticity

implications in our further analysis.

3 Costly Trade under CCP vs. PCP

In this section, we compare across our *invoicing-specific* model versions and under *float* and *symmetry* the optimization problems agents solve and the resulting equilibrium. In particular, the outcomes for the exchange-rate level, international relative prices, cross-country consumption and leisure allocations and, ultimately, some key measures of trade flows are derived and interpreted.

3.1 Optimization and Equilibrium

Invoicing-Specific Notation The two major invoicing practices in the open economy whose implications we highlight next have imposed a specific notation, which we now summarize.[17] All our quantity variables are denoted by lowercase Latin letters. These quantities can be indexed by up to two subscripts and up to two superscripts. A first *sub*script H or F indicates the origin of the respective variable at the national-economy level, i.e. the country where a particular good i or i^* (first *sub*scripts again but at the individual-firm level) has been produced. Following the tradition, we use an asterisk ($*$) as a first *super*script to denote that a particular quantity variable has been consumed in Foreign. The second *sub*script, 0 for ex-ante quantities and s for ex-post quantities, indexes the state of nature whereas the second *super*script, C (for CCP) or P (for PCP), indicates the assumed price setting. The same notational rules apply to the (money) prices or nominal variables that correspond to all respective quantities in our model, the only difference being that these are denoted by uppercase Latin letters. Greek letters, in turn, designate model parameters and shocks.

Consumption Demands and Price Levels Standard cost minimization à la Dixit-Stiglitz (1977) of (2) defines the optimal demands of the Home representative household for H- (equations (10) below) and F-produced ((11) below) goods and the respective Home price indices at the domestic absorption (equations (12)), import demand (13) and consumer (14) levels for the CCP vs. PCP model versions as follows:

$$c_{H,s}^{C} = \frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C} \quad \text{vs.} \quad c_{H,s}^{P} = \frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}; \tag{10}$$

of demand for foodstuffs with respect to income changes in developed countries is 0.6% and of agricultural raw materials such as rubber and vegetable oils 0.5%.

[17] Since we do not explicitly distinguish an intermediary import/export sector in the two-country economy we study, as Tille (2000) has first done within NOEM, CCP and PCP are equivalent here to, respectively, *importer*'s (buyer's) and *exporter*'s (seller's) currency pricing.

$$c_{F,s}^C = \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C} \quad \text{vs.} \quad c_{F,s}^P = \frac{1}{2}\left(\frac{\overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\equiv P_{F,s}^P}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}; \tag{11}$$

with

$$P_H^C \equiv \left[\int_0^1 \left(P_i^C\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad P_H^P \equiv \left[\int_0^1 \left(P_i^P\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}; \tag{12}$$

$$P_F^C \equiv \left[\int_1^2 \left(P_{i^*}^C\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad \underbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}_{\equiv P_{F,s}^P} \equiv \left[\int_1^2 \underbrace{\left(\frac{S_s^P P_{i^*}^{*,P}}{1-\tau}\right)}_{\equiv P_{i^*,s}^P}^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}; \tag{13}$$

$$\begin{aligned} P^C &\equiv \left[\frac{1}{2}\left(P_H^C\right)^{1-\nu} + \frac{1}{2}\left(P_F^C\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \quad \text{vs.} \\ P_s^P &\equiv \left[\frac{1}{2}\left(P_H^P\right)^{1-\nu} + \frac{1}{2}\underbrace{\left(\frac{S_s^P P_F^{*,P}}{1-\tau}\right)}_{\equiv P_{F,s}^P}^{1-\nu}\right]^{\frac{1}{1-\nu}}. \end{aligned} \tag{14}$$

Output Prices The expected market value of real profits which a H firm $i \in [0,1]$ maximizes under CCP vs. PCP is defined by:

$$\underset{P_i^C, P_i^{*,C}}{Max} E_0 \left[\frac{u_{c,s}}{P^C} \underbrace{\left(P_i^C c_{i,s}^C + S_s^C P_i^{*,C} c_{i,s}^{*,C} - W_s^C c_{i,s}^C - \frac{W_s^C c_{i,s}^{*,C}}{1-\tau}\right)}_{\equiv \Pi_{i,s}^C}\right], s \in S \tag{15}$$

$$\text{vs. } \underset{P_i^P}{Max} E_0 \left[\frac{u_{c,s}}{P_s^P} \underbrace{\left(P_i^P c_{i,s}^P + \frac{P_i^P c_{i,s}^{*,P}}{1-\tau} - W_s^P c_{i,s}^P - \frac{W_s^P c_{i,s}^{*,P}}{1-\tau} \right)}_{\equiv \Pi_{i,s}^P} \right], s \in S. \quad (16)$$

Using the first order conditions of the two problems, the CCP vs. PCP optimal prices preset by the Home firm i, which is also the representative Home firm, and relevant for consumer households in the domestic and foreign market are thus, respectively:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s \right]}{E_0 \left[u_{c,s} M_s \right]} \text{ vs.} \quad (17)$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[\frac{u_{c,s}}{P_s} W_s \frac{M_s}{P_s^{1-\nu}} \right] + (1-\tau)^{\nu-1} E_0 \left[\frac{u_{c,s}}{P_s} W_s \frac{S_s M_s^*}{(S_s P_s^*)^{1-\nu}} \right]}{E_0 \left[\frac{u_{c,s}}{P_s} \frac{M_s}{P_s^{1-\nu}} \right] + (1-\tau)^{\nu-1} E_0 \left[\frac{u_{c,s}}{P_s} \frac{S_s M_s^*}{(S_s P_s^*)^{1-\nu}} \right]}; \quad (18)$$

$$P_i^{*,C} = P_H^{*,C} = \frac{1}{1-\tau} \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s^* \right]}{E_0 \left[u_{c,s} S_s^C M_s^* \right]} \text{ vs.} \quad (19)$$

$$P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P (1-\tau)} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \left[\frac{1 + \left(\frac{1}{1-\tau} \frac{1}{S_s^P} \right)^{1-\nu}}{1 + \left(\frac{1}{1-\tau} S_s^P \right)^{1-\nu}} \right]^{\frac{1}{1-\nu}} P_s^P.}_{\text{PPP-}reminiscent \text{ equation}} \quad (20)$$

As it would be in a NOEM baseline with $\tau = 0$ and $\nu \equiv \varphi > 1$, the exchange-rate pass-through to import prices – i.e. the elasticity of the import price index, $P_{H,s}^*$ above for Foreign (and, symmetrically, $P_{F,s}$ for Home), with respect to the nominal exchange rate,[18] S_s – is operating under PCP but not under CCP.[19] That is why import price indexes and, hence, CPIs are constant under CCP (P^C and $P^{*,C}$) but state-dependent under PCP (P_s^P and $P_s^{*,P}$). With transport cost and distinct cross-country substitutability incorporated in the present extended model, equations (11) show that, irrespective of the invoicing assumption, (Home) import demand now optimally depends on τ as well, via the prices P_F^C and $P_{F,s}^P$.[20] Moreover, optimal consumer demands (10) and (11)

[18] Defined in the usual way as the *Home*-currency price of *Foreign* money.

[19] To see it, compare the invoicing-specific equations in (13), bearing in mind the optimally preset prices in Foreign, symmetric to those in Home as given in (19) under CCP and (20) under PCP.

[20] Observe that the CCP export market price for Foreign, P_F^C, is optimally preannounced at a level symmetric to expression (19) for the analogous price for Home, $P_H^{*,C}$.

reveal that it is now ν that matters for cross-country substitution, although φ is still important in the determination of CPIs.[21]

The cost of international exchange, τ, is thus ultimately passed on to consumers, via the effective consumer-relevant price, but in a different way under the alternative invoicing conventions we study. Under *CCP* it is passed on to importing foreign consumer-households via the price charged directly in *foreign* currency. The exchange-rate risk is nevertheless borne by domestic producing firms, because of their preset export-market prices. Under *PCP* the trade cost is passed on to importing foreign consumer-households too, but now the mechanism is not the same. It consists in buying, at the price charged in the *seller's* currency, the equivalent – including the output to be lost in transit – of the quantity of imports optimally demanded. The buyer then loses τ% of the shipped quantity, so that he effectively consumes less in *real* terms than the amount paid for.[22]

Equilibrium The constrained optimization problems agents solve and the market clearing conditions for the world economy, given the invoicing and timing assumptions of our stochastic NOEM framework, lead to an equilibrium concept consistent with the described environment. Its formal definition is given in Appendix A.1, while the equilibrium solutions for the macrovariables we are interested in are interpreted in the following subsections.

3.2 Equilibrium Nominal Exchange Rate

The equilibrium nominal exchange rate (NER) solves the international *forex market clearing* condition in any state of nature $s \in S$. Given the *full symmetry* we assumed, i.e. with $P_H^C = P_F^{*,C}$, $P_F^C = P_H^{*,C}$, $P^C = P^{*,C}$ under CCP vs. $P_H^P = P_F^{*,P}$, $P_{F,s}^P \equiv \frac{S_s^P P_F^{*,P}}{1-\tau}$, $P_{H,s}^{*,P} \equiv \frac{P_H^P}{(1-\tau)S_s^P}$, $P_s^P = \left[\frac{1+\left(\frac{1}{1-\tau} S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}} \right]^{\frac{1}{1-\nu}} P_s^{*,P}$ under PCP[23] it can be derived to be[24]

$$S_s^C = \frac{M_s}{M_s^*} \text{ vs. } S_s^P = \left[\frac{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}} \right]^{\frac{1}{\nu}} \left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\nu}}. \tag{21}$$

The exchange rate expression in (21) under CCP, S_s^C, is exactly the same as in the frictionless benchmark of Bacchetta and van Wincoop (2000 a), so under full symmetry neither transport cost nor distinct substitutability considerations affect CCP NER determination in equilibrium. The reason is that import prices, relevant to consumers, are preset independently from the ex-post NER at the

[21] Which becomes clear from the price level formulas (12) through (14) above.

[22] An alternative interpretation could be that importing households pay a *higher* "true" price for the consumed quantity, because they also buy the quantity lost in transit and thus not consumed.

[23] With also $\nu < \varphi \equiv \varphi^* > 1$ and $0 < \tau \equiv \tau^* < 1$ in both model versions.

[24] See Appendix A.2.

same level relative to domestic prices in Home and in Foreign. However, the equilibrium PCP exchange rate, S_s^P, is now given by a more complicated (implicit) function reflecting the role of τ and ν in household decisions on how to split-up their state s budget across the national good-types. Our brief explanation here will soon become clearer.

With a fixed exchange-rate regime, i.e. when $M_s \equiv M_s^*$ for $\forall s \in S$, the CCP NER obviously becomes 1. The PCP NER expression in (21) also reduces to a unique solution of 1 once S_s^P is restricted to be a positive real number (as it should be for an exchange rate), yet this is not directly evident from the formula above.[25] So in the present context with an iceberg friction and two distinct substitutabilities a *peg* implies again that – under CCP as well as under PCP and ex-post as well ex-ante – the exchange rate can be substituted by 1 in all expressions which contain it. We shall exploit this finding further on, in discussing the effects of a fixed exchange-rate regime on trade prices and flows.

Optimal Firm Prices under Full Symmetry Using (7) and its equivalent for Foreign as well as (21) to substitute for the endogenous variables W_s, W_s^* and S_s in (17) through (20), the optimal firm prices derived earlier can now be fully determined under CCP and (via the implicit function giving the equilibrium NER) PCP. The final model solutions for prices in terms of exogenous variables and parameters are thus, respectively:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} P^C \frac{E_0\left[u_{l,s} M_s\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0\left[u_{l,s}\dfrac{M_s}{P_s^{1-\nu}}\right] + (1-\tau)^{\nu-1} E_0\left[u_{l,s}\dfrac{S_s M_s^*}{\left(S_s P_s^*\right)^{1-\nu}}\right]}{E_0\left[\dfrac{u_{c,s}}{P_s}\dfrac{M_s}{P_s^{1-\nu}}\right] + (1-\tau)^{\nu-1} E_0\left[\dfrac{u_{c,s}}{P_s}\dfrac{S_s M_s^*}{\left(S_s P_s^*\right)^{1-\nu}}\right]};$$

$$P_i^{*,C} = P_H^{*,C} = \frac{1}{1-\tau}\frac{\varphi}{\varphi - 1} P^{*,C} \frac{E_0\left[u_{l,s} M_s^*\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P\,(1-\tau)} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \left[\frac{1 + \left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}{1 + \left(\frac{1}{1-\tau} S_s^P\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} P_s^P}_{\text{PPP-}\textit{reminiscent}\text{ equation}}.$$

It is easily seen that under *CCP* the prices set by the Home representative firm domestically, P_H^C, and abroad, $P_H^{*,C}$, will generally not be the same with now nonzero iceberg costs $0 < \tau < 1$ even if $E_0\left[u_{l,s} M_s\right] = E_0\left[u_{l,s} M_s^*\right]$ is true,

[25] See Appendix A.2.

as it is under *separable* utility in consumption and leisure.[26] With $P_H^P = P_F^{*,P}$ due to symmetry, the respective ex-post *PCP* prices in the foreign currency, $P_{H,s}^{*,P}$ and $P_{F,s}^P$, will in general not be equal to those preset domestically either, as obvious from the last equation above. Observe as well that in the presence of iceberg costs ($0 < \tau < 1$) a *peg* will *never* guarantee that the relevant (ex-post) prices of home and foreign goods agents in both symmetric countries face under CCP as well as under PCP are the same, i.e. that $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$ and $P_H^P = P_H^{*,P} = P_F^P = P_F^{*,P}$. This is a result very different from – in a sense, opposite to – what one would obtain in the usual frictionless NOEM set-up.

We are now ready to derive – under full symmetry and separable utility – expressions for some traditional characteristics of international trade, which we interpret below.

3.3 Equilibrium Relative Prices

Let us begin by comparing across our invoicing conventions the most important pair of international relative prices. This analysis will help us later in understanding the channel along which optimal consumption – and, hence, trade – flows are determined in the extended NOEM framework with iceberg costs and distinct cross-country substitutability we are analyzing here.

With costly trade under (full) CCP, the relative price of foreign-produced goods in terms of domestically-produced ones in both countries is still *preannounced* at the *same* level, as it would be in the frictionless case of $\tau = 0$. But instead of being 1, its level is now $\frac{1}{1-\tau}$, once a symmetric iceberg friction has also been introduced. In such a way, no matter whether trade is costless or costly, any effects of the ex-post values of this key international relative price on consumer behavior are *precluded* under CCP:

$$p_H^C \equiv \frac{P_F^C}{P_H^C} = \frac{1}{1-\tau} = \frac{P_H^{*,C}}{P_F^{*,C}} \equiv p_F^{*,C} \text{ for } \forall s \in S. \tag{22}$$

Under PCP, the relative price of foreign-produced goods in terms of domestic ones is generally *not reciprocal* across countries anymore, as it would be without trade frictions, just because of the *nonzero* iceberg costs ($\tau \neq 0$):

$$p_{H,s}^P \equiv \frac{\overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\equiv P_{F,s}^P}}{P_H^P} = \frac{S_s^P}{1-\tau} \neq \frac{1}{S_s^P(1-\tau)} = \frac{\overbrace{\frac{P_H^P}{S_s^P(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_F^{*,P}} \equiv p_{F,s}^{*,P} \text{ unless } S_s^P = 1. \tag{23}$$

A depreciation under PCP is therefore passed on, no matter whether trade is costless or costly, to the relative price of foreign in terms of domestic brands. This pass-through induces, in turn, *expenditure switching*, an international spillover channel largely debated in the Mundell-Fleming-Dornbusch tradition and

[26] As formally shown in Bacchetta and van Wincoop (2000 a), Lemma 1 and Proposition 1.

crucial in understanding the different implications of PCP vs. CCP on equilibrium consumption allocations across national outputs. The transmission mechanism of money shocks abroad uncovered in (23) plays an important role in the interpretation of the principal contribution of the present paper, as we shall explain later.

3.4 Equilibrium Consumption and Leisure across Countries

Having looked at CCP vs. PCP international relative prices in equilibrium, we now turn to the corresponding cross-country real allocations. Our main results are summarized below in logical order. Their proofs are straightforward, based largely on earlier definitions and derivations, and are not included.

Relative Consumption Dividing the invoicing-specific equilibrium consumption expressions, c_s and c_s^*, one finds that under costly trade *relative* real consumption is ultimately determined by the *relative* money stock, as it was in our baseline without trade frictions. But under *PCP* and not CCP, *trade costs* and *import demand elasticity* influence as well the equilibrium allocation across countries of the quantities consumed. An important implication of the extended set-up is that it is now a *richer* parameter set, (τ, ν) compared to only φ earlier, which pins down relative consumption under PCP in any state of nature $s \in S$. In particular, the elasticity of consumption demand relevant to imports is the *cross-country* one, ν with $0 \leq \nu < \varphi > 1$, and not the substitutability across the homogeneous product brands, $\varphi > 1$. What is novel here, as already mentioned, is that ν is defined over a larger domain, including in addition the region of import demand *in*elasticity as well as the case of *unit* elasticity. This finding has insightful consequences for our analysis to which we shall return in more detail later.

Home Bias and Consumption Switching Dividing now our invoicing-specific equilibrium expressions for $c_{H,s}$ and $c_{F,s}$, we arrive at a result with prime importance for our study. With *positive* (symmetric) trade costs and *non-zero* cross-country output substitutability, the optimal split-up of real consumption between demand for domestic ($c_{H,s}$, for Home) and foreign ($c_{F,s}$, for Home) goods always results under CCP in a (symmetric) *home bias* in both countries, $(1-\tau)^{-\nu} > 1$, *invariant* across states of nature. Under PCP, by contrast, this split-up is ultimately determined by the *equilibrium nominal exchange rate*, through the induced optimal consumption switching in any state that has materialized, $\left(S_s^P\right)^{\nu}(1-\tau)^{-\nu}$ for Home and $\left(S_s^P\right)^{-\nu}(1-\tau)^{-\nu}$ for Foreign.

Now with iceberg costs, *import substitution* is generally *optimal* not only under PCP when there is exchange rate pass-through but also – and much more – under CCP when there isn't. Consequently, under CCP with costly trade in *similar* output mixes and even full symmetry, there will always be a *home* bias (identical for the two countries), unless (i) $\tau = 0$ or (ii) $\nu = 0$. This CCP home bias under *elastic* import demand and *unique* consumption substitutability, across brands of a *homogeneous* good-type, $(1-\tau)^{-\varphi}$ is a positive function of τ and $\varphi(\equiv \nu) > 1$. Due to the trading friction, $\tau \neq 0$, foreign-produced goods become more expensive, hence less demanded, than their nationally-produced (close) substitutes. These conclusions are also valid, to a lesser extent, under costly trade in *different* composite outputs (when $\nu < \varphi > 1$) with CCP, but not with PCP. In that latter case, the consumption bias is not necessarily also a home bias for both countries in all states of nature.

Evidence for a home bias in goods consumption has often been found in applied work, and is thus empirically relevant. The theoretical reasons proposed to explain it have usually been associated with either transaction costs or structural or informational asymmetries. The NOEM literature has only recently started to integrate such a feature into its mainstream set-up. Warnock (2003), for example, imposes it via *heterogeneous preferences* of households. In our analytical framework here the home bias originates in the optimal behavior of economic agents when facing a trade friction, as Obstfeld and Rogoff (2001) first did within NOEM (under unit cross-country substitutability).

Relative Leisure It follows from our two results highlighted above that under costly trade equilibrium output, employment and leisure are not generally equal across nations, no matter the price-setting convention. The intuition is that, since output is demand-determined up to *exhausting* the CiA constraint in any state of nature and technologies are assumed *identical*, the two countries do not generally produce the same quantities and do not employ the same labor in equilibrium. Hence, the hours of leisure the representative households residually enjoy in Home and in Foreign are in general not the same either.

3.5 Equilibrium Trade Flows

In this subsection we interpret *equilibrium trade-to-output*, derived under our alternative invoicing assumptions in Appendix A.3.

Trade Shares by Country It is shown in the mentioned appendix that the trade share for Home under CCP vs. PCP is given by

$$(ft)_H^C = \frac{2}{(1-\tau)^{1-\nu}+1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1 \text{ vs.} \tag{24}$$

$$(ft)_{H,s}^P = \frac{2}{(1-\tau)^{1-\nu}\underbrace{\left[\left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}\right]^{\nu-1}}_{=S_s^P}+1} \neq const \text{ unless } S_s^P = 1. \tag{25}$$

The corresponding expressions for Foreign are, of course, symmetric:

$$(ft)_F^C = \frac{2}{(1-\tau)^{1-\nu}+1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1 \text{ vs.} \tag{26}$$

$$(ft)_{F,s}^P = \frac{2}{(1-\tau)^{1-\nu} \underbrace{\left[\left(\frac{M_s^*}{M_s} \right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}} \right)^{\frac{1-\nu}{1-2\nu}} \right]}_{=S_s^P}^{1-\nu} + 1} \neq const \text{ unless } S_s^P = 1. \tag{27}$$

The above equations compare directly the impact of our alternative price-setting assumptions on the ratio of nominal trade to nominal output.

Under CCP, (24) and (26) show that the equilibrium trade share is a state-*invariant* function (implicitly) of the relative price of domestic goods in terms of foreign ones, $1-\tau$ (cf. (22)) and the home bias, $(1-\tau)^{-\nu}$, and (explicitly) of their deeper "fundamentals", τ and ν. Moreover, CCP trade-to-output is the *same* for the two countries in any state of nature that has materialized. However, with positive iceberg costs it is not 1 anymore, as in the frictionless baseline, unless cross-country output substitutability is *unitary*. For *elastic* import demand, $\nu > 1$, CCP trade-to-output ratios are smaller than 1 in both economies, due to costly trade and high substitutability. If import demand is instead *inelastic*, $\nu < 1$, CCP trade shares are larger than 1, because of the identical taste for both goods which are now, under national specialization of production, practically not substitutable no matter the costs of their exchange.

Under PCP, by contrast, equilibrium trade-to-output ratios by country are *not* state-invariant, as clear from (25) and (27). Trade-to-output can be 1 in both economies only with *unitary* import substitutability, similarly to the CCP case. Otherwise, PCP trade shares are *decreasing* in the *own* NER under elastic import demand but *increasing* in the *own* NER under demand inelasticity.

This analysis helps highlighting the role played by each of our three key ingredients of the NOEM model developed here, namely the NER, trade costs and cross-country substitutability. In addition to the home bias originating in costly trade under CCP, $(1-\tau)^{-\nu}$, it is the equilibrium exchange rate, S_s^P, that induces under PCP and float – by means of its *pass-through* on import and, ultimately, consumer relative prices, $\frac{S_s^P}{1-\tau}$ for Home and $\frac{1}{S_s^P(1-\tau)}$ for Foreign – the optimally arising *expenditure switching* , $\left(S_s^P\right)^{\nu}(1-\tau)^{-\nu}$ for Home and $\left(S_s^P\right)^{-\nu}(1-\tau)^{-\nu}$ for Foreign, away from the home bias allocation under CCP: toward domestic products in the country having experienced currency depreciation (or relative monetary expansion) and toward foreign products in the country with appreciating currency (or relative monetary contraction). But just because of τ (for any given ν), demand for domestic goods in the country of depreciation is now *stronger* than that for foreign goods in the country of appreciation, which would not be the case in a frictionless model. Finally, ν (for any given τ) determines *how strong* (or rather how feasible) this cross-country output substitution is, in any state of nature that has materialized.

World Trade Share We now state an important result from our analysis as a first proposition. A proof is provided in Appendix B.

Proposition 1 *(Equilibrium World Trade-to-Output) With iceberg costs and flexible exchange rates, equilibrium trade-to-output for the world economy as a whole is constant under CCP; but state-dependent under* PCP*, and* always lower *than its peg level when import demand is* inelastic*:*

$$(ft)^P_{W,s} = \frac{1}{1+(1-\tau)^{1-\nu}\left(S^P_s\right)^{1-\nu}} + \frac{\left(S^P_s\right)^{1-\nu}}{(1-\tau)^{1-\nu}+\left(S^P_s\right)^{1-\nu}}, \forall s \in S \neq M_s = M^*_s.$$

The formula in Proposition 1 makes it obvious that under PCP and float the world trade/GDP ratio, $(ft)^P_{W,s}$, is a function of the NER, S^P_s, in any state of nature, whereas it would be state-invariant at 1 in the frictionless case. Note also that a constant PCP world trade share, $\frac{2}{(1-\tau)^{1-\nu}+1}$ as under costly trade with CCP or peg, obtains only under a *fixed* exchange-rate regime, i.e. $S^P_s = 1, \forall s$. In two other, analytically important cases, much exploited in the NOEM literature, the above constant further reduces to 1: (i) *frictionless* trade, i.e. $\tau = 0$, and (ii) *unit* cross-country output substitutability, i.e. $\nu = 1$.

From the formula above it becomes clear too that under *inelastic* import demand, i.e. for $0 < \nu < 1$, with PCP and float the world trade share in output integrated over the symmetric distribution of monetary shocks is always lower than 1 and will therefore be *less* than the same expected trade measure under peg, itself always higher than 1. In other words, a peg would increase expected trade relative to float whenever demand is inelastic. An analogous conclusion for the elastic case is, however, not evident from the formula in Proposition 1. We return to the issue of whether and how the exchange-rate regime matters for *expected* trade in Proposition 2.

4 Does the Exchange-Rate Regime Matter for Trade?

Making further use of the CCP vs. PCP equilibrium solutions under a symmetric iceberg friction and two distinct substitutabilities affecting consumption demand we characterized thus far, the present section focuses on the implications of the alternative *monetary* arrangements we study for international trade prices and flows. We analyze both the *expected* level of trade-to-output ratios (by country and for the world economy as a whole), the relevant measure of trade under *uncertainty*, and their *variability* across states of nature.

4.1 When Does a Peg Increase Trade-to-Output?

Trade-to-Output under CCP (with Float or Peg) Under (full) *CCP* with *float*, we derived national trade shares to be independent of the nominal

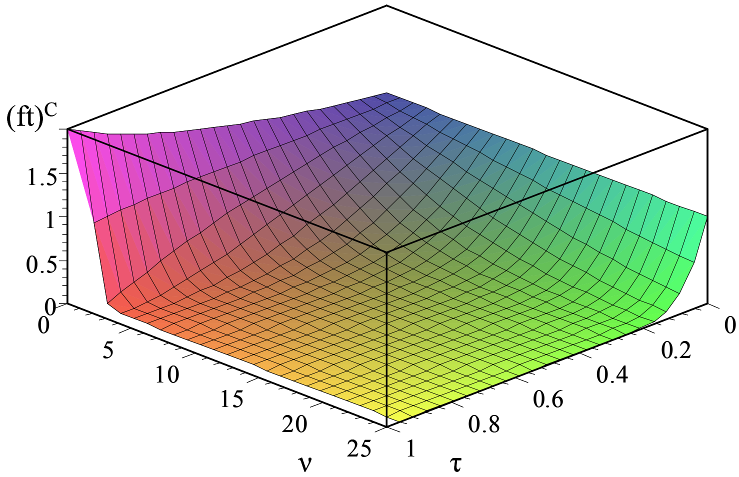

Figure 1: Peg Trade Share Surface across Iceberg Costs and Substitutabilities

exchange rate and, ultimately, of relative money stocks. CCP equilibrium trade-to-output ratios are thus invariant across states of nature and coincide with their expected level. A *peg* under (full) *CCP* will therefore not change anything directly related to trade shares or their volatility.

Mind however that, under float, CCP by itself does not generally imply equal equilibrium consumption, hence leisure and utility across countries. This will be the case only in the much less probable states of monetary shocks having the same magnitude. A peg under (full) CCP, by equalizing equilibrium cross-country utility, will bring about this additional effect in all states of the world.

Trade-to-Output under PCP with Float By contrast, a *peg* under (some degree of) *PCP* will equalize the *ex-post* Home and Foreign trade shares, thus leading to a result that is essentially the same – concerning trade only, not consumption and leisure – as the one implied by (full) CCP under float. The peg trade share function in Figure 1 summarizes across trade costs and cross-country output substitutabilities the *identical* under CCP (with either float or peg) or peg (with either CCP or PCP) Home and Foreign *equilibrium* trade/GDP ratios. It will be interpreted in more detail along the dimensions of its two arguments in section 5.

Our principal result in this paper is stated in the following proposition.

Proposition 2 *(Expected Trade-to-Output under PCP) Under (some degree of)* PCP*, a peg reduces expected trade-to-output if import demand is* elastic *but increases it if import demand is* inelastic*.*

A proof is given in Appendix B. In interpreting Proposition 2 we first note that our trade measure is a *ratio*, not the value or volume of trade in general, as in many studies usually claiming that a peg would increase trade. Second, we stress that in a *frictionless*, *unique-substitutability* setting with trade of highly similar brands of a homogeneous good-type imposing *elastic* demands for imports, the expected trade share would be the same no matter the price setting and the exchange-rate regime, as shown in Mihailov (2003). The introduction here of more realistic features – such as costs of trade and a distinct substitutability between good-types lower than that among brands (within each type) which is, furthermore, not restricted outside the inelastic zone – has thus helped improve our understanding on the effects of the exchange-rate regime on trade, measured in terms of output. Finally, the presence of any one of our two real trade "fundamentals" *alone* in the extended NOEM model we developed is not sufficient to produce the reversal effect of interest here: it is precisely the *interaction* of ν and τ that drives our main result.[27] The combination of the wedge driven between the cost of the domestic vs. the foreign good-type, intervening in decisions on import substitution, and the particular magnitude of the substitutability between these good-types, embodying the wish to trade and inducing, in consequence, a certain level of the equilibrium PCP NER in each state, thus ultimately matters in explaining when a peg would increase trade and when a float would do it instead. Without the richer setting of the present set-up, this channel of interaction could not have been uncovered, and in that consists our principal import to the related NOEM literature.

The intuition for the result in Proposition 2 we would provide is the following.

Consider first a *fixed* exchange-rate regime, $S_s = 1, \forall s$. CCP vs. PCP is then irrelevant for trade: the nominal exchange rate is constant so the pass-through and expenditure-switching channel is inoperative. Trade-to-output in each of the countries and, hence, for the world as a whole would be state-invariant, at $\frac{2}{(1-\tau)^{1-\nu}+1}$. Take as a first benchmark the case of *zero* cross-country output substitutability, $\nu = 0$: the home bias, otherwise identical for both trading nations at $(1-\tau)^{-\nu}$, now vanishes, so equilibrium as well as expected trade-to-GDP equals $\frac{2}{2-\tau} > 1$: the higher the trade cost, the higher the trade share, because countries are "doomed" to expensive trade by their identical preferences but differing endowments. As a second benchmark, consider *unitary* import demand elasticity, $\nu = 1$: there is now a bias in favour of the domestic product consumption, $\frac{1}{1-\tau} > 1$, but the price of the foreign good consumer-households are facing is $1-\tau$ times higher than that of the home good so the *values* of imports and domestic absorption in each country match exactly and, hence, expected trade-to-output becomes 1, i.e. lower than when ν is zero. Now let us turn to the two more general cases of interest in the present paper. In the *elastic* zone, when $\nu > 1$, there is a home bias, $\frac{1}{(1-\tau)^{\nu}} > 1$, weaker than with unit elasticity but increasing in ν (and τ), which is intuitive; expected trade is thus lower than 1, the more so the higher ν (and τ); at the extreme of $\nu \to \infty$, the

[27] As clear from the changing sign of the second term of $F''(1)$ in the proof of Proposition 2 in Appendix B, the first one being always positive.

home bias becomes so huge that trade vanishes completely. Conversely, in the *inelastic* zone, when $0 < \nu < 1$, there is again a home bias, $\frac{1}{(1-\tau)^{\nu}} > 1$, increasing in ν (and τ) again but much weaker than with elastic import demand, which is intuitive too; yet now import substitution by consuming domestic products is much less possible and the (cif) value of expected trade-to-GDP is ultimately higher than 1, due to expensive imports; at the extreme of $\nu \to 0$ the home bias is forced to a negligible magnitude so that the expected trade share is further "inflated" and approaches $\frac{2}{2-\tau} > 1$.

In the preceding paragraph, we analyzed four interesting cases. But all of them assumed a fixed exchange-rate regime. Our argumentation has not, as yet, taken into account the effect of *exchange-rate variability*.[28] We now incorporate in the interpretation of our principal result, Proposition 2, the role played by a float. In what a float differs from a peg regime, as far as trade-to-GDP is concerned, is the volatility of the latter ratio across countries (in any state of nature of differing money shocks, $\mu_s \neq \mu_s^* \Rightarrow M_s \neq M_s^* \Rightarrow S_s^P \neq 1$).

Now, with $\tau \neq 0$, the symmetry – or rather reciprocity – on which the NER regime irrelevance for expected trade rested in the frictionless baseline of $\tau = 0$ in Mihailov (2003) is gone. Given float and PCP, this is obvious from comparing the equilibrium world trade expressions under costless (with $\nu \equiv \varphi > 1$) vs. costly trade, respectively:

$$\frac{1}{2}\left[\frac{2}{\left(S_s^P\right)^{\varphi-1}+1}+\frac{2}{\left(\frac{1}{S_s^P}\right)^{\varphi-1}+1}\right]=1, \forall S_s^P,$$

but

$$\frac{1}{2}\left[\frac{2}{\left(\frac{1-\tau}{S_s^P}\right)^{1-\nu}+1}+\frac{2}{\left[(1-\tau)\,S_s^P\right]^{1-\nu}+1}\right]\neq\frac{2}{(1-\tau)^{1-\nu}+1}\neq 1, \forall S_s^P \neq 1.$$

More precisely, the intuition behind Proposition 2 is related to the definition of elastic and inelastic demand and to key results we derived earlier. Under *elastic* import demand and float with PCP, any change in the relative price of domestic to foreign goods, $\frac{1-\tau}{S_s^P}$ for Home (see (23)), following the realization of the equilibrium NER, induces an even *larger* change in the relative quantity of cross-country output demanded, i.e. in consumption switching , $\left(S_s^P\right)^{\nu}(1-\tau)^{-\nu} = \left(\frac{1-\tau}{S_s^P}\right)^{-\nu}$ for Home. The product then of the relative price and quantity under float, $\left(\frac{1-\tau}{S_s^P}\right)^{1-\nu}$, is higher, on average, than that under peg, $(1-\tau)^{1-\nu} > 1$, when $S_s^P = 1$ and there is no consumption

[28] Recall that a peg is consistent with *monetary uncertainty*, i.e. it is equivalent to (or, rather, a special case of) a common monetary shock hitting the world economy across states of nature, as noted in the beginning.

switching. That is why trade-to-output is ultimately lower under peg than under float with elastic cross-country substitutability and (some degree of) PCP. The case of *inelastic* demand reverses our interpretation above, in that now $\left(\frac{1-\tau}{S_s^P}\right)^{1-\nu} < (1-\tau)^{1-\nu} < 1$, on average, so a peg would increase expected trade-to-GDP relative to a float. What drives the result is the intensity of consumption switching, determined by $\left(S_s^P\right)^{\nu}$ for Home and $\left(S_s^P\right)^{-\nu}$ for Foreign. Taking Home, the function $\left(S_s^P\right)^{\nu}$ is increasing convex for $\nu > 1$ but increasing concave for $0 < \nu < 1$.[29] Therefore, trade is lost under peg relative to float when consumption switching, $\left(S_s^P\right)^{\nu}(1-\tau)^{-\nu}$, is strong under $\nu > 1$. In other words, a flexible exchange-rate regime increases trade-to-output for elastic import demand. Conversely, trade is gained under peg relative to float when consumption switching, $\left(S_s^P\right)^{\nu}(1-\tau)^{-\nu}$, is weak under $0 < \nu < 1$, due to low substitutability. To put it differently, a fixed exchange-rate regime increases trade-to-output for inelastic import demand. It is also easy to see why unit substitutability leads to nominal trade equal to nominal output in any state of nature: simply the relative price change and the corresponding change in the relative quantity demanded cancel out when $\nu = 1$, resulting in $\frac{1-\tau}{S_s^P}\left(\frac{1-\tau}{S_s^P}\right)^{-1} = 1$.

Overall then, under costly trade with elastic import demand of similar output mixes, a float mitigates, on average, the home bias inherent to a fixed exchange-rate regime and plays a trade-promoting role, once some degree of PCP is allowed for; by contrast, under inelastic demand because of differing but equally-valued national good-types, a float makes the exchange of goods too costly, on average, relative to a peg and thus plays a trade-reducing role, unless there is full CCP.

4.2 How Much Does a Peg Increase Trade-to-Output?

To further judge about the likely magnitude of the effect of the exchange-rate regime on expected trade-to-output we established in Proposition 2 and at the same time to explore how money-stock volatility translates into variability of the resulting trade shares, we next simulated our model under jointly symmetric national money-growth disturbances. Having in mind that our framework was set-up assuming *price stickiness*, in line with the NOEM approach we follow here, we were interested in, and imposed in the simulation, *low* monetary uncertainty. The outcomes across a few sets of parameter constellations are reported in Table 1.[30]

[29] As was formally shown in Appendix A.2.

[30] The money-stock growth shocks (μ_s, μ_s^*) underlying the numbers in Table 1 were simulated 100 times from two independent (continuous) uniform distributions on the unit interval, $\mathcal{U}(0,1)$, one for Home and the other for Foreign. We then *centered* the shocks around 0, according to what we assumed in our no-(productivity)-growth NOEM model here, and *discretized* them. In discretizing, we used a *small* step to obtain "realistic" uncertainty, i.e. with a great (in fact, $10201 = 101 \times 101$) number of possible states, but at the same time limited the range of the latter to comply with price rigidity, namely by using $\mathcal{U}_l(-5.0, -4.9, ..., -0.1, 0, 0.1, ..., 4.9, 5.0)$. The equally-spaced values inside the parentheses defining the uniform distribution, the same for Home and Foreign, we simulated are directly

$(\mu_s, \mu_s^*) \in \mathcal{U}_l(-5,5)$	$(M_s, M_s^*)\,; \nu, \tau$ - determined NER: *PCP-Float* Sample		Cif *Trade Shares* in Output, %					*Peg Gain* for *World* Trade over Float, %
			PCP-Float Sample				*CCP*⇔*Peg* *constant*	
			Mean		SD			
	Mean	SD	*H*	*F*	*H*	*F*	$H = F$	
Panel I: (very) *low* trade costs: $\tau = 0.01$								
$\nu = 11$	0.9997	0.0035	95.14	94.82	1.74	1.74	94.98	−0.0016
$\nu = 2$	0.9983	0.0200	99.59	99.40	1.00	1.00	99.50	−0.0001
$\nu = 0.5$	0.9957	0.0800	100.06	100.44	2.00	2.00	100.25	0.0001
$\nu = 0.2$	1.0008	0.2005	99.66	101.14	7.84	7.84	100.40	0.0025
Panel II: *moderate* trade costs: $\tau = 0.2$								
$\nu = 11$	0.9998	0.0021	19.43	19.36	0.37	0.37	19.39	−0.0161
$\nu = 2$	0.9984	0.0190	88.98	88.80	0.94	0.94	88.89	−0.0011
$\nu = 0.5$	0.9957	0.0760	105.39	105.75	1.89	1.89	105.57	0.0019
$\nu = 0.2$	0.9970	0.1491	108.32	109.42	5.85	5.85	108.90	0.0281
Panel III: (very) *high* trade costs: $\tau = 0.6$								
$\nu = 11$	0.9998	0.0019	0.02	0.02	0.00	0.00	0.02	−0.0223
$\nu = 2$	0.9986	0.0165	57.21	57.08	0.67	0.67	57.14	−0.0042
$\nu = 0.5$	0.9960	0.0654	122.36	122.65	1.55	1.55	122.51	0.0047
$\nu = 0.2$	0.9956	0.0835	134.79	135.33	2.92	2.92	135.09	0.0252

Table 1: Gains from Peg/Float for World Trade: Simulation Summary

The second and third columns of Table 1 report the sample mean and standard deviation of the equilibrium nominal exchange rate, as calculated in the simulation. The next four columns indicate, analogously, the sample mean and standard deviation of the Home and Foreign trade shares in GDP. The last-but-one column gives the *state-invariant* trade share in output attained under CCP with float (as well as with peg) or, alternatively, under peg with PCP (but also with CCP) and identical for both national economies. We interpret the reported results on trade share volatility in the next section.

Now looking at the last column of Table 1, the first regularity one notices is related to the sign of what we have defined as the gain for expected world trade as a share in world output from a peg regime relative to a float. This measure is simply the percentage difference between the constant world trade share under peg (or CCP) and the "expected" world trade-to-GDP under float with PCP, the latter approximated by the (equally-weighted) *sample* means for Home and Foreign and taken as a base (i.e. normalized to 100). A positive difference is thus a trade gain from a fixed exchange-rate regime, whereas a negative sign means the opposite, namely that a flexible exchange-rate regime would, on average, bring about more international trade relative to world output than a peg. The simulation has thus, first of all, cross-checked and confirmed

interpretable as *growth rates* of the money stock in *percentages*, that is, as −0.1% or 4.9%, for instance. The GAUSS programs as well as more details on the algorithm of computations and on the results are available upon request.

our analytically derived conclusions in Proposition 2: under *elastic* demand (i.e. for $\nu = 11$ and $\nu = 2$ in Table 1, no matter what the particular value of τ is) a peg does reduce the expected world trade share in GDP, but *only slightly* – and this is the new point here, coming out from the simulation; and under *inelastic* demand, it does increase expected trade-to-output for the world economy as a whole, but – again – *only slightly.* "Only slightly" means, more precisely, by less than 1%, i.e. up to about 3 basis points,[31] as clear from the table.

Our qualitatively important theoretical finding in Proposition 2 has thus "crashed" into a quantitatively insignificant magnitude: for all practical or policy purposes, it will therefore be difficult to rely much on the above result... Yet this particular quantification of the *first*-moment effect, on *expected* trade-to-output, of a *second*-moment model feature, namely monetary *uncertainty* embodied in the driving shocks, does not go astray from similar conclusions in related NOEM papers. It is true that the exchange-rate regime effect on expected trade-to-output is tiny compared to the impact of trade costs or import demand elasticity themselves. However, that is a point common to the whole literature about the impact of uncertainty: on trade shares, in our case, but also on the conduct of monetary policy or on the welfare implications of peg vs. float. Once uncertainty is driven by *monetary* shocks – like in our present work as well as in NOEM research by others, for instance, Devereux and Engel (1998, 1999, 2000) – these shocks are just not that large empirically (or when simulated, as we did here, to comply with our assumption of sticky prices), hence there is no way that they will result in a large impact on the expected levels of any endogenous variable.

The key import of our extended NOEM analysis is thus in the conclusion that under PCP with trade costs interacting with somewhat more structured preferences (and, ultimately, import demand), monetary uncertainty does have an effect on expected trade-to-GDP, be it a tiny one, whereas in a frictionless, single-substitutability model – e.g. Mihailov (2003) under PCP (and CCP) or Bacchetta and van Wincoop (2000 a) under CCP – such a channel cannot be captured and explained. Moreover, our quantification of the effect of peg vs. float on expected trade-to-output is completely of the order of magnitude of similar effects, such as those recently reported in a policy-oriented paper by Devereux, Engel and Tille (2003), for example.[32]

5 The Role of Trade Costs and Import Demand Elasticity

We finally turn to the role of the *real* determinants of trade/GDP ratios which were explicitly modelled here. This role is reflected in Figure 1 and Table 1 and

[31] A basis point is $\frac{1}{100}$ of 1%.

[32] To quote them exactly, on page 237 of the cited reference they write: "The actual gains in expected consumption and reduction in expected employment are small (e.g. when $\rho = 2$, the gain in expected consumption in Europe is 0.02%, and the reduction in expected employment in Europe and the United States is 0.004%)".

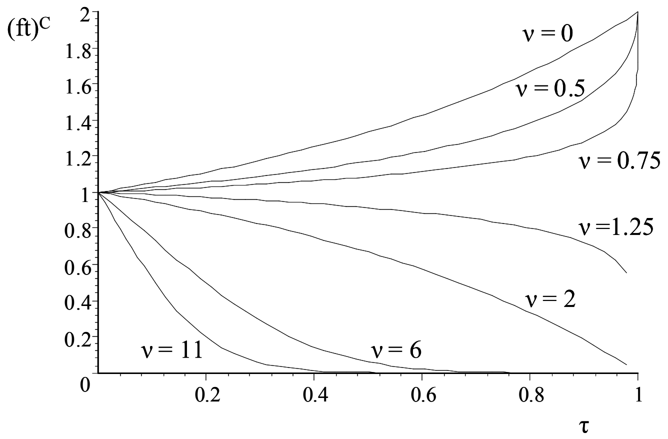

Figure 2: Peg Trade Share Curves across Iceberg Costs for Given Substitutability

relates to both expected trade-to-output and its variability, as will be discussed below.

Indeed, we have seen that the importance of our real trade fundamentals is crucial for the findings we reported. Recall that the magnitude of the trade friction τ in combination with that of import demand (in)elasticity ν defines the home bias, $(1-\tau)^{-\nu} > 1$, and ultimately the state-invariant and equal trade shares in Home and Foreign, under CCP or *peg.* Moreover, our Proposition 2 has shown that under PCP and *float* expected trade-to-output is slightly higher or lower than under peg (or CCP), depending on whether import demand is elastic ($\nu > 1$) or inelastic ($0 < \nu < 1$), respectively.

5.1 Trade Frictions

We first examine the impact of trade costs. Figure 2, in fact a two-dimensional variation of Figure 1, plots the peg (or CCP) trade share in any of the countries as a function of τ for different levels of ν. One can see that for given *elastic* import demand, *higher* transport costs *decrease* – *decreasingly* (for $\nu = 11$ or $\nu = 6$) or *increasingly* (for $\nu = 2$ or $\nu = 1.25$) – the *expected* level of trade-to-output.[33] But *inelastic* imports ($\nu = 0.75$ or $\nu = 0.5$ or $\nu = 0$) reverse this conclusion: higher trade frictions always lead to an increasing – but "inflated" – trade share (see also the last but one column in all three panels of Table 1).

[33] It follows from our analysis that *space* (or geography) matters, as in *gravity* models of trade, in particular if transport costs are modelled to be some positive function of distance (as we have implicitly assumed here).

The above reversal result needs a word of comment. As we noted earlier, in the inelastic ν region national output mixes are so *poor substitutes* that both countries are doomed – by their taste (or need) for diversity under complete specialization – to trade even when the international exchange of goods is extremely expensive, i.e. when shipping losses are very high (or exogenously rise). For a given level of transport costs under float, the trade share in output would thus be almost insensitive to (potential) ex-ante price discrimination under CCP or ex-post expenditure switching under PCP, since households practically cannot substitute away from imports into the (now completely different) home-produced good-type. Our finding that under peg with inelastic import demand national trade shares would both be higher than 1 is, in essence, explained by the fact that we used to derive them the *value* of *nominal* trade taking account of transport costs (*cif*) divided by *nominal* output. This latter ratio is thus highly inflated by the "true" price to the consumer of the huge percentage of output lost in transit.[34]

For moderate ($\tau = 0.2$) to high ($\tau = 0.6$) iceberg costs, simulations have furthermore indicated that lower shipment losses tend to increase trade share *volatility* (which becomes clear from comparing the standard deviation columns of panels II and III in Table 1). When tiny transport frictions ($\tau = 0.01$) are allowed for as well (see Panel I of Table 1), there is, however, no monotone function describing the relation discussed here, so trade variability generally depends on the particular parameter constellation.

5.2 Cross-Country Substitutability

We now summarize how the degree of import substitutability affects trade. Figure 3 – which is another two-dimensional perspective on Figure 1 – shows that, for any given iceberg cost, *lower* substitutability *increases* the *expected* value of the trade-to-output ratio common to both countries (see also the last but one column of panels I, II and III in Table 1). The intuition is that in this case consumers cannot substitute as much as they like to and would, *ceteris paribus*, so the resulting costly trade inflates trade shares in output. Another finding which stands out clearly in this graph is that as τ increases from 0 to 1 *(near-)linearity* of the peg trade share as a function of ν gradually transforms itself into a *steeper* and *more convex* curve.

For moderate to high shipment losses, lower substitutability also increases the PCP *volatility* of trade shares across states of nature (cf. the standard deviation columns in each of panels II and III of Table 1). Note, however, that for tiny costs of transport ($\tau = 0.01$) as in Panel I of Table 1 the relation

[34] Yet there is no way to measure instead trade-to-GDP in terms of the exchanged quantities ultimately consumed relative to the produced ones in each country, $\frac{c^*_{H,s}+c_{F,s}}{c_{H,s}+\frac{c^*_{H,s}}{1-\tau}}$ for Home and $\frac{c_{F,s}+c^*_{H,s}}{c^*_{F,s}+\frac{c^*_{F,s}}{1-\tau}}$ for Foreign, because one cannot add up "apples to oranges"...

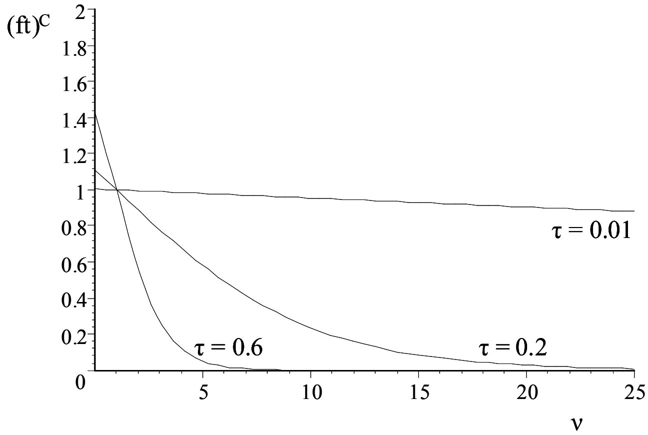

Figure 3: Peg Trade Share Curves across Substitutabilities for Given Iceberg Costs

in question appears not to be monotone. Instead, we have a divergence of simulated trade volatility under PCP away from 0 – which corresponds to the widely-exploited in NOEM unit substitutability special case ($\nu = 1$) – into higher magnitudes in both directions: as $\nu \to \infty$ and as $\nu \to 0$.

To sum-up, the simulations we performed have also indicated *how much* trade stabilization would be achieved by a shift from a fixed to a flexible exchange-rate regime. This depends on monetary as well as real trade determinants. As a lesson for policy, the *degree* of trade share variability thus eliminated would be greater for (symmetric) nations, or currency unions, which (i) have a larger proportion of PCP in their (bilateral) trade, (ii) are exposed to higher monetary uncertainty and – for moderate to high costs of international exchange – (iii) produce less substitutable outputs and (iv) are located closer to one another or apply weaker (reciprocal) tariff and non-tariff restrictions. Therefore, the lesser the extent to which the above-enumerated four conditions are met, the less efficient would a peg be as an instrument to stabilize trade.

6 Concluding Comments

The extended NOEM model we developed is useful to study the role of monetary uncertainty under trade frictions, distinct brand and type demand elasticity and alternative currency of price rigidity in determining the effects of the exchange-rate regime on trade, measured in terms of output. Given symmetry of structures and shocks, we distinguished *two types* of effects, namely an *expected level* (or *first*-moment) effect and a *volatility* (or *second*-moment) effect.

Expected Level Effects Our unified NOEM framework designed to nest trade in both similar and different output mixes has clearly indicated when a peg would increase *expected* trade, the relevant measure in a stochastic setting as ours here, relative to a float and when not. A peg increases expected trade-to-output under *inelastic* import demand for a different foreign good-type valued equally as the domestic one and decreases it under elastic demand for similar composites produced in various brands across two symmetric economies. We have also concluded that this effect, although qualitatively novel and important, is quantitatively very *weak*, as already established in other models within the NOEM literature examining welfare issues. By contrast, a *strong* effect on the magnitude of the expected trade share in GDP has been found for some deeper trade "fundamentals" such as transport or tariff frictions and cross-country output substitutability, which is another contribution of the present paper. More precisely, such real determinants affect – via the optimally arising stronger or weaker home bias – both the expected level of trade-to-output and its volatility across states, in a different way under elastic vs. inelastic import demand. Some of these trade fundamentals, e.g. tariffs, can relatively quickly be affected by policy. Changing the structural underpinnings of other, e.g. transportation technologies or preferences, would require much more time.

Variability Effects What fixing the exchange rate also attains under (some degree of) *PCP* although not (full) CCP – by shutting down the pass-through and expenditure-switching channel – is to stabilize (across states of nature) and equalize (across countries) trade shares in GDP at their expected level. Our simulations have indicated that *how much* trade stabilization would be achieved by a policy change from a fixed to a flexible exchange-rate regime ultimately depends on both monetary and non-monetary trade determinants. Within the perspective of real-world economies, it seems worth concluding that the *degree* of trade share variability eliminated by a shift to peg would be greater for (symmetric) nations, or currency unions, which (i) have a larger proportion of PCP in their trade, (ii) are exposed to higher monetary uncertainty and – for moderate to high costs of international exchange – (iii) produce less substitutable outputs and (iv) are located closer to one another or apply weaker tariff and non-tariff restrictions.

Limitations of Our Analysis We do not have illusions about the limitations of the NOEM set-up employed throughout the paper. But we would not repeat them here, since we have duly noted them in the relevant parts of our study. These limitations, certainly, constitute avenues for future research and could be addressed with more realism in subsequent extensions. This is how an insightful but thus far oversimplified analytical approach like NOEM will gradually be enriched to become more useful in empirical work or policy-oriented applications.

A Derivation of Equilibrium Results

A.1 Definition of Equilibrium

We here formally define an equilibrium concept that corresponds to the set-up described in the main text.

Definition 1 *In the context of the model versions we presented, an* equilibrium *is a set of quantities and prices, such that:*

1. ***[Ex-Ante Conditions]*** before *the resolution of monetary uncertainty but under common knowledge of the joint symmetric distribution of money growth shocks* (μ_s, μ_s^*)*;*

   (a) *[Firms Stochastic Optimization] given the technology constraint and the* expected *quantities demanded in the* goods *market,* $\left\{E_0\left[c_{H,s}^{C}\right],\right.$ $E_0\left[c_{H,s}^{*,C}\right]$, $E_0\left[c_{F,s}^{*,C}\right]$, $\left.E_0\left[c_{F,s}^{C}\right]\right\}$ *under CCP or* $\left\{E_0\left[c_{H,s}^{P}\right], E_0\left[c_{H,s}^{*,P}\right],\right.$ $E_0\left[c_{F,s}^{*,P}\right]$, $\left.E_0\left[c_{F,s}^{P}\right]\right\}$ *under PCP, the prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, that are optimally preset ex-ante (i.e. in state* 0*) and bindingly posted to consumer households for transactions ex-post (in state* s *for* $\forall s \in S$*) solve the profit maximization problem of the representative producer firm in* Home *as well as in* Foreign*;*

2. ***[Ex-Post Conditions]*** following *the resolution of monetary uncertainty and in any state of nature* $s \in S$ *that has materialized;*

   (a) *[Households Labor-Leisure Trade-Off] given its constraints and the posted prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, the representative consumer household in* Home *as well as in* Foreign *spends up all available cash on its* total *real consumption* $\{c_s, c_s^*\}$*; hours of work (employment)* $\{1-l_s\,,\ 1-l_s^*\}$ *are supplied by households until firms demand labor to equilibrate ex-post consumption demand for their differentiated products at the resulting equilibrium real wage rates* $\left\{\frac{W_s^C}{P^C}, \frac{W_s^{*,C}}{P^{*,C}}\right\}$ *under CCP and* $\left\{\frac{W_s^P}{P_s^P}, \frac{W_s^{*,P}}{P_s^{*,P}}\right\}$ *under PCP;*

   (b) *[Households Consumer Basket Allocation] given the posted prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, the consumption quantities* $\left\{c_{H,s}^C, c_{H,s}^{*,C}, c_{F,s}^{*,C}, c_{F,s}^C\right\}$ *under CCP or* $\left\{c_{H,s}^P,\right.$ $\left.c_{H,s}^{*,P}, c_{F,s}^{*,P}, c_{F,s}^P\right\}$ *under PCP solve the cost minimization problem à la Dixit-Stiglitz (1977) of the representative consumer household in* Home *as well as in* Foreign*;*

(c) *[Goods Market Clearing] all quantities under CCP or PCP satisfy the feasibility conditions for each* differentiated brand *so that all product-brand markets – and, hence, the* international *product-type market as a whole – clear;*

(d) *[Forex Market Clearing] the* international *forex market clears as well.*

A.2 Equilibrium Nominal Exchange Rate

CCP Under CCP, S_s^C is determined by:

$$\underbrace{P_F^C c_{F,s}^C}_{F \text{ export revenues} \Leftrightarrow HCsupply} \quad - \quad S_s^C \cdot \underbrace{P_H^{*,C} c_{H,s}^{*,C}}_{H \text{ export revenues} \Leftrightarrow HCdemand} \quad = \quad 0$$

Substituting for optimal $c_{F,s}^C$ and $c_{H,s}^{*,C}$ above as well as for the H and F CPI definitions further on in the algebraic manipulation derives:

$$S_s^C = \frac{\left(\frac{P_F^C}{P^C}\right)^{1-\nu}}{\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{1-\nu}} \frac{M_s}{M_s^*} = \frac{\frac{\left(P_F^C\right)^{1-\nu}}{\left(P_H^C\right)^{1-\nu} + \left(P_F^C\right)^{1-\nu}}}{\frac{\left(P_H^{*,C}\right)^{1-\nu}}{\left(P_F^{*,C}\right)^{1-\nu} + \left(P_H^{*,C}\right)^{1-\nu}}} \frac{M_s}{M_s^*} =$$

$$= \frac{\frac{1}{\frac{\left(P_H^C\right)^{1-\nu}}{\left(P_F^C\right)^{1-\nu}} + 1}}{\frac{1}{\frac{\left(P_F^{*,C}\right)^{1-\nu}}{\left(P_H^{*,C}\right)^{1-\nu}} + 1}} \frac{M_s}{M_s^*} = \frac{1 + \left(\frac{P_F^{*,C}}{P_H^{*,C}}\right)^{1-\nu}}{1 + \left(\frac{P_H^C}{P_F^C}\right)^{1-\nu} + 1} \frac{M_s}{M_s^*}$$

Now using the price equalities established earlier, namely $P_H = P_H^* = P_F^* = P_F^*$ to substitute above, one obtains the CCP expression in (21).

PCP Under PCP, S_s^P is determined by:

$$S_s^P \cdot \underbrace{\frac{P_F^* c_{F,s}^P}{1-\tau}}_{F \text{ export revenues} \Leftrightarrow HCsupply} \quad - \quad \underbrace{\frac{P_H c_{H,s}^{*,P}}{1-\tau}}_{H \text{ export revenues} \Leftrightarrow HCdemand} = 0$$

Hence:

$$S_s^P c_{F,s}^P = c_{H,s}^{*,P}$$

Substituting for optimal $c_{F,s}^P$ and $c_{H,s}^{*,P}$ and rearranging, we get:

$$S_s^P \left(\frac{\frac{S_s^P P_F^*}{1-\tau}}{P_s^P} \right)^{-\nu} \frac{M_s}{P_s^P} = \left(\frac{\frac{P_H}{S_s^P(1-\tau)}}{P_s^{*,P}} \right)^{-\nu} \frac{M_s^*}{P_s^{*,P}}$$

$$S_s^P \left(\frac{\frac{S_s^P P_F^*}{1-\tau}}{P_s^P} \frac{P_s^{*,P}}{\frac{P_H}{S_s^P(1-\tau)}} \right)^{-\nu} = \frac{M_s^*}{M_s} \frac{P_s^P}{P_s^{*,P}}$$

$$\left(S_s^P\right)^{1-2\nu} \left(\frac{P_s^P}{P_s^{*,P}} \right)^{\nu} = \frac{M_s^*}{M_s} \frac{P_s^P}{P_s^{*,P}}$$

$$S_s^P = \left(\frac{M_s^*}{M_s} \right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}} \right)^{\frac{1-\nu}{1-2\nu}} \tag{28}$$

Now we use the CPI definitions derived earlier to substitute for their ratio above:

$$\frac{P_s^P}{P_s^{*,P}} = \left[\frac{\left(1 + \frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\left(1 + \frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}} \right]^{\frac{1}{1-\nu}}$$

So that:

$$S_s^P = \left(\frac{M_s^*}{M_s} \right)^{\frac{1}{1-2\nu}} \left\{ \left[\frac{\left(1 + \frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\left(1 + \frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}} \right]^{\frac{1}{1-\nu}} \right\}^{\frac{1-\nu}{1-2\nu}}$$

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s} (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu} \frac{\left(1 + \frac{S_s^P}{1-\tau}\right)^{1-\nu}}{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}$$

$$\left(S_s^P\right)^{-\nu} = \frac{M_s^*}{M_s} \frac{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}}{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}$$

$$S_s^P = \left[\frac{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}} \right]^{\frac{1}{\nu}} \left(\frac{M_s}{M_s^*} \right)^{\frac{1}{\nu}},$$

which is the PCP expression in (21).
Under a peg, i.e. with $M_s = M_s^*$ for any $s \in S$, one would further on obtain:

$$\left(S_s^P\right)^{\nu} = \frac{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}}$$

An obvious solution is $S_s^P = 1$. Is it unique, more precisely within the domains for our variables, $0 < S_s^P << \infty$, and parameters $0 < \tau < 1$ and

$0 < \nu << \infty$? To prove it, we define two functions (skipping the state s subscript and the PCP P superscript, for convenience here, since we work with our PCP model version and invoicing is thus not ambiguous):

$$g(S) \equiv S^{\nu};$$

$$h(S) \equiv \frac{1 + (1-\tau)^{1-\nu} S^{1-\nu}}{(1-\tau)^{1-\nu} + S^{1-\nu}}.$$

We then analyze these functions, essentially in the vicinity of 1, as follows:

$$g'(S) = \nu S^{\nu-1} > 0,$$

hence $g(S)$ is monotone *increasing* in its domain;

$$g''(S) = \nu (\nu - 1) S^{\nu-2}:$$

with elastic demand, $\nu > 1 \Leftrightarrow g''(S) > 0$, hence $g(S)$ is convex;
with inelastic demand, $0 < \nu < 1 \Leftrightarrow g''(S) < 0$, hence $g(S)$ is concave.
Moreover,
when $S \to 0$, $\lim_{S \to 0} g(S) = 0$;
and when $S \to \infty$, $\lim_{S \to \infty} g(S) = \infty$.

We thus have that $g(1) = 1$ and that $g(S < 1) < 1$ and $g(S > 1) > 1$, as the function $g(S)$ increases from close to zero to infinity.

Now,

$$h'(S) = \frac{(1-\nu) S^{-\nu} \left\{ \left[(1-\tau)^{1-\nu} \right]^2 - 1 \right\}}{\left[(1-\tau)^{1-\nu} + S^{1-\nu} \right]^2}.$$

Before being able to conclude about the sign of the above derivative, we need to consider two cases:
with elastic demand, $\nu > 1 \Leftrightarrow h'(S) < 0$, because:
$\left[(1-\tau)^{1-\nu} \right]^2 - 1 > 0$ and $1 - \nu < 0$,
with $S^{-\nu} > 0, \forall \nu$ and $\left[(1-\tau)^{1-\nu} + S^{1-\nu} \right]^2 > 0, \forall \nu$;
hence, $h(S)$ is monotone *decreasing* in its domain when $\nu > 1$;
with inelastic demand, $\nu < 1 \Leftrightarrow h'(S) < 0$ again, because:
$\left[(1-\tau)^{1-\nu} \right]^2 - 1 < 0$ and $1 - \nu > 0$,
with $S^{-\nu} > 0, \forall \nu$ and $\left[(1-\tau)^{1-\nu} + S^{1-\nu} \right]^2 > 0, \forall \nu$;
hence, $h(S)$ is monotone *decreasing* in its domain when $\nu < 1$ as well.

Therefore, no matter what demand is (elastic, i.e. $\nu > 1$, or inelastic, i.e. $\nu < 1$), $h(S)$ is a monotone decreasing function. Since we have shown above that $g(S)$ is monotone increasing (again, no matter whether demand is elastic or inelastic), the two functions will have a unique crossing point, at $S_s^P = 1$. This proves our claim in the main text that, similarly to the CCP model version, under peg implying $M_s = M_s^*, \forall s \in S$ (or whenever there occurs a state of relative monetary equilibrium under float) we can always write $S_s^P = 1$.

A.3 Equilibrium Trade Shares

With iceberg costs $0 < \tau < 1$ taken into account, the Home[35] CCP vs. PCP equilibrium trade/GDP ratio is defined by

$$(ft)_{H,s}^C \equiv \frac{(FT)_{H,s}^C}{Y_{H,s}^C} = \frac{(Ex)_{H,s}^{C,cif} + (Im)_{H,s}^{C,cif}}{(DA)_{H,s}^C + (Ex)_{H,s}^{C,cif}} = \frac{S_s^C \cdot \overbrace{P_H^{*,C}}^{\text{FC } cif} \cdot \overbrace{c_{H,s}^{*,C}}^{\text{consumed}} + \overbrace{P_F^C}^{\text{HC } cif} \cdot \overbrace{c_{F,s}^C}^{\text{consumed}}}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot \underbrace{P_H^{*,C}}_{\text{FC } cif} \cdot \underbrace{c_{H,s}^{*,C}}_{\text{consumed}}} =$$

$$= \frac{S_s^C \cdot \overbrace{(1-\tau) P_H^{*,C}}^{\text{FC } fob} \cdot \overbrace{\frac{c_{H,s}^{*,C}}{1-\tau}}^{\text{produced}} + \overbrace{(1-\tau) P_F^C}^{\text{HC } fob} \cdot \overbrace{\frac{c_{F,s}^C}{1-\tau}}^{\text{produced}}}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot \underbrace{(1-\tau) P_H^{*,C}}_{\text{FC } fob} \cdot \underbrace{\frac{c_{H,s}^{*,C}}{1-\tau}}_{\text{produced}}} \text{ vs.} \tag{29}$$

$$(ft)_{H,s}^P \equiv \frac{(FT)_{H,s}^P}{Y_{H,s}^P} = \frac{\overbrace{(Ex)_{H,s}^{P,fob}}^{\equiv (Ex)_{H,s}^{P,cif}} + \overbrace{(Im)_{H,s}^{P,fob}}^{\equiv (Im)_{H,s}^{P,cif}}}{(DA)_{H,s}^P + \underbrace{(Ex)_{H,s}^{P,fob}}_{\equiv (Ex)_{H,s}^{P,cif}}} = \frac{\overbrace{P_H^P}^{\text{HC } fob} \overbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}^{\text{produced}} + S_s^P \overbrace{P_F^{*,P}}^{\text{FC } fob} \overbrace{\frac{c_{F,s}^P}{1-\tau}}^{\text{produced}}}{P_H^P c_{H,s}^P + \underbrace{P_H^P}_{\text{HC } fob} \underbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}_{\text{produced}}}$$

$$= \frac{\overbrace{\frac{P_H^P}{1-\tau}}^{\text{HC } cif} \overbrace{c_{H,s}^{*,P}}^{\text{consumed}} + \overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\text{HC } cif} \overbrace{c_{F,s}^P}^{\text{consumed}}}{P_H^P c_{H,s}^P + \underbrace{\frac{P_H^P}{1-\tau}}_{\text{HC } cif} \underbrace{c_{H,s}^{*,P}}_{\text{consumed}}}, \tag{30}$$

where $(Ex)_{H,s}^{C,cif}$ denotes Home exports at *cif* prices, $(Im)_{H,s}^{C,cif}$ Home imports at *cif* prices and $(DA)_{H,s}^C$ Home domestic absorption, with all these three *values* (prices multiplied by quantities) expressed in Home currency under CCP for any state $s \in S$ that has materialized. $(Ex)_{H,s}^{P,fob}$, $(Im)_{H,s}^{P,fob}$ and $(DA)_{H,s}^P$ are, of course, the respective Home-currency values under PCP, with Home exports and imports now measured at *fob* prices. It is important to recall at this point that once a transport and/or tariff friction is considered in our extended

[35] For Foreign, the respective expressions are symmetric.

NOEM model, the relevant prices for equilibrium trade flows as implied by the invoicing conventions we analyze become the *cif* ones under CCP and the *fob* ones under PCP. However, due to our *symmetric* iceberg costs assumption, we have shown by the last equalities in (29) and (30) above that the fob values are *exactly equal* to their respective cif values in both our CCP and PCP model versions, so that trade shares can be meaningfully compared across alternative price setting as if calculated on the *same*, *cif* basis. This latter, *cif domestic*-currency value is, furthermore, the appropriate measure to use, since it duly accounts for the difference between quantities *bought* and quantities *consumed* arising from the output lost in transit and thus reflects the *true cost* to the representative consumer.

Substitutions for optimal domestic and external demands for H and F output and use of the CPI definitions derive – under *full symmetry* and *separable preferences* – the CCP vs. PCP equilibrium trade shares in the main text.

The derivation under CCP for Home is:

$$(ft)^C_{H,s} \equiv \frac{(FT)^C_{H,s}}{Y^C_{H,s}} = \frac{(Ex)^{C,cif}_{H,s}+(Im)^{C,cif}_{H,s}}{(DA)^C_{H,s}+(Ex)^{C,cif}_{H,s}} = \frac{S^C_s \cdot \overbrace{P^{*,C}_H}^{\text{FC } cif} \cdot \overbrace{c^{*,C}_{H,s}}^{\text{consumed}} + \overbrace{P^C_F}^{\text{HC } cif} \cdot \overbrace{c^C_{F,s}}^{\text{consumed}}}{P^C_H \cdot c^C_{H,s} + S^C_s \cdot \underbrace{P^{*,C}_H}_{\text{FC } cif} \cdot \underbrace{c^{*,C}_{H,s}}_{\text{consumed}}} =$$

$$= \frac{\overbrace{\frac{M_s}{M^*_s}}^{S^C_s} P^{*,C}_H \overbrace{\frac{1}{2}\left(\frac{P^{*,C}_H}{P^{*,C}}\right)^{-\nu} \frac{M^*_s}{P^{*,C}}}^{c^{*,C}_{H,s}} + P^C_F \overbrace{\frac{1}{2}\left(\frac{P^C_F}{P^C}\right)^{-\nu} \frac{M_s}{P^C}}^{c^C_{F,s}}}{P^C_H \underbrace{\frac{1}{2}\left(\frac{P^C_H}{P^C}\right)^{-\nu} \frac{M_s}{P^C}}_{c^C_{H,s}} + \underbrace{\frac{M_s}{M^*_s}}_{S^C_s} P^{*,C}_H \underbrace{\frac{1}{2}\left(\frac{P^{*,C}_H}{P^{*,C}}\right)^{-\nu} \frac{M^*_s}{P^{*,C}}}_{c^{*,C}_{H,s}}} =$$

Using our earlier result that, under CCP, *Home* and *Foreign* price levels are *equal*, due to the symmetry in the model, i.e. $P^C = P^{*,C}$, and dividing through by $\frac{M_s}{(P^C)^{1-\nu}}$, we obtain:

$$= \frac{P^{*,C}_H \left(P^{*,C}_H\right)^{-\nu} + P^C_F \left(P^C_F\right)^{-\nu}}{P^C_H \left(P^C_H\right)^{-\nu} + P^{*,C}_H \left(P^{*,C}_H\right)^{-\nu}} = \frac{(P^{*,C}_H)^{1-\nu} + (P^C_F)^{1-\nu}}{(P^C_H)^{1-\nu} + (P^{*,C}_H)^{1-\nu}} =$$

Recalling that $P^{*,C}_H = P^C_F$, due to the symmetry again, one can write:

$$= \frac{(P^{*,C}_H)^{1-\nu} + (P^{*,C}_H)^{1-\nu}}{(P^C_H)^{1-\nu} + (P^{*,C}_H)^{1-\nu}} = \frac{2(P^{*,C}_H)^{1-\nu}}{(P^C_H)^{1-\nu} + (P^{*,C}_H)^{1-\nu}} =$$

Now dividing through by $(P^{*,C}_H)^{1-\nu}$, we finally get:

$$= \frac{\dfrac{2(P_H^{*,C})^{1-\nu}}{(P_H^{*,C})^{1-\nu}}}{\dfrac{(P_H^{C})^{1-\nu}}{(P_H^{*,C})^{1-\nu}} + \dfrac{(P_H^{*,C})^{1-\nu}}{(P_H^{*,C})^{1-\nu}}} = \frac{2}{\left(\dfrac{P_H^C}{P_H^{*,C}}\right)^{1-\nu} + 1}$$

So under CCP

$$(ft)_H^C = \frac{2}{\left(\frac{P_H^C}{P_H^{*,C}}\right)^{1-\nu} + 1} = \frac{2}{\left(\frac{E_0[u_{l,s} M_s]}{\frac{1}{1-\tau} E_0[u_{l,s} M_s^*]}\right)^{1-\nu} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu} + 1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1,$$

which is (24) in the main text.
The derivation under PCP for Home is:

$$(ft)_{H,s}^P \equiv \frac{(FT)_{H,s}^P}{Y_{H,s}^P} = \frac{\overbrace{(Ex)_{H,s}^{P,fob}}^{\equiv (Ex)_{H,s}^{P,cif}} + \overbrace{(Im)_{H,s}^{P,fob}}^{\equiv (Im)_{H,s}^{P,cif}}}{(DA)_{H,s}^P + \underbrace{(Ex)_{H,s}^{P,fob}}_{\equiv (Ex)_{H,s}^{P,cif}}} = \frac{\overbrace{P_H^P}^{\text{HC } fob} \overbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}^{\text{produced}} + S_s^P \overbrace{P_F^{*,P}}^{\text{FC } fob} \overbrace{\frac{c_{F,s}^P}{1-\tau}}^{\text{produced}}}{P_H^P c_{H,s}^P + \underbrace{P_H^P}_{\text{HC } fob} \underbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}_{\text{produced}}} =$$

$$= \frac{\dfrac{P_H^P}{1-\tau} \overbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{P_H^P}{S_s^P (1-\tau)}}{P_s^{*,P}}\right)^{-\nu} \dfrac{M_s^*}{P_s^{*,P}}}^{\equiv c_{H,s}^{*,P}} + \dfrac{S_s^P P_F^*}{1-\tau} \overbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu} \dfrac{M_s}{P_s^P}}^{\equiv c_{F,s}^P}}{P_H^P \underbrace{\dfrac{1}{2} \left(\dfrac{P_H^P}{P_s^P}\right)^{-\nu} \dfrac{M_s}{P_s^P}}_{\equiv c_H^P} + \dfrac{P_H^P}{1-\tau} \underbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{P_H^P}{S_s^P (1-\tau)}}{P_s^{*,P}}\right)^{-\nu} \dfrac{M_s^*}{P_s^{*,P}}}_{\equiv c_{H,s}^{*,P}}} =$$

Using that, due to symmetry, $P_H^P = P_F^*$,we get:

$$= \frac{1 + S_s^P \left(\dfrac{S_s^P}{P_s^P} \dfrac{P_s^{*,P}}{\frac{1}{S_s^P}}\right)^{-\nu} \dfrac{M_s}{P_s^P} \dfrac{P_s^{*,P}}{M_s^*}}{(1-\tau)^{\nu-1} \left(\dfrac{1}{P_s^P} \dfrac{P_s^{*,P}}{\frac{1}{S_s^P}}\right)^{-\nu} \dfrac{M_s}{P_s^P} \dfrac{P_s^{*,P}}{M_s^*} + 1} = \frac{1 + \left(S_s^P\right)^{1-2\nu} \dfrac{M_s}{M_s^*} \left(\dfrac{P_s^{*,P}}{P_s^P}\right)^{1-\nu}}{(1-\tau)^{1-\nu} (S_s^P)^{-\nu} \dfrac{M_s}{M_s^*} \left(\dfrac{P_s^{*,P}}{P_s^P}\right)^{1-\nu} + 1}.$$

Now recall our earlier result (28) that

$$S_s^P = \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}.$$

Rearranging, we can write it as:

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{1-\nu}.$$

Hence:

$$\frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu} = \frac{1}{(S_s^P)^{1-2\nu}},$$

which we now use to substitute $\frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu}$ out in our PCP Home trade share derivation, as we continue it below:

$$(ft)_{H,s}^P = \frac{1 + \left(S_s^P\right)^{1-2\nu} \frac{1}{(S_s^P)^{1-2\nu}}}{(1-\tau)^{1-\nu}(S_s^P)^{-\nu}\frac{1}{(S_s^P)^{1-2\nu}} + 1} = \frac{2}{(1-\tau)^{1-\nu}(S_s^P)^{\nu-1} + 1}$$

We can finally write:

$$(ft)_{H,s}^P = \frac{2}{(1-\tau)^{1-\nu}(S_s^P)^{\nu-1} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu}\underbrace{\left\{\left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}\right\}}_{=S_s^P}^{\nu-1} + 1} \neq const \text{ unless } S_s^P = 1,$$

which is (25) in the main text.

The respective expressions for Foreign, (26) under CCP and (27) under PCP, can be derived by analogy.

B Proofs of Propositions

B.1 Proof of Proposition 1 (Equilibrium World Trade-to-Output)

Proof. Under *CCP*, the proof is immediate from the *constant* H and F trade shares, in (24) and (26) respectively.

Under *PCP*, from (25) for Home and (27) for Foreign, and using as a shorthand below $c \equiv (1-\tau)^{1-\nu}$, one obtains:

$$(ft)_{H,s}^{P} + (ft)_{F,s}^{*,P} = \frac{2}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{1-\nu}+1} + \frac{2}{(1-\tau)^{1-\nu}\left(\frac{1}{S_s^P}\right)^{1-\nu}+1} =$$

$$= \frac{2}{c\left(S_s^P\right)^{1-\nu} + \frac{\left(S_s^P\right)^{1-\nu}}{\left(S_s^P\right)^{1-\nu}}} + \frac{2}{c\frac{1}{\left(S_s^P\right)^{1-\nu}} + \frac{\left(S_s^P\right)^{1-\nu}}{\left(S_s^P\right)^{1-\nu}}} = 2\left[\frac{1}{1+c\left(S_s^P\right)^{1-\nu}} + \frac{\left(S_s^P\right)^{1-\nu}}{c+\left(S_s^P\right)^{1-\nu}}\right] =$$

$$= 2\left[\frac{1}{1+(1-\tau)^{1-\nu}\left(S_s^P\right)^{1-\nu}} + \frac{\left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}\right].$$

Thus, in any state of nature that has materialized (equally-weighted) world trade is:

$$(ft)_{W,s}^{P} \equiv \frac{1}{2}(ft)_{H,s}^{P} + \frac{1}{2}(ft)_{F,s}^{*,P} = \frac{1}{2}\left[(ft)_{H,s}^{P} + (ft)_{F,s}^{*,P}\right] =$$

$$\frac{1}{1+(1-\tau)^{1-\nu}\left(S_s^P\right)^{1-\nu}} + \frac{\left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}, \text{ for } \forall s \in S.$$

This completes our proof. ■

B.2 Proof of Proposition 2 (Expected Trade-to-Output under PCP)

Proof. Write the equilibrium trade shares we derived for Home, (25), and Foreign, (27), under PCP as functions of the exchange rate (skipping below the P superscript for convenience since now, in the PCP case, there is no ambiguity on invoicing):

$$ft_H(S_s) = \frac{2}{(1-\tau)^{1-\nu} S_s^{\nu-1}+1} \qquad \text{and} \qquad ft_F(S_s) = \frac{2}{(1-\tau)^{1-\nu} S_s^{1-\nu}+1}.$$

With symmetry, as assumed:

$$ft_H(S_s) = ft_F\left(\frac{1}{S_s}\right).$$

Symmetry in our particular context here implies that for each state of nature s there is a symmetric state s' such that:

1. the exchange rate is inverse: $S_{s'} = \frac{1}{S_s}$;
2. the two states have the same probability: $\pi_s = \pi_{s'}$.

Let us first focus on Home. The expected trade share *across the two symmetric states in the pair* is:

$$\begin{aligned} E_0 \left[ft_{H,(s,s')} (S_s) \right] &= \pi_s ft_H (S_s) + \pi_s ft_H (S_{s'}) = \\ &= \pi_s \tfrac{2}{(1-\tau)^{1-\nu} S_s^{\nu-1}+1} + \pi_s \tfrac{2}{(1-\tau)^{1-\nu} S_s^{1-\nu}+1} = \\ &= \pi_s \tfrac{2}{(1-\tau)^{1-\nu}+1} \left[\tfrac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{\nu-1}+1} + \tfrac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{1-\nu}+1} \right] = \\ &= \pi_s ft_{H,peg} F (S_s) . \end{aligned}$$

The expectation is thus equal to the *identical* (and fixed for any given jointly symmetric distribution) probability of s and s' occurring, $\pi_s = \pi_{s'}$, times the *constant* trade share under peg (or CCP), $ft_{H,peg} = \frac{2}{(1-\tau)^{1-\nu}+1}$, times the *function*

$$F (S_s) \equiv \frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{\nu-1}+1} + \frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{1-\nu}+1}.$$

For a benchmark, consider what would be the value of the above expectation if the trade share was a constant, as under PCP with peg implying $M_s = M_s^*, \forall s$ so that $ft_H (S_s) = ft_H (S_{s'}) = ft_{H,peg}$. Then one would have:

$$E_0 \left[ft_{H,(s,s')} (S_s) \right] = 2\pi_s ft_{H,peg}.$$

Making use of local analysis around $F(1)$, we shall now show that $F(S_s) \geq 2$, which would mean that the expected trade share for each *pair of symmetric states* of relative monetary disequilibrium under float and PCP exceeds the corresponding trade share under peg.

One can easily prove that:

$$F (S_s) = F \left(\frac{1}{S_s} \right) \qquad \text{and} \qquad F(1) = 2,$$

i.e. that $F(S_s)$ is a symmetric function equal to 2 when the exchange rate is 1. We further write:

$$\begin{aligned} F' (S_s) &= (\nu - 1) (1-\tau)^{1-\nu} \left[(1-\tau)^{1-\nu} + 1 \right] \times \\ &\quad \times \left(- \frac{S_s^{\nu-2}}{\left[(1-\tau)^{1-\nu} S_s^{\nu-1} + 1 \right]^2} + \frac{S_s^{-\nu}}{\left[(1-\tau)^{1-\nu} S_s^{1-\nu} + 1 \right]^2} \right), \\ F'(1) &= 0, \end{aligned}$$

so that $F(S_s)$ is flat at 1, i.e. it attains a local extremum at 1. Moreover:

$$
\begin{aligned}
F''(S_s) &= (\nu-1)(1-\tau)^{1-\nu}\left[(1-\tau)^{1-\nu}+1\right]\times \\
&\times\left[\begin{array}{c}
-(\nu-2)\frac{S_s^{\nu-3}}{\left[(1-\tau)^{1-\nu}S_s^{\nu-1}+1\right]^2}+2(\nu-1)\frac{S_s^{\nu-2}(1-\tau)^{1-\nu}S_s^{\nu-2}}{\left[(1-\tau)^{1-\nu}S_s^{\nu-1}+1\right]^3}- \\
-\nu\frac{S_s^{-\nu-1}}{\left[(1-\tau)^{1-\nu}S_s^{1-\nu}+1\right]^2}+2(\nu-1)\frac{(1-\tau)^{1-\nu}S_s^{-\nu}S_s^{-\nu}}{\left[(1-\tau)^{1-\nu}S_s^{1-\nu}+1\right]^3}
\end{array}\right],
\end{aligned}
$$

$$
F''(1)=\frac{2(\nu-1)^2(1-\tau)^{2(1-\nu)}}{\left[(1-\tau)^{1-\nu}+1\right]^2}\left[1-(1-\tau)^{\nu-1}\right]=\left[\frac{\sqrt{2}(\nu-1)(1-\tau)^{1-\nu}}{(1-\tau)^{1-\nu}+1}\right]^2\left[1-(1-\tau)^{\nu-1}\right].
$$

$\left[\frac{\sqrt{2}(\nu-1)(1-\tau)^{1-\nu}}{(1-\tau)^{1-\nu}+1}\right]^2$ being always positive, we now have to consider two cases, in addition to the trivial third case of unit substitutability when the trade share is constant at 1.

- *Elastic* import demand, $\nu > 1$. In this case $1-(1-\tau)^{\nu-1} > 0 \Rightarrow F''(S_s) > 0$ so that $F(S_s)$ is *convex* around $S_s = 1$, which proves that the function $F(S_s)$ attains a local *minimum* $F(1) = 2$. Then it follows that $F(S_s) \geq 2$, at least around $S_s = 1$ (the region in which we are interested in, particularly under price rigidity compatible with relatively small money shocks as assumed in this paper). Finally, summing over *all* pairs of symmetric states, we obtain:

  $$E_0\left[ft_{H,(s,s')}(S_s)\right] \geq ft_{H,peg} \Leftrightarrow F(S_s) \geq 2.$$

  The same arguments apply for Foreign's expected trade share. Adding up for the two countries in the model leads to the conclusion that expected trade-to-output for the world is *lower under peg* than under float, with trade costs and import demand elasticity accounted for.

- *Inelastic* import demand, $0 < \nu < 1$. In this case $1-(1-\tau)^{\nu-1} < 0 \Rightarrow F''(S_s) < 0$ so that $F(S_s)$ is *concave* around $S_s = 1$, which proves that the function $F(S_s)$ attains a local *maximum* $F(1) = 2$. Then it follows that $F(S_s) \leq 2$, at least around $S_s = 1$ (the region in which we are interested in). Finally, summing over *all* pairs of symmetric states, we obtain:

  $$E_0\left[ft_{H,(s,s')}(S_s)\right] \leq ft_{H,peg} \Leftrightarrow F(S_s) \leq 2.$$

  The same arguments apply for Foreign's expected trade share. Adding up, again, expected world trade-to-output is *higher under peg* than under float, once accounting for trade costs and import demand inelasticity as in our extended NOEM framework here.

This completes our proof. ■

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