

***Profits, Confidence and Public Deficits :  
Modeling Minsky's Institutional Dynamics*** (\*)

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**Abstract**

The aim of this paper is to present a “Minskian” model which explicitly deals with the influence of the institutional dynamics on the relation between finance, investment and economic fluctuations. First, the Minskian foundations of the proposed analytical framework are highlighted. Second the dynamical properties of the model are studied, drawing the inferences of a stabilization policy. It is shown that the economy is unstable when the budget policy is not very sensitive to variations in private investment. On the contrary, when, the counter cyclical deficit constraint is flexible enough, the economy is stabilized. These results, that echo recent debates and proposals on budget deficits rules in the EMU, are fully consistent with the way Minsky considers that public authorities may “stabilize an unstable economy”.

## ***Introduction***

Starting in the middle of the fifties, and for the next forty years, Hyman P. Minsky developed an original business cycle theory based on an endogenous and financial conception of economic fluctuations, and more specifically, on the "financial instability hypothesis" (Minsky, 1982, 1986). This hypothesis relates to two types of phenomena characterizing the changes in "financially sophisticated economies". The first phenomenon refers to an endogenous process of transition toward greater financial fragilization of the economy. The second characterizes the transition from a financially fragile situation to a situation of recession and then of large amplitude economic crisis.

The absence of modeling has sometimes made the consistency of Minsky's arguments seem difficult to check. Indeed the implications of the interaction that takes place between real and financial factors are closely dependent on the specification of the dynamic structure of Minsky's 'financially sophisticated economies', the shapes of the functions and the values of the parameters describing them, all aspects that appear difficult to take into account without the support of at least some sort of formalization. In the past ten years some scholars have sized up the problem and endeavored to propose formalized interpretations of the financial instability hypothesis.

The main characteristic of these models is that they embed financial structure variables (such as indebtedness ratios) into standard Keynesian macroeconomic frameworks. Under certain circumstances, these models (e.g. Delli Gatti and Gallegati 2000, Skott 1994, Keen 1995, Arena and Raybaut 2000) produce fluctuations analogous to those imagined but not modeled by Minsky. Financial factors are indeed capable, in systems that are otherwise stable, to be at the origin of unstable endogenous dynamics. Exclusively divergent in linear models, such dynamics can be more complex in non-linear models and lead to periodical or a-periodic trajectories (limit cycles or deterministic chaos).

However, a closer examination of Minsky's analysis leads to consider these nonlinear models with a critical eye or at least to suggest a substantial enrichment of their assumptions. Indeed, these models neglect another essential aspect of the financial instability hypothesis, an aspect we propose to call "institutional dynamics". The latter characterizes the influence of institutional mechanisms and of the interventions of public authorities on the dynamics of market economies.

For Minsky the various institutional mechanisms that are present in contemporary market economies play a central role in the unfolding of economic fluctuations. Their function is to slow down and adjust the dynamic process at the origin of the economy's endogenous and 'incoherent' behavior. Two types of institutional agents exert a crucial influence on the dynamics of market economies: financial institutions (especially commercial banks), on the one hand, and public authorities, on the other.

This paper is centered upon the role of public authorities<sup>1</sup>. In Minsky's approach, stabilizing economic activity is essentially the concern of the government, via its fiscal policy, and of the central bank, through its role as lender of last resort. Minsky indeed views budget deficits and interventions by the central bank as lender-of-last-resort as extremely effective instruments for stabilizing economic fluctuations. Even if full employment is not achieved, these instruments help limit the drop in income and in liquidity during economic recessions and during the onset of a financial crisis.

In this perspective, the aim of this paper is to present a model which extends and completes recent models of financial instability by explicitly examining the influence of the institutional dynamics on the relation between finance, investment and economic fluctuations. The paper is organized as follows. First, the Minskian foundations of the proposed analytical framework are highlighted (Section 1). Second, the dynamical properties of the model are studied, drawing the inferences of a stabilization policy (Section 2).

## *1. Some Minskian foundations*

As is well known Minsky's theory of endogenous and financial instability is mainly based on his "financial theory of investment" founded on the 'two-price' approach. In the model, investment decisions are thus depicted by the following function :

$$I_t = h(p_{k_t} - p_{I_t}) \quad (0.1)$$

where  $p_{k_t}$  and  $p_{I_t}$  stand respectively for the price of capital assets and of current production. Function  $h(\cdot)$  is increasing, continuous and positive in  $y_t = p_{k_t} - p_{I_t}$ , satisfying

$$\lim_{y \rightarrow -\infty} h(y) = 0^+ \text{ and } \lim_{y \rightarrow +\infty} h'(y) = 0.$$

The first price system (the supply price of investment output) is a mark-up on costs. In the aggregate, the main out-of-pocket costs that need to be recovered are wage costs. Thus, the price of current production will move as labor costs move and as the ability of the suppliers of investment outputs to maintain discipline in determining mark ups changes. We assume in this paper that labor costs and mark ups are constant so that the price of current output is; hence we write  $p_{I_t} = \widehat{p}_I$ .

The second price system refers to the prices of capital assets and reflects the views of agents dealing in such assets with regard to the levels of gross profit flows. These prices thus reflect the expectations of market participants about the future of the economy, i.e. future achievements of its various actors and future performances of financial markets. For Minsky, such future conditions are uncertain and cannot be appraised in terms of probabilistic risk. As a result, current asset prices mirror both portfolio managers' views on the future evolution of aggregate profits and the margins of safety they require.

For simplicity reasons, the inter-temporal framework we retain is rather sketchy since we consider that present investment decisions in period  $t$  relate only on discounted expected profits for the next period,  $\Pi_{t+1}^e$ . Therefore, we have

$$p_{k_t} = \frac{\Pi_{t+1}^e}{1 + i_t} \quad (0.2)$$

where  $i_t$  is the discount rate that entrepreneurs apply to their expected profits.

Expectations dynamics is described by the following adaptive motion:

$$\Pi_{t+1}^e = \Pi_t^e + \alpha(\Pi_t - \Pi_t^e) \quad (0.3)$$

where,  $0 \leq \alpha \leq 1$ .

The adaptive framework used here provides a rather simplified but relevant representation of Minsky's expectations analysis. It describes, as the author does, a sequential economy where, as agents make mistakes, they will be required to correct them over time, on the basis of what their actual observations are.

The discount rate used by business men is not constant, but endogenous and is given by:

$$i_t = i^* + ax_t + b\varphi_t \quad (0.4)$$

where,  $i^*$  is the risk less interest rate determined by the central bank policy,  $\varphi_t$  refers to the borrower's risk and  $x_t$  is an indicator of the state of confidence; and where

$a < 0$  and  $b > 0$  are two parameters.

Central bank policy, borrower's risk and state of confidence play a central role in Minsky's theory of investment. The characteristics of each of these variables and the way they influence the discount rate (and thus the price of capital assets) may briefly be recalled.

Let us begin by the central bank. In Minsky's approach, one of the main purposes of this institution is to offset debt-deflation phenomena or the different forms of financial instability that market economies have been experiencing, especially since the middle of the 1980s.

According to Minsky, these phenomena underline the need for an extended interpretation of the role of lender of last resort (Minsky, 1986). For simplicity reasons, in the model we focus only upon one type of lender of last resort intervention by the central bank: the stabilization of the price of capital assets (the value of the claims agents exchange for liquidity) by modifying the amount of money in circulation and thus the degree of liquidity of the economy. This role of the central bank is captured indirectly via the interest rate  $i^*$  that is supposed to depend negatively of the amount of money in circulation.<sup>2</sup>

The second term  $b\varphi$  stands for the influence of the borrower's risk on the discount rate. The fact that the parameter  $b$  is positive means that the greater is the borrower's risk, the greater the discount rate ( $i$ ) that the firms apply to their expected profits. In Minsky's approach, the borrower's risk is related to the fact that beyond the amount that can be self-financed, investment implies financial costs that are inescapable, whereas the cash flows generated by production are unquestionably uncertain. Therefore 'the borrower's risk will increase as the weight of external or liquidity diminishing financing increases' (1986, p. 191).

In other words, the greater is the  $(I_t - \rho\Pi_t)$  gap (where  $\rho$  is the rate of retention of profits), the greater the borrower's risk (the smaller the margin of safety for managers and equity owners). The dynamics of the borrower's risk is thus depicted by the following equation<sup>3</sup>:

$$\varphi_t = \Phi(I_t - \rho\Pi_t) \quad (0.5)$$

where function  $\Phi$  shapes as follows:

### Insert Fig 1

The last variable,  $x_t$ , influencing the discount rate is an indicator of the state of the degree of optimism or pessimism of entrepreneurs, i.e. of the state of confidence. Since it is assumed in relation (0.4) that  $a < 0$ , the higher (smaller) is the confidence, the smaller (higher) the discount rate applied to expected returns.

This variable  $x_t$  captures the role played by changes in long-term expectations in the emergence and recurrence of economic fluctuations. This role is of utmost importance according to many post Keynesian authors.<sup>4</sup>

However, far from being pure psychological and subjective phenomena, Minsky conceives the modifications of the state of confidence as based both on ‘objective’ endogenous economic magnitudes and on aspects determined in a conventional or ‘subjective’ way.

This is the reason why the motion of  $x_t$  can be modeled by the following relation :

$$x_{t+1} = \theta x_t + (1 - \theta)(\Pi_t - \Pi_t^e) \quad (0.6)$$

with,  $0 < \theta < 1$ .

The term  $(1 - \theta)(\Pi_t - \Pi_t^e)$  indicates the influence of objective factors, represented here by profit expectations errors. Such an influence is highlighted by Minsky (1986 p. 193-194). In his approach, when investment and past decisions to invest are on the whole validated (i.e. when,  $\Pi_t \geq \Pi_t^e$ ), confidence of economic agents is reinforced. Indeed actual leverage of the aggregate balance sheet structure will be smaller than expected. Consequently investors will come upon sounder balance sheets than predicted, meaning that there will be an ‘unused borrowing power’, and subsequent financing conditions will become more favorable (1986, p. 194). In Minsky’s model, this leads to an increase in the maximum level of indebtedness that agents deem prudent, to a rise of investment and to a boom.

Conversely, when actual profits are smaller than expected ones ( $\Pi_t < \Pi_t^e$ ), confidence declines: indebtedness ratios formerly judged prudent are henceforth considered dangerous. Adoption by businesses and by financial institutions of various defensive measures then contributes to diminishing the level of investment, thereby causing a recession and, possibly, economic depression.

The term  $\theta x_t$  refers to the role played by subjective elements in the formation of the state of

confidence. It captures the idea that agents can, to some extent, modify their long term expectations independently of how their realized outcomes fit in with their expectations, i.e. independently of economic fundamentals. According to Minsky, the longer the period during which no financial crisis occurs, the more economic agents are confident about the future and the more these agents will be likely to raise their estimates of the maximum level of indebtedness to which it is prudent to agree.

Let us finally suppose that total consumption is determined by  $C_t = c[W + (1 - \rho)\Pi_t]$ , where  $W$  refers to exogenous total wages,  $c$  is the propensity to consume and  $\rho$  is the rate of retained profits. Then, the macroeconomic linkage relation given by Kalecki's (1971) accounting identity writes:

$$\Pi_t = \frac{I_t + D_t - (1 - c)W}{(1 - c(1 - \rho))} \quad (0.7)$$

where,  $D_t = G_t - T_t$ , stands for the public budget deficit.

Consequently, a deficit, by upholding aggregate demand when private investment flags, establishes a lower limit, a floor, for profits.

According to Minsky, such stabilization of actual and expected profits is crucial to ensuring the continuity of the economic system. It is utilized in particular to maintain the viability of debt structures and therefore the level of private investment. In fact 'once rational bankers and business men learn from experience that actual profits do not fall when private investment declines, they will modify their preferred portfolios to take advantage of the stability of profits' (Minsky, 1992, p. 12).

The importance of "Big Government" (that is a public sector that constitutes a significant share of aggregate demand) in economic dynamics is so fundamental for Minsky that he divides the performance of the US economy into two periods: a "small government" era from the end of the Civil War to the Depression and a "big government" era dating from World

War II (Minsky 1986). He provides a detailed analysis of how the deficit of “Big Government” was crucial in maintaining profits during the 1974-75 recession (1986, chapter 2).

For Minsky, the government reacts in an endogenous fashion to the behavior of private agents. This implies that the budget policy is very sensitive to variations in investment:

‘policy will be stabilizing if a shortfall of private investment quickly leads to a government deficit, and a burst of investment quickly leads to a budget surplus’ (Fazzari and Minsky, 1984, p. 107).

This negative relation between budget deficit and private investment is explicitly taken into account in our model. Accordingly, we assume

$$D_t = \gamma(I)(C_t + I_t) \quad (0.8)$$

where,  $\gamma$  is a continuous, decreasing function of  $I_t$ , satisfying  $-\sigma \leq \gamma(I) \leq \sigma$ , with  $0 \leq \sigma < 1$ . The budget deficit-merchant GDP ratio  $\gamma$  is not exogenous but negatively related to investment. In this perspective,  $\gamma'(I)$  can be seen as the degree of flexibility of the budget deficit constraint  $\gamma(I)$ . The rest of the paper is devoted to the analysis of the role played by  $\gamma'$  on the dynamics of the economy.

## ***2. Profits expectations, confidence and economic stabilization***

From the previous discussion we derive two first order differences equations in  $\Pi_t^e$  and  $x_t$  describing the dynamics of expected profits and of the state of confidence.

Accordingly, combining relations (0.1), (0.2), (0.4) and (0.5) we obtain:

$$I_t - h \left( \frac{\Pi_{t+1}^e}{1 + ax_t + b\Phi(I_t - \rho\Pi_t)} - \hat{p}_I \right) = 0 \quad (0.9)$$

This expression determines implicitly the level of investment as a function of the level of expected profits  $\Pi_{t+1}^e$  for  $t + 1$  and of the current state of confidence,  $x_t$ . That is:

$$I_t = g(\Pi_{t+1}^e, x_t) \quad (0.10)$$

Substituting (0.10) into the definition of current profits given by Kalecki's relation, the adaptive dynamics of expectations writes as

$$\Pi_{t+1}^e - (1 - \alpha)\Pi_t^e - \alpha \left\{ \frac{g(\Pi_{t+1}^e, x_t) + W[c(1 + \gamma(g(\Pi_{t+1}^e, x_t)) - 1)]}{1 - c(1 - \rho)[1 + \gamma(g(\Pi_{t+1}^e, x_t))]} \right\} = 0 \quad (0.11)$$

This implicit first order difference equation in  $\Pi_{t+1}^e, \Pi_t^e, x_t$  determines the dynamics of expected profits  $\Pi_{t+1}^e$ . Hence, we have <sup>5</sup>:

$$\Pi_{t+1}^e = f(\Pi_t^e, x_t) \quad (0.12)$$

Similarly, substituting (0.7), (0.8), (0.10) and (0.12) into (0.6) we obtain a first order difference equation which gives the dynamics of the state of confidence  $x_t$ :

$$x_{t+1} = \theta x_t + (1 - \theta) \left( \frac{\Psi(\Pi_t^e, x_t) + W[c(1 + \gamma(\Psi(\Pi_t^e, x_t)) - 1)]}{1 - c(1 - \rho)[1 + \gamma(\Psi(\Pi_t^e, x_t))]} - \Pi_t^e \right) \quad (0.13)$$

where,  $\Psi(\Pi_t^e, x_t) = g(f(\Pi_{t+1}^e, x_t), x_t)$ .

It is therefore possible to characterize the stationary state  $(\bar{\Pi}^e, \bar{x})$  of system (0.12)-(0.13). The following proposition obtains.

**Proposition 1**

Assume that parameters  $\rho, i^*, \hat{p}_I$  satisfy  $h(0) = \rho(1 + i^*)\hat{p}_I$ , then the dynamical system (0.12)-(0.13) admits one stationary state such as  $\bar{p}_k = \hat{p}_I$  and  $I_t = \bar{I}$ .

This “Nirvana” stationary state, is also characterized by perfect foresights,  $\bar{\Pi}^e = \bar{\Pi}$ , and no risk,  $\bar{\varphi} = 0$ . Consequently, agents are neither optimistic nor pessimistic, that is to say  $\bar{x} = 0$ .

Then, we have,  $\bar{\Pi}^e = \bar{\Pi} = \hat{p}_I(1 + i^*)$ .

***(See proof in Annex 2)***

Let us now consider the dynamical properties of the model in a neighborhood of this stationary point. As will be shown, these dynamical properties are closely dependent on the values of  $\gamma'(\cdot)$  which captures the degree of flexibility of the counter-cyclical public deficits constraint; the following result obtains <sup>6</sup> :

***Proposition 2***

The stationary state is locally asymptotically stable if and only if the public deficits constraint is counter-cyclical enough. That is to say if:

$$\gamma'_I(\cdot) < \Xi \tag{0.14}$$

where  $\gamma'_I(\cdot) < 0$ , and  $\Xi = \frac{[1 - c(1 - \rho)][(1 - c(1 - \rho))\tilde{\Gamma} - 1]}{c\rho(W + (1 - \rho)\bar{\Pi})}$ .

***(See proof in Annex 3)***

Therefore, according to proposition 2, an efficient stabilization policy should be associated with a flexible counter cyclical deficit constraint. These analytical findings are finally illustrated with the following numerical example.

In this example, the investment function, the borrower’s risk function and budget deficit constraint are specified by:

$$h(p_{k_t} - \hat{p}_I) = \text{Arc tan}[p_{k_t} - \hat{p}_I] + \frac{\pi}{2}$$

$$\Phi(I_t - \rho\Pi_t) = \text{Tanh}[I_t - \rho\Pi_t]$$

$$\gamma(I_t) = -\sigma\text{Tanh}[I_t - \bar{I}]$$

This latter function shapes as follows for  $\sigma = .15$ :

**Insert Fig 2**

Wages are normalized to unity,  $\hat{p}_I = \frac{h(0)}{\rho(1+i^*)}$ , and the other values of parameters are:

$$\{a, b, \alpha, \rho, c, i^*\} = \{-.25, .75, .15, .8, .8, .05\}$$

Then, for  $0 \leq \sigma \leq .25$  and  $0 < \theta < 1$ , the following graphics illustrating proposition 2 obtain:

**Insert Fig 3**

**Insert Fig 4**

As we may recall, the economy is locally stable (unstable), when the criterion is negative (positive).<sup>7</sup> Thus, it clearly appears that it exists a threshold value of  $\sigma$ ,  $\tilde{\sigma}$ . When  $\sigma < \tilde{\sigma}$ , the economy is unstable and stable otherwise.

Our findings argue in favor of the implementation of an institutional design allowing large deficits in periods of economic slumps; leaving budget deficits cuts off and possible surpluses to periods of sustained expansion. This result is along the same lines as Minsky's argument on the role played by institutional thwarting systems<sup>8</sup>. It also echoes recent debates and proposals on budget deficits rules in the EMU.

Indeed, Budgetary policies in Economic and Monetary Union (EMU) countries are constrained by the "excessive deficit procedure" of the article 104 of the Maastricht Treaty,

prescribing upper public debt and deficits limits. Accordingly, fiscal discipline was held to be vital for ensuring macroeconomic stability and dismissing the risks of spillovers. The classical underlying principle was that each member state might otherwise face incentives to spend and borrow too much, generating upward pressures on the union-wide interest rates and downward pressures on the external value of the common currency. It was also acknowledged that the loss of the exchange rate instrument at the national level would be accompanied by a greater role for automatic stabilizers to help economies to adjust to asymmetric shocks, and would make it “*necessary to ensure that national budgetary policies support stability oriented monetary policies*” (Eur-Lex, European Commission). This was the rationale underlying the core commitment of the so called “Stability and Growth Pact” (SGP) procedure introduced in 1997, i.e. to set the “*medium-term objective of budgetary positions close to balance or in surplus ... [which] will allow all Member States to deal with normal cyclical fluctuations while keeping the government deficit within the reference value of 3 % of GDP*” (Ibid). From this perspective, member states implement and regularly update since 1999 national “stability programs”, laying out a stability-oriented route to a balanced budget by, originally, 2004<sup>9</sup>. The Pact was designed assuming that governments would accumulate surpluses in good times to ensure the smooth running of automatic stabilizers in bad times. However, several countries – particularly the largest ones – did not reduce their deficits during the years of expansion. Indeed, the high growth years (1999-2000) have not been put to good use by France and Germany to reduce significantly their structural (cyclically-adjusted) deficits that have reached 1.6 % of GDP in 1999 and 1.4 % in 2000 in France, and respectively 0.6 % and 0.9 % in Germany. At the same time, in the US, during the 1992-2000 expansion period, the federal budget balance comes through a serious deficit (5 % of GDP) to a healthy surplus (1.7 % of GDP) (from a 4% deficit to a 1.3% surplus in cyclically-adjusted terms) (INSEE, December 2001).

After the US went into recession in the first half of 2001, the federal fiscal policy became extremely expansive. The federal budget deficit attained 0.5 % of GDP in 2001, 3.4% in 2002 and would be around 3.5% in 2003 (INSEE, October 2003). According to a recent OFCE study, from 2001 to 2003, the structural primary deficit variation – measuring the discretionary part of fiscal policies – reached 5.3 points of GDP in United States (OFCE, October 2003). As a result of this contra cyclical fiscal policy the US recession was surprisingly short and mild. According to the Bureau of Economic Analysis, real gross domestic product increased by 2.4% in 2002 (compared to 0.3% in 2001); it grew at an annual rate of 7.2 percent in the third quarter of 2003 (the best economic performance for 19 years) and is expected to reach 2.4% for the 2003. In contrast the Euro-area fiscal policies constrained by the Stability and Growth Pact were less expansive. On the one hand, from 2001 to 2003 the structural primary deficit variation was only equal to 0.8 % of GDP (Ibid). On the other hand, for 2003 fiscal policies are projected to be broadly neutral: the structural primary deficit variation in the euro-zone is even expected to be negative (-0.2 % of GDP) (Idem). Consequently, it is thus not surprising that the Euro-area growth fell from 3.5% in 2000 to 1% in 2002 and should roughly reach 0.5 % in 2003 (Idem). According to the European Commission and OECD, what fundamentally explains current difficulties of larger Euro-area countries was their failure to reduce structural deficits during the years of high growth, insuring themselves against a cyclical downturn. This is the reason why the 2003 Spring EcoFin Council endorsed the following new principles: The close-to-balance or in surplus rule should apply not only at a medium-term horizon, but in cyclically-adjusted terms each year; countries that have yet to comply with this requirement will be committed to consolidate their fiscal position by at least ½ per cent of GDP per year in cyclically-adjusted terms; pro-cyclical budgetary policies should be avoided, especially when growth conditions are favourable. This adjustment of the Pact does not indeed resolve short term problems of

high deficit-GDP ratios and low growth countries such as France and Germany, which will have to implement pro-cyclical – that is restrictive – fiscal policies to fulfil these new requirements.

### *Concluding remarks*

The aim of this paper was to propose a formalized version of Minsky's conception of institutional dynamics. The originality of our approach consisted in taking into account not only the financial aspects but also the institutional dimension of the financial instability hypothesis since this issue has received scant treatment in recent formalizations. With this aim in view, we considered the role of the government, via its budget policy, and of the central bank, through its role as lender of last resort. The analysis of the dynamical properties of the model shows that the economy is unstable when the budget policy is not very sensitive to variations in private investment. On the contrary, when, the counter cyclical deficit constraint is flexible enough, the economy is stabilized. The model could obviously be extended to take into consideration other important aspects of Minsky's writings, such as the existence of endogenous financial cycles or the ambivalence of institutional thwarting systems.<sup>10</sup> However, our main result, namely that an efficient stabilization policy requires the implementation of an institutional set-up allowing large deficits in periods of economic slumps; leaving budget deficits cuts off and surpluses to periods of sustained expansion, is fully consistent with the way Minsky considers that public authorities may "stabilize an unstable economy".

### *Annex 1 a complement on the borrower's risk*

The first derivative of retained profits with respect of investment is given by the following expression:

$$\frac{\rho (1 - c + c\rho + (1 + 2c(-1 + \rho)) \gamma[\text{Invest}] + c(-1 + \rho) \gamma[\text{Invest}]^2 + (\text{Invest} + cW\rho) \gamma'[\text{Invest}])}{(1 + c(-1 + \rho) + c(-1 + \rho) \gamma[\text{Invest}])^2}$$

In the figure below, this derivative evaluated in a neighborhood of the stationary state is plotted for the different relevant values of the rate of retention of profits  $\rho$  and of the degree of flexibility of the counter-cyclical public deficits constraint,  $\sigma$ . We get:

#### **Insert Fig 5**

It clearly appears that in a neighborhood of the stationary state, the first derivative of retained profits with respect of investment is smaller than unity. (See Annex) In this perspective, the financial gap  $I - \rho\Pi$  is pro-cyclical, and in accordance with Minsky's financial instability hypothesis the borrower's risk will increase during an investment boom.

### **Annex 2 proof of proposition 1**

-First, for  $\bar{\Pi}^e \neq 0$  and  $\bar{\Pi} \neq 0$ , relation (0.11) evaluated at a stationary state directly implies  $\bar{\Pi}^e = \bar{\Pi}$ . Substituting this result into (0.13) evaluated at a stationary state, gives  $\bar{x} = 0$ .

-Second, using the fact that at the stationary state  $\bar{\varphi} = 0$ , we have  $\bar{i} = i^*$ . In addition, since  $\bar{p}_k = \hat{p}_I$  at a stationary state, we get  $\bar{\Pi} = (1 + i^*)\hat{p}_I$

-Finally, notice that  $\bar{p}_k = \hat{p}_I$  means that  $\bar{I} = h(0)$ , while  $\bar{\varphi} = 0$  means that  $\bar{I} = \rho\bar{\Pi}$ . Thus, condition  $h(0) = \rho(1 + i^*)\hat{p}_I$  is required for consistency reasons ■

### **Annex 3 proof of proposition 2**

The modulus of the two eigenvalues of the  $2 \times 2$  Jacobian matrix  $J^*$  of the dynamical system, evaluated at the stationary state, lie inside the unit circle if and only :

$$|Det(J^*)| < 1 \quad (0.15)$$

$$|Tr(J^*)| - 1 - Det(J^*) < 0 \quad (0.16)$$

The elements of  $J^*$  evaluated at the stationary state are the following:

$$J_{11}^* = \frac{1 - \alpha}{1 - \frac{\alpha(1 + i^*)h'(0)\Gamma}{(1 + i^*)^2 + b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0)}}; J_{12}^* = \frac{-a\alpha\bar{\Pi}h'(0)\Gamma}{(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma + b(1 - \rho\Gamma)\Phi'(0))}$$

$$J_{21}^* = (1 - \theta)(\Gamma I'_{\Pi_{t+1}^e} J_{11}^* - 1); J_{22}^* = \theta + (1 - \theta)\Gamma(I'_{\Pi_{t+1}^e} J_{12}^* + I'_{x_t})$$

$$\text{where, } I'_{\Pi_{t+1}^e} = \frac{h'(0)}{(1 + i^*)\left(1 + \frac{b\bar{\Pi}h'(0)(1 - \rho\Gamma)\Phi'(0)}{(1 + i^*)^2}\right)}; I'_{x_t} = \frac{-a\bar{\Pi}h'(0)}{(1 + i^*)\left(1 + \frac{b\bar{\Pi}h'(0)(1 - \rho\Gamma)\Phi'(0)}{(1 + i^*)^2}\right)}$$

$$\text{and where, } \Gamma = \Pi'_I = \frac{1 - c(1 - \rho) + \gamma'_I(\cdot)(Wc\rho + c(1 - \rho)\rho\bar{\Pi})}{(1 - c(1 - \rho))^2}$$

Thus, we have :  $Det(J^*) = J^*_{11}(\theta + (1 - \theta)\Gamma I'_{x_t}) + (1 - \theta)J^*_{12}$  (0.17)

$$Tr(J^*) = J^*_{11} + \theta + (1 - \theta)\Gamma(I'_{\Pi_{t+1}} J^*_{12} + I'_{x_t}) \quad (0.18)$$

Assuming that  $\Gamma > 0$ , necessitating that  $\gamma'(\cdot) > \frac{c(1 - \rho) - 1}{c\rho(W + (1 - \rho)\bar{\Pi})}$ , with

$$a < 0, b > 0, \Phi'(0) > 0, \text{ one can verify that}^{11}: \quad 0 < \Gamma < Min\left\{\frac{1}{\rho}, \frac{1 + i^*}{\alpha h'(0)}\right\} \Leftrightarrow$$

$$\{ I'_{\Pi_{t+1}} > 0, I'_{x_t} > 0, J^*_{11} > 0, J^*_{12} > 0 \} \quad (0.19)$$

Consequently, it is obvious, since  $0 < \theta < 1$  that condition (0.19) implies :

$$Det(J^*) > 0 \text{ and } Tra(J^*) > 0$$

Hence, the stability criterion writes:

$$Det(J^*) < 1 \quad (0.20) \text{ and } Tr(J^*) - 1 - Det(J^*) < 0 \quad (0.21)$$

Substituting the values of  $J^*_{11}$ ,  $J^*_{12}$ ,  $I'_{\Pi_{t+1}}$  and of  $I'_{x_t}$  into (0.20) and (0.21), we obtain the respective values of the determinant and of the trace at the stationary state:

$$Det(J^*) = \frac{(1 + i^*)^2(1 - \alpha)\theta + \bar{\Pi}h'(0)(a(\theta - 1)\Gamma + b(1 - \alpha)\theta(1 - \rho\Gamma)\Phi'(0))}{(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma - b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0))}$$

$$Tr(J^*) = \frac{(1 + i^*)^2(\alpha - 1 - \theta) + h'(0)[\Gamma(\alpha\theta(1 + i^*) + a(1 - \theta)\bar{\Pi}) + b(\alpha - 1 - \theta)\bar{\Pi}(1 - \rho\Gamma)\Phi'(0)]}{(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma - b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0))}$$

and the two stability conditions become:

$$\frac{(1 + i^*)^2(1 - \alpha)\theta + \bar{\Pi}h'(0)(a(\theta - 1)\Gamma + b(1 - \alpha)\theta(1 - \rho\Gamma)\Phi'(0))}{(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma - b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0))} < 1 \quad (0.22)$$

$$\frac{\alpha(\theta - 1)[(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma - b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0))]}{(1 + i^*)^2 - h'(0)(\alpha(1 + i^*)\Gamma - b\bar{\Pi}(1 - \rho\Gamma)\Phi'(0))} < 0 \quad (0.23)$$

Recalling we have assumed above that  $0 < \Gamma < \text{Min} \left\{ \frac{1}{\rho}, \frac{1+i^*}{\alpha h'(0)} \right\}$ , the denominator in the two

conditions is positive. Thus, (0.22) and (0.23) are equivalent to:

$$(1+i^*)^2((1-\alpha)\theta-1) - h'(0)b\bar{\Pi}\Phi'(0) < -h'(0)\left[\alpha(1+i^*) + a(\theta-1) + b\bar{\Pi}\rho\Phi'(0)(1-(1-\alpha)\theta)\right]\Gamma$$

$$\alpha(\theta-1)\left[(1+i^*)^2 + h'(0)b\bar{\Pi}\Phi'(0) - h'(0)(1+i^* + b\bar{\Pi}\Phi'(0))\Gamma\right] < 0$$

The first condition can be rewritten:  $0 < \Gamma < \Gamma_1$  (0.24)

$$\text{With, } \Gamma_1 = -\frac{(1+i^*)^2((1-\alpha)\theta-1) - h'(0)b\bar{\Pi}\Phi'(0)}{h'(0)\left[\alpha(1+i^*) + a(\theta-1) + b\bar{\Pi}\rho\Phi'(0)(1-(1-\alpha)\theta)\right]}$$

The second condition can be rewritten

$$0 < \Gamma < \Gamma_2 \quad \mathbf{(0.25)},$$

$$\text{with } \Gamma_2 = \frac{(1+i^*)^2 + h'(0)b\bar{\Pi}\Phi'(0)}{h'(0)(1+i^* + b\bar{\Pi}\Phi'(0))}$$

Consequently  $\Gamma$  meets conditions (0.24) and (0.25) if and only if

$$0 < \Gamma < \text{Min}\{\Gamma_1, \Gamma_2\} \quad \mathbf{(0.26)}$$

Therefore, one can deduce that it exists a positive threshold value of  $\Gamma$ ,

$$\tilde{\Gamma} = \text{Min}\left\{\frac{1}{\rho}, \frac{1+i^*}{\alpha h'(0)}, \Gamma_1, \Gamma_2\right\}, \text{ such that } J^* \text{ has two stable roots, if and only if:}$$

$$0 < \Gamma < \tilde{\Gamma} \quad \mathbf{(0.27)}$$

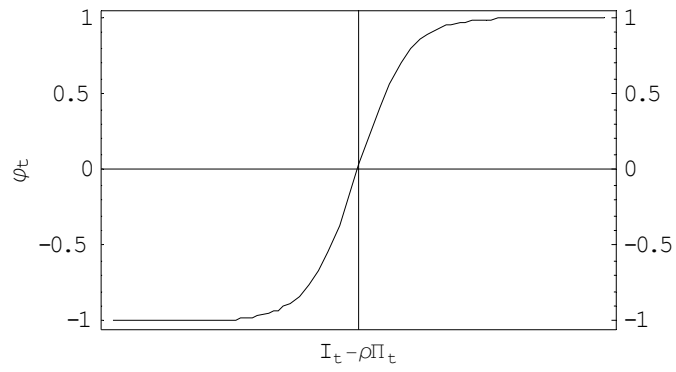
Accordingly, recalling that  $\Gamma = \frac{1 - c(1-\rho) + \gamma'_I(\cdot)(Wc\rho + c(1-\rho)\rho\bar{\Pi})}{(1-c(1-\rho))^2}$ , condition (0.27)

finally becomes:  $\gamma'(\cdot) < \Xi$  (0.28)

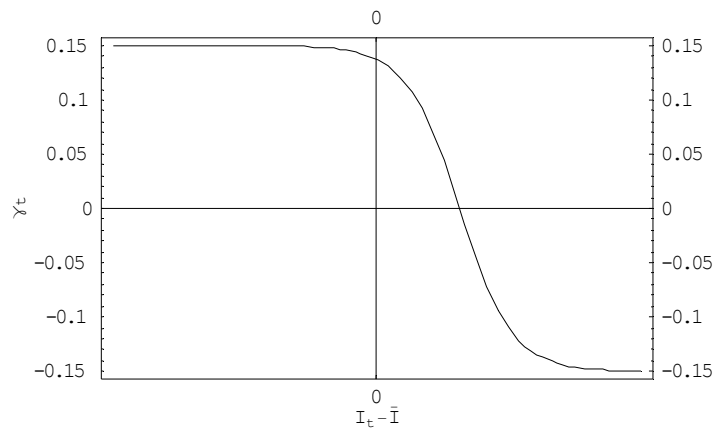
Where  $\Xi = \frac{[1-c(1-\rho)][(1-c(1-\rho))\tilde{\Gamma}-1]}{c\rho[W-(1-\rho)\bar{\Pi}]}$ . Which completes the proof of **Proposition 2** ■

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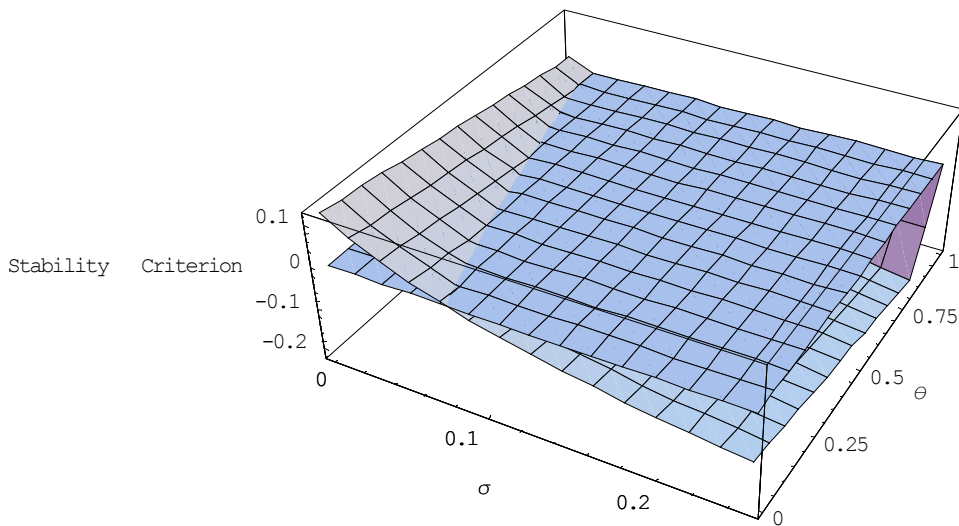
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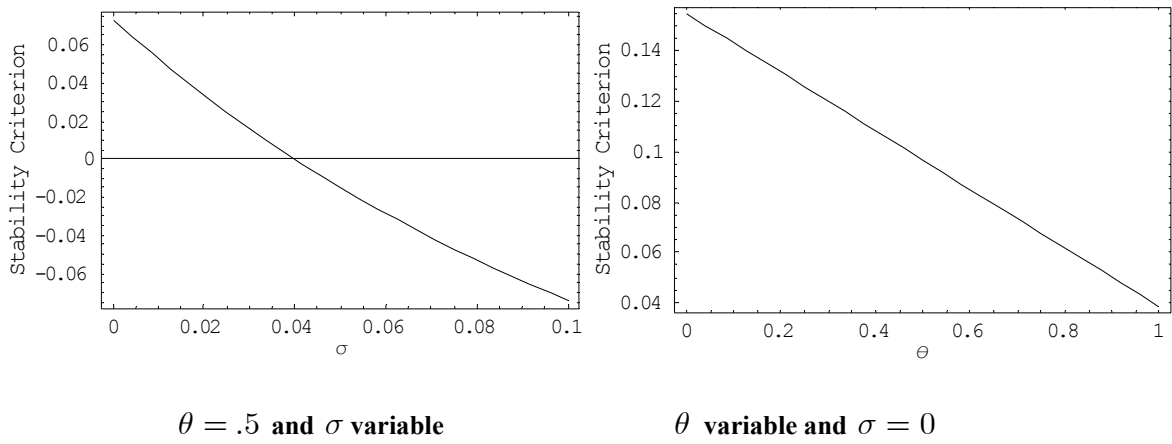
**Fig.1**



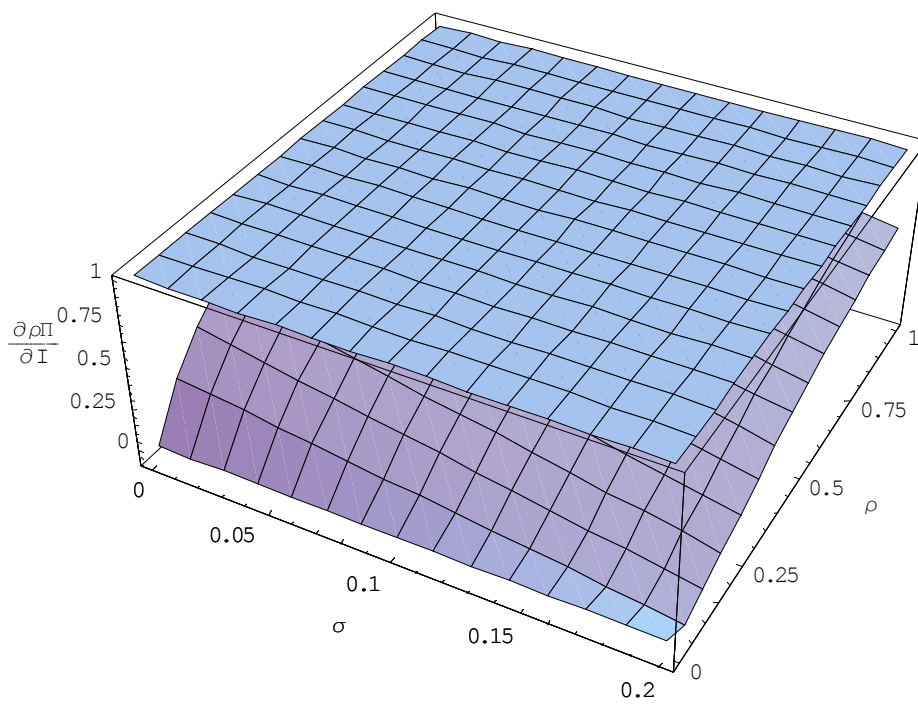
**Fig 2**



**Fig 3**



**Fig 4**



**Fig 5**

## Notes

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(\*) We want to thank Marc Lavoie for comments and suggestions on a first version of this paper. Usual caveats apply.

<sup>1</sup> Minsky's policy analysis is based on an "agenda for reform" that addresses four areas: "Big Government (size, spending, and taxing), an employment strategy, financial reform, and market power" (Minsky, 1986, p. 295). Our paper is focused on the first area. For an analysis of other aspects of this agenda, see for instance the article of Papadimitriou et Wray (1998) that also examines other proposals of Minsky regarding bank regulation, tax reform and poverty.

<sup>2</sup> This is a rather simplified view of Minsky's analysis. In the latter the way commercial banks react to profit opportunities and to the policy conducted by the central bank can prevent the latter from setting the interest rate at the level it deems desirable. Indeed the evolution of this rate depends strongly on the succession of phases of institutional stability and instability induced by the active behaviour of the commercial banks (For a more complete approach, see Nasica, 1997).

<sup>3</sup> It can be shown that the first derivative of retained profits with respect of investment is smaller than unity in a neighbourhood of the stationary state of the model (See Annex 1). In this perspective, the financial gap  $I_t - \rho\Pi_t$  is procyclical, and in accordance with Minsky's financial instability hypothesis, the borrower's risk will increase during an investment boom.

<sup>4</sup> For Kregel (1976) considering long-period expectations as given is synonymous with reasoning in terms of a static and not a dynamic equilibrium.

<sup>5</sup> Assuming that the implicit function theorem applies.

<sup>6</sup> Notice that since  $Det(J^*) > 0$ , the model can admit two complex eigenvalues, which may generate, for the relevant set of parameters, endogenous cycles. This issue will not be dealt with in this paper.

<sup>7</sup> The criterion plotted here is  $|Tr(J^*)| - 1 - Det(J^*) < 0$ . It can be shown in these examples that the first condition  $|Det(J^*)| < 1$  is always satisfied for the retained set of parameters.

<sup>8</sup> See e.g. Ferri and Minsky (1992).

<sup>9</sup> As it was judged impossible to rectify the situation quickly in view of the poor growth prospects, the achievement of a balanced budget was put off by three years – from 2003 to 2006 – in several countries (Germany, Italy, Portugal, and France) if these countries reduced their budget deficits by 0.5 percent of GDP per year starting in 2003.

<sup>10</sup> The impact of an institutional structure is not immutable: its capacity to stabilize the amplitude of economic fluctuations and to constrain market agents to undertake only moderately risky actions varies greatly over time. This means that some institutional

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interventions and mechanisms that were initially stabilizing may turn into factors of instability and inefficiency. About this point, see for instance Nasica (1999).

<sup>11</sup> Condition  $\gamma'(\cdot) > \frac{c(1-\rho)-1}{c\rho(W+(1-\rho)\bar{\Pi})}$  is not restrictive since it simply means that the percentage of deficit to GDP is bounded from above. In addition we have,  $\frac{1}{\rho} > 1$ . Indeed, specifying the investment function as in the numerical example below by:  $h(p_k - p_I) = \text{Arc tan}(p_k - p_I) + \frac{\pi}{2}$ , we obtain, at the stationary state,  $h'(0) = 1$ . Then with,  $0 < \alpha < 1$ , we have  $\frac{1+i^*}{\alpha h'(0)} > 1$ .