Heterogeneous Expectations, Currency Options and the Euro/Dollar Exchange rate

Bronka Rzepkowski*

CEPII, 9 rue Georges Pitard, 75740 PARIS cedex 15
tél : 01-53-68-55-54, email: rzepkowski@cepii.fr

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Abstract

An exchange rate model with heterogeneous expectations is developed in which agents are subject to mutual mimetic contagion in their portfolio decisions. Two alternative sources of heterogeneity are tested in order to explain the short-term dynamics of the euro/dollar since January 1999. Information conveyed by over-the-counter currency options allows the time-varying proportions of each category of agents to be inferred, as well as their respective exchange rate expectations and standard deviations. The proportion of optimistic agents in the evolution of the euro and the proportion of confident agents in their exchange rate anticipations induce portfolio reallocations, which generate euro/dollar forecasts.

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Introduction

Standard economic and financial theory is based on the hypotheses of rational and homogeneous expectations. Agents are thus supposed to take all the available information into account when optimizing according to a common model. This allows a simple aggregation of microeconomic behavior, via the representative agent method. However, the empirical evidence rejects the rational expectations assumption. Theories grounded on concepts such as bounded rationality and heterogeneous expectations appear more relevant in explaining observed patterns in financial data.

Two strands of the literature focus on the impact of heterogeneous beliefs on market dynamics. First, heterogeneity in expectations and in behavior may be due to some specific characteristics of market participants: initial endowment, risk aversion, time horizon, information set. Such an approach leads to models with rational and irrational agents (De Long *et al.* 1990, 1991), informed versus uninformed agents (Genotte and Leland, 1990; Lyons, 1991), chartists versus fundamentalists (Frankel and Froot, 1988; De Grauwe, 1993) sophisticated versus naive agents (Day and Huang, 1990). According to the second branch of the literature, market participants recognize that they are not perfectly informed and that their knowledge of the underlying model of the economy is incomplete (Topol, 1991). Agents then behave differently according to their own information set without relying totally on their evaluation of the economic fundamentals. The proportion of each class of agents fluctuates due to mimetic contagion phenomena. The probability to switch from one group to the other can be formalize as a stochastic process of random meetings (Kirman, 1993; Topol, 1991) or can be grounded on microeconomic foundations (Orlean, 1995, Lux, 1995,1998, Laurent, 1995).[1]

The aim of this paper is to construct a model of exchange rate dynamics that explicitly includes mimetic contagion of opinion among traders. The model highlights the key role of heterogeneous expectations in portfolio decisions and ultimately in the determination of the exchange rate. Agents are all the more likely to mimic the behavior of others, the less they are confident in their forecast and the

[1] Note that the proportion of each category of investors in De Long (1990, 1991) and in DeGrauwe (1993) is not constant, but do not derive from some mimetic dynamics.

more the others obtain superior *ex-post* return on their investments. As in Brock and LeBaron (1996) and Lux (1995, 1998), the market is composed of heterogeneous agents which are then regrouped in different classes. Two groups of investors are differentiated here, according to two competing sources of heterogeneity. The first distinction rests on pessimistic/optimistic agents, whereas the second relies on confident/unconfident agents. Under these two alternative assumptions, model-based forecasts of the future exchange rate result from portfolio reallocations induced by the time-varying proportion of each class of agents.

In this study, this proportion, the average expectations of both groups and their respective standard deviations are inferred from the over-the-counter options market. The methodology consists first in recovering the probability density function (PDF) of the future exchange rate on the assumption that the PDF results from a weighted mixture of two lognormal distributions (Melick and Thomas, 1997). This flexible specification allows some stylized fact of exchange rate returns to be caught, which are the presence of skewness and leptokurtosis (Hsieh, 1988, Boothe and Glassman, 1987a). Intuitively these two lognormal densities can characterize the beliefs of the two classes of agents. Their relative weight is hence supposed to determine the time-varying proportion of each category of investors. Unlike most exchange rate models with heterogeneous expectations, the present specification enables us to assess the relevance of the heterogeneity assumptions by testing the out-of-sample predictive power of model-based forecasts. Such an empirical investigation is crucial when the aim is to achieve a good description of the short-term dynamics of the euro/dollar.

The paper is organized as follows: the first part describes the exchange rate model with heterogeneous beliefs, the second section presents the methodology used to infer from option prices, the heterogeneous expectations and their standard deviations as well as the proportions of each category of agents. The third section tests the out-of-sample predictive power of the model in order to discriminate between the heterogeneity assumptions and the final part concludes.

1- An exchange rate model with heterogeneous expectations

The economy is populated with N risk neutral agents, who face similar investment choices. At the beginning of the initial period, each agent is endowed with one resource unit denominated in US dollars[2]. In each period, all investors simultaneously have to decide to put their wealth either in the United States or in Europe. The US asset is risk free and pays a known rate r. The European asset pays a known rate r^*, but the expected return is uncertain. It depends on the change of the euro/dollar exchange rate over the investment period. The interest rates can be thought of as short-term inter-bank deposits.

To describe market dynamics arising from heterogeneous investment behavior, two alternative sources of heterogeneity among investors are successively introduced. In a first place, one category of agents is supposed to be systematically pessimistic about the evolution of the euro: they expect the euro to be lower than the forward exchange rate at maturity. This distinguishes optimistic versus pessimistic investors. Then, the second way to differentiate agents rests on the degree of confidence they have in their exchange rate anticipations. The group of confident agents displays a low dispersion in their expectations relative to other market participants. By convention, pessimistic and confident agents are denoted as type 1 agents, whereas optimistic and unconfident agents are of type 2.

Let θ_t be the proportion of agents 1 and $(1-\theta_t)$ the proportion of type 2 agents. $N_{1,t} = \theta_t N$ and $N_{2,t} = (1-\theta_t)N$ are the number of members of the first and the second group respectively. Furthermore $S_{i,t}^{t+\tau}$ and $\sigma_{i,t}^{t+\tau}$ denote the average expectation and the standard deviation of the euro/dollar formed at time t by agent i, $(i = 1,2)$, for maturity $(t+\tau)$, where the exchange rate is quoted as the amount of US dollars per one unit of euro. Agents are supposed to be able to observe the average forecast of both groups, but the forecasts' standard deviation of one group is an information available only for its own members.

[2] The results do not change with the denomination of wealth in US dollar or in euro.

Under the assumptions of perfect capital mobility and perfect substitutability of US and European assets, investment decisions depend on which expected return is greater. Individuals being risk neutral, the forward exchange rate for maturity $(t+\tau)$, denoted $F_t^{t+\tau}$, is thus an average of the expectations of both groups weighted by the parameter θ_t. If type 1 agents intend to invest all their wealth in the United States, expecting a lower appreciation (or possibly a higher depreciation) of the euro than implied in the forward exchange rate ($S_{1,t}^{t+\tau} < F_t^{t+\tau}$), type 2 individuals will invest in Europe ($S_{2,t}^{t+\tau} > F_t^{t+\tau}$). The induced flows of dollar sales or purchases depend thus on the proportion θ_t of pessimistic or alternatively of confident agents, and thus crucially on the heterogeneity of expectations.

This proportion fluctuates due to the possibility for all agents to switch from one category to the other at the end of each day. Although the time horizon for investments is $(t+\tau)$, investment decisions are reversible and agents are allowed to change their mind at any period *t*. Let $p_{1\to 2,t}$ (respectively $p_{2\to 1,t}$) be the probability for agents 1 (agents 2) of joining the group of agents 2 (agents 1) at the end of the period *t*, that is the probability of adopting the expectations and the behavior of agents 1 (agents 2).

The numbers of agents 1 at time *t* is equal to the number of agents 1 at time (*t*-1) minus the members who have left the group at the end of (*t*-1), plus the investors previously belonging to group 2 who have joined group 1: $N_{1,t} = N_{1,t-1}(1-p_{1\to 2,t-1}) + N_{2,t-1}p_{2\to 1,t-1}$. The same logic applies for agents 2: $N_{2,t} = N_{2,t-1}(1-p_{2\to 1,t-1}) + N_{1,t-1}p_{1\to 2,t-1}$. Hence the number of migrations from one group to the other can be approximated by the product of the members from one category at the beginning of the period by its probability to change (Lux, 1995). The dynamics of the market opinion can hence be expressed as follows:[3]

[3] Lux (1995) shows that equation (1) can be derived more formally as an approximate mean value equation for the original stochastic system using the Master equation approach. Equation (1) focuses only to the mean values and to the more probable development; it thus neglects the higher moments dynamics.

$$\theta_t = \theta_{t-1} + \frac{1}{N}\left[N_{2,t-1}p_{2\to1,t-1} - N_{1,t-1}p_{1\to2,t-1}\right] \quad (1)$$

The opinion of market participants is furthermore supposed to depend on two distinct sources of information. The first one is exogenous to the market and derives from their information set, which they know to be incomplete and from their exchange rate model, which they also know to be imperfect. The second source of information is endogenous to the market and derives from the observation of the market dynamics and from the comparison of the profitability of investment strategies related to the ability to form good forecasts.

The transition probabilities evolve according to the two following criteria. When deciding to stay in or go out of its group, each investor is influenced by the past average performances achieved by each group. The benefits of investment strategies are then considered as a measure of expected future profits. This induces herd behavior insofar as successful strategies will tend to be imitated by a growing number of investors (Sentana and Wadhwani, 1992; De Long *et al.*, 1990, 1991; Laurent, 1995; Lux, 1995, 1998). It is thus assumed that agents are able to observe past forecast errors of both groups and to compare the *ex post* portfolio returns. Let $\sigma_{i,t}^{pe}$ be the standard deviation of past forecast errors of group i's members. The following specification intends to highlight the importance of large forecast errors in the measure of forecasts accuracy.

$$\sigma_{1,t}^{pe} = \sqrt{\frac{1}{n}\sum_{k=0}^{n}\left(\ln(S_{1,t-k-\tau}^{t-k}) - \ln(S_{t-k})\right)^2} \quad (2)$$

$$\sigma_{2,t}^{pe} = \sqrt{\frac{1}{n}\sum_{k=0}^{n}\left(\ln(S_{2,t-k-\tau}^{t-k}) - \ln(S_{t-k})\right)^2}$$

The better the exchange rate forecasts, the lower the standard deviation. Agents displaying the greater dispersion $\sigma_{i,t}^{pe}$ will naturally be tempted to join the other group, which achieves better performances.

The second relevant criterion concerns the degree of confidence investors put in their exchange rate forecasts. Let $\sigma_{i,t}^{t+\tau}$ be the measure of the opinion diversity among a group of investors, that is the inverse of the precision attached to the average forecast. The standard deviation of expectations of one group is unknown for its competitors. Thus, the higher $\sigma_{1,t}^{t+\tau}$ for type 1 agents, the greater the incentive to discard their own average forecast and to rely on that of the other group, intending to gain some possible additional information not available to them. All agents are then potentially likely to adopt mimetic behavior. Finally, the weighting scheme entering the transition probabilities is supposed to be symmetrical for the two groups and to ensure that the probabilities lie between 0 and 1.

$$p_{1\to 2,t} = \lambda_{1,t}\sigma_{1,t}^{t+\tau} + \lambda_{2,t}\sigma_{1,t}^{pe} \qquad (3)$$
$$p_{2\to 1,t} = \lambda_{1,t}\sigma_{2,t}^{t+\tau} + \lambda_{2,t}\sigma_{2,t}^{pe}$$

These probabilities enter the calculus of capita wealth. At any period t, the wealth per capita is directly related to the history of both the past investments in Europe and in the United States and to the past migrations among the two categories. The aggregate resources of each group at the beginning of period $t+1$ depend on their previous capitalized wealth and on migrations operated at the end of the preceding period. Equations (4) describe the wealth accumulation process for both groups.

$$N_{1,t+1}W_{1,t+1} = \begin{cases} N_{1,t}W_{1,t}(1+r_t)^{\frac{1}{365}}(1-p_{1\to 2,t}) + N_{2,t}W_{2,t}(1+r_t^*)^{\frac{1}{365}}p_{2\to 1,t}\dfrac{S_{t+1}}{S_t} & if \quad S_{1,t}^{t+\tau} < F_t^{t+\tau} < S_{2,t}^{t+\tau} \\ N_{1,t}W_{1,t}(1+r_t^*)^{\frac{1}{365}}(1-p_{1\to 2,t})\dfrac{S_{t+1}}{S_t} + N_{2,t}W_{2,t}(1+r_t)^{\frac{1}{365}}p_{2\to 1,t} & if \quad S_{2,t}^{t+\tau} < F_t^{t+\tau} < S_{1,t}^{t+\tau} \end{cases}$$

(4)

$$N_{2,t+1}W_{2,t+1} = \begin{cases} N_{2,t}W_{2,t}(1+r_t^*)^{\frac{1}{365}}(1-p_{2\to 1,t})\dfrac{S_{t+1}}{S_t} + N_{1,t}W_{1,t}(1+r_t)^{\frac{1}{365}}p_{1\to 2,t} & if \quad S_{1,t}^{t+\tau} < F_t^{t+\tau} < S_{2,t}^{t+\tau} \\ N_{2,t}W_{2,t}(1+r_t)^{\frac{1}{365}}(1-p_{2\to 1,t}) + N_{1,t}W_{1,t}(1+r_t^*)^{\frac{1}{365}}p_{1\to 2,t}\dfrac{S_{t+1}}{S_t} & if \quad S_{2,t}^{t+\tau} < F_t^{t+\tau} < S_{1,t}^{t+\tau} \end{cases}$$

Portfolio reallocations at each period induce net selling or buying waves of US dollars. Insofar as all trades have to be carried out within the two classes of agents, the supply of dollars from one group at any period t must exactly match the demand of the other group. A market clearing device prevents net excess supply or demand of dollars[4], that is:

$$D_t = \left(N_{1,t+1}W_{1,t+1} - N_{1,t}W_{1,t}\right) - \left(N_{2,t+1}W_{2,t+1} - N_{2,t}W_{2,t}\right) = 0 \tag{5}$$

This leads to a forecast of the exchange rate for the next period:

$$\widetilde{S}_t^{t+1} = \begin{cases} S_t \dfrac{\left[N_{1,t}(2p_{1\to 2,t} - 1)W_{1,t}(1+r_t)^{\frac{1}{365}} + N_{1,t}W_{1,t} - N_{2,t}W_{2,t}\right]}{N_{2,t}(2p_{2\to 1,t} - 1)W_{2,t}(1+r_t^*)^{\frac{1}{365}}} & if \quad S_{1,t}^{t+\tau} < F_t^{t+\tau} < S_{2,t}^{t+\tau} \\ \\ S_t \dfrac{\left[N_{2,t}(1 - 2p_{2\to 1,t})W_{2,t}(1+r_t)^{\frac{1}{365}} + N_{1,t}W_{1,t} - N_{2,t}W_{2,t}\right]}{N_{1,t}(1 - 2p_{1\to 2,t})W_{1,t}(1+r_t^*)^{\frac{1}{365}}} & if \quad S_{2,t}^{t+\tau} < F_t^{t+\tau} < S_{1,t}^{t+\upsilon} \end{cases} \tag{6}$$

where $\widetilde{S}_t^{t+1}$ denotes the exchange rate predicted by the model at time t for t+1. Equation (6) describes how the opinion of different groups of investors may impact on the short term market dynamics. Tests of predictive power can thus be undertaken in order to assess the validity of the two alternative hypotheses about the heterogeneity of agents. Whereas most exchange rate models with interacting agents, heterogeneous beliefs or mimetic contagion can only be simulated, the model proposed here allows empirical investigations. Using information from the over-the-counter euro/dollar options market, heterogeneous expectations of the future euro/dollar exchange rate, their respective standard deviations as well as the time-varying proportions of each category of agents can then be recovered.

2- Options and expectations of the future exchange rate

Options, whose payoff depends on a limited range of the expected exchange rate, offer broader information about market expectations than the forward exchange rate. Whereas the latter provides an indication about the mean of the distribution of the expected exchange rate, the entire probability

[4] Lux (1995) supposes that prices adjust in finite time in the presence of excess demand or supply and relies on a market

density function can be inferred from option prices. Intuitively, let's take two options with the same maturity but with different adjacent strike prices. The difference between the two option prices gives information about the probability of the asset price lying in the interval defined by the two strike prices at the expiration of the options. Combining all the option quotes on the same exchange rate for a given maturity reveals the risk neutral probability associated with each terminal expected value, and thus allows the probability density function of the future exchange rate to be recovered.

2-1 Risk neutral probability density function of the future exchange rate

A call (put) option gives the right but not the obligation to its holder to buy (to sell) a certain amount of foreign currency, at a given strike price and at a predetermined date. Unlike the American options, which can be exercised any time prior to their expiration date, European ones, used here, can solely be exercised at the maturity of the option. A European call option is said to be out-of-the-money (respectively in-the-money) if the underlying asset price at maturity lies below (beyond) the strike price or if the delta is inferior (superior) to 0.5.[5] For at-the-money options, the strike price is equal to the forward exchange rate and the delta is roughly equal to 0.5.

Recovering the PDF of the expected exchange rate from option prices consists in deriving a relationship between the option valuation formula and the underlying PDF. Breeden and Litzenberger (1978) show that the second derivative of a call valuation formula is directly proportional to the risk neutral PDF; the formula must verify some monotonicity and convexity conditions and must be twice differentiable in strikes.

The payoff at maturity T of a European call with a strike price K is $\max(S_T - K, 0)$, where S_T is the exchange rate at time T. Let $C(S_t, t, T, K)$ be the value of a European call:

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maker to avoid temporary rationing.

[5] The delta is a measure of the sensitivity of the option price with respect to a small variation of the underlying price. Mathematically, the delta of a currency option is the first derivative of the call formula with respect to the exchange rate. In other words, the delta of an option is a metric for moneyness, that is, it provides a measure of the amount by which the option is away from the money.

$$C(S_t,t,T,K)=e^{-r(T-t)}E^*[\max(S_T-K,0)]$$
$$=e^{-r(T-t)}\int_K^{\infty}(S_T-K)f(S_T,t,T,S_t)dS_T \qquad (7)$$

r is the (annualized) risk free domestic interest rate over the period (*T-t*), E^* denotes expected value in a risk neutral world and $f(S_T,t,T,S_t)$ is the risk neutral density function of the future exchange rate S_T, conditional on the current exchange rate S_t. The first derivative of equation (8) is proportional to the risk neutral cumulative distribution function $F(K)$:

$$\frac{\partial C(S_t,t,T,K)}{\partial K}=-e^{-r(T-t)}\int_K^{\infty}f(S_T,t,T,S_t)dS=-e^{-r(T-t)}(1-F(K)) \qquad (8)$$

Finally, the PDF of the future exchange rate, evaluated at $K=S_T$, can be expressed as being proportional to the second derivative of the call valuation formula. The coefficient of proportionality is the present value of a zero coupon bond paying one domestic monetary unit at maturity, the discount rate being the risk free interest rate.

$$f(S_T,t,T,S_t)=e^{r(T-t)}\frac{\partial^2 C(S_t,t,T,S_T)}{\partial K^2}\bigg|_{K=S_T} \qquad (9)$$

This risk neutral PDF differs from the true PDF the market participants have in mind when they quote option prices, because it incorporates attitudes towards risk, in addition to beliefs about future outcomes. The PDF recovered from option prices is influenced by risk premia just as forward exchange rates are, so that the interpretation of the PDF's changes over time can be distorted by changes in risk premia. However, on the assumption that the risk premium is relatively constant over time, the changes of expectations are quite well approximated.

Equation (9) shows that the PDF can be derived without any assumption about the exchange rate dynamics and with no restriction about agents' preferences and beliefs. It requires either a continuous, twice-differentiable call valuation formula or at least a continuum of option prices with the same time-to-expiration with strike prices from zero to infinity. Unfortunately, the range of observed option prices is limited and some procedures must be implemented to interpolate between the observed quotes and extrapolate beyond the highest available strike for calls, and below the lowest strike for puts.

Unlike the Black and Scholes (1973) model, several methods based on the Breeden and Litzenberger Theorem (1978) are able to take into account the skewness and kurtosis of the expected exchange rate implicit in the option prices. Bahra (1997) surveys different approaches that can be found in the literature whose aim is the inference of the risk neutral PDF. Here the applied method consists in postulating a functional form for the PDF and minimizing the distance between the theoretical prices stemming from such a hypothesis and the observed option prices.

2-2 A mixture of two lognormal densities as a candidate for the risk neutral PDF

Any candidates can be specified *a priori* for the PDF in equation (7). For example, the implicit density can be issued from a Burr distribution (Sherrick, Garcia and Tirrapur, 1996) or from a mixture of independent lognormal densities (Ritchey, 1990; Melick and Thomas, 1997; Leahy and Thomas, 1996; Mizrach, 1996; Bahra, 1996, 1997). As Melick and Thomas note, this method is more general than those which postulate a process on the underlying asset dynamics: a given stochastic process involves a unique terminal distribution, but the reciprocal is not true; any PDF is consistent with several different processes for the underlying asset.

The call valuation formula under the assumption that the risk neutral PDF implicit in observed option prices is a weighted sum of k lognormal densities can be written as follows:

$$C(K,t,T)=e^{-r(T-t)}\int_K^{\infty}\left\{\sum_{i=1}^{k}\theta_i\frac{1}{\sqrt{2\pi}K\beta_i}e^{-\frac{1}{2}\left[\frac{\ln K-\alpha_i}{\beta_i}\right]^2}\right\}(S_T-K)dS_T \qquad (10)$$

where α_i and β_i are the parameters defining the lognormal distributions. The factor weighting the distributions meets the following condition $\sum_{i=1}^{k}\theta_i=1$, where $0\leq\theta_i\leq1$, $\forall i$. To avoid numerical integration errors due to the upper limit of infinity, the call valuation formula can be rewritten with the cumulative normal distribution rather than with the lognormal density (Bahra, 1997).

$$C(K,t,T) = e^{-r(T-t)} \sum_{i=1}^{k} \theta_i \left\{ e^{\alpha_i + \frac{1}{2}\beta_i^2} N\left(\frac{-\ln K + \alpha_i + \beta_i^2}{\beta_i} \right) - KN\left(\frac{-\ln K + \alpha_i}{\beta_i} \right) \right\} \tag{11}$$

The choice between a mixture of two or three lognormal densities depends to a large extent on the availability and reliability of option quotes on the euro/dollar with the same maturity. Söderlind and Svensson (1997) highlight some problems raised by the estimation of the PDF when the quality of data is questionable. Relative illiquidity for deep in and out-of-the money options or lack of synchronization across strike prices can lead to an impossible sharply spiked PDF (Bahra, 1996, 1997). Such a problem arises because the convergence of the optimization program can be achieved for different sets of parameters. To limit these shortcomings, two-lognormal distributions would be better, to the extent that only five parameters have to be estimated as opposed to eight in the case of a mixture of three lognormal densities. The value of the five parameters changes daily, reflecting changes in beliefs about the future euro/dollar exchange rate and possibly also changes in risk premia. On the assumption that the latter is negligible or at least constant, the variation of the estimated parameters will be interpreted as revealing a modification of expectations.

2-3 Data and estimation procedure

The data used to infer the probability density functions cover the period from January 4, 1999 to September 26, 2000 and consist of European-style OTC options drawn from the J.P. Morgan Internet site.[6] The choice of OTC options relative to Exchange-traded options stems from the superior liquidity of OTC currency trading. Another advantage with OTC options stems from the fixed time to expiration, unlike Exchange-traded options for which the time to maturity progressively decreases with time. These OTC options are quoted in implied volatilities according to different deltas. These volatilities are a convenient way to express the options' price, the volatility being the only

[6] http://www.jpmorgan/fx.com.

unobservable parameter in the Garman and Kohlhagen (1983) formula. On a day-to-day basis, options are quoted for several times to expiration (i.e. from overnight to one year). For each maturity, the available data are strangles and risk reversals[7] with delta 0.10 and 0.25 and at-the-money-forward implied volatility. These quotes allow a volatility smile with five observations to be recovered, the implied volatility of call options ranging from delta 0.1, 0.25, 0.5, 0.75 to 0.9. These indicative quotes are averages of the bid and ask spread and do not coincide with transaction prices. The one and three-month maturity is considered here, the liquidity of shorter-term options being of significantly less magnitude. The series of the euro/dollar exchange rate, US and European interest rates are those prevailing when the option prices were reported by J.P. Morgan.

Linear least squares have been used to estimate the five parameters $\{\alpha_1, \beta_1, \alpha_2, \beta_2, \theta\}$ with the GAUSS software and the OPTMUM module. The procedure minimizes the squared distance between the observed option prices and the theoretical prices stemming from the mixture of lognormal densities. Because OTC options are quoted in implied volatilities relative to deltas, in a first stage, each delta has to be converted into a strike price and each implied volatility quote into an option price. The delta is a strictly increasing function with respect to the strike price, so that a unique strike price corresponds to a given delta. The strike prices have thus been recovered using a non-linear optimization procedure (NLSYS module). To obtain the option prices from the volatility quotes, the Garman and Kohlhagen (1983) formula is used, not as a pricing model but as conversion formula, just like the common practice of traders.

In the absence of arbitrage opportunities, the forward exchange rate should equal the mean of the PDF, given by the weighted average of the two lognormal distributions means:

$$F_t^{t+\tau} = S_t e^{(r-r^*)(T-t)} = \theta e^{\alpha_1+\frac{1}{2}\beta_1^2} + (1-\theta)e^{\alpha_2+\frac{1}{2}\beta_2^2} \qquad (12)$$

[7] The risk reversal quote is the difference between the call and the put volatilities, both options being out-of-the-money (delta 0.10 or 0.25) and with the same expiration date. The strangle quote is calculated as the average volatility of the out-of-the-money call and put volatilities less the delta neutral volatility. Strangles refers to the strategy of buying a put and a call with the same expiration date and different strike prices (delta 0.10 or 0.25).

This constraint is used in the minimization process, so that at least four implied volatility quotes are necessary to solve for the five parameters. Although five option quotes on the euro/dollar exchange rate are available, the convergence is difficult to achieve. For numerical purposes starting values have thus been derived from a first set of estimations involving cubic splines.[8]

Furthermore, in order to deal with the two possible sources of heterogeneity, the optimization procedure is successively implemented under one of the two following constraints. First because type 1 agents are assumed to be more pessimistic about the future value of the euro than type 2 agents, the constraint $S_{1,t}^{t+\tau} \leq S_{2,t}^{t+\tau}$ is imposed, where $S_{1,t}^{t+\tau}$ and $S_{2,t}^{t+\tau}$ are the means of the two lognormal distributions of the euro/dollar expected at maturity T, defined by equations (13)[9]:

$$
\begin{aligned}
S_{1,t}^{t+\tau} &= E_{1,t}(S_T) = e^{\alpha_1+\frac{1}{2}\beta_1^2} \\
S_{2,t}^{t+\tau} &= E_{2,t}(S_T) = e^{\alpha_2+\frac{1}{2}\beta_2^2}
\end{aligned}
\qquad (13)
$$

The second constraint takes into account the fact that agents 1' expectations are supposed to display a lower dispersion than agents 2' anticipations: $\sigma_{1,t}^{t+\tau} < \sigma_{2,t}^{t+\tau}$, where $\sigma_{1,t}^{t+\tau}$ and $\sigma_{2,t}^{t+\tau}$ are standard deviations of the two lognormal densities:

$$
\begin{aligned}
\sigma_{1,t}^{t+\tau} &= \sqrt{Var_{1,t}(S_T)} = \sqrt{S_t^2 e^{2\left(\alpha_1+\frac{1}{2}\beta_1^2\right)}\left[e^{\beta_1^2}-1\right]} \\
\sigma_{2,t}^{t+\tau} &= \sqrt{Var_{2,t}(S_T)} = \sqrt{S_t^2 e^{2\left(\alpha_2+\frac{1}{2}\beta_2^2\right)}\left[e^{\beta_2^2}-1\right]}
\end{aligned}
\qquad (14)
$$

Figure 1 (Appendix1) displays the one-month risk neutral PDF of the euro/dollar as well as its two lognormal components on January 4, 1999.[10] The lognormal distribution with the higher weight

[8] The cubic splines method makes it possible to interpolate between the implied volatility quotes, thus providing more observations for the minimisation procedure. It connects two points on the smile by a polynomial function of order three, imposing that at each point where the two polynomial functions meet, the first derivatives of the two functions are equal and differentiable. This ensures that the first derivative is continuous over the range of the strike prices. Furthermore, the second derivative at the first and last points is supposed to be equal to zero (Campa, Chang and Reider, 1997).

[9] When x follows a normal distribution, $E\left(e^x\right) = e^{E(x)+\frac{1}{2}\sigma^2(x)}$.

[10] The more distant the horizon, the flatter the PDF and the more diffuse the expectations of the future exchange rate. This results from a market convention: OTC options are quoted according to constant deltas instead of strike prices, so that when maturity grows, strikes corresponding to a given delta go further away from the forward exchange rate. The range of the future exchange rate widens mechanically with the maturity.

($\theta_i = 0.8$) localizes the PDF. It has a low standard deviation, whereas the density with the lower weight that catches the skewness and kurtosis of the PDF presents a greater dispersion of the expected exchange rates. This latter distribution being located to the right of the former, the PDF is skewed to the right: the median lies below the forward exchange rate. A depreciation of the euro relative to the forward is expected with a probability superior to ½.

Figures 2 (Appendix 1) display the proportion of type 1 agents according to the two distinctions about heterogeneity at a one and three-month horizon. Over the period January 4, 1999 to September 26, 2000, this average proportion is superior to ½ indicating that pessimistic and confident agents were predominant in the euro/dollar market.[11]

3- Out-of-sample exchange rate forecasting

In this section, we compare the *ex ante* forecasting performance of model-based forecasts of the exchange rate under the two competing hypotheses about heterogeneous expectations for the two maturities. The out-of-sample framework appears the most appropriate to compare models or alternative assumptions. Model-based forecast at time t for time t+1 has thus been estimated on the basis of the most up-to-date available information at the time of a given forecast. To generate exchange rate forecasts based on equation (6), the only unknown parameters at each time t are the weights λ_1 and λ_2 entering the transition probabilities. Indeed, the information set at time t is composed of variables directly observable, like the interest rates, the exchange rate, the standard deviations of forecast errors and of exchange rate expectations inferred from the options market, while the other variables, like the migration probabilities and the wealth can easily be calculated once the λ parameters have been estimated. These latter parameters are initially estimated using data up through the first forecasting period, November 4, 1999. Actually, the full sample, January 4, 1999 to

[11] Average proportions are respectively 56.06% and 56.69% for the pessimistic investors at a one and three month horizon, and 58.46% and 56.93% for the confident agents. The variability of the series is more pronounced for the one-month maturity than for the tree-month horizon.

September 26, 2000, has been divided by two in order to have sufficient degree of freedom for the initial parameter estimates. The function to be minimized under the constraint that the probabilities are positive and inferior to 1 is given by equations (1) and (3) :

$$\underset{\lambda_{1,t-1},\lambda_{2,t-1}}{Min} \sum_{j=t_0+\tau+1}^{t} \left\{\theta_j - \theta_{j-1} - \lambda_{1,j-1}\left[(1-\theta_{j-1})\sigma_{2,j-1}^{j+\tau-1} - \theta_{j-1}\sigma_{1,j-1}^{j+\tau-1}\right] - \lambda_{2,j-1}\left[(1-\theta_{j-1})\sigma_{2,j-1}^{pe} - \theta_j\sigma_{1,j-1}^{pe}\right]\right\}^2 \quad (15)$$

where t_0 denotes the initial date, January 4, 1999. The size of the estimation sample depends on the considered maturity, τ, due to the calculus of forecast errors. It begins in February 4, 1999 for the one-month maturity and in April 4, 1999 for the three-month horizon. The λ parameters are then re-estimated over a growing sample accounting for new observations each day. This leads to a series of 209 estimates. The λ_1 and λ_2 parameters, as well as the other variables entering the migration probabilities (eq. 3) are all indexed by *t*. However, according to equation (15), their estimation based on information available at time *t* leads to parameters indexed by (*t*-1) and not by *t*. To generate exchange rate forecasts by equation (6), it is thus assumed that the λ parameters are those estimated accounting for information until time *t*, that is those indexed by (*t*-1).

In a first stage to assess out-of-sample predictive accuracy, three statistics are calculated for each forecaster: the mean forecast error (*ME*), the mean absolute error *(MAE)* and the root mean squared error (*RMSE*).

$$ME = \frac{1}{N}\sum_{t=n_0}^{N}\left(\tilde{S}_t^{t+1} - S_{t+1}\right)$$

$$MAE = \frac{1}{N}\sum_{t=n_0}^{N}\left|\tilde{S}_t^{t+1} - S_{t+1}\right| \quad (16)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=n_0}^{N}\left(\tilde{S}_t^{t+1} - S_{t+1}\right)^2}$$

where n_0 is the first forecasting period, November 4, 1999, $\tilde{S}_t^{t+1}$ denotes the exchange rate predicted at time *t* for time *t*+1 and S_{t+1} is the observed rate. Results are summarized in Table 1. It is straightforward for both maturities that the model based on confident vs. unconfident agents achieves better accuracy than if we consider the market as being composed of optimistic vs. pessimistic individuals. The results between maturities are more ambiguous. The *MAE* and the *RMSE* appear slightly lower for the three-month horizon, but it is not possible to conclude if the performances are significantly different with these criteria. Diebold and Mariano (1995) suggest to use the sign test instead of the *RMSE*. The null hypothesis is a zero median loss differential. Assuming that the latter series is independently and identically distributed, the number of positive loss differential observations in a sample of size N has a binomial distribution with parameters N and ½ under the null hypothesis. The test statistics is defined as follows:

$$S = \sum_{t=n_0}^{N} I_+(d_t), \quad I_+(d_t) = \begin{cases} 1 & if \quad d_t > 0 \\ 0 & otherwise \end{cases} \tag{17}$$

where $d_t = (\widetilde{S}'_{1M,t-1} - S_t)^2 - (\widetilde{S}'_{3M,t-1} - S_t)^2$, $\widetilde{S}'_{1M,t-1}$ and $\widetilde{S}'_{3M,t-1}$ denote the exchange rate predicted by the model where agents form their expectations at a one and three-month maturity respectively. The cumulative binomial distribution can be used to assess the significance or the normal distribution in large sample for the studentized version of the sign test.

$$S_a = \frac{S - \frac{1}{2}N}{\sqrt{\frac{1}{4}N}} \sim \mathrm{N}(0,1) \tag{18}$$

The S and S_a statistics indicate that the accuracy performances between the one and three month horizon for both distinctions about heterogeneity are not significantly different. Furthermore, if we compare the forecast precision of model-based forecasts to that of a random walk, it appears that the random walk is much more accurate than the model with the pessimistic vs. optimistic agents, for both maturities. However, no significant difference emerges between the random walk and the model based on the confident vs. unconfident agents.[12]

[12] The S_a statistics for the one-month horizon is 0.76 and 8.92 when the distinction relies respectively on confident vs. unconfident and on pessimistic vs. optimistic investors. At a three-month horizon, this statistics is 1.72 and 9.2 respectively.

Table 1: Comparisons of alternative out-of-sample exchange rate forecasts

	Model-based Forecasts			
	One-month		Three-month	
	Confident/ Unconfident	Pessimistic/ Optimistic	Confident/ Unconfident	Pessimistic/ Optimistic
ME	0.00081	0.00004	0.00046	-0.00154
MAE	0.00545	0.03758	0.00541	0.02520
RMSE	0.00709	0.05921	0.00696	0.03536
S	103	115		
S_a	-0,2075	1,4526		
F	53.11%	51.20%	50.24%	46.41%
G1	0.10095	0.10068	0.01361	-0.07795
G2	1.00259	1.00674	0.96013	0.91690
G3	1.00258	0.99978	0.96013	0.91055

Note : 209 out-of-sample forecasts, from November 4, 1999 to September 26, 2000.

Although comparisons of forecasts accuracy is important to discriminate among alternative models or hypotheses, an accurate forecast in the *RMSE* sense may be unprofitable in financial transactions, where the speculative profit depends on the direction of the forecast error. Model-based forecasts can also be used as a guide to speculative opportunities (MacDonald and Marsh, 1996). In addition to the frequency with which the model predicts the right variation of the exchange rate at time t for t+1, denoted F, we also consider simple measures of profitability. Let consider an investor who uses the exchange rate predicted by the model to speculate in the foreign exchange market. Two speculative strategies are considered. The first one involves buying or selling the current forward exchange rate, expressed in units of US dollar per euro, depending on the model-based forecast (Boothe and Glassman, 1987b). The size of the forward contract is assumed to be one euro, whatever the expected payoff. The investor is supposed to buy the overnight forward exchange rate if the exchange rate forecast is greater than the forward, expecting to sell the euro with a positive profit the next period. He will sale the euro forward in the opposite case. Its final gain denoted G1 over the investment period, November 1999 to September 2000, results from the aggregation of its daily benefits or losses from each forward contract.

In the second strategy, we assume that the investor is initially endowed with 1 US dollar. He invests in Europe (respectively in the United States), that is, he adopts a buy and hold strategy, if the exchange rate forecast is higher (lower) than the forward exchange rate. Its final return at the end of the investment period can be calculated in two ways. First, the continuously compounded return, denoted G2, is converted daily in US dollar. Alternatively, if the strategic rule indicates to invest in the same currency for consecutive periods, the return calculation accounts for the variation of the exchange rate only when one switches from one currency to the other. In this case, the final return expressed in US dollar is denoted G3.

Forecasts based on the model where agents form expectations at a one-month horizon allow to predict the right sign of the exchange rate variation the next day with a frequency equal to 53.11% when the distinction confident vs. unconfident agents is considered. This proportion turns out to 51.20% when heterogeneity rests on pessimistic vs. optimistic investors. At the three-month maturity, the frequency of well predicted exchange rate changes falls respectively to 50.21% and 46.41%. Furthermore, using model-based forecasts to speculate in the foreign exchange market appears profitable essentially when the model involves the one-month maturity. Actually, the profit related to the three-month horizon remains positive only for the first strategy and when heterogeneity relies on confident vs. unconfident agents. In all the other cases, the investor looses money following the predicted exchange rate arising from the three-month horizon.

Whereas the accuracy performance is clearly better for the model with confident vs. unconfident agents, the speculative profits do not differ significantly with respect to the heterogeneity assumptions for the one-month maturity. Hence, the model which best fits the short term dynamics of the euro/dollar market is based on confident vs. unconfident agents forming their expectations at a one-month maturity. Figure 3 (Appendix 1) displays their migration probabilities. Over the period November 4, 1999 to September 26, 2000, less confident agents display a higher average probability of discarding their own exchange rate forecast and of joining the competitors than confident agents: 5.3% as opposed to 7.2%.

Conclusion

The paper presents a model of exchange rate dynamics that explicitly includes mimetic contagion of opinion among two groups of agents. The model highlights the key role of heterogeneous expectations in portfolio decisions. Agents know that they are imperfectly informed and that their knowledge of the underlying model of the economy is incomplete. They are all the more likely to mimic the behavior of their competitors, the less they are confident in their forecast of the future exchange rate and the more the others agents obtain superior *ex-post* return on their investments. Hence mimetic contagion induces a time-varying proportion of the two classes of agents that leads to portfolio reallocations

Two alternative sources of heterogeneity are furthermore considered. The first distinction rests on pessimistic/optimistic agents, whereas the second relies on confident/unconfident agents. Unlike most exchange rate models with heterogeneous expectations and mimetic contagion, the present specification makes it possible to directly test the predictive power of the model and the relevance of the heterogeneity assumptions. The paper uses information from the over-the-counter currency options market to infer heterogeneous expectations of both classes of agents. The approach consists first in estimating the probability density function of the future exchange rate on the assumption that it results from a weighted mixture of two lognormal densities. These two distributions are supposed to characterize the beliefs of the two groups, the weighting factor catching the respective proportion of each group. The estimated proportions of the two classes of agents are far from being constant. This result shows up the weakness of specifications relying on fixed proportions of traders to describe short-term dynamics of market prices (Frankel and Froot, 1988).

In order to discriminate between the two alternate assumptions about heterogeneity, we compare the out-of-sample predictive power of model-based forecasts over the period November 4, 1999 to September 26, 2000, resulting from portfolio reallocations. It clearly appears that the model with confident vs. unconfident investors forming their expectations at a one-month horizon (against a three-month horizon) is the most relevant hypothesis to describe the short-term dynamics of the euro/dollar market. The estimated migration probability is proved to be much higher for unconfident agents than for confident investors. This result tends to validate theoretical choices on microeconomic foundations of mimetic contagion (Orléan, 1992) : agents are all the more prone to adopt herd behavior as their confidence in their information and in their assessment of the fundamentals is weak.

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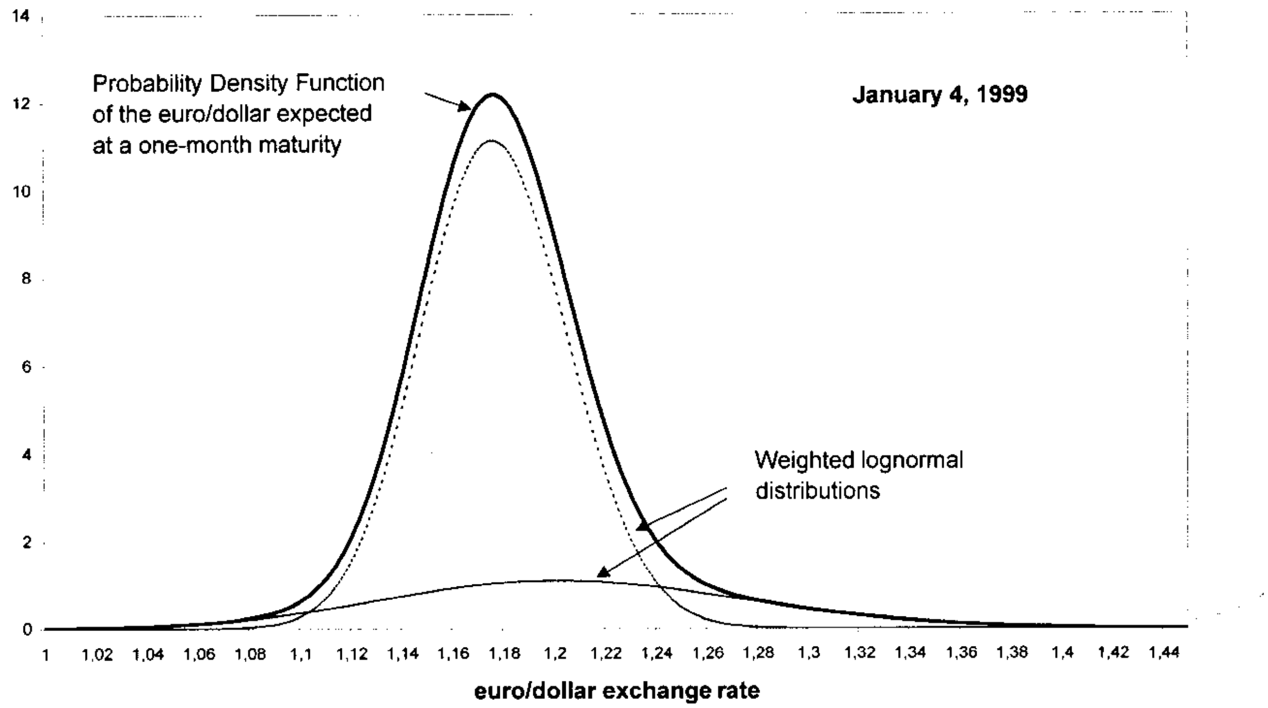
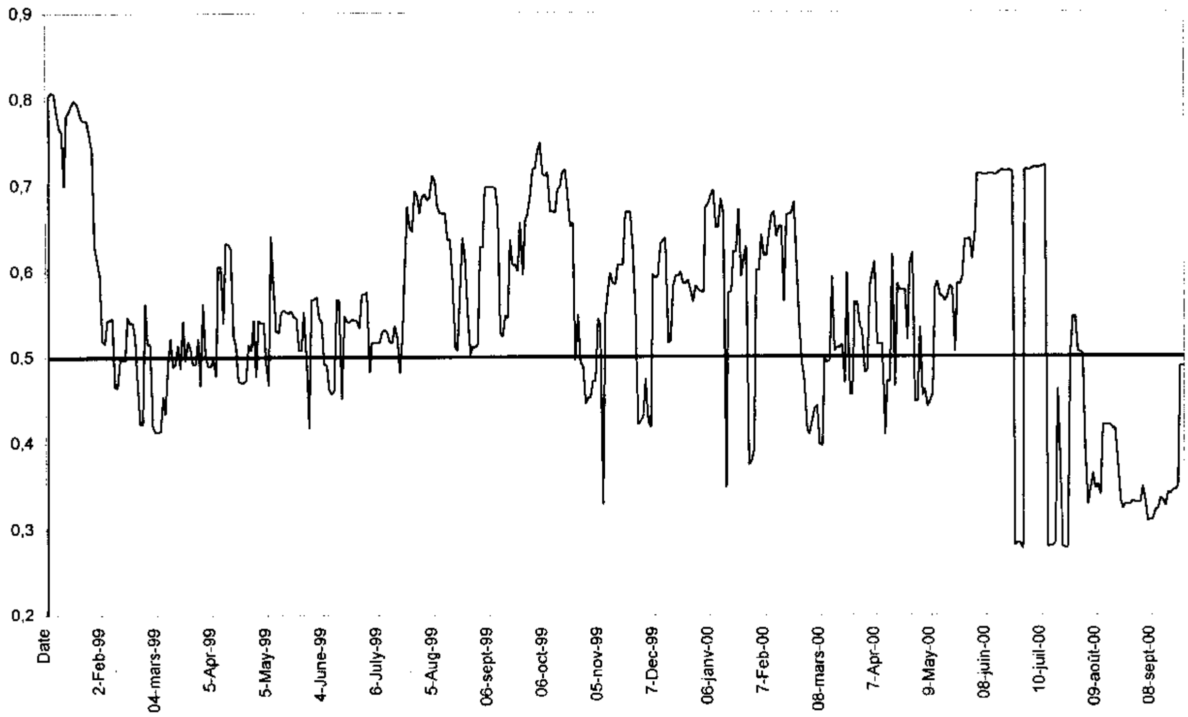
Appendix 1

Figure 1- Risk neutral Probability Density Function of the euro/dollar

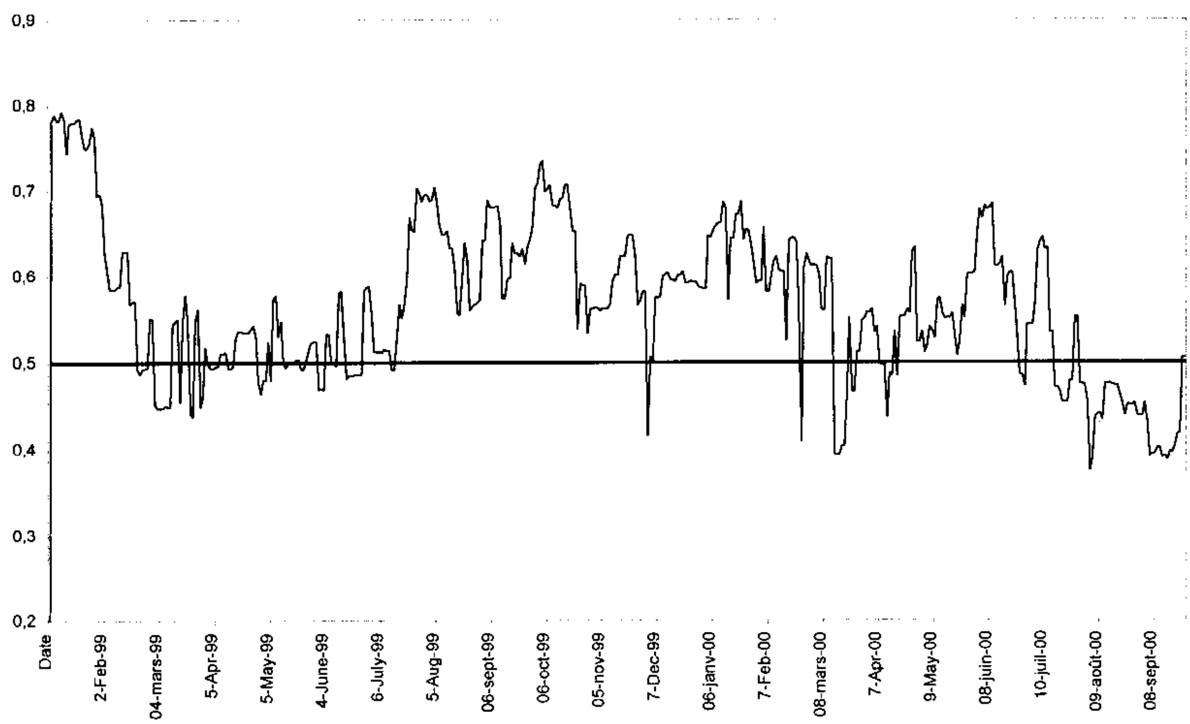


Figures 2- Proportion of pessimistic agents

One-month maturity

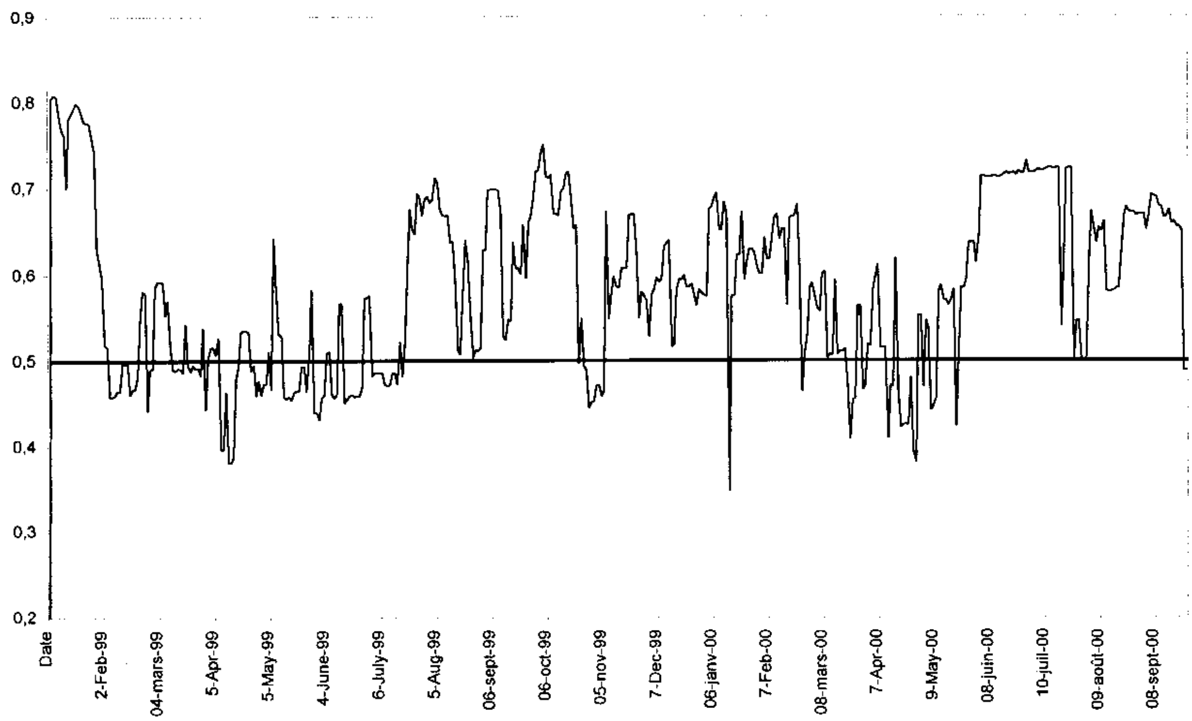


Three-month maturity



Proportion of confident agents

One-month maturity



Three-month maturity

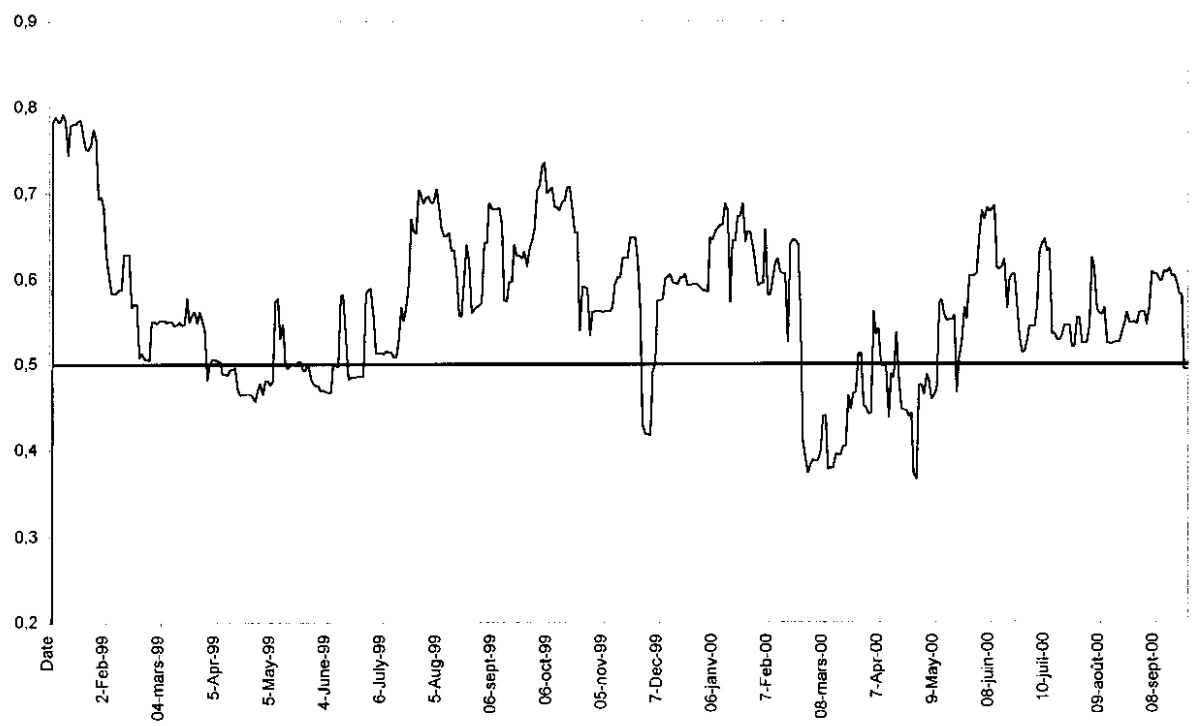


Figure 3- Migration probabilities
Confident vs. unconfident agents
One-month maturity