June 3, 2004

Optimal Monetary Policy for the ECB

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Draft version: 3 June 2004

Abstract

The effectiveness of monetary policy in the euro area may be enhanced by paying attention to national information, as opposed to reacting exclusively to aggregate area-wide variables. Even though the objectives of the ECB are framed exclusively in area-wide terms, the fact that the economies of the euro-area are characterized by significant structural differences and asymmetric shocks across countries can make neglecting national developments very costly. The results of this paper suggest that an optimal monetary policy rule should take into consideration not only the relative size of the countries (relative output or population), but also the structural characteristics of the economies. In order to substantiate this claim, we analytically derive an optimal interest rate reaction function of the central bank of the union by minimizing its loss function subject to different structural model in each country.

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1 Monetary Policy in EMU

On January 1, 1999, the third and final stage of the EMU began with the establishment of a currency union encompassing 11 of the 15 member countries. Greece became the 12th member on January 1, 2001. Participating countries are subject to a centralized monetary policy conducted by the European Central Bank (ECB) and to a common currency. On announcing the monetary policy strategy in October and December 1998, the ECB expressed its intention to adhere to a strict interpretation of its mandate to guarantee price stability, as defined in the Maastricht Treaty (article 105 (1)).[1] The Treaty (article 2)[2] further stipulates other objectives of the common monetary policy, which, however, should not conflict with the primary objective of price stability. The present official view is that in order to pursue its objectives, the ECB should only consider aggregate (i.e. euro-area wide) data, neglecting national idiosyncrasies. According to the ECB Monthly Bulletin, January 1999, "*its decisions will be based on an assessment of developments in the euro area as a whole. [...] policy decisions must be made in a manner that reflects conditions across the euro area in its entirety, rather than specific regional or national developments*".

Research on the monetary policy strategy of the ECB has increased in recent years. Most researchers focus mainly on the specification of the appropriate monetary policy rule and the improvement of efficiency that can be achieved with this specification. However, little attention has been given to the method of data aggregation and the importance of national idiosyncrasies for the success of the common monetary policy. Given the asymmetries found among the EMU member countries, the choice of the ECB to react to weighted aggregate data, though fair, seems inappropriate. A thorough examination of the proper incorporation of national information in monetary policy decisions, turns out to be very significant.

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[1] "*The primary objective of the ESCB shall be to maintain price stability. Without prejudice to the objective of price stability, the ESCB shall support the general economic policies in the Community with a view to contributing to the objectives of the Community as laid down in Article 2.*"

[2] "*The Community shall have as its task ... to promote throughout the Community a harmonious and balanced development of economic activities, sustainable and non-inflationary growth respecting the environment, a high degree of convergence of economic performance, a high level of employment and of social protection, the raising of the standard of living and quality of life, and economic and social cohesion and solidarity among Member States.*"

The importance of considering aggregate information when the ECB decides on monetary policy, has been studied empirically by Wieland (1996) who compares an asymmetric regime, under which one country (i.e. Germany) conducts monetary policy for the whole union (which includes France, Italy and the UK) by targeting domestic variables (nationalistic perspective), and a symmetric regime, under which the center country (Germany) targets union-wide variables (consensus perspective). The simulation results show that the asymmetric regime allows the reduction of price and output variability in the German economy, but the other countries bear the full adjustment burden. In contrast, under the symmetric regime, the negative impact on the other countries of the union is alleviated, at the cost of larger output and inflation variability in Germany. Similar results are found in Taylor (1999), who examines the efficiency of the ECB's monetary policy, by simulating two different policy rules, the first with country-specific data (i.e. of Germany) and the other with union weighted average data (i.e. of Germany, France and Italy). The author finds that the more symmetric rule increases slightly the impact of shocks in Germany, but reduces it in France and Italy, compared with the country-specific rule. Thus, the more symmetric rule has a minor effect on German inflation, but improves inflation performance in the rest of the countries.

Further research by Aksoy, De Grauwe and Dewachter (2001) and Aksoy, De Grauwe and Dewachter (2002) emphasized the prominent role played by national central bank Governors within the Governing Council of the ECB and examine welfare implications of alternative decision procedures, namely whether the governors adopt a nationalistic or consensus (union-wide) perspective. Similar studies by De Grauwe (2000), De Grauwe and Piskorski (2001), Angelini, Del Giovane, Siviero and Terlizzesse (2002) and Monteforte and Siviero (2002) study the usefulness of country-specific information to monetary policy within a monetary union. In particular, the authors evaluate, using the framework proposed by Rudebusch and Svensson (1999), the performance (relative losses) of national data and aggregate data targeting rules for calibrated aggregate equations and find that the second type of rules may deliver large welfare gains.

Using two-country optimizing models, Benigno (forthcoming), Lombardo (2002) and Lombardo (2004) focus on investigating optimal monetary policy in a currency area characterized by asymmetric shocks across regions, like EMU. According to these authors, a nearly optimal inflation targeting policy should put higher weight on the inflation of the region with higher degree of nominal rigidity Benigno (forthcoming) or on the inflation of the region with higher market competition, Lombardo (2002) and Lombardo (2004). The rationale behind these results is that less flexible prices or a higher degree of competition affect relative prices, cause dispersion of output and worsen the welfare of the currency area.

A common conclusion from the above literature is that the effectiveness of monetary policy in the euro area may be enhanced by paying attention to national information, as opposed to reacting exclusively to aggregate area-wide variables. Even though the objectives of the ECB are framed exclusively in area-wide terms, the fact that the economies of the euro-area are characterized by significant structural differences and asymmetric shocks across countries can make neglecting national developments very costly. The results of this paper suggest that an optimal monetary policy rule should take into consideration not only the relative size of the countries (relative output or population), but also the structural characteristics of the economies. In order to substantiate this claim, we extend the analysis of De Grauwe and Piskorski (2001), Angelini, Del Giovane, Siviero and Terlizzesse (2002) and Monteforte and Siviero (2002), and analytically derive an optimal interest rate reaction function of the central bank of the union by minimizing its loss function subject to different structural model in each country. The paper contributes to the literature on the optimal design of monetary policy in a union in the presence of structural asymmetries by studying the dependence of the coefficients of the interest rate rule on the parameters of the aggregate relationships of each economy, the central bank's preferences in the loss function and the relative size of each country, and by assessing the welfare gains that would be produced by such a rule compared with the case where the central bank considers only union-wide variables.

The baseline model used to derive the optimal monetary policy is described in Section 2 and is a dynamic, general equilibrium model, with the aggregate demand equation resulting from the consumer's welfare maximization problem and the New Keynesian Phillips curve based on Calvo price setting. Following Svensson (1997), Svensson (1999) and Giannoni and Wooford

(2002b), we first present in Section 3, the optimal reaction function of the interest rate, which results from the minimization of a union loss function, subject to the union aggregate demand equation and Phillips curve. The relevant variables are the corresponding variables of two countries consisting the union, weighted according to the country size. In the next section, we develop a two-country framework, which consists of an aggregate loss function to be minimized by the monetary authority, subject to the equations of aggregate demand and supply for each country, having different structural parameters. The minimization results in a reaction function in which the singe interest rate responds to the variables of both countries, with coefficients that depend on the parameters of the aggregate demand and supply curves of both countries and may be different from the weight given to each country according to its size. Moreover, we show what is the appropriate reaction of the single interest rate to the variables of each country according to the effectiveness of the transmission mechanism, the intertemporal elasticity of substitution, the market competition parameter, and other structural parameters that characterize each economy. In Section 4, we estimate the structural equations of both models, the first with data from a hypothetical union between Germany and France, and the second with individual country data for Germany and France. Using these estimates, we calculate in the next section the optimal coefficients of the single interest rate reaction function, the volatility of the variables of interest and the value of the loss function for both versions. Of course, the calculations depend on the choice of the countries, on the empirical estimates of their parameters and on the assumptions about the loss function of the central bank. However, the comparison of the results can serve as an example that could give us some indication about the welfare improvement to be achieved if the central bank of a monetary union focuses on the variables of each member country, and not only on union-wide variables.

2 Baseline Model

The New Keynesian model used is a dynamic, stochastic, general equilibrium model, based on optimizing behavior combined with some form of nominal price rigidity. Early examples of such models include Goodfriend and King (1997), Rotemberg and Woodford (1997), Rotemberg and Woodford (1999) and McCallum and Nelson (1999). The equations of the model are derived from well-specified optimization problems, i.e. the representative agent's decision problem and the pricing decisions of individual firms. Traditional aggregate demand - aggregate supply equations are often criticized of being too *ad-hoc*. However, this criticism does not apply in the New Keynesian framework, since the parameters appearing in these equations are explicit functions of the underlying structural parameters of the consumer's utility function, the production function and the assumed price adjustment process. Furthermore, both equations contain expectations of future variables, which was a critical shortcoming of traditional aggregate demand - aggregate supply models.

2.1 Derivation of the aggregate demand equation

Suppose there is a continuum of households indexed by i $(0 < i < 1)$. Each of these households produces a single good, while it consumes the composite good. The utility of household i at period t is given by:

$$\max_{C_t^i} E_t \sum_{j=0}^{\infty} \delta^j \left[U(C_{t+j}^i, \xi_{t+j}) - V(N_{t+j}^i, \xi_{t+j}) \right] \tag{1}$$

where δ is the discount rate $(0 < \delta < 1)$, C_t is consumption, ξ_t is a vector of preference or technological disturbances, $U(C_t^i, \xi_t) = \frac{(C_t^i)^{1-\sigma}}{1-\sigma}$ is utility from consumption, V is disutility from work N_{t+j}^i, and $\sigma = \frac{-U_{cc}\overline{C}}{U_c}$ is the intertemporal elasticity of substitution of aggregate expenditure.

Following Dixit and Stiglitz (1977) the index of the representative household's consumption of the continuum of differentiated goods produced in the economy is given by:

$$C_t^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

where $c_t^i(z)$ is the quantity of good z consumed by household i and θ is the price elasticity of demand $(\theta > 1)$. The total demand for the differentiated good z is given by a constant elasticity demand function:

$$y_t(z) = Y_t \left(\frac{p_t(z)}{P_t} \right)^{-\theta} \tag{3}$$

where $y_t(z)$ is household's production of the differentiated good z, Y_t is aggregate demand for the composite good, $p_t(z)$ is the price of good z and P_t is the price index defined as:

$$P_t = \left[\int_0^1 p_t(z)^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}} \tag{4}$$

The household's decision problem involves maximizing its utility of consuming C_t^i subject to its budget constraint given by:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{M_{t-1}}{P_t} + \frac{I_{t-1} B_{t-1}}{P_t} + \Pi_t$$

where M_t is household's nominal holdings of money, B_t is household's holding of bonds, which pay a grow nominal interest rate of interest I_t, W_t is nominal wage, N_t is labor supply and Π_t is real profits received from the production of the good.

Log-linear approximations of the Euler condition of the household's utility maximization around the steady state gives:

$$c_t = c_{t+1/t} - \frac{1}{\sigma} \left[i_t - \pi_{t+1/t} \right] - \frac{1}{\sigma} \ln \delta$$

where $c_t = \ln\left(\frac{C_t}{\overline{C}_t}\right)$, $i_t = \ln\left(\frac{I_t}{\overline{I}_t}\right)$, $\pi_{t+1} = p_t - p_{t-1} = \ln\left(\frac{P_t}{\overline{P}_t}\right) - \ln\left(\frac{P_{t-1}}{\overline{P}_{t-1}}\right)$ denote log deviation of the variables around their steady states (marked with bars).

Disregarding government consumption and investment and identifying consumption as aggregate demand ($\widehat{y}_t = c_t$), where $\widehat{y}_t = \ln\left(\frac{Y_t}{\overline{Y}_t}\right)$, gives:

$$\widehat{y}_t = \widehat{y}_{t+1/t} - \frac{1}{\sigma} \left[i_t - \pi_{t+1/t} \right] - \frac{1}{\sigma} \ln \delta$$

Next, we assume that flexible-prices output is exogenous $\left(Y_{t+1}^n = Y_t^n + \varepsilon_{t+1}\right)$, with growth rate ω_t and productivity shocks ε_t. Following Woodford (1999), the Wicksellian natural rate of interest $(\overline{r})$ is defined as the equilibrium real rate of return in case of fully flexible prices, which is required to bring output to the flexible-prices level, and is assumed to follow an exogenous stochastic process:

$\overline{r} = -\ln \delta$. Thus, we can write aggregate demand in terms of output gap ($\widetilde{y}_t = \widehat{y}_t - \widehat{y}_t^n$), where $\widehat{y}_t^n = \ln\left(\frac{Y_t^n}{\overline{Y}_t}\right)$, as follows:

$$\widetilde{y}_t = \widetilde{y}_{t+1/t} - \frac{1}{\sigma}\left[i_t - \pi_{t+1/t} - \overline{r}\right] + \varepsilon_{t+1} \tag{5}$$

This equation corresponds to an expectational, forward-looking aggregate demand curve derived from the Euler condition of a representative agent's optimization problem and it relates the output gap to the expected future output gap and the real interest rate. Thus, it includes the effect on the output due to the central bank's control on nominal interest rate. Changes in the latter, affect the real interest rate, and this alters the optimal time path of consumption.

2.2 Derivation of the New Keynesian Phillips curve

The source of the real effects of monetary policy in this model is that prices are adjusted at exogenous random intervals (Calvo (1983)). Assuming perfect competition in the market for composite goods and monopolistic competition in the market for intermediate inputs, a fraction $(1-\alpha)$ of producers charge a new price at the end of a period, whereas the rest (α) must continue charging their old price. The parameter (α) is a measure of the degree of nominal rigidity. For those firms who do adjust their prices at time t, they do so by maximizing the expected discounted value of current and future profits resulting from sales revenues minus the cost of output supply, i.e.:

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \alpha^j \delta^j \left[\frac{P_t^*(z)}{P_{t+j}} c_{t+j}(z) - \Phi_{t+j} c_{t+j}(z)\right] \Rightarrow$$

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \alpha^j \delta^j \left[\frac{P_t^*(z)}{P_{t+j}} C_{t+j} \left(\frac{P_t^*(z)}{P_{t+j}}\right)^{-\theta} - \Phi_{t+j} C_{t+j} \left(\frac{P_t^*(z)}{P_{t+j}}\right)^{-\theta}\right]$$

where P_t^* is the optimal price chosen by those firms who are chosen to adjust their prices and $\Phi_t = \frac{W_t}{P_t Z_t}$ is the real marginal cost of producing the differentiated good z, using labor $N_t(z)$ with productivity Z_t.

Although individual firms produce differentiated products, they face common production function and demand curve. Therefore, all firms that have the chance to adjust their prices in period t, they face the same optimization problem and they will set the same price, denoted by P_t^*. Recall that θ is the elasticity of substitution and $\frac{\theta}{\theta-1}$ is the markup over nominal marginal cost that each firm sets its price on, standard in monopolistic competition models.

The first order condition is given by:

$$\frac{1}{1-\alpha\delta}\frac{P_t^*}{P_t} = E_t \sum_{j=0}^{\infty} \alpha^j \delta^j \frac{\theta}{\theta-1} \Phi_{t+j} \frac{P_{t+j}}{P_t} \tag{6}$$

The log linear approximation of this first order condition around the steady state of flexible prices equilibrium using $\frac{P_t^*(z)}{P_t} = \frac{\theta}{\theta-1}\Phi_t = 1$, gives:

$$p_t^* = (1-\alpha\delta) \sum_{j=0}^{\infty} \alpha^j \delta^j \left(E_t \phi_{t+j} + E_t p_{t+j} \right) \tag{7}$$

where $p_t^* = \ln\left(\frac{P_t^*}{\overline{P}_t}\right)$, $p_t = \ln\left(\frac{P_t}{\overline{P}_t}\right)$, $\phi_t = \ln\left(\frac{\Phi_t}{\overline{\Phi}_t}\right)$, i.e. the log deviation of the variables around their steady states.

The price index evolves according to (4) and is equal to:

$$P_t = \left[\alpha \left(P_{t-1}\right)^{1-\theta} + (1-\alpha)\left(P_t^*\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Log-linearization around steady state gives:

$$\pi_t = \delta \pi_{t+1/t} + \rho \phi_t \tag{8}$$

where $\rho = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha}$.

This New Keynesian Phillips curve differs from traditional Phillips curves because it relates inflation to expected future inflation and to deviations of real marginal cost from its steady state. It has been derived explicitly from the price setters optimization problem, given monopolistic competition, constant elasticity demand equations and random adjustment of prices.

However, real marginal cost can be related to an output gap measure. As before, the firm's real marginal cost is equal to the real wage divided by the marginal product of labor:

$$\Phi_t = \frac{W_t}{P_t Z_t}$$

given the following production function: $Y_t = C_t = Z_t N_t\left(z\right).$

Given flexible wages, real wage must be equal to the marginal rate of substitution between leisure and consumption, i.e.:

$$\frac{W_t}{P_t} = \frac{N_t^{\eta}}{C_t^{-\sigma}} e^{\epsilon_t}$$

where the cost shock added $\left(e^{\epsilon_t}\right)$ represents a stochastic wage markup, which could arise for example from changes in tastes that affect leisure or stochastic shifts in the markup of wages over the marginal rate of substitution.

Log-linearization of:

$$\Phi_t = \frac{W_t}{P_t Z_t} = \frac{N_t^{\eta}}{C_t^{-\sigma} Z_t} e^{\epsilon_t}$$

gives:

$$\phi_t = \left(\eta + \sigma\right) \widehat{y}_t - \left(\eta + 1\right) \zeta_t + \epsilon_t$$

where $\sigma = -\frac{\overline{C} U_{cc}}{U_c}$, $\eta = \frac{\overline{Y} V_{yy}}{V_y}$, while $\widehat{y}_t = \ln\left(\frac{Y_t}{\overline{Y}_t}\right)$ and $\zeta_t = \ln\left(\frac{Z_t}{\overline{Z}_t}\right)$ i.e. the deviations of variables around their steady states.

Also, given flexible prices equilibrium: $\frac{W_t}{P_t Z_t} = \Phi_t = \frac{\theta - 1}{\theta}$,

$$\widehat{y}_t^n = \frac{\eta + 1}{\eta + \sigma} \zeta_t^n$$

where $\widehat{y}_t^n = \ln\left(\frac{Y_t^n}{\overline{Y}_t}\right)$ and $\zeta_t^n = \ln\left(\frac{Z_t^n}{\overline{Z}_t^n}\right)$ are the deviations around their steady states, given flexible prices.

Thus,

$$\phi_t = \gamma\left(\widehat{y}_t - \widehat{y}_t^n\right) + \epsilon_t \tag{9}$$

where $\gamma = \eta + \sigma$.

Using this result, the New Keynesian Phillips curve (8) may be written:

$$\begin{aligned}\pi_t &= \delta\pi_{t+1/t} + \rho(\eta+\sigma)(\widehat{y}_t - \widehat{y}_t^n) + \epsilon_t \Rightarrow \\ \pi_t &= \delta\pi_{t+1/t} + \rho\gamma(\widetilde{y}_t) + \epsilon_t \end{aligned} \tag{10}$$

where $\widetilde{y}_t = \widehat{y}_t - \widehat{y}_t^n$ denotes the deviation of actual output from the flexible prices equilibrium level.

This form of the New Keynesian Phillips curve, relates inflation to expected future inflation as above, but also to the deviation of output from the potential output that could be attained under flexible prices.

2.3 Derivation of the loss function

A common and plausible assumption widely used in the literature, is to consider that a central bank aims at minimizing a quadratic loss function that depends on inflation and the output gap. Although this assumption may seem rather *ad-hoc*, Woodford (1999) has provided an important justification for using such a quadratic loss function, derived as a log-linear approximation of the representative agent's welfare functions.

As before, the average level of welfare is given by:

$$\max_{C_t^i} E_t \sum_{j=0}^{\infty} \delta^j \left[U(C_{t+j}^i, \xi_{t+j}) - V(N_{t+j}^i, \xi_{t+j})\right] \tag{11}$$

This welfare function can have an especially simple form by taking second order Taylor series approximation around steady state values, disregarding government consumption, as follows:

$$V = \left(-\frac{1}{2}\right)(\sigma+\eta)U_c\overline{Y}\left\{(\widetilde{y}_t - y^*)^2 + \left(\frac{\frac{1}{\theta}+\eta}{\sigma+\eta}\right) var_z \widetilde{y}_t\,(z)\right\} + t.i.p \tag{12}$$

where $\sigma = -\frac{\overline{C}U_{cc}}{U_c}$, $g_t = -\frac{U_{c\xi}}{\overline{Y}U_{cc}}\xi_t$, $\eta = \frac{\overline{Y}V_{yy}}{V_y}$, $q_t = -\frac{v_{y\xi}}{\overline{y}v_{yy}}\xi_t$.

Furthermore, $\overline{Y}$ is the steady state level of output associated with zero inflation, in the absence of real shocks, $\widehat{y}_t(z) = \ln\left(\frac{y_t(z)}{\overline{Y}}\right)$ is the log deviation of $y_t(z)$ around its steady state, $\widehat{y}_t^n = \ln\left(\frac{Y_t^n}{\overline{Y}}\right) = \frac{\sigma g_t + \eta q_t}{\sigma+\eta}$ is the log deviation of output under complete price flexibility (Y_t^n) from its steady state level, $\widetilde{y}_t = \widehat{y}_t - \widehat{y}_t^n$ is the output gap between actual output and the flexible prices output, $y^* = \ln\left(\frac{Y^*}{\overline{Y}}\right) = \frac{\frac{1}{\theta}}{(\sigma+\eta)}$ is the log deviation of efficient level of output (Y^*) around its steady state and represents a measure of the distortion caused by the presence of monopolistic competition, and $t.i.p$ denotes terms independent of policy.

This welfare measure (12) depends on the degree to which aggregate output deviates from flexible-prices output, but also on the degree of allocation of real resources, expressed as the dispersion of output across the different varieties of goods being produced at each point in time. This dispersion of output levels corresponds directly in equilibrium to the degree of dispersion of output prices, which in turn depends on variation in the overall price level.

From the demand function (3), one can get:

$$\begin{aligned} \ln y_t(z) &= \ln Y_t - \theta\left(\ln p_t(z) - \ln P_t\right) \Rightarrow \\ var_z \ln y_t(z) &= \theta^2 var_z \ln p_t(z) \end{aligned}$$

Therefore, the welfare function (12) may be written as follows:

$$V = \left(-\frac{1}{2}\right) U_c\overline{Y}\left\{(\sigma+\eta)\left(\widetilde{y}_t - y^*\right)^2 + \left(\frac{1}{\theta}+\eta\right)\theta^2 var_z \ln p_t(z)\right\} + t.i.p \tag{13}$$

Given Calvo (1983) price setting model,

$$\begin{aligned} var_z \ln p_t(z) &= \alpha var_z \ln p_{t-1}(z) + \frac{\alpha}{1-\alpha}\left(\pi_t\right)^2 \Rightarrow \\ \sum_{j=0}^{\infty}\delta^j var_z \ln p_{t+j}(z) &= \frac{\alpha}{(1-\alpha)(1-\alpha\delta)}\sum_{j=0}^{\infty}\delta^j\left(\pi_{t+j}\right)^2 \end{aligned}$$

Thus, the intertemporal welfare measure may be expressed as follows:

$$E_t\sum_{j=0}^{\infty}\delta^j V_{t+j} = E_t\sum_{j=0}^{\infty}\delta^j\left(-\frac{1}{2}\right)U_c\overline{Y}\left\{(\sigma+\eta)\left(\widetilde{y}_t - y^*\right)^2 + \left(\frac{1}{\theta}+\eta\right)\theta^2\frac{\alpha}{(1-\alpha)(1-\alpha\delta)}\left(\pi_{t+j}\right)^2\right\} + t.i.p$$

$$= \left(-\frac{1}{2}\right) U_c \overline{Y}\left(\theta + \theta^2 \eta\right) \frac{\alpha}{(1-\alpha)(1-\alpha\delta)} E_t \sum_{j=0}^{\infty} \delta^j \left\{ \left(\pi_{t+j}\right)^2 + \frac{(\sigma+\eta)}{\left(\theta+\theta^2\eta\right)\frac{\alpha}{(1-\alpha)(1-\alpha\delta)}} \left(\widetilde{y}_t - y^*\right)^2 \right\} + t.i.p \Rightarrow$$

$$E_t \sum_{j=0}^{\infty} \delta^j V_{t+j} = \Omega E_t \sum_{j=0}^{\infty} \delta^j \left\{ \left(\pi_{t+j}\right)^2 + \lambda \left(\widetilde{y}_{t+j} - y^*\right)^2 \right\} + t.i.p \tag{14}$$

where $\Omega = \left(-\frac{1}{2}\right) U_c \overline{Y}\left(\theta + \theta^2\eta\right) \frac{\alpha}{(1-\alpha)(1-\alpha\delta)}$ and $\lambda = \frac{(\sigma+\eta)}{\left(\theta+\theta^2\eta\right)\frac{\alpha}{(1-\alpha)(1-\alpha\delta)}}$

This loss measure collects the terms that depend solely on the degree of variability of inflation and the output gap and is similar in form to an *ad-hoc* loss function, which is commonly assumed in many analyses of optimal monetary policy.

2.4 Derivation of the loss function in the presence of transactions frictions

A useful modification of the above described loss function (14) by Woodford forthcoming, can be derived if we include real money balances as an additional argument in the consumer's utility function $U(C, m)$. This can be justified by acknowledging that money balances facilitate transactions, and thus add to consumer's utility. The marginal utility of household depends on the level of real money balances in addition to the level of real expenditure.

In this case, money demand is given by:

$$\frac{M_t}{P_t} = L\left(Y_t, \Delta_i, \xi_t\right)$$

where $\Delta_i = \frac{i_t - i_t^m}{1+i_t}$ is the interest rate differential between non-monetary (i_t) and monetary assets (i_t^m).

Log-linearization of money demand around steady state of zero inflation, gives:

$$m_t = l_y \widehat{y}_t - l_i\left(i_t - i_t^m\right) + \varepsilon_t^m$$

where $m_t = \ln\left(\frac{M_t}{\overline{M}_t}\right)$ is the deviation of money demand from its steady state, $l_y = -\frac{\overline{Y} U_{cm}}{\overline{m} U_{mm}}$, and $l_i = \frac{-U_c}{\overline{m} U_{mm}}$ denotes interest rate semi-elasticity of money demand and ε_t^m is an exogenous shock.

The log deviation of output under complete price flexibility and fixed interest rate differential $\left(\overline{\Delta}\right)$, from its steady state level becomes: $\widehat{y}_t^n = \ln\left(\frac{Y_t^n}{\overline{Y}}\right) = \frac{\sigma g_t + \eta q_t + \chi \varepsilon_t^m}{\sigma + \eta}$, while the log deviation of efficient level of output from its steady state becomes: $y^* = \ln\left(\frac{Y^*}{\overline{Y}}\right) = \frac{\frac{1}{\theta} + \frac{\overline{m} U_m}{\overline{c} U_c} l_y}{(\eta + \sigma - l_i \chi)}$, where $\chi = \frac{m U_{cm}}{U_c}$ denotes the complementarity between private expenditure and real balances.

Now, welfare is given by:

$$V = \left(-\frac{1}{2}\right) U_c \overline{Y} \left\{ (\eta + \sigma - l_i \chi) \left(\widetilde{y}_t - y^*\right)^2 + \left(\theta + \theta^2 \eta\right) var_z \ln p_t(z) + \frac{1}{v} l_i \left(\widehat{i}_t - \widehat{i}_t^m + \overline{\Delta}\right)^2 \right\} + t.i.p \tag{15}$$

where $v = \frac{\overline{Y}}{\overline{m}}$ is the velocity of money, $\widehat{i}_t = \ln\left(\frac{1+i_t}{1+\overline{i}}\right)$, $\widehat{i}_t^m = \ln\left(\frac{1+i_t^m}{1+\overline{i}}\right)$ and $\overline{i}$ is the interest rate consistent with inflation rate equal to $\overline{\pi}$.

Assuming that the interest rate paid on the monetary base (i_t^m) is constant (close to zero), then the intertemporal loss function to be minimized can be written as:

$$E_t \sum_{j=0}^{\infty} \delta^j V_{t+j} = \Omega E_t \sum_{j=0}^{\infty} \delta^j \left\{ \left(\pi_{t+j}\right)^2 + \lambda_y \left(\widetilde{y}_{t+j} - y^*\right)^2 + \kappa \left(i_{t+j} - \overline{i}\right)^2 \right\} + t.i.p \tag{16}$$

where $\lambda_y = \frac{(1-\alpha)(1-\alpha\delta)}{\alpha} \frac{\sigma + \eta - l_i \chi}{\theta + \theta^2 \eta}$ and $\kappa = \frac{l_i}{v} \frac{(1-\alpha)(1-\alpha\delta)}{\alpha(\theta + \theta^2 \eta)}$.

Therefore, considering transaction frictions adds an additional term to the loss function, namely the squared deviations of the interest rate from a constant interest rate $\left(i_t - \overline{i}\right)$. According to Woodford forthcoming, the interest rate smoothing objective in the central bank's loss function may be desirable, even if it's not a social objective in itself, because it helps steer the expectations of the private sector about future policy. Introducing interest rate smoothing affects the anticipations about future policy and adds history-dependence in the policy conduct of the central bank, which succeeds in stabilizing inflation and output gap, with less variability in the interest rates.

3 Derivation of the optimal rule under union loss function

Using the baseline model[3] described in the previous section and following Giannoni and Woodford (2002a), we can derive the optimal reaction function of the central bank. We start by assuming

[3] We recognize that this model treats the union like a closed economy and disregards features present in currency areas, like terms-of -trade, relative prices effects, etc. However, it enables us to get manageable and straightforward solutions and trace on monetary policy implications.

that the central bank decides on the interest rate for the union, taking into account the weighted aggregate variables of all countries in the union. The central bank is assumed to minimize a modified loss function, with positive weights 1, λ and κ on squared deviations of inflation from the inflation target ($\widehat{\pi}$), squared output gap and squared interest rate deviation from a constant rate $\left(\widehat{i}\right)$ consistent with the inflation target ($\widehat{\pi}$), as follows:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j L_{t+j} = \min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \frac{1}{2} \left[\left(\pi_{t+j}^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_t^U\right)^2 + \kappa \left(i_t - \widehat{i}\right)^2 \right] \tag{17}$$

subject to the union's forward-looking Phillips curve and aggregate demand equation:

$$\pi_t^U = \delta \pi_{t+1/t}^U + \rho \widetilde{y}_t^U + \epsilon_{t+1} \tag{18}$$

$$\widetilde{y}_t^U = \widetilde{y}_{t+1/t}^U - \frac{1}{\sigma} \left(i_t - \pi_{t+1/t}^U - \overline{r}\right) + æ_{t+1} \tag{19}$$

where $\pi_t^U = w\pi_t^H + (1-w)\pi_t^F$, $\widetilde{y}_t^U = w\widetilde{y}_t^H + (1-w)\widetilde{y}_t^F$ and w is the weight given to each country according to its size.

The Lagrangian is given:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left\{ \begin{array}{c} \frac{1}{2} \left[\left(\pi_{t+j}^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_{t+j}^U\right)^2 + \kappa \left(i_{t+j} - \widehat{i}\right)^2 \right] \\ +\phi_{t+j} \left[\pi_{t+j}^U - \delta \pi_{t+1+j/t}^U - \rho \widetilde{y}_{t+j}^U \right] \\ +\psi_{t+j} \left[\widetilde{y}_{t+j}^U - \widetilde{y}_{t+1+j/t}^U + \frac{1}{\sigma} \left(i_{t+j} - \pi_{t+1+j/t}^U - \overline{r} \right) \right] \end{array} \right\}$$

For two periods, the Lagrangian becomes:

$$\min_{i_t} E_t \left\{ \begin{array}{c} \frac{1}{2} \left[\left(\pi_t^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_t^U\right)^2 + \kappa \left(i_t - \widehat{i}\right)^2 \right] \\ +\phi_t \left[\pi_t^U - \delta \pi_{t+1/t}^U - \rho \widetilde{y}_t^U \right] + \psi_t \left[\widetilde{y}_t^U - \widetilde{y}_{t+1/t}^U + \frac{1}{\sigma} \left(i_t - \pi_{t+1/t}^U - \overline{r} \right) \right] \\ +\frac{\delta}{2} \left[\left(\pi_{t+1/t}^U - \widehat{\pi}\right)^2 + \lambda \left(\widetilde{y}_{t+1/t}^U\right)^2 + \kappa \left(i_{t+1/t} - \widehat{i}\right)^2 \right] \\ +\delta\phi_{t+1} \left[\pi_{t+1/t}^U - \delta \pi_{t+2/t}^U - \rho \widetilde{y}_{t+1/t}^U \right] + \delta\psi_{t+1} \left[\widetilde{y}_{t+1/t}^U - \widetilde{y}_{t+2/t}^U + \frac{1}{\sigma} \left(i_{t+1/t} - \pi_{t+2/t}^U - \overline{r} \right) \right] \end{array} \right\}$$

The first order conditions can be written as follows:

with respect to $\pi^U_{t+1/t}$:

$$\delta \left(\pi^U_{t+1/t} - \widehat{\pi} \right) - \phi_t \delta + \phi_{t+1/t} \delta - \psi_t \frac{1}{\sigma} = 0 \tag{20}$$

with respect to $\widetilde{y}^U_{t+1/t}$:

$$\delta \lambda \left(\widetilde{y}^U_{t+1/t} \right) - \psi_t - \delta \phi_{t+1/t} \rho + \delta \psi_{t+1/t} = 0 \tag{21}$$

with respect to i_t :

$$\kappa \left(i_t - \widehat{i} \right) + \psi_t \frac{1}{\sigma} = 0 \Rightarrow \psi_t = -\kappa \sigma \left(i_t - \widehat{i} \right) \tag{22}$$

After some algebraic manipulations of the first order conditions we get:

$$\begin{aligned} &\left(\pi^U_{t+1/t} - \widehat{\pi} \right) + \frac{\lambda}{\rho} \left(\widetilde{y}^U_{t+1/t} - \widetilde{y}^U_t \right) \\ &= -\frac{\kappa}{\delta} \left(i_t - \widehat{i} \right) + \frac{\kappa \sigma}{\delta \rho} \left(i_{t-1} - \widehat{i} \right) - \frac{\kappa \sigma}{\rho} \left(i_t - \widehat{i} \right) - \frac{\kappa \sigma}{\delta \rho} \left(i_t - \widehat{i} \right) + \frac{\kappa \sigma}{\rho} \left(i_{t+1/t} - \widehat{i} \right) \\ &= \frac{\kappa \sigma}{\rho} \left\{ i_{t+1/t} + i_t \left[-\frac{\rho}{\sigma \delta} - 1 \right] + (i_t - i_{t-1}) \left[-\frac{1}{\delta} \right] + \widehat{i} \left[-\frac{\rho}{\sigma \delta} \right] \right\} \end{aligned}$$

or

$$i_t = \frac{\rho}{\kappa \sigma} \left(\pi^U_t - \widehat{\pi} \right) + \frac{\lambda}{\kappa \sigma} \Delta \widetilde{y}^U_t + i_{t-1} \left[\frac{\rho}{\sigma \delta} + 1 \right] + (i_{t-1} - i_{t-2}) \left[\frac{1}{\delta} \right] + \widehat{i} \left[\frac{\rho}{\sigma \delta} \right] \tag{23}$$

This direct, implicit instrument rule involves a positive contemporaneous response of the interest rate to deviations of union inflation from the target and to changes in union output gap. Furthermore, it involves history dependence, due to the positive response to past interest rate and past output gap. The coefficients of the rule satisfy the generalized Taylor Principle of determinacy, as proposed by Giannoni and Wooford (2002b).[4]

Examining the response of interest rate to the variables of both countries, we can draw the following conclusions:

i) i_t and ρ are positively related, thus the interest rate should respond positively to changes of the Phillips curve's slope. The lower is nominal rigidity (α), i.e. the percentage of firms not adjusting their prices, the steeper is the Phillips curve and the stronger must be the interest rate reaction.

[4] $coef(\pi_t) + \frac{1-\delta}{4\rho} coef\left(\widetilde{y}_t\right) > 1 - coef\left(i_{t-1}\right)$

ii) i_t and $\frac{1}{\sigma}$ are positively related, thus the interest rate should also respond positively to changes of the slope of the aggregate demand equation. The lower the intertemporal elasticity of substitution of aggregate expenditure, the steeper is the aggregate demand curve, and the stronger must be the interest rate reaction.

The rationale behind these two results, is that the steeper the two curves are, the stronger the effect of the real interest rate on the output gap, and/or that of the output gap on inflation, the more aggressive must be the interest rate adjustment to stabilize inflation towards the target and output gap towards zero.

iii) i_t and δ are inversely related, so the more importance consumers assign to the future level of the variables (higher δ means reduced inertia in the variables), the stronger is the monetary policy leverage. As a result, the interest rate should adjust less to past interest rates.

iv) i_t and λ are positively related, so the higher the weight given by the central bank on the output gap, the stronger must be the interest rate adjustment to output deviations from potential.

v) i_t and κ are inversely related, thus the more the central bank is concerned about interest rate variability, the smoother must she adjust the interest rate in response to changes in the economic conditions.

vi) λ and κ are inversely related to θ, so the higher the price elasticity of demand (θ) and the market competition parameter, the higher the disutility from deviations of inflation from the target, compared with that from the output gap variation (λ) and from the interest rate differential (κ). Thus, the stronger must be the interest rate adjustment to inflation deviations from target.

4 Derivation of the optimal rule under two-country aggregate loss function

Given the existence of asymmetries between member countries of a monetary union, it is important to derive the optimal reaction function of the central bank, taking into account national information explicitly. For the simplest case of a two-country union, the central bank should minimize the sum of both countries' loss functions, weighted according to their size (w):

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \left[w L_{t+j} + (1-w)\, L^*_{t+j} \right] \tag{24}$$

The loss functions of the two countries share the same features, namely the same discount factor (δ), the inflation target ($\widehat{\pi}$) and the weights assigned to output gap (λ) and to interest rate deviations (κ), while the interest rate (i_t) is assumed common in both countries.

The loss function of the first country is given by:

$$L_t = \frac{1}{2} \left[(\pi_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^2 + \kappa (i_t - \widehat{i})^2 \right]$$

and that of the second country (whose variables are denoted with asterisks) is given by:

$$L^*_t = \frac{1}{2} \left[(\pi^*_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^{*2} + \kappa (i_t - \widehat{i})^2 \right]$$

Assume forward-looking Phillips curves and aggregate demand equations for both countries, i.e.:

$$\pi_t = \delta \pi_{t+1/t} + \beta \widetilde{y}_t + \varepsilon_{t+1} \tag{25}$$

$$\widetilde{y}_t = \widetilde{y}_{t+1/t} - \frac{1}{\tau} \left(i_t - \pi_{t+1/t} - \overline{r} \right) + \upsilon_{t+1} \tag{26}$$

$$\pi^*_t = \delta \pi^*_{t+1/t} + b \widetilde{y}^*_t + e_{t+1} \tag{27}$$

$$\widetilde{y}^*_t = \widetilde{y}^*_{t+1/t} - \frac{1}{t} \left(i_t - \pi^*_{t+1/t} - \overline{r} \right) + \upsilon_{t+1} \tag{28}$$

The Lagrangian for two periods is given:

$$\begin{aligned}
&\min w \frac{1}{2} \left[(\pi_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^2 \right] + (1-w) \frac{1}{2} \left[(\pi^*_t - \widehat{\pi})^2 + \lambda \widetilde{y}_t^{*2} \right] + \frac{1}{2} \kappa (i_t - \widehat{i})^2 \\
&+ \phi_t \left[\pi_t - \delta \pi_{t+1/t} - \beta \widetilde{y}_t \right] + \chi_t \left[\widetilde{y}_t - \widetilde{y}_{t+1/t} + \frac{1}{\tau} \left(i_t - \pi_{t+1/t} - \overline{r} \right) \right] \\
&+ \psi_t \left[\pi^*_t - \delta \pi^*_{t+1/t} - b \widetilde{y}^*_t \right] + \omega_t \left[\widetilde{y}^*_t - \widetilde{y}^*_{t+1/t} + \frac{1}{t} \left(i_t - \pi^*_{t+1/t} - \overline{r} \right) \right]
\end{aligned}$$

$$+\delta\{w\frac{1}{2}\left[\left(\pi_{t+1/t}-\widehat{\pi}\right)^2+\lambda\widetilde{y}^2_{t+1/t}\right]+(1-w)\frac{1}{2}\left[\left(\pi^*_{t+1/t}-\widehat{\pi}\right)^2+\lambda\widetilde{y}^{*2}_{t+1/t}\right]+\frac{1}{2}\kappa(i_{t+1}-\widehat{i})^2$$

$$+\phi_{t+1}\left[\pi_{t+1/t}-\delta\pi_{t+2/t+1}-\beta\widetilde{y}_{t+1/t}\right]+\chi_{t+1}\left[\widetilde{y}_{t+1/t}-\widetilde{y}_{t+2/t+2}+\frac{1}{\tau}\left(i_{t+1/t}-\pi_{t+2/t+1}-\bar{r}\right)\right]$$

$$+\psi_{t+1}\left[\pi^*_{t+1/t}-\delta\pi^*_{t+2/t+1}-b\widetilde{y}^*_{t+1/t}\right]+\omega_{t+1}\left[\widetilde{y}^*_{t+1/t}-\widetilde{y}^*_{t+2/t+1}+\frac{1}{t}\left(i_{t+1/t}-\pi^*_{t+2/t+1}-\bar{r}\right)\right]\}$$

The first order conditions are as follows:

with respect to $\pi_{t+1/t}$:

$$\delta w\left(\pi_{t+1/t}-\widehat{\pi}\right)-\delta\phi_t-\frac{1}{\tau}\chi_t+\delta\phi_{t+1/t}=0 \tag{29}$$

with respect to $\pi^*_{t+1/t}$:

$$\delta\left(1-w\right)\left(\pi^*_{t+1/t}-\widehat{\pi}\right)-\delta\psi_t-\frac{1}{t}\omega_t+\delta\psi_{t+1/t}=0 \tag{30}$$

with respect to $\widetilde{y}_{t+1/t}$:

$$\delta w\lambda\widetilde{y}_{t+1/t}-\chi_t-\delta\beta\phi_{t+1/t}+\delta\chi_{t+1/t}=0 \tag{31}$$

with respect to $\widetilde{y}^*_{t+1/t}$:

$$\delta\left(1-w\right)\lambda\widetilde{y}^*_{t+1/t}-\omega_t-\delta b\psi_{t+1/t}+\delta\omega_{t+1/t}=0 \tag{32}$$

with respect to i_t:

$$\kappa(i_t-\widehat{i})+\frac{1}{\tau}\chi_t+\frac{1}{t}\omega_t=0 \tag{33}$$

After some algebraic manipulations of the first order conditions we get the following condition:

$$t\beta w\left(\pi_{t+1/t}-\widehat{\pi}\right)+t\lambda w\Delta\widetilde{y}_{t+1/t}+\tau b\left(1-w\right)\left(\pi^*_{t+1/t}-\widehat{\pi}\right)+\tau\lambda\left(1-w\right)\Delta\widetilde{y}^*_{t+1/t}=$$

$$-t\chi_{t+1/t}+t[1+\frac{\beta}{\delta\tau}+\frac{1}{\delta}]\chi_t-t\frac{1}{\delta}\chi_{t-1}-\tau\omega_{t+1/t}+\tau[1+\frac{b}{\delta t}+\frac{1}{\delta}]\omega_t-\tau\frac{1}{\delta}\omega_{t-1}=$$

$$t\tau\kappa(i_{t+1/t}-\widehat{i})-[1+\frac{1}{\delta}]t\tau\kappa(i_t-\widehat{i})+\frac{1}{\delta}t\tau\kappa(i_{t-1}-\widehat{i})-\frac{1}{\delta}\left[\frac{\beta w}{\tau}+\frac{b\left(1-w\right)}{t}\right]t\tau\kappa((i_t-\widehat{i})$$

or

$$
\begin{aligned}
i_t \quad = \quad & \frac{\beta w}{\tau \kappa}\left(\pi_t-\widehat{\pi}\right)+\frac{\lambda w}{\tau \kappa} \Delta \widetilde{y}_t+\frac{b(1-w)}{t \kappa}\left(\pi_t^*-\widehat{\pi}\right)+\frac{\lambda(1-w)}{t \kappa} \Delta \widetilde{y}_t^* \\
& +[1+\frac{1}{\delta}] i_{t-1}-\frac{1}{\delta} i_{t-2}+\frac{1}{\delta}\left[\frac{\beta w}{\tau}+\frac{b(1-w)}{t}\right]\left(i_{t-1}-\widehat{i}\right)
\end{aligned} \tag{34}
$$

This implicit instrument rule also involves positive interest rate reaction to contemporaneous deviations of inflation from the target and to output gap changes of both countries, as well as to past interest rates, introducing history dependence. Examining the response of interest rate to the variables of both countries, we can draw the following conclusions:

i) given that i_t, π_t and π_t^* are positively related, the interest rate must respond positively to the deviations of inflation from target in each country, depending on the slope of their Phillips curves (β and b). Therefore, the interest rate must adjust more to the variables of the country with a steeper Phillip curve and lower price rigidity.

ii) given that i_t, π_t , $\Delta\widetilde{y}_t$, π_t^* and $\Delta\widetilde{y}_t^*$ are positively related, the interest rate must adjust positively both to output gap changes and to deviations of inflation from target, in proportion to the slope of the aggregate demand of each country $\left(\frac{1}{t}, \frac{1}{\tau}\right)$. Thus, the interest rate responds more strongly to changes of the variables of the country with a steeper aggregate demand curve and a lower intertemporal elasticity of substitution.
The strong effect of the real interest rate on the output gap, and/or that of the output gap on inflation in a country, requires the central bank to react more aggressively in stabilizing this country's variables. Otherwise, any monetary policy decisions will strongly affect this country.

Given our assumptions on the existence of a unique discount factor (δ) and the same central bank preferences for output and interest rate stabilization parameters (κ and λ) in both countries, the reaction of the interest rate to changes in these parameters does not vary between countries.

5 Model Estimation

In order to evaluate the optimal interest rate reaction functions described in the previous sections, we need to estimate the structural parameters of the models.[5] In the case of the first model[6] (eqs. 18 and 19), EMU was approximated by a hypothetical monetary union between Germany and France using quarterly aggregate data, weighted according to the OECD weighting scheme,[7] for the period 1999:1-2003:4. Due to a limited number of time series observations, it was found necessary to calculate Bayesian estimates, using the prior assumption that the parameters will be close to the weighted average of the parameters of the individual countries' estimated equations. The second model, which consists of the individual country equations (eqs. 25, 26, 27 and 28), was estimated using quarterly data for Germany and France for the period 1970:01-1998:4.

All data series used for the estimation of both models have a quarterly frequency and are drawn from the OECD Economic Outlook, December 2003. The annual percentage change of the Harmonized Index of Consumer Prices is used to measure inflation. The output gap used is the deviation of actual real GDP from its potential level, the calculation of which is based on a production function approach.[8] The nominal interest rate used is the official discount rate (set by the Bundesbank for Germany and the Banque de France for France until 1998:4, and the minimum bid rate set by the European Central Bank after 1999). The parameters are estimated using the Generalized Method of Moments (GMM), which is widely used in models with forward looking variables. The instruments chosen are lagged values of the explanatory variables (inflation, output gap and interest rate), so that they are predetermined at the time the central banks decides on the level of the interest rate. Furthermore, they are uncorrelated with the residuals, but strongly correlated with the forward looking variables.

Table 1. GMM estimates of the Phillips curve and the aggregate demand equation

[5] Previous research on empirical estimation of New Keynesian equations include Fuhrer and Rudebusch (2003), Galí, Gertler and López-Salido (2001) and Jondeau and Le Bihan (2001).

[6] Most researchers justify the use of synthetic data for the euro area constructed by aggregating weighted national data before EMU, given the gradual process of monetary convergence and the participation in the Exchange Rate Mechanism (ERM) since the beginning of the 90s. Moreover, statistical evidence by Mihov 2001 rejects the presence of a structural break in January 1999 and suggests that the use of pooled data from euro area member countries can be a good approximation to the euro area data.

[7] Namely, the weight given to France is 0.42 and to Germany 0.58.

[8] For explicit details, see Giorno et al (1995).

Germany		France		Union of France and Germany	GMM	Bayesian
δ	0.99 (0.016)	δ	0.99 (0.014)	δ	0.99 (0.014)	0.99
β	0.04 (0.025)	b	0.29 (0.069)	ρ	0.01 (0.031)	0.14
$\frac{1}{\tau}$	0.06 (0.032)	$\frac{1}{t}$	0.03 (0.014)	$\frac{1}{\sigma}$	0.02 (0.013)	0.03
$adjR^2(PC)$	0.85	$adjR^2(PC)$	0.98	$adjR^2(PC)$	0.69	
$adjR^2(AD)$	0.87	$adjR^2(AD)$	0.94	$adjR^2(AD)$	0.83	
$SE\ (PC)$	0.72	$SE\ (PC)$	0.67	$SE\ (PC)$	0.44	
$SE\ (AD)$	1.16	$SE\ (AD)$	0.58	$SE\ (AD)$	0.49	
$J-stat\ (PC)$	0.10	$J-stat\ (PC)$	0.09	$J-stat\ (PC)$	0.23	
$J-stat\ (AD)$	0.09	$J-stat\ (AD)$	0.13	$J-stat\ (AD)$	0.38	
Numbers in parentheses correspond to standard errors (SE) of the estimated parameters.						

Our estimates show almost flat aggregate demand equations for both countries, similarly to Fuhrer and Rudebusch (2003) estimates for the US.[9] The Phillips curve estimates show high values of the discount factor (δ), as found in most empirical studies. Furthermore, we obtain very low estimates for the slope of the Phillips curve for Germany and the Union (β and ρ), which are significant only at the 10 percent level, as in Jondeau and Le Bihan (2001).[10] In contrast, the Phillips curve in France (with slope b) is found comparatively steeper, revealing a greater effect of the output gap on inflation in that country. The values of the $J-statistics$ verify the validity of the instruments used, and the values for $adjR^2$ are reasonably high.

6 Monetary policy performance under the alternative rules

The central bank takes as given the models estimated in the previous section, observes the current state of the economy and then sets its policy instrument, in order to minimize the following

[9] Fuhrer and Rudebusch (2003) estimate a modified output Euler equation which includes both past and future values of the output gap.

[10] Jondeau and Le Bihan (2001) estimate a hybrid Phillips curve, which includes a lag and a lead of inflation in addition to output gap.

objective function:

$$\min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j L_{t+j} = \min_{i_t} E_t \sum_{j=0}^{\infty} \delta^j \frac{1}{2} \left[(\pi_{t+j} - \widehat{\pi})^2 + \lambda \left(\widetilde{y}_t \right)^2 + \kappa \left(i_t - \widehat{i} \right)^2 \right]$$

In the case $\delta \to 1$, Rudebusch and Svensson (1999) showed that the intertemporal loss function can be interpreted as the unconditional mean of the period loss function:

$$E(L_t) = var(\pi_t) + \lambda var(\widetilde{y}_t) + \kappa var(i_t)$$

where the parameters λ and κ represent the dislike by an inflation-targeting central bank of variations in the output gap and interest rate changes.

In order to compare the performance of the optimal interest rate reaction functions, derived in Sections 3 and 4, we compute the optimal coefficients of the interest rate reaction function over the state variables and then, following Soderlind (1999) we calculate the variance of inflation deviations from the target, the variance of the output gap and of interest rate differentials, assuming that the parameters of the loss function (λ and κ) take values ranging from 0.1 to 0.5. The state-space representation of the structural equations and the loss function to be minimized for both versions, is presented in the Appendix.

The minimization of the union loss function (17), resulted in the following optimal rules:

Table 2: Coefficients of the optimal interest rate reaction function

parameters	π_t^U	$\widetilde{y}_t^U$	$\widetilde{y}_{t-1}^U$	i_{t-1}	i_{t-2}	constant
$\lambda = 0.1, \kappa = 0.1$	0.0414	0.0329	-0.0316	1.934	-0.9406	0.0044
$\lambda = 0.1, \kappa = 0.5$	0.0084	0.0063	-0.0060	2.0141	-1.0175	0.0042
$\lambda = 0.5, \kappa = 0.1$	0.0429	0.1600	-0.1519	1.9698	-0.9792	0.0041
$\lambda = 0.5, \kappa = 0.5$	0.0084	0.0315	-0.0300	2.0054	-1.0093	0.0043

In the case of the two-country aggregate loss function (24), the optimal rules are given by:

Table 3: Coefficients of the optimal interest rate reaction function

parameters	π_t^G	$\widetilde{y}_t^G$	$\widetilde{y}_{t-1}^G$	π_t^F	$\widetilde{y}_t^F$	$\widetilde{y}_{t-1}^F$	i_{t-1}	i_{t-2}	constant
$\lambda = 0.1, \kappa = 0.1$	0.0182	0.0483	-0.1050	0.0408	0.0290	-0.0102	1.9834	-1.0300	0.0100
$\lambda = 0.1, \kappa = 0.5$	0.0180	0.0590	-0.1082	0.0500	0.0660	-0.0604	1.9340	-0.9900	0.1003
$\lambda = 0.5, \kappa = 0.1$	0.0300	0.7206	-0.0770	0.0473	0.0136	-0.0108	1.7950	-1.0400	0.0321
$\lambda = 0.5, \kappa = 0.5$	0.0320	0.0446	-0.0360	0.0964	0.0227	-0.0097	1.9300	-0.9800	0.7546

Both estimated rules share the same features with the rules derived from the minimization of the loss functions. Namely, they exhibit positive responses to inflation and output gap changes and present increased interest smoothing, which is due to the inclusion of the interest rate changes in the loss function. Furthermore, increasing the weight given to the output gap (λ) in the loss function leads to a higher interest rate response to output gap changes, while increasing the weight given to deviations of the interest rate from the interest rate consistent with inflation target (κ) reduces the interest rate reaction both to inflation and to output gap changes.

The comparison of the optimal coefficients of the rules supports our proposition that the central bank should take into account the national structural characteristics. In particular, the rule (23) that reacts to the union variables, corresponds to each country's variables according to their weight in the aggregate variable. By contrast, the rule (34) that reacts explicitly to national variables, suggests an adjustment to each country's variables according to the structural parameters of the economy. This is the reason why the interest rate adjusts more aggressively to inflation in France, and to the output gap in Germany. Indeed, the steeper Phillips curve in France, makes it necessary for the single interest rate to react stronger to inflation in France, while the fact that aggregate demand is slightly flatter in France, and a higher weight is given to Germany's variables, justifies the stronger response to the output gap changes in Germany.

A Appendix

A.1 State space representation of the union loss function minimization problem

The structural equations of the state-variables can be represented by:

$X_{t+1}^U = A^U X_t^U + B^U i_t + V_t^U$, i.e.

$$\begin{bmatrix} \pi_{t+1}^U \\ \widetilde{y}_{t+1}^U \\ \widetilde{y}_t^U \\ i_t \\ i_{t-1} \\ 1 \\ E_t\pi_{t+2}^U \\ E_t\widetilde{y}_{t+2}^U \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{\rho}{\delta} \\ 0 & 0 & 0 & 0 & 0 & -\frac{\overline{r}}{\sigma} & -\frac{1}{\sigma\delta} & 1+\frac{\rho}{\sigma\delta} \end{bmatrix} * \begin{bmatrix} \pi_t^U \\ \widetilde{y}_t^U \\ \widetilde{y}_{t-1}^U \\ i_{t-1} \\ i_{t-2} \\ 1 \\ E_t\pi_{t+1}^U \\ E_t\widetilde{y}_{t+1}^U \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sigma} \end{bmatrix} * i_t + \begin{bmatrix} \varepsilon_{t+1} \\ æ_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The target variables are given by:

$Y_t^U = C_X^U X_t + C_I^U i_t$, i.e.

$$\begin{bmatrix} \pi_t^U - \widehat{\pi} \\ \widetilde{y}_t^U \\ i_t - \widehat{i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\widehat{i} & 0 & 0 \end{bmatrix} * \begin{bmatrix} \pi_t^U \\ \widetilde{y}_t^U \\ \widetilde{y}_{t-1}^U \\ i_{t-1} \\ i_{t-2} \\ 1 \\ E_t\pi_{t+1}^U \\ E_t\widetilde{y}_{t+1}^U \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} * i_t.$$

The loss function can be found by:

$$L_t = Y_t^{U\prime} K Y_t^U = \begin{bmatrix} \pi_t^U - \widehat{\pi} \\ \widetilde{y}_t^U \\ i_t - \widehat{i} \end{bmatrix}' * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \kappa \end{bmatrix} * \begin{bmatrix} \pi_t^U - \widehat{\pi} \\ \widetilde{y}_t^U \\ i_t - \widehat{i} \end{bmatrix}.$$

A.2 State space representation of the two-country aggregate loss function minimization problem

The structural equations are represented by:

$X_{t+1}^T = A^T X_t^T + B^T i_t + V_t^T$, i.e.

$$\begin{bmatrix} \pi_{t+1} \\ \widetilde{y}_{t+1} \\ \widetilde{y}_t \\ \pi^*_{t+1} \\ \widetilde{y}^*_{t+1} \\ \widetilde{y}^*_t \\ i_t \\ i_{t-1} \\ 1 \\ E_t\pi_{t+2} \\ E_t\widetilde{y}_{t+2} \\ E_t\pi^*_{t+2} \\ E_t\widetilde{y}^*_{t+2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{\beta}{\delta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\overline{r}}{\tau} & -\frac{1}{\tau\delta} & 1 + \frac{\beta}{\tau\delta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\delta} & -\frac{b}{\delta} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\overline{r}}{t} & 0 & 0 & -\frac{1}{t\delta} & 1 + \frac{b}{t\delta} \end{bmatrix} * \begin{bmatrix} \pi_t \\ \widetilde{y}_t \\ \widetilde{y}_{t-1} \\ \pi^*_t \\ \widetilde{y}^*_t \\ \widetilde{y}^*_{t-1} \\ i_{t-1} \\ i_{t-2} \\ 1 \\ E_t\pi_{t+1} \\ E_t\widetilde{y}_{t+1} \\ E_t\pi^*_{t+1} \\ E_t\widetilde{y}^*_{t+1} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\tau} \\ 0 \\ \frac{1}{t} \end{bmatrix} * i_t + \begin{bmatrix} \varepsilon_{t+1} \\ v_{t+1} \\ 0 \\ e_{t+1} \\ v_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The target variables are given by:

$Y_t^T = C_X^T X_t^T + C_I^T i_t$, i.e.

$$\begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\widehat{\pi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\widehat{i} & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \pi_t \\ \widetilde{y}_t \\ \widetilde{y}_{t-1} \\ \pi_t^* \\ \widetilde{y}_t^* \\ \widetilde{y}_{t-1}^* \\ i_{t-1} \\ i_{t-2} \\ 1 \\ E_t\pi_{t+1} \\ E_t\widetilde{y}_{t+1} \\ E_t\pi_{t+1}^* \\ E_t\widetilde{y}_{t+1}^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} * i_t.$$

The loss function can then be found by:

$$L_t = Y_t^{T\prime} K^T Y_t^T = \begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix}' * \begin{bmatrix} w & 0 & 0 & 0 & 0 \\ 0 & w\lambda & 0 & 0 & 0 \\ 0 & 0 & (1-w) & 0 & 0 \\ 0 & 0 & 0 & (1-w)\lambda & 0 \\ 0 & 0 & 0 & 0 & \kappa \end{bmatrix} * \begin{bmatrix} \pi_t - \widehat{\pi} \\ \widetilde{y}_t \\ \pi_t^* - \widehat{\pi} \\ \widetilde{y}_t^* \\ i_t - \widehat{i} \end{bmatrix}.$$

A.3 Estimation of loss

The evaluation of the loss function relies on the optimization framework used by Rudebusch and Svensson (1999) and Soderlind (1999) and is based on the dynamic stochastic regulator problem (Chow (1970) and Sargent (1987)).

Given an optimal rule of the following form:

$$i_t = fX_t$$

the state variables are given by:

$$X_{t+1} = (A + Bf)\, X_t + V_t$$

and the target variables by:

$$Y_t = (C_X + C_I f) X_t$$

The expected value of the loss function is then given by:

$$E\,(L_t) = X_t^{'} V X_t + \frac{\delta}{1-\delta} trace\,(V \Sigma_{VV})$$

In case $\delta \to 1$, Rudebusch and Svensson (1999) showed that the loss function can then be approximated by:

$$E\,(L_t) = trace\,(V \Sigma_{VV})$$

where $V = [I - \delta M' \otimes M]^{-1} * W$

$$W = \left[\begin{bmatrix} I \\ G \\ f \begin{bmatrix} I \\ G \end{bmatrix} \end{bmatrix}' \begin{bmatrix} C_X' K C_X & C_X' K C_I \\ \left(C_X' K C_I'\right)' & C_I' K C \end{bmatrix} \begin{bmatrix} I \\ G \\ f \begin{bmatrix} I \\ G \end{bmatrix} \end{bmatrix} \right],$$

M and G correspond to the dynamic solution of

$$X_{P,t+1} = M X_{P,t} + V_t$$

$$X_{F,t+1} = G X_{F,t}$$

and Σ_{VV} is the variance-covariance matrix of disturbances.

Alternatively, the value of the loss function can be derived by:

$$E(L_t) = trace(K\Sigma_{YY})$$

where $\Sigma_{YY} = (C_X + C_I f) \begin{bmatrix} I \\ G \end{bmatrix} \Sigma_{X_P X_P} \begin{bmatrix} I \\ G \end{bmatrix}' (C_X + C_I f)'$

and $vec(\Sigma_{X_P X_P}) = [I - M \otimes M]^{-1} vec(\Sigma_{VV})$

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