Energy Demand Models : A Threshold Panel
Specification of the “Kuznets Curve”

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Abstract

This paper proposes an original panel specification of the energy demand model. Based on panel threshold regression models, we derive country-specific and time-specific energy elasticity. We find a fall of the elasticity when the income level increase.

Keywords : Energy Elasticity, Kuznets Curve, Panel Smooth Transition Regression model.

JEL classification : C23, Q41, Q43

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1. Introduction

With substantial increases in world oil prices, the analysis of the national energy intensity measurement becomes an important political issue. To serve these purposes, many energy efficiency indicators were developed and applied for explain differences in energy performance between countries and for international benchmarking. The energy elasticities is one solution often found in energy studies in the 1970s and 1980s.

With these indicators, a set of empirical model finds that the energy intensity of a country have an inverted-U curve shaped. In general, the intensity passes through a more or less strong and long growth phase, before reaching a turning point which is sometimes marked, sometimes in the form of a plateau, and then decreases. This curve has been popularized during the nineties under the name of “Kuznets curve” by environment economists who found the same type of relation between environmental pollutants and economic activity. Many studies explain this phenomena by the different stages of economic development and inter-energy substitutions. The energy intensity of a country first rises along with the economic development process, in the industrialization phase, and then declines in the post-industrialization phase because of the increase of services and high technology industries, despite the development of transportation.

Usually for estimate the energy elasticity, the authors consider a double-log specification of the energy demand equation. They regress the logarithm of the consumption of primary energy per capita by the logarithm of the per
capita GDP in cross-section model. Unfortunately, the log-log structural relationship has the great disadvantage of being intrinsically unable to reproduce the empirical evidence of the inverted-U curve. One way to deal with the problem is to use a quadratic logarithmic specification (Ang, 1987). This quadratic form function can be viewed as an approximation to a more complex function and constitutes an alternative solution to non-parametric approaches. However in a panel data context, these methods suffer to two major drawbacks. It implies that, for a given level of per capita GDP, the energy-GDP ratio is the same for all the countries of the sample. If the panel includes Canada and Mexico, this assumption may be doubtful. Moreover, they do not make explicit the time dimension.

The main purpose of this paper is therefore to propose an original solution to consider the “U inverted” shaped, the heterogeneity and the time instability of the energy demand model by introducing regime switching approach. More specifically, we apply a panel smooth transition model (PSTR). This choice is justified by two reasons. Firstly, the fact that income elasticity of energy demand depends on income level, clearly corresponds to the definition of a threshold regression model. Secondly, we justify this methodology by showing that the quadratic polynomial model widely used to examine the energy elasticity can be viewed as an approximation of the PSTR.

This paper is organized as follows: the next section introduces the econometric methodology of smooth threshold model. Section 3 reports empirical results. Section 4 concludes.
2. methodology

2.1. Model

The basis of our empirical approach is exactly the same as that used by many authors in this literature. It consists in evaluating the energy demand equation for a panel of \(N\) countries. However, we modify this approach by using PSTR model recently developed by González et al. (2005). The corresponding model with fixed effects \(\alpha_i\) is then define as follows:

\[
c_{it} = \alpha_i + \beta_0 y_{it} + \beta_1 y_{it} g(q_{it}; \gamma, \delta) + \varepsilon_{it} \tag{1}
\]

where \(c_{it}\) is the logarithm of the consumption of primary energy per capita of the \(i^{th}\) country for the year \(t\), \(y_{it}\) is the logarithm of the corresponding per capita GDP and \(\varepsilon_{it}\) is \(i.i.d. (0, \sigma^2\)). We assume that the transition function \(g(q_{it}; \gamma, \delta)\) follows a logistic function:

\[
g(q_{it}; \gamma, \delta) = \left[1 + \exp(-\gamma(q_{it} - \delta))\right]^{-1}, \quad \gamma > 0, \tag{2}
\]

This function is continuous and bounded between \([0;1]\). It depends of a transition variable \(q_{it}\), a threshold \(\delta\) and a smooth parameter \(\gamma\). If the parameter \(\gamma\) tends to infinity, the transition function \(g(q_{it}; \gamma, \delta)\) tends to the indicator function, the transition is sharp as a Panel Threshold Regression (PTR) model (Hansen, 1999). When \(\gamma\) tends to zero the transition function \(g(q_{it}; \gamma, \delta)\) is constant and the model corresponds to a standard linear model with individual effects (so-called “within” model), that is with constant and homogenous elasticities. We assume here that the transition variable is the logarithm of per capita GDP i.e. \(q_{it} = y_{it}\). This choice points out the fact that income elasticity of energy demand depends on income level.
In our context, the PSTR energy demand model has two main advantages. Firstly, it provides a parametric approach to the cross-country heterogeneity and the time instability of the slope coefficients (and consequently the income elasticity). It allows the parameters to change smoothly as a function of the threshold variable $y_u$. More precisely, the income elasticity for the $i^{th}$ country at time $t$ is defined by the weighted average of the parameters $\beta_0$ and $\beta_1$:

$$ e_u = \frac{\partial c_u}{\partial y_u} = \beta_0 + \beta_1 g(y_u; y, c) + \beta_1 y_u \frac{\partial g(y_u; y, c)}{\partial y_u} $$  \hspace{1cm} (3) $$

Secondly, the slope of the PSTR model can be different from the estimated parameters for the extreme regimes ($\beta_0$ for the first regime and $\beta_0 + \beta_1$ for the second). Between these two extremes, there exists a “continuum” of intermediate regime, and the slopes are defined as a weighted average of the parameters $\beta_0$ and $\beta_1$. Therefore, it is important to note that it is often difficult to directly interpret the values of these parameters (as in a probit or logit model). It is generally preferable to interpret (i) the sign of these parameters which indicates an increase or a decrease of the energy demand with the value of the threshold variable and (ii) the time varying and individual elasticity given by the equation (3). We hope to find here that $\beta_0 > 0$ and $\beta_1 < 0$, because in this case, the slope of the second regime is thus lower than the first, which makes it possible to take account of the inverted-U curve.
2.2. Estimation and Linearity Test

The estimation of the parameters of the PSTR model consists of eliminating the individual effects $\alpha_i$ by removing individual-specific means and then by applying non-linear least squares to the transformed model (see for details González et al., 2005). This method is equivalent to the maximum likelihood estimation in the case of normal errors.

However, before estimating the PSTR model, it is necessary to determine whether the regime-switching effect is statistically significant. Testing the linearity can be done by testing $H_0 : \gamma = 0$ or $H_0 : \beta_0 = \beta_i$. But in both cases, the test will be non-standard since, under $H_0$, the PSTR model contains unidentified nuisance parameters. A solution consists in replacing the transition function $g(y_u; \gamma, c)$ by its first-order Taylor expansion around $\gamma = 0$ and by testing an equivalent hypothesis in an auxiliary regression. Then, we obtain:

$$c_u = \alpha_i + \beta_0 y_u + \theta_i y_u^2 + \epsilon_u$$

(4)

In this first-order Taylor expansion, the parameter $\theta_i$ is proportional to the slope parameter $\gamma$. Thus, testing the linearity against the PSTR model simply consists of testing $H_0 : \theta_i = 0$ in this linear panel model. For this objective, we can apply standard tests like the F-statistics.

As we can notice, equation (4) corresponding the quadratic polynomial model, which is the econometric specification used in most energy demand studies for represent “the Kuznets curve”. Therefore, this point empirically
justify the idea of regime-switching in the analysis of energy intensity by showing that the quadratic model derives from a PSTR specification.

If we have rejected the linearity hypothesis, we can check that there is no remaining non-linearity. The issue is then to test whether there is one transition function or whether there are at least two transition functions defined as:

\[
c_{i} = \alpha + \beta_{0} y_{i} + \beta_{1} y_{i} g_{1}(y_{i}; \gamma_{1}, c_{1}) + \beta_{2} y_{i} g_{2}(y_{i}; \gamma_{2}, c_{2}) + \epsilon_{i} \tag{5}
\]

The logic of the test consists of replacing the second transition function by its first-order Taylor expansion around \( \gamma_{2} = 0 \) and then testing linear constraints on the parameters. If we use the first-order Taylor approximation of \( g_{2}(y_{i}; \gamma_{2}, c_{2}) \), the model becomes:

\[
c_{i} = \alpha + \beta_{0} y_{i} + \beta_{1} y_{i} g_{1}(y_{i}; \gamma_{1}, c_{1}) + \theta_{1} y_{i}^{2} + \epsilon_{i} \tag{6}
\]

and the test of no remaining non-linearity is simply defined by \( H_{0} : \theta_{1} = 0 \). If we reject \( H_{0} \), we must check if there exist a third transition function etc.

4. Data and Results

In this study, we consider a balanced panel of 44 countries over the period 1950-99. Total consumption is the sum by source aggregated on the basis of its net calorific value (expressed in tones of oil equivalent: 1 toe = 42 GJ).

For primary electricity, the consumption equivalence is 1 kWh = 860 kcal (except for the nuclear 2600 kcal). Population and GDP data have been gathered from the last publication of A. Maddison in OECD (2003).
The first step consists of testing the log-linear specification of energy demand against a specification with threshold effects. The table 1 shows that the linearity hypothesis is strongly rejected. This first result confirms the non-linearity of the energy demand, but more originally shows the presence of strong threshold effects determined by income level. It will be therefore necessary, in a second step, to determine the number of transition functions required to capture all the non-linearity of the energy demand. In our second test of no remaining non-linearity, the null hypothesis isn’t rejected. Thus, our model needs only one transition function.

In second step, we analyze the parameter estimates of the final PSTR models. The small smooth parameter (1.296) shows that the estimated transition function is not sharp. This point is particularly important, since it implies that the heterogeneity of the energy elasticities cannot be reduced to a limited number of regimes with different slope parameters. Now, with regard to the slope coefficients, we observe a negative and significant parameter $\beta_1$. Our model thus takes well account of the “U inverted” shape of the energy demand.

Given the parameter estimates in a third step, it is possible to compute, for each country of the sample and for each date, the time varying income elasticity\(^2\), denoted $e_{it}, i = 1, \ldots, N$ and $t = 1, \ldots, T$. Given equation (1), we obtain:
\[ e_u = \frac{\partial c_u}{\partial y_u} = 1.569 - \frac{0.800}{1 + \exp(-1.296(y_u - 3.055))} - 0.800 y_u \frac{1.296 \times \exp[-1.296(y_u - 3.055)]}{[1 + \exp(-1.296(y_u - 3.055))]^2} \]  

(7)

On the figure 1, this elasticity is displayed for all possible value of the transition variable. We confirm that when the income level increase, the estimated energy elasticity decreases. The transition between theses two extremes is not linear but smooth. More precisely, it has a half U inverted shaped. Elasticity does not decrease in regular way according to the income level. The fall is more important when the income level is relatively high.

5. Conclusion

The main purpose of this paper is to justify the idea of regime switching as a more general specification than the quadratic polynomial model widely used in the literature. Second, we propose an original solution to consider the heterogeneity and the time instability of the energy elasticity.
### Table 1. Parameter Estimates for the PSTR Models

<table>
<thead>
<tr>
<th>Specification</th>
<th>PSTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold Variable</td>
<td>( y_{it} )</td>
</tr>
<tr>
<td>Fisher type test of linearity</td>
<td>551.5</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>Fisher type test of no remaining</td>
<td>2.36</td>
</tr>
<tr>
<td>non-linearity</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Income Parameter ( \beta_0 )</td>
<td>1.569</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>Income Parameter ( \beta_1 )</td>
<td>-0.800</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
</tr>
<tr>
<td>Location Parameter ( \delta )</td>
<td>3.055</td>
</tr>
<tr>
<td>Smooth Parameter ( \gamma )</td>
<td>1.296</td>
</tr>
<tr>
<td>Residual Sum of Squares</td>
<td>108.9</td>
</tr>
</tbody>
</table>

Notes: The test of linearity has an asymptotic F(1, TN-N-1) distribution under \( H_0 \) and F(1, TN-N-2) for the no remaining non-linearity test where N is the number of individuals and T the number of periods. The standard errors in parentheses are corrected for heteroskedasticity.
Fig. 1: Estimated Energy Elasticity

Notes: Energy Elasticity are displayed for all possible value of income level (equation [6]).
References


Notes:

(1) : Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Colombia, EX-Czechoslovakia, Denmark, Egypt, Finland, France, Germany, Hungary, India, Indonesia, Iran, Italy, Japan, South Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Nigeria, Peru, Philippines, Poland, Romania, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, USA, EX URSS, Venezuela.

(2) : For individual elasticities see Destais, Fouquau, Hurlin (2006)