Estimating the Contribution of Public Capital with Times Series Production Functions : a Case of Unreliable Inference

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Abstract

This paper assesses the magnitude of bias in reported estimates of the productive contribution of public capital stock to private factors productivity and growth. We replicate the standard approach, based on a production function, in order to estimate structural parameters on simulated pseudo samples generated from a stochastic general equilibrium model.

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1 Introduction

The production function approach has often been used to evaluate the impact of the public capital stock on productivity and output. This approach gives very different results depending on the econometric methodology and the specification that have been used. With time series in level, the estimated elasticity of public infrastructures then ranges from 20% to 40% in Aschauer (1989), Munnell (1990) or Eisner (1994). But Tatom (1991) or Sturm and De Haan (1995) showed that when the production function is taken in first differences, very low or insignificant estimated values of the public capital elasticity are obtained.

This large range observed in empirical studies leads us to propose a sensibility analysis of these alternative strategies based on the simulation of a stochastic general equilibrium model with endogenous public capital. In this model, the value of the public capital elasticity is set to a moderate, but positive level. We replicate the Aschauer's methodology on 10 000 pseudo samples. A test of the ability of alternatives econometric methodologies to correct the biases and to bring more reasonable estimates is then proposed.

First, this paper shows that the standard approach applied to simulated series issued from our model leads systematically to overestimate the productive contribution of public infrastructures. More precisely, in our exercise an estimated elasticity near to 40% is found (close to Aschauer's results), whereas the calibrated value in data generating process is only 5%.

Second, the results suggest that the main correction methodologies proposed in the literature may not be able to provide reliable estimates of the production elasticities at least in our context. It implies that the specification of an econometric model in partial equilibrium leads to spurious measures of the productive contribution of the public capital when this contribution is determined by general equilibrium mechanisms.

The paper is organized as follows. In section 1, the benchmark model and the parameters values are presented. In section 2 and 3 the methodology and the results are discussed. A last section concludes.

2 The benchmark economy

The only constraint imposed by our exercise on the model is the stochastic dimension. Indeed, the estimation of a production function augmented by the introduction of public capital requires at least the introduction of two independent stochastic regressors. So, in order to avoid the multicolinearity problems, our model must contain two stochastic shocks.

A reference satisfying this requirement is provided by the Kocherlakota and Yin's (1996) model which includes a standard technological shock and introduces a second shock affecting the government's decision rules of investment. The only difference is that we introduce an endogenous supply of labor as in the Baxter and King's (1993) model in order to allow for richer dynamics.

So let us consider a single good economy, composed by an infinitely lived representative agent who maximizes the following lifetime expected utility

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \theta_L \log(1 - N_t) \right] \tag{1}$$

subject

$$C_t + K_{t+1} - (1-\delta) K_t \leq Y_t - T_t$$

where C_t,N_t, Y_t and K_t denote respectively consumption, labor, product and private capital. The term T_t represents a lump–sum tax.

The private inputs are capital K_t and efficient labor. As it was generally done in the literature, we suppose that the stock of public capital KG_t, which is considered as given by the firm, improves the private factors productivity. The technology is

$$Y_t = A_t F(K_t, X_{ht} N_t, KG_t) = A_t N_t^{1-e_k} X_{ht}^{1-e_k-e_g} K_t^{e_k} KG_t^{e_g} \tag{2}$$

where $X_{h,t}$ represents a neutral Harrodian technical progress which grows at the factor $\gamma_{x,h}$. The total factor productivity, A_t, follows an $AR(1)$ process.

$$\log(A_{t+1}) = \rho_A \log(A_t) + \epsilon_{A,t+1} \qquad |\rho_A| \leq 1 \tag{3}$$

The innovations $\epsilon_{A,t}$ are iid and $V(\epsilon_{A,t}) = \sigma^2_{\epsilon_a}$. For the reasons presented above, we suppose that public decision rules are affected by a shock defined as in the Kocherlakota and Yin's model :

$$\frac{KG_{t+1}}{Y_t} = \mu v g_t \tag{4}$$

with $\log(vg_{t+1}) = \rho_g \log(vg_t) + \epsilon_{g,t+1}$ and $|\rho_g| < 1, V(\epsilon_{g,t}) = \sigma^2_{\epsilon_g}$. This assumption means that government provides a constant ratio of public capital which is only affected by stationary disturbances vg_t. Public investment is then determined by :

$$IG_t = KG_{t+1} - (1 - \delta_g)\, KG_t = \mu\, v g_t Y_t - (1 - \delta_g)\, KG_t \qquad (5)$$

The lump–sum tax T_t finances public investment IG_t and consumption G_t which does not affect the agent's utility. In each and every period, the government's budget is balanced

$$T_t = G_t + IG_t = s_g Y_t + IG_t \qquad (6)$$

In order to obtain constant ratio at the steady state, we deflate all the growing variables for growth. We consider two cases: the first case in which $\rho_a = 1$ and $\gamma_{xh} = 1$, corresponds to an exogenous non-stationary growth model, the second one in which $|\rho_a| < 1$ and $\gamma_{xh} > 1$ corresponds to a stationary case. Thus, when growth is stochastic we divide all the variables that grow by $A_t^{1/(1-e_k-e_g)}$ to obtain constant variables in a steady state, whereas in the case of the deterministic growth all variables are divided by $X_{h,t}$.

Given the public decision rules, the first order conditions are log-linearized around the deterministic steady state. The model is then solved using the standard forward–backward method for rational expectations models.

The log-linearized version of the model is simulated 10,000 times using the Monte–Carlo method for a sample of 50 years which is roughly the average sample size used in empirical studies. Then, we rebuilt the gross series in order to perform empirical estimates as in the literature.

The calibration of the structural parameters is the same as in the Kocherlakota and Yin's (1996) and Baxter and King's (1993) models for US annual data (See table 1).

Table 1 : Calibration

In order to get a non stationary representation (model 1), we suppose that the technological shock follows a random walk and the deterministic rate of growth is zero ($\gamma_{xh} = 1$). Conversely, in the second model the technological shock is stationary but highly persistent ($\rho_a = 0.95$) and the deterministic factor of growth is equal to 1.0161 as in King, Plosser and Rebelo's (1988).

The crucial assumption in this calibration is the value of the public capital elasticity e_g (5% as in Baxter and King 1993) which is very low compared to the Aschauer's results (39%), but not inconsistent with the actual rate of public investment.

3 Methodology

The pseudo samples generated by the model are used to estimate the public capital elasticity. We replicate the standard methodology used in the empirical studies based on the production function approach.

First, we consider a Cobb Douglas specification. As in the literature, the Solow residual filtered for the effects of public externalities is approximated by a constant and a deterministic trend. Second, as in the literature, we consider and test several hypothesis on the returns–to–scale. The first is the private factors constant returns–to–scale (PFCRS), which corresponds to our model. The second is the overall constant returns–to–scale (OCRS).[1]

$$PFCRS \ : \ y_t - n_t = \alpha_c + \alpha_t t + \alpha_k \left(k_t - n_t\right) + \alpha_g kg_t + \mu_t \tag{7}$$

$$OCRS \ : \ y_t - n_t = \alpha_c + \alpha_t t + \alpha_k \left(k_t - n_t\right) + \alpha_g \left(kg_t - n_t\right) + \mu_t \tag{8}$$

We estimate the elasticities α_i according to the estimation methods used in the literature. The first is the OLS used among others by Aschauer (1989) or Munnel (1990). The second is the OLS applied to first differenced series, noted OLSΔ, considered by Tatom (1991), Sturm and De Haan (1995) or Ford and Poret (1993). The Cochrane–Orcutt procedure is then explored, as in Ratner (1983).

The last two methods are fairly recent. The Fully Modified method (FM-OLS), proposed by Phillips and Hansen (1990), is set up to correct the endogeneity bias due to the long term relationship between variables. It was applied in this context by Otto and Voss in 1997. We also use the Dynamic Least Squared method (DOLS) proposed by Stock and Watson (1994), which exhibits better properties on small samples than the FM-OLS (Montalvo 1995).

4 Results

Model 1 : non stationary case

For the model 1 (non stationary case), the average results are presented in table 2.

Table 2 : Model 1 (non-stationary case)

In both specification the elasticity $\widehat{\alpha}_g$ of public capital is drastically and significantly overestimated (35% or 40%) by the OLS. These results mean

[1] Lower case letters denote the natural logarithm of the corresponding aggregates

that a low elasticity of public capital in the structural model (5%) is compatible with Aschauer's results (39%) when the traditional production function approach is adopted for econometric inference.

But, as it was stressed in Granger and Newbold (1974), given the non stationarity of our variables, the accurate strategy is to run regressions on first differentiated data. However, in this case, no clear–cut conclusion can be made concerning the productive impact of the public capital. On the one hand, the estimated parameter $\widehat{\alpha}_g$ is not significantly different from zero on average, and on the other hand the elasticity of the stock of private capital is always negative, as in Sturm and De Haan (1995). The results of Cochrane-Orcutt's method ($AR(1)$) are very close to those obtained by the OLS applied to first differenced series, since the estimated autoregressive parameter of residuals is close to unity.

These results show that the OLS applied to first differenced series are quantitatively better, but qualitatively worse than the Aschauer's approach. It leads to a spurious rejection of the hypothesis of a productive contribution of public capital.

The Fully Modified method does not improve the quality of the results, and lead to estimated values very similar to OLS ones. Only the DOLS brings a small improvement, since the public capital elasticity is estimated around 20%.

The approximated density functions of these estimators under the PFCRS hypothesis are presented in figure (1). We can observe the relative importance of the simulated bias associated with the main estimating procedures applied in the empirical literature.

Fig 1 : Approximated density functions of $\widehat{\alpha}_g$ under the hypothesis of PFCRS

Then, it seems that the production function approach, whatever the estimation method used, is not valid to bring an answer to Aschauer's (1989) question : *is public expenditure productive ?* Indeed, the OLSΔ are quantitatively better (no bias) than the OLS on level, but are qualitatively worse (spurious non rejection of the null $\alpha_g = 0$).

In our study, this approach gives biased results for several reasons.[2] The first one, is the endogeneity of the factors of production.[3] Indeed, it's

[2] These results are not due to a multicolinearity problem since the theoretical correlation of the variables of our model are close to those observed by Sturm and De Haan (1949-1985) or Ford and Poret (1956-1989).

[3] These observations reinforce the critics commonly addressed in the literature (see Gramlich 1994 for a survey) to Aschauer's approach.

easy to see that the residual μ_t of the econometric model (7) corresponds to the structural process of the total factor productivity A_t. Given that this autocorrelated process is one of our state variable in the model 1, the regressors $\log(K_t/N_t)$ and $\log(KG_t)$ are correlated with this residual.

The fact that the output and the inputs are not cointegrated, constitutes the second major issue of this approach, as it was stressed by Tatom (1991) or Sturm and De Haan (1995)[4]. In our model, all variables (except vg_t and N_t) are non stationary given that the Solow residual, A_t, is governed by a $I(1)$ process. As, under the PFCRS hypothesis, the residual of the production function regression corresponds to A_t, there is no cointegration between output and inputs. So OLS, applied on variables in levels, lead to the well-known problem of spurious regressions.

Nevertheless, the OLSΔ leads to non significant production elasticities of the public capital stock and to negatives ones for the private capital stock[5]. These results are mainly due to the misspecification of the dynamics of labor which is $I(0)$ and of the capital stocks which are cointegrated with vector $(1, -1)$. As it was stressed by Munnel (1992) or Gramlich (1994), first differentiating the series may be too radical and could destroy the long term relationships in the data.

Model 2 : stationary case

Most of the econometric criticisms addressed to the Aschauer's approach deal with the presence of a common trend among the series. So we proposed here a variant of our sensibility analysis in which all the variables are trend stationary (TS) but very persistent ($\rho_a = 0.95$), in order to explore to which extent the inconsistency of the production function approach is due to the non stationarity properties and associated with the spurious regression argument. The results of the model 2 are presented in table 3.

Table 3 : Model 2 (stationary case)

We can observe that when the Solow residual is very close to the unit root process, we obtain approximately the same conclusions as in the non-stationary case. There exists here a kind of continuity between the unit root case and the near unit root one. The distributions of our estimators are not well centered, since the OLS, the FM-OLS and the DOLS are biased in the case of a near root residual. Moreover, as in model 1, the endogeneity

[4]On US data, the authors show that there is no cointegration relationship between output and inputs.

[5]Sturm and De Haan (1995) also found a negative estimated elasticity for the private capital under several specifications of the production function.

of the regressors used in the production function approach leads to biased estimated production elasticities of the public capital. It can be checked, using a Hausman's (1978) test, that the null hypothesis of exogeneity is rejected in 72% of the replications under the PFCRS specification, and for 75% under the CR one.

5 Conclusion

This exercise shows that the production function approach (applied to time series at least) commonly used in econometric studies, does not provide a reliable inference strategy and might not be the best way to determine the genuine rate of return of the public infrastructures.

Currently, most of the empirical studies devoted to this subject, adopt a dual approach (usually using panel data). Studies which keep the production function approach are based on pooled time series or cross-sectional data. The question of the reliability of such inferences based on panel data remains open.

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Figure 1: Empirical density function of $\widehat{\alpha}_g$ under the PFCRS hypothesis

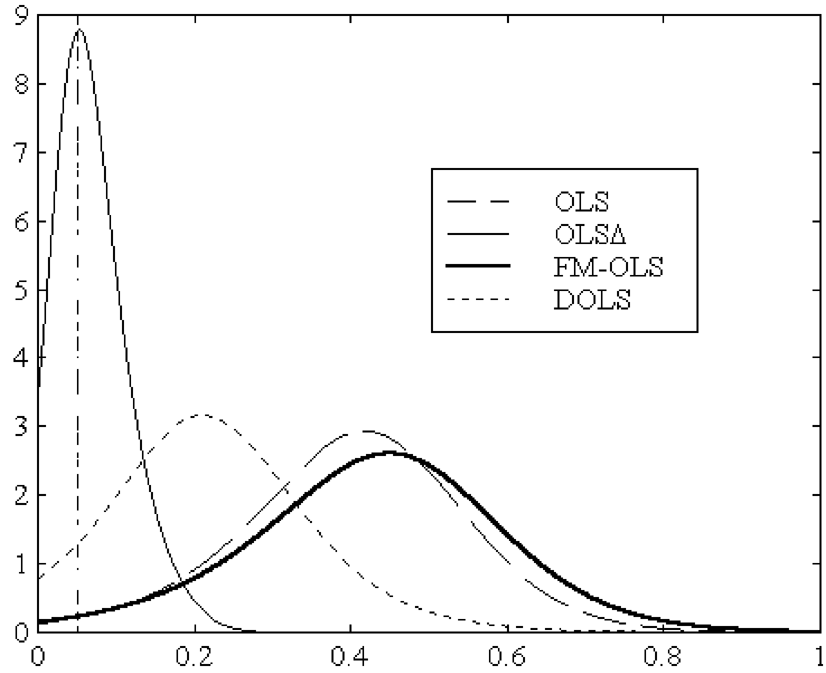

Table 1: Calibration

β	δ	δ_g	e_k	e_g	ρ_g	σ_a	σ_g	μ	s_g
0.96	0.1	0.1	0.42	0.05	0.9	0.05	0.06	0.5	0.15

Table 2: Model 1 (non-stationary case)

	True Value	OLS	$OLS\Delta$	$AR(1)$	FM	$DOLS$
			(A) $PFCRS$ specification			
$\widehat{\alpha}_k$	0.42	0.23 (3.31)	−0.37 (−4.25)	−0.41 (−0.63)	0.29 (3.26)	0.63 (12.89)
$\widehat{\alpha}_{kg}$	0.05	0.40 (7.71)	0.06 (1.33)	0.07 (0.78)	0.42 (6.81)	0.21 (4.87)
$CI_{95\%}$	–	$[0.14, 0.63]$	$[-0.008, 0.15]$	$[-0.01, 0.19]$	$[0.12, 0.69]$	$[-0.03, 0.43]$
ρ	–	–	–	0.92	–	–
			(B) $OCRS$ specification			
		OLS	$OLS\Delta$	$AR(1)$	FM	$DOLS$
$\widehat{\alpha}_k$	0.42	0.14 (1.55)	−0.37 (−3.93)	−0.44 (−0.81)	0.20 (1.63)	0.53 (7.90)
$\widehat{\alpha}_{kg}$	0.05	0.35 (4.85)	0.01 (0.26)	0.01 (0.11)	0.39 (4.37)	0.21 (3.41)
$CI_{95\%}$	–	$[0.02, 0.67]$	$[-0.04, 0.08]$	$[-0.04, 0.08]$	$[0.002, 0.76]$	$[-0.18, 0.57]$
ρ	–	–	–	0.95	–	–

t-statistics in parentheses. All the statistics are average over 10 000 replications. For each method, the confidence interval at 95% is presented. The parameter ρ denotes the autoregressive root estimated in the Cochrane-Orchutt procedure.

Table 3: Model 2 (stationary case)

	True value	OLS	$OLS\Delta$	$AR(1)$	FM	$DOLS$
			(A) $PFCRS$ specification			
$\widehat{\alpha}_k$	0.42	0.23 (3.47)	−0.29 (−3.93)	−0.31 (−0.65)	0.28 (3.44)	0.64 (14.72)
$\widehat{\alpha}_{kg}$	0.05	0.38 (7.67)	0.08 (1.66)	0.08 (0.80)	0.41 (6.86)	0.19 (4.98)
$CI_{95\%}$	–	[0.14, 0.60]	[0.004, 0.16]	[0.002, 0.19]	[−0.03, 0.60]	[−0.02, 0.39]
ρ	–	–	–	0.92	–	–
			(B) $OCRS$ specification			
	True value	OLS	$OLS\Delta$	$AR(1)$	FM	$DOLS$
$\widehat{\alpha}_k$	0.42	0.14 (1.59)	−0.30 (−3.60)	−0.34 (−0.79)	0.20 (1.68)	0.53 (8.75)
$\widehat{\alpha}_{kg}$	0.05	0.33 (4.53)	0.02 (0.48)	0.02 (0.14)	0.37 (4.11)	0.19 (3.28)
$CI_{95\%}$	–	[0.01, 0.64]	[−0.03, 0.08]	[−0.03, 0.08]	[−0.04, 0.73]	[−0.18, 0.53]
ρ	–	–	–	0.95	–	–

t-statistics in parentheses. All the statistics are average over 10 000 replications. For each method, the confidence interval at 95% is presented. The parameter ρ denotes the autoregressive root estimated in the Cochrane-Orchutt procedure.