Compatibility Between European Securities Settlement Systems: A Spatial Competition Approach

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Abstract

This paper studies the question of Securities Settlement System compatibility in Europe through price competition in post-trading market. We study the incentives of CSD to migrate to TARGET 2 Securities when they are competitors in the market for depository. In the spirit of Matutes and Padilla (1994), we construct a stylized model of CSD competition which emphasises the distinctive features of SSS compatibility. We investigate the trade-off between the network and the substitution effects in a setting where, rival CSD first agree on a compatibility regime and then compete in Securities Settlement Services and depository. Therefore, we derive the implications for CSD compatibility of post-trading cost, entry, and banks switching costs. Finally, we investigate the normative implications of our model and draw some policy conclusions.

JEL CODES: D43, G29, G19, D43.

1 Introduction

The liberalization of financial markets in the 90’s induced a stronger competition between the market institutions. The European Securities Clearing and Settlement industry was the theater of important mergers as NYSE, Euronext and Deutsche Börse, Euroclear and Sicovam in France (See Hamon, Jacquillat, Saint-Étienne, 2007). This vast movement of consolidation has been encouraged

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by the European Commission who seeks to establish a unified market for the issuance and trade of financial securities across the EU.

Most studies focus either on the integration forms taken by the clear or Links between SSS by Kanko (2004 and 2007), Paradox of the creation of interlinking securities settlement systems the central securities depositories. In order to save cost, (CSDs) establish a mutual link that enables investors to make transactions with foreign securities.

Our study deals with two important activities of post-trading systems named depository and securities settlement services. These services are located at the end of the trading treatment. Thus, all securities bought on financial market have to be transferred from the account of the seller to the account of the buyer and recorded in the account of a central securities depositary. The first operation of settlement is managed by Securities Settlement Services and the second is a typical notary function named depository. Generally, in every European country, there is one national central securities depositary (CSD), which runs both SSS and depository. A CSD is the central bank for securities.

Such industry faces high fix cost due to important investment in software (up-grade), network security and backup of data. Hence, Securities Settlement Services (SSS) are often highly centralized. In the last decade, market forces failed to harmonized securities settlement system in Europe. As a consequence, securities are issued by domestic CSD and institutions. This fragmented European post-trading systems creates a bottleneck which discourage European financial market development (see 2001 and 2003 Giovannini Reports) by generating high post-trading costs in comparison with the US (see Lanno and Levin, 2001, for empirical studies). The constant research of saving cost, large volume of payments and the management of cross-border operations lead to the modernization of the settlement securities system in euro area. ECB will extend TARGET, the European real-time gross settlement platform, to securities settlement with TARGET 2 securities\footnote{1} (also named T2S) in 2008. This common post-market infrastructures named should answer to these inefficiencies.

As mention in TARGET annual report (2007), TARGET 2 Securities “(...) organisation would leave the current legal relationships between the SSSs and their “home” central bank untouched, but would facilitate the remote access to foreign SSSs by virtue of additional legal agreements/guarantees between the central bank concerned”. So, depository services remained the monopoly of domestic CSD, but SSS activity becomes competitive.

In the spirit of Matutes Padilla (1994), we investigate the trade-off between the network and the substitution effects in a setting where, rival CSD first agree on a compatibility regime and then compete in Securities Settlement Service. We analyses competition between CSD for Securities Settlement System and depository. For a bank the value of a SSS network depends on a better access to foreign market institution, when their need to settle cross-border transactions. Our model deals with the following questions: Are the banks willing to pay more for a larger network (network effect) as CSD compatibility makes them better

\footnote{1}{i.e. Trans-European Automated Real-time Gross settlement Express Transfer}
substitute for each others? Does substitution effect enhanced price rivalry in the SSS market?

The remainder of the paper is organized as follows. Section 2 presents the game theoretical analysis of post-trading services which is based on Matutes and Padilla (1994). Section 3 then reports the theoretical predictions of compatibility agreements. The compatibility agreements results are compared in Section 4, and conclusions are drawn with respect to the factors contributing to it. The implications of these results are used finally in Section 5 to discuss the potential role of public regulation through T2S.

2 The model

Our model is based on Salop (1979) spatial modelization of product differentiation. We consider that, after each transaction on stock exchange, dealers need two complementary services named depository and settlement. Contrary to Matutes Padilla (1994), the two services delivered by CSD are perfect complements, each deal need to be settled and conserved in a CSD account.

We assume that each stock exchange has a national CSD such that the circle represents the financial markets of a unified geographic area such as EU. Consider that there is N deals uniformly distributed over the unit circle with CSD 1, 2 and 3 in arbitrary location. Each CSD competes for post-trade services of the deals located in these two neighborhoods. Thus, \( q_{i,j} \) indicates the market share of CSD \( i \) competing with CSD \( j \) with \( i \neq j \).

The total quantity of depository services sold by a CSD \( i \) at price \( p_i^D \) is noted \( q_i^D \) with \( q_i^C = q_{i,j}^C + q_{i,m}^C \) with \( m \neq j \). For securities settlement, these values are noted \( p_i^S \) and \( q_i^S \) respectively. In addition, banks pay an access fee noted \( f_i \) to the CSD \( i \). Thus, the mean price of securities settlement decrease for using a same CSD settlement system. Each CSD \( i \) bears a high fix cost, \( C_i \), and a variable cost \( c_i \), which can be separated by activity. Therefore, the proportion of \( c_i \) generated by the production of depositary is noted \( u \) and \( v \) for securities settlement services, with \( u + v = 1 \).

Supposing that \( n_i \) banks use the services of the CSD \( i \), his total profit correspond to:

\[
\pi_i = p_i^D q_i^D + p_i^S q_i^S + n_i f_i - c_i(uq_i^D + vq_i^S) - C_i
\]

for \( i = 1, 2, 3 \).

In the spirit of Matutes Padilla (1994), CSDs play a two stage game. In the first stage, each CSD proposes a compatibility agreement on their securities settlement system which states the CSD with which it is willing to share its settlement system. The set of feasible compatibility proposals for CSD \( i \) is, first, full compatibility with CSDs \( j \) and \( m \) with \( i \neq j \neq m \), second, partial compatibility either with CSD \( j \) or with CSD \( m \). Compatibility between CSD requires the agreement of all CSD in the subset. One last case prevails, in which there is none compatible agreement, such that securities settlement systems remained incompatible.
At stage two, given the compatibility scheme that arises in the first stage, CSD simultaneously (and independently) choose depository and settlement prices. Every deal need depository and security settlement services. We simply assume that a bank will ask those services to a CSD in his neighborhood. Given the absence of counterparty risk, and for a given CSD network size, banks always prefer lower prices.

We assume that banks regularly need CSD services of depository and settlement services, and visit the CSD for both services at the same frequency. Contrary to depository, which can only be done by the national CSD of the security, banks choose any compatible CSD for securities settlement. Transportation costs associated to each services are $t^D$ and $t^S$ respectively with higher transportation cost for SSS, i.e. $t^D < t^S$. Depository is a service which refers to legislative and fiscal constraints which are similar, but remain specific to each country. Settlement service may be more costly because of safety constraints and complexity of technical connections.

If banks buy some “foreign” securities, this cost is growing up. This specific extra cost is noted $C_s$. Each bank sets deals on different stock exchanges (i.e. on different location on the circle). Hence, banks have to settle and depository foreign securities. So, they need post-trading services from foreign CSD with positive probability, $p$. Suppose, for instance, that each banks has a positive probability of dealing some time at any location of the circle (i.e. an English bank having a deal either on the german or french stock-exchange), in any of the three CSD neighborhoods. If CSD’s securities settlement systems are compatible, the expected travelling cost is lower because she can visit the nearest CSD instead of having to go back to his national CSD. The network effect reflects the fact that the more CSDs share their SSS, the greater the utility bank derives from his national CSD.

Let $k(i)$ be the expected costs of demanding to a CSD post-trading services, where $i$ is the number of CSD at which the bank can settle foreign securities. These costs are equal to the probability of foreign deals, $p$, multiply by the extra cost $C_s$, and the expected . This expected distance falls with each additional compatibility agreement established by CSD $i$. Therefore, the lower $k_i$, the more CSD are compatible with CSD $i$, i.e. $k(1) > k(2) > k(3)$.

There is a network externality taking into account the expected benefit of a CSD with a larger network. This network externality is one way of differentiation among otherwise identical CSD. The extra quality, $K = k(1) - k(2)$, of a CSD compatible with an incompatible CSD corresponds to saving transportation costs. If CSDs have the same network sizes, then $K$ has no impact on the choice of banks since they are both equal.

\footnote{i.e. for each deal on a Stock-Exchange}
In the following, we investigate market sharing, optimal tariffication of post-trading services and corresponding profit for CSD in the three compatible agreements. First, incompatible case represents the actual organization of securities settlement system in euro area, i.e. banks make operations of securities settlement services and depository to the same CSD. Cross-border deals are possible at high post-trading cost. Second, compatible case corresponds to the migration of all CSD to T2S. In our model, CSDs mutually agree to make compatible their securities settlement services. Hence, banks can settle a security either with the CSD of the security, or settle it with an other. Finally, partial compatible case implies that the migration of CSD to T2S is incomplet, i.e. only countries of the euro area migrate to T2S.

3.1 Incompatibility of the securities settlement services

3.1.1 Market sharing with three competitive CSD

According to our previous assumptions, identical CSDs expect the same market shares. Suppose that $q_{i,j}$ is the share of the market that firm $i$ has when $i$ is in competition with $j$. For CSD 1 and 2, \(^3\)

$$k(1 + p_1^D + p_1^S + f_1 + (t^S + t^D) x = k(1 + p_2^D + p_2^S + f_2 + (t^S + t^D) \left( \frac{1}{3} - x \right)$$

with $t^S + t^D = T$. At the equilibrium, the quantity of post-trade services sold by CSD 1, $q_1^T$, results from the market sharing with CSD 2 and 3, respectively.

\(^3\) with $q = xN$ and $q_{i,j} = \frac{N}{x} - q_{i,j}$
and access fees get lower market shares and profit per operation between '0rs'. Variable costs and access fees '0r with higher cost post-trading services of each '0r are called relatively to each other. The profit function of '0r 

\[ \pi_i = q_i^T (p_i^S + p_i^D) + n_i f_i - c_i q_i^T - C_i \]

with \( i = 1, 2, 3 \). By equalizing to zero the partial derivative of the CSD 'i's profit relatively to \( p_i^S \), we obtain the reaction functions of rational CSD 'i':

\[
\begin{align*}
    p_1^S(p_2^S, p_3^S) &= \frac{T}{6} + \frac{p_2^S + p_3^S + f_2 + f_3}{4} + \frac{c_1 - f_1 - p_1^D}{2} \\
    p_2^S(p_1^S, p_3^S) &= \frac{T}{6} + \frac{p_1^S + p_3^S + f_1 + f_3}{4} + \frac{c_2 - f_2 - p_2^D}{2} \\
    p_3^S(p_1^S, p_2^S) &= \frac{T}{6} + \frac{p_2^S + p_1^S + f_2 + f_1}{4} + \frac{c_3 - f_3 - p_3^D}{2}
\end{align*}
\]

The resolution of the system allows to compute the optimal price of securities settlement, \( a_i^* \), relative to depository price, \( p_i^* \):

\[
\begin{align*}
    p_1^* &= \frac{T}{3} + \frac{3c_1 + c_2 + c_3 - 2f_1 + f_2 + f_3}{5} - p_1^D \\
    p_2^* &= \frac{T}{3} + \frac{c_1 + 3c_2 + c_3 - 2f_2 + f_1 + f_3}{5} - p_2^D \\
    p_3^* &= \frac{T}{3} + \frac{c_1 + c_2 + 3c_3 - 2f_3 + f_2 + f_1}{5} - p_3^D
\end{align*}
\]

Thus, the optimal quantities sold by the three CSD will depend on the difference between CSDs' variable costs and access fees. CSD with higher cost and access fee gets lower market shares and profit:

\[
q_i^T = N \left( \frac{c_2 - 2c_1 + c_3 - 2f_1 + f_2 + f_3}{5T} \right) + \frac{N}{3}
\]

\[
\pi_i = TN \left( \frac{c_2 + c_3 - 2c_1 + f_2 + f_3 - 2f_1}{5T} + \frac{1}{3} \right)^2 + n_1 f_1 - C_i
\]

Note that identical CSD (i.e. \( c_3 = c_1 = c_2 \) and \( f_3 = f_1 = f_2 \)) leads to an equal share of the market and profits.
3.2 Compatibility of the securities settlement services

3.2.1 Market sharing and tariffication

The case of compatible CSDs give us two market sharing conditions. On security settlement market, we set \( p_1^D + t_D y = p_2^D + t_D (\frac{1}{3} - y) \) for CSD 1 and 2. Note the absence of network effect because of the monopolistic nature of depository services. For securities settlement, we have \( k(3) + p_1^S + f_1 + t^S \min(x; \frac{1}{3} - x) = k(3) + p_2^S + f_2 + t^S \min(x; \frac{1}{3} - x) \). The last condition implies that CSDs face a fierce price competition (Bertrand competition type). Hence, they set the same price menu, \( p_1^S + f_1 = p_2^S + f_2 \), and get one third of the total market.

The total quantities are:

\[
q_1^D = \frac{N}{3} + \frac{N}{2D} (p_2^D + p_3^D - 2p_1^D) \quad \text{and} \quad q_1^S = \frac{N}{3}
\]

\[
q_2^D = \frac{N}{3} + \frac{N}{2D} (p_1^D + p_3^D - 2p_2^D) \quad \text{and} \quad q_2^S = \frac{N}{3}
\]

\[
q_3^D = \frac{N}{3} + \frac{N}{2D} (p_2^D + p_1^D - 2p_3^D) \quad \text{and} \quad q_3^S = \frac{N}{3}
\]

3.2.2 Optimal tariffication

First order conditions give us the following reaction function of rational CSD i :

\[
p_1^D(p_2^D, p_3^D) = \frac{t_D}{6} + \frac{p_2^D + p_3^D}{4} + \frac{uc_1}{2}
\]

\[
p_2^D(p_1^D, p_3^D) = \frac{t_D}{6} + \frac{p_1^D + p_3^D}{4} + \frac{uc_2}{2}
\]

\[
p_3^D(p_1^D, p_2^D) = \frac{t_D}{6} + \frac{p_2^D + p_1^D}{4} + \frac{uc_3}{2}
\]

The resulting optimal prices of depository depends only of weighted average of depository variable cost and banks’ transportation costs for depository :

\[
p_1^{D*} = \frac{t_D}{3} + \frac{u(3c_1 + c_2 + c_3)}{5}
\]

\[
p_2^{D*} = \frac{t_D}{3} + \frac{u(c_1 + 3c_2 + c_3)}{5}
\]

\[
p_3^{D*} = \frac{t_D}{3} + \frac{u(c_1 + c_2 + 3c_3)}{5}
\]

The optimal post-trading services sold by a CSD decrease at constant rate \( \frac{N}{5t_D} \), when his variable cost of depository is higher than its rival:

\[
q_1^T = \frac{N}{5t_D} u(c_2 + c_3 - 2c_1) + \frac{N}{3}
\]
It is interesting to underline that, equal market sharing on settlement leads to an increasing profit function in $a_i$; thus the higher settlement prices, the higher benefits. But, they must be equal otherwise the CSD with the higher price loses all his market share.

$$\pi^*_1 = t^D N \left( \frac{u}{5t^D} (c_2 + c_3 - 2c_1) + \frac{1}{3} \right)^2 + \frac{N}{3} (p^1_S - v_c_1) + n_1 f_1 - C_1$$

In a contestable market, settlement prices’ should be close to the marginal cost $d_i$ and uncompetitive or leave the market. The remaining profit or being corresponding to the activity of depository.

### 3.3 Partial compatibility of the securities settlement services

#### 3.3.1 Market share with two compatible firms

For the two compatible CSD, the market sharing corresponds to:

$$k(2) + p_1^D + t^D y = k(2) + p_2^D + t^D \left( \frac{1}{3} - y \right)$$

$$k(2) + p_1^S + f_1 + t^S \min(x; \frac{1}{3} - x) = k(2) + p_2^S + f_2 + t^S \min(x; \frac{1}{3} - x)$$

where $\beta$ and $\gamma$ represent respectively the costs of the incompatible (depository) and compatible (settlement security services) operations. We denote the number of securities settlement and depository respectively $q^S$ and $q^D$. with $q = q^D = q^S = q^{1,2}$, we find the number of operations.

$$q^D_{1,2} = \frac{N}{6} - \frac{N}{2t^D} (p_1^D - p_2^D)$$

$$q^D_{2,1} = \frac{N}{6} - \frac{N}{2t^D} (p_2^D - p_1^D)$$

$$q^S_{1,2} = \frac{N}{6}$$

with $p_1^D + f_1 = p_2^S + f_2$.

#### 3.3.2 Market share between compatible and non compatible CSDs

For both incompatible CSD, the market sharing leads to the following results, with $x = q^S$ and $y = q^D$. Note that depository and securities settlement are budding goods, such that banks must buy them to a same CSD, $q^S = q^D$.

$$k(2) + p_1^D + p_1^S + f_1 + t^D \frac{q^{1,3}}{N} + t^S \min \left( \frac{q_1}{N}; \frac{1}{3} - \frac{q_1}{N} \right) = k(1) + p_3^S + f_3 + \left( t^S + t^D \right) \left( \frac{1}{3} - \frac{q_1}{N} \right)$$

with $K = k(1) - k(2)$, the network externality of compatibility. This network externality reflects the expected benefit of settlement and depository to a larger network. This network externality constitutes a source of differentiation among otherwise identical CSDs. The extra quality, of a CSD compatible with a rival relative to an incompatible CSD equals the savings in the transportation costs.
paid for cross-border transactions. A bank can either decide to settle and deposit his security to the compatible CSD (case 1) or to buy these services to the incompatible CSD. Thus, these two cases must be investigated:

- If \( \min \left( \frac{q_{1,3}}{N} : \frac{1}{3} - \frac{q_{1,3}}{N} \right) = \frac{q_{1,3}}{N} \), then

\[
K + p_3^D - p_1^D + p_3^S - p_1^S + f_3 - f_1 + \frac{t^S + t^D}{3} = 2 \left( t^S + t^D \right) \frac{q_{1,3}}{N}
\]

- If \( \min \left( \frac{q_{1,3}}{N} : \frac{1}{3} - \frac{q_{1,3}}{N} \right) = 1 - \frac{q_{1,3}}{N} \), then

\[
K + p_3^D - p_1^D + p_3^S - p_1^S + f_3 - f_1 + \frac{t^D}{3} = 2t^D \frac{q_{1,3}}{N}
\]

The resulting quantities of depository and securities settlement services are reported in the Appendix.

### 3.3.3 Optimal tarification

**First case** Then, optimal prices value \( p_i^S \) and \( p_i^D \) are:

\[
\begin{align*}
p_1^{S^*} &= \frac{K}{3} + \frac{1}{12} (7c_1 + c_2 + 4c_3 + f_2 + 4f_3 - 5f_1) + \frac{5}{9} T \\
p_2^{S^*} &= \frac{K}{3} + \frac{1}{12} (7c_2 + c_1 + 4c_3 + f_1 + 4f_3 - 5f_2) + \frac{5}{9} T \\
p_3^{S^*} &= -\frac{K}{3} - p + \frac{1}{6} (c_1 + c_2 + 4c_3 + f_1 + f_2 - 2f_3) + \frac{4}{9} T \\
p_1^{D^*} &= \frac{u}{3} (2c_1 + c_2) \\
p_2^{D^*} &= \frac{u}{3} (2c_2 + c_1) \\
p_3^{D^*} &= p^D
\end{align*}
\]

The equilibrium quantities and profit:

\[
\begin{align*}
q_1^{S^*} &= \frac{N}{2T} \left[ \frac{K}{3} + \frac{5T}{9} + \frac{1}{12} (c_2 + 4c_3 - 5c_1 + f_2 + 4f_3 - 5f_1) \right] + \frac{N}{3} \\
q_1^{D^*} &= q_1^{S^*} + \frac{uN}{6T} (c_2 - c_1) \\
q_3^{S^*} &= q_3^{D^*} = \frac{4N}{9} + \frac{N}{2T} \left[ -\frac{2K}{3} + \frac{1}{3} (c_1 + c_2 - 2c_3 + 4f_1 + 4f_2 - 8f_3) \right] \\
\pi_1^* &= \frac{N}{2T} \left\{ \left[ \frac{K}{3} + \frac{c_2 + 4c_3 - 5c_1 + f_2 + 4f_3 - 5f_1}{12} + \frac{5}{9} T \right]^2 + \frac{u^2}{9} (c_2 - c_1)^2 \right\} + n_1 f_1 - C_1 \\
\pi_3^* &= \frac{N}{T} \left[ \frac{c_1 + c_2 - 2c_3 + f_1 + f_2 - 2f_3}{6} - \frac{2K}{3} + \frac{4}{9} T \right] \left[ \frac{c_1 + c_2 - 2c_3 + f_1 + f_2 - 2f_3}{6} - \frac{K}{3} + \frac{4}{9} T \right] + n_3 f_3 - C_3
\end{align*}
\]
Second case To determined the optimal prices $p_i^S^*$ and $p_i^D^*$, we fix $p_3 = p$, such that:

$$p_1^S^* = \frac{K}{3} + \frac{1}{12} (7c_1 + c_2 + 4c_3 + 4f_3 + f_2 - 5f_1) + \frac{5}{9}t^D - \frac{2c_1 + c_2}{3}$$

$$p_2^S^* = \frac{K}{3} + \frac{1}{12} (7c_2 + c_1 + 4c_3 + 4f_3 + f_1 - 5f_2) + \frac{5}{9}t^D - \frac{2c_2 + c_1}{3}$$

$$p_3^S^* = -\frac{K}{3} - p + \frac{1}{6} (f_1 + f_2 - 2f_3 + c_1 + c_2 + 4c_3) + \frac{4}{9}t^D$$

$$p_1^D^* = \frac{u}{3} (2c_1 + c_2)$$

$$p_2^D^* = \frac{u}{3} (2c_2 + c_1)$$

$$p_3^D^* = p^D$$

The equilibrium quantities and profit:

$$q_1^S^* = \frac{N}{2t^D} \left[ \frac{K}{3} - \frac{t^D}{9} + \frac{1}{12} (c_2 + 4c_3 + 7c_1 + f_2 + 4f_3 - 5f_1) \right] + \frac{N}{3}$$

$$q_1^D^* = q_1^S^* + \frac{uN}{6t^D} (c_2 - c_1)$$

$$q_3^S^* = q_3^D^* = \frac{4N}{9} + \frac{N}{2t^D} \left[ -\frac{2K}{3} + \frac{1}{3} (c_1 + c_2 - 2c_3 + f_1 + f_2 - 2f_3) \right]$$

$$\pi_i^* = \frac{N}{2t^D} \left\{ \left[ \frac{K}{3} + \frac{c_2 + 4c_3 - 5c_1 + f_2 + 4f_3 - 5f_1}{12} + \frac{5}{9}t^D \right]^2 + \frac{u^2}{9} (c_2 - c_1)^2 \right\} + n_1f_1 - C_1$$

$$\pi_3^* = \frac{N}{t^D} \left[ \frac{c_1 + c_2 - 2c_3 + f_1 + f_2 - 2f_3}{6} - \frac{2K}{3} + \frac{4}{9}t^D \right] \left[ \frac{c_1 + c_2 - 2c_3 + f_1 + f_2 - 2f_3}{6} - \frac{K}{3} + \frac{4}{9}t^D \right] + n_3f_3 - C_3$$

In both cases, if a compatible CSD $i$ has a lower cost than his compatible and incompatible rivals, then his profit is higher because of the dominant effect of quantity increase over his price decrease (with $u < \frac{7}{8}$ in case 2). Higher access fee makes price and market share decrease generating lower profit. These cost differences will be reinforced by the size of the transportation costs.

4 Compatibility Agreement at the Equilibrium

Stage two: Competition for deals settlement and depository

Equilibrium concept use to induce CSD behavior in the second stage is based on the concept of Coalition-Proof Nash Equilibrium introduced by Bernheim et al. (1987). The main feature of this equilibrium concept deals with collective
rationality. As explained in Bernheim et al. (1983): ‘An agreement is coalition-proof if and only if it is Pareto efficient within the class of self-enforcing agreements. In turn, an agreement is self-enforcing if and only if no proper subset (coalition) of players, taking the actions of its complement as fixed, can agree to deviate in a way that makes all of its members better off.’

Assuming identical CSDs, (i.e. $c_1 = c_2 = c_3$ and $f_1 = f_2 = f_3$), tables 2, 3 and 4 present the optimal results of prices, quantities and profits respectively. In the following, we assume that only CSD 3 is either completely compatible or not, contrary to CSD 1 and 2, who can be partially compatible.

<table>
<thead>
<tr>
<th>Compatibility</th>
<th>$p^D_1$</th>
<th>$p^D_2$</th>
<th>$p^D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>$\frac{T}{3} + c - p^D_3$</td>
<td>$p^D_2$</td>
<td>$p^D_3$</td>
</tr>
<tr>
<td>All</td>
<td>$p^S$</td>
<td>$\frac{T}{3} + uc$</td>
<td></td>
</tr>
<tr>
<td>Partial</td>
<td>case 1</td>
<td>$\frac{K}{3} + c + \frac{\theta}{3}T - uc$</td>
<td>$-\frac{K}{3} + c + \frac{\theta}{3}T - p^D_3$</td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>$\frac{K}{3} + c + \frac{\theta}{3}T - uc$</td>
<td>$-\frac{K}{3} + c + \frac{\theta}{3}T - p^D_3$</td>
</tr>
</tbody>
</table>

Table 1: Price comparison in different compatibility agreements

Price comparison:
- For none compatible agreement, if $p^D_1 = p^D_2 > p^D_3$, then $p^S_1 = p^S_2 > p^S_3$.
- For compatible agreement, $p^D_1 = p^D_2 = p^D_3$ and $p^S_1 = p^S_2 = p^S_3$.
- For partial compatibility agreements if $p^D_i = p^S_i < p^S_3$, one obtain for case 1 $p^D_1 - p^D_3 > \frac{2K}{3} + \frac{\theta}{3}$ and $p^D_1 - p^D_3 > \frac{2K}{3} + \frac{\theta}{3}$ for case 2.

When comparing compatibility agreements,
- For CSD 1 and 2, prices can be ranked as follows for SSS operations $PC_2 < NC < PC_1$ if $K < \frac{T}{3}$ and for depository $PC_1 = PC_2 < C$.
- For CSD 3, the ranking of price is $PC_2 < NC < PC_1$ if $\frac{4}{3}T - T < K < \frac{T}{3}$ for SSS operations and for depository $PC_1 = PC_2 = NC$.

<table>
<thead>
<tr>
<th>Compatibility</th>
<th>Securities Settlement System</th>
<th>Depository</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal quantities</td>
<td>$q^T_i$ with $i = 1, 2.$</td>
<td>$q^D_3$</td>
</tr>
<tr>
<td>None</td>
<td>$\frac{N}{T}$</td>
<td>$\frac{N}{T}$</td>
</tr>
<tr>
<td>All</td>
<td>$\frac{N}{T}$</td>
<td>$\frac{N}{T}$</td>
</tr>
<tr>
<td>Partial</td>
<td>case 1</td>
<td>$\frac{N}{2T} \left( \frac{K}{3} + \frac{2T}{3} \right)$</td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>$\frac{N}{2T} \left( \frac{K}{3} - \frac{\theta}{3} \right)$</td>
</tr>
</tbody>
</table>

Table 2: Quantities comparison in different compatibility agreements.

For both compatible and none compatible agreements, the CSDs share equally the market, $\frac{N}{3}$.
In partial compatibility cases, the quantity sold by CSD 1 or 2 is higher than CSD 3 quantities, if \( K > \frac{5}{3} T \) in first case and \( K > \frac{5}{6} t_D \) in the second case.

In addition if \( K > \frac{5}{6} t_D \), we obtain for each CSD according to compatibility agreement, the following ranking among quantities for CSD 1 and 2, \( C < PC_2 < PC_1 \) and \( PC_1 < PC_2 < C \) for CSD 3.

<table>
<thead>
<tr>
<th>Compatibility</th>
<th>( \pi_i ) with ( i = 1, 2 )</th>
<th>( \pi_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>( \pi_i = \frac{N}{T} N_i + n_i f_i - C_i )</td>
<td>( \pi_3 = \frac{N}{T} \left( \frac{5}{6} T + \frac{4}{9} t_D \right) - \frac{K}{T} + \frac{4}{9} t_D + n_3 f_3 - C_3 )</td>
</tr>
<tr>
<td>All</td>
<td>( \pi_i^* = \frac{N}{T} \left( K + \frac{5}{6} T \right)^2 + n_i f_i - C_i )</td>
<td>( \pi_3^* = \frac{N}{T} \left( \frac{5}{6} T - \frac{1}{3} \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3} T} \right) - \frac{K}{T} + \frac{4}{9} t_D + n_3 f_3 - C_3 )</td>
</tr>
<tr>
<td>Partial</td>
<td>( \pi_1^* = \frac{N}{T} \left( K + \frac{5}{6} t_D \right)^2 + n_1 f_1 - C_1 )</td>
<td>( \pi_2^* = \frac{N}{T} \left( \frac{1}{3} \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3} T} \right) - \frac{K}{T} + \frac{4}{9} t_D + n_3 f_3 - C_3 )</td>
</tr>
</tbody>
</table>

Table 3: Profits comparison in different compatibility agreements

The comparison of profit function in compatible and incompatible agreements leads to the following conclusions:

\[
\pi^C > \pi^{IN} \iff p_i^s - vc_i > \frac{t^s}{3}
\]

Thus, the transportation cost of banks for securities settlement has to be lower than the CSD marginal revenue for settlement.

If the following conditions hold, all CSDs prefer partial compatibility agreement on SSS to incompatibility:

\[
\begin{align*}
\pi^IN & > \pi^{PC_1} \iff K > T \left( \frac{5}{3} - \sqrt{\frac{2}{3}} \right) \text{ for the two compatible CSDs} \\
\pi^IN & > \pi^{PC_1} \iff T \left( 1 - \sqrt{\frac{2}{3}} \right) < K < T \left( 1 + \sqrt{\frac{2}{3}} \right) \text{ for the incompatible CSDs}
\end{align*}
\]

\[
\begin{align*}
\pi^IN & > \pi^{PC_2} \iff K > \frac{5}{3} t_D - \sqrt{\frac{2}{3} t_D} \text{ for the two compatible CSDs} \\
\pi^IN & > \pi^{PC_2} \iff t_D - \frac{1}{6} \sqrt{2 t_D^2 (2 t_D^2 + 9)} < K < t_D + \frac{1}{6} \sqrt{2 t_D^2 (2 t_D^2 + 9)} \text{ for the incompatible CSD}
\end{align*}
\]

If the following conditions hold, all CSDs prefer compatibility agreement on SSS to partial compatibility:

\[
\begin{align*}
\pi^C & > \pi^{PC_1} \\
\iff K & > \frac{5}{3} T - \sqrt{6 T \left( \frac{t_D}{3} + p_i^s - vc \right)} \text{ for the CSD } i = 1, 2,
\end{align*}
\]

\[
\begin{align*}
\pi^C & > \pi^{PC_1} \\
\iff T - \frac{3}{2} \sqrt{\frac{2}{3} T \left( \frac{2 T}{27} + \frac{4}{3} t_D + p_i^s - vc \right)} < K < T + \frac{3}{2} \sqrt{\frac{2}{3} T \left( \frac{2 T}{27} + \frac{4}{3} t_D + p_i^s - vc \right)} \text{ for the CSD 3}
\end{align*}
\]

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\[ \pi^C > \pi^{PC2} \]
\[ \iff K > \frac{5}{3} t^D - \sqrt{6t^D \left( \frac{t^D}{3} + p^S - vc \right)} \] for the CSD i=1,2

\[ \pi^C > \pi^{PC2} \]
\[ \iff t^D - \frac{3}{2} \sqrt{\frac{2t^D}{3} \left( \frac{11t^D}{27} + p^S - vc \right)} < K < t^D + \frac{3}{2} \sqrt{\frac{2t^D}{3} \left( \frac{11t^D}{27} + p^S - vc \right)} \] for the CSD 3

5 Conclusion

In this paper, we analyze the theoretical properties of compatibility agreement on securities settlement.

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References

Appendix

6.1 Depository and settlement quantities in Partial compatibility agreement.

- With \( A = (\beta + \gamma) \) for case 1 and \( A = \beta \) for case 2, we obtain the following quantities of depository and securities settlement services:

\[
q_1^C = N \left( \frac{1}{3} + \frac{K + p_3 - p_1 + a_3 - a_1 + s_3 - s_1}{2A} + \frac{p_2 - p_1}{2\beta} \right)
\]

\[
q_2^C = N \left( \frac{1}{3} + \frac{K + p_3 - p_2 + a_3 - a_2 + s_3 - s_2}{2A} + \frac{p_1 - p_2}{2\beta} \right)
\]

\[
q_3^C = N \left( \frac{1}{3} + \frac{2(K - p_3 - a_3 - s_3) + p_3 + p_2 + a_1 + a_2 + s_1 + s_2}{2A} \right)
\]

\[
q_1^S = N \left( \frac{1}{3} + \frac{K + p_3 - p_1 + a_3 - a_1 + s_3 - s_1}{2A} \right)
\]

\[
q_2^S = N \left( \frac{1}{3} + \frac{K + p_3 - p_2 + a_3 - a_2 + s_3 - s_2}{2A} \right)
\]

\[
q_3^S = N \left( \frac{1}{3} + \frac{2(K - p_3 - a_3 - s_3) + p_1 + p_2 + a_1 + a_2 + s_1 + s_2}{2A} \right)
\]