Debt Overhang and Bank Bailouts

Linus Wilson
Assistant Professor of Finance
University of Louisiana at Lafayette
B. I. Moody III College of Business
Department of Economics & Finance
214 Hebrard Boulevard, Moody Hall 253
P. O. Box 44570
Lafayette, LA 70504-4570
Phone: (337) 482-6209
Fax: (337) 482-6675
E-mail: linuswilson [at] louisiana [dot] edu
Web: http://www.linuswilson.com

February 1, 2009
Debt Overhang and Bank Bailouts

*When a bank is deemed “too-big-to-fail” by regulators, it may be tempted to buy risky assets. This paper analyzes bank bailouts involving the purchases of toxic assets, preferred stock, and common stock when the government wants to encourage efficient lending. It finds that preferred stock recapitalizations are the least efficient in correcting debt overhang problems from both an ex post and ex ante perspective. In contrast, efficient lending and voluntary participation can be best achieved without subsidy by purchasing either toxic assets or common stock. Nevertheless, troubled banks must be subsidized if they will voluntarily participate in any recapitalization.*

*Journal of Economic Literature Codes: G21, G28, G38*

*Keywords: bailout, banking, debt overhang, common stock, Capital Purchase Program, lending, preferred stock, TARP, too-big-to-fail, toxic assets*
1. **Introduction**

This paper asks, “To what extent are all bank bailouts the same?” The financial crisis of 2008 has led many governments to buy troubled assets, common stock, and preferred stock from financial institutions. In table 1, the author has listed many of the government funded capital infusions in developed nations since the failure of the investment bank Lehman Brothers on September 15, 2008.\(^1\) It is clear from the table that there is little consensus among government policy makers about what types of securities or asset purchases are most effective in improving lending incentives. This is in part because there is little formal research into the best securities for governments to buy when they attempt to recapitalize troubled banks.\(^2\) This is the first paper to analyze what securities most efficiently counteract the problems of debt overhang in a bank recapitalization. The problem of debt overhang was first illustrated by Myers (1977). For banks, this means that many positive net present value loans will not be undertaken because the gains from those loans will be partially shared with creditors. The common equity claimants will not get enough of the returns from good loans to justify raising new equity to fund them.

This paper finds that government purchases of toxic assets or common stock are equally efficient forms or recapitalizations. Yet, the dollar value of the recapitalizations

---


\(^2\) Bebchuck and Goldstein (2008) and Wilson and Wu (2008) are notable exceptions. Diamond (2001), for example, does discuss how to optimally bailout a relationship lender. Yet, that study does not discuss the merits of different types of securities.
will be much lower if common stock is used to correct the banks’ lending incentives. Preferred stock recapitalizations are only effective in correcting bad lending incentives if the government greatly overpays for its preferred stock stake.\(^3\) Anecdotal evidence indicates that the U.S. Treasury’s Troubled Asset Relief Program (TARP), which has been used to primarily buy preferred stock in U.S. banks, has not induced many recipients to increase lending.\(^4\) One news report writes, “Fifty-one lenders who got TARP money reduced total loans by $92.9 billion, or 2.5 percent, in the fourth quarter [of 2008] from the prior quarter… Their smaller peers who didn’t apply for capital or declined cash infusions curtailed lending by $1.87 billion, or 1.3 percent.”\(^5\) The Duke University CFO survey released in December 10, 2008, found that 62 percent of chief financial officers surveyed said they cannot access the credit that their firms need.\(^6\) This is especially troubling since Ivanisha and Sharfstein (2008) found that commercial lending was down 68 percent in the financial crisis period from the peak of the lending boom in May to August 2007.

This paper finds that preferred stock acts too much like debt and thus contributes to debt overhang instead of helping to alleviate the problem. Unfortunately, most of the securities purchased by the U.S. Treasury have been preferred stock. The Interim

\(^3\) The Congressional Budget Office report “The Troubled Asset Relief Program: Report on Transactions through December 31, 2008,” dated January 16, 2009, estimates that of the first $178 billion of the Capital Purchase Program that was used to buy preferred stock and warrants represented a $32 billion subsidy to the institutions receiving funds. This is similar to the estimates of Veronesi and Zingales, (2008) which argued that the first nine institutions receiving $125 billion in the Emergency Economic Stabilization Act funds represented a $13 billion to $36 billion subsidy.


Director of the U.S. Treasury Office of Financial Stability Neel Kashkari, who oversaw the first $350 billion dollars of the TARP funds, recently spoke at the Brookings Institution. Mr. Kashkari said that policymakers chose to buy preferred stock because it is senior to common stock and they were uncomfortable with the government owning common equity in private enterprises.\(^7\)

None of the result of the present paper, or Wilson and Wu (2008), which derives similar results when the troubled banks can engage in risk-shifting, rest on the government’s exercising the residual control rights that usually are bundled with common stock. The government need not exercise control with its common equity stakes beyond prohibitions against leverage increasing transactions\(^8\) to get banks to lend efficiently. Indeed, the benefit of buying common stock illuminated here is that it has the same level of seniority that other common stock holders have. Thus, the government could immediately sell its common stock stake in secondary markets and this paper’s results would still hold.


\(^8\) Allowing insolvent banks to pay large dividends did not work out well in the last major banking crisis in the United States. Kroszner and Strahan (1996) found that insolvent thrifts were allowed to pay out aggressive dividends at the ultimate expense of taxpayer funded deposit insurance during the savings and loan crisis. More recently some editorials have argued that any capital infusion must be accompanied with tighter restrictions on dividends. One such example is the op-ed by David S. Scharfstein and Jeremy C. Stein, October 20, 2008, “This Bailout Doesn’t Pay Dividends,” New York Times, http://www.nytimes.com/2008/10/21/opinion/21stein.html (Accessed online: January 30, 2009).

Lawrence Summers, incoming National Economics Council director-designate for President Obama’s administration urged in a public letter to Congressional leaders to authorize the second $350 billion dollars of monies from the Emergency Economic Stabilization Act (EESA) of 2008. He promised that, as a condition of taxpayer investments, recipient banks will need regulatory approval to issue dividends. Dividends increase a firm’s leverage by taking cash and equity capital out of the firm. A similar provision was included in “TARP Capital Purchase Program, Senior Preferred Stock and Warrants, Summary of Senior Preferred Terms” U.S. Department of the Treasury, which was used for most of the capital purchases with the first $350 billion dollars of the package. In addition, in his letter to Congress, the director-designate Summers argued that the new administration would “Preclude use of government funds the purchase of healthy firms rather than to boost lending.” Cash acquisitions can undo the leverage decreasing effects of any common equity purchase by the government. What was unclear was what type of securities that would be bought with the second half of the EESA monies.
This paper demonstrates that a troubled bank will not voluntarily sell its common stock or toxic assets for its fair market value. Thus, any voluntary reduction in the riskiness of the bank’s share price must be subsidized. This will be familiar to anyone familiar with the basic determinants of an option’s price in the model of Black and Scholes (1973) and Merton (1973). The at-the-money or out-of-the-money option often owes most of its value to the volatility of the underlying asset. Reducing the volatility of the bank’s share price through the sale of risky assets or by issuing more shares will hurt the bank’s current shareholders.

The present paper supports both the buying of distressed assets and common equity stakes in getting banks to lend efficiently. Common stock capital infusions are weakly the most effective means to improve lending incentives in terms of minimizing the taxpayer subsidy to recipient banks. In addition, common stock purchases require a much smaller initial investment than toxic asset purchases. In our numerical example, we find that the minimum size of the government’s common equity stake necessary for efficient lending is 10 percent of assets. Yet the government would have to purchase 100 percent of assets to get the same risk reduction if it bought toxic assets in the undercapitalized bank.

Kane and Klingebiel (2004) study the policy responses in twelve major banking crises over the last two decades. They argue that effective policy responses such as quickly closing deeply insolvent banks and providing liquidity for more viable institutions can mitigate the negative impacts of banking crises.\(^9\) One “success story”

\(^9\) One of the most troubling aspects of current policy response is that policy makers have not put more emphasis on closing down insolvent banks which are too small to pose any systematic risk. The sale of smaller banks’ assets at distressed prices and the bankruptcy style cramming down of their unsecured creditors should improve the profitability and efficiency of the banking sector. Yet, Zingales (2008) argues
they cite is the Swedish response to its banking crisis from 1991 to 1994. According to Ergunogor (2007), Sweden assumed the liabilities of the banks that were not shuttered and in exchange became the 100 percent shareholder in many institutions. In addition, a “bad bank” was set up that bought troubled assets it recovered. According to Ergunogor (2007), the bad bank sold all its assets within four years. It recovered just 58 percent of what it paid for the assets.

Loan guarantees are often granted by governments. Chaney and Thakor (1985)’s analysis sharply criticizes the *ex post* and *ex ante* incentives created by government loan guarantees to non-financial institutions. While loan guarantees have been popular among policymakers in both earlier crises and the recent crisis, the author believes the moral hazard problems created by such policies are well understood and are not explored here.

There are some similarities in the results of this paper and Bebchuck and Goldstein (2008), in that both papers find that the government’s purchase of equity stakes can be effective in inducing banks to lend. In contrast to the present paper, that paper argues that banks refuse to lend because of coordination problems not debt overhang. Bebchuck and Goldstein (2008) consider a global game where participants in the banking sector observe noisy signals of the fundamentals. If the economic fundamentals fall too low without government intervention, an inefficient credit freeze can occur. Banks’ self-fulfilling expectations that lending opportunities are poor can be corrected by government intervention. They find that credit freezes are less likely as the government buys larger equity stakes in the banks. Nevertheless, unlike this paper, they do not find that lending will be always socially efficient if the government buys a large enough equity stake.

---

that political considerations may prevent cram downs in the current crisis since the bankers and their creditors may be a more effective lobby than taxpayers. Similarly, Brown and Dinc (2005) find evidence that political considerations may have a role in bank closure decisions in developing countries.
Instead, Bebchuck and Goldstein (2008) argue that lenders must commit to lend out all their government capital to prevent completely the possibility of an inefficient macroeconomic credit freeze.

This paper is most similar to the paper by Wilson and Wu (2008). Nevertheless, Wilson and Wu (2008) focuses on the problem of risk-shifting. Banks can only engage in risk shifting if they have liquid assets and the ability select speculative projects. A speculative, negative net present value project will not be able to find external capital under symmetric information. If banks have no cash on hand or they cannot undertake speculative loans because of regulatory constraints, then risk shifting is not a problem. When risk-shifting is not possible, debt overhang may still prevent over levered banks from making good loans.

The model is introduced in section 2. It is analyzed in section 3. In section 4, a numerical example is pursued, and, in section 5, the author concludes.
2. Model

The sequence of events is summarized in figure 1 above.

Suppose that there are two banks Bank B and Bank C. Bank B is a bad bank which is tempted to take on a speculative investment. That is what makes it a “bad” bank. It is also probably pretty “big” and is a bank that is potentially “too-big-to-fail.” The other bank, Bank C, acts as the “counterparty” to Bank B. Bank B can only make this speculative investment in toxic mortgages with the help of Bank C. Bank C represents the wider systematic risk of the failure of Bank B. The exogenous social cost on defaulting on agreement to Bank C is denoted by $K > 0$. Rochet and Tirole (1996) argue that banks that become “too-big-to-fail” when their default creates a crisis through the interbank lending market. Freixas et al. (2000) argue that the failure of a money center bank caused by consumers’ liquidity shocks can induce regulators to bail out an insolvent money center bank to prevent all banks from becoming insolvent. In contrast to those papers, here this cost is exogenous. Thus, $K$ can be seen as the cost of this systematic crisis. Alternatively, $K$ can be seen as the costs of a disorderly restructuring of
the claims on Bank B’s assets. For smaller, less complex financial institutions this cost may be relatively trivial. Yet, for large complex institutions like Lehman Brothers, the cost of resolving the claims on the former investment bank’s assets may be substantial.

Both Bank B and Bank C have deposits worth $D$. All depositors are insured by the government and are unconcerned about the bank’s solvency. Deposits are assumed to stay in the bank until the end of the game. Bank C’s equity is worth zero prior to the start of the game. Bank B has cash owned by its common equity investors worth $\beta + \gamma$, where $\beta > 0$ and $\gamma > 0$. Bank B would like to purchase toxic mortgages that have a present value of $M = 2D$. Yet, the cost of buying these mortgages is $M + \beta$. This cost, $\beta$, can reflect the transaction cost of buying these securities. Further, to borrow Bank C’s deposits, Bank B must promise to pay back Bank C a sum of interest and principal of $D + \gamma$. $\gamma$ can represent the transaction cost of borrowing Bank C’s deposits and is in excess of the risk-free rate. The mortgage securities must be purchased in period 0, and they will be liquidated in period 2b.

In period 1a, Bank B is presented with the opportunity to undertake a positive net present value loan. The new loan requires capital of $N$ and the present value of its safe return is $N + \pi$. Thus, the net present value of this lending opportunity is $\pi$. It is assumed that Bank B will raise new equity with a present value of $N$ to fund this loan. The social value of this loan being extended that does not accrue to the lender is $\Pi > 0$.

This paper assumes that the government and all investors are risk-neutral and that the risk-free rate of interest is normalized to zero.

The mortgage securities can be worth either $M_H$ or $M_L$ in period 2a.
\[ M = pM_H + (1 - p)M_L, \text{ where } M_H > M > M_L > 0. \]  \hspace{1cm} (1)

The subscript “L” denotes that the period 2a mortgage returns are low. The subscript “H” denotes that the mortgage portfolio increased in value in period 2a. Figure 2 below summarizes this relationship.

**Figure 2: Evolution of the mortgage portfolio**

\[
\begin{array}{c}
\text{Period 1} \\
\downarrow \\
\text{Period 2} \\
\end{array}
\begin{array}{c}
M \\
\downarrow \\
1 - p \\
\end{array}
\begin{array}{c}
M_H \\
p \\
M_L \\
\end{array}
\]

Let us assume that Bank B’s equity is wiped out if it both buys mortgage securities and mortgage returns are low in period 2a. In particular, let us assume that

\[ M_L + N - D - F + \pi < 0. \]  \hspace{1cm} (2)

Thus, even with the extra cash, \( N \), from the equity raised to fund the good loan and the increase in shareholder value, \( \pi \), the bank’s common equity is still wiped out if the mortgage portfolio has low returns in period 2a.

Further, since \( M = 2D = F + D, N > 0 \), and \( \pi > 0 \), it must be the case that
Thus the bank has positive value in period 1b if it takes on the good loan and toxic mortgages.

Let us assume that the government can raise capital at no social cost. At the same time, the troubled bank cannot raise equity in excess of $N$, the magnitude of $N$ its new lending opportunity. This reflects the fact that in a financial crisis equity investors are likely to be scarce. The government is assumed to maximize *ex post* social welfare. It is assumed to be unable to commit to any policy that differs from that objective. In particular, the government cannot commit to maximize *ex ante* welfare. If the government offers any subsidies, $S$, to Bank $B$, the government suffers a disutility

\[ \tau |S| \geq 0, \quad \tau \in (0,1]. \]

This represents the deadweight social costs of taxation. Thus taxing, $S < 0$, or subsidizing Bank $B$, $S > 0$, is frowned upon by the government. Ballard *et al.* (1985) estimate that the marginal deadweight losses from taxation are about 40 percent of revenue. Goolsbee (1998) puts the marginal cost at closer to 25 percent. Either way, social welfare is reduced by the subsidy to the troubled bank. The benevolent government will intervene to help Bank $B$ if the sum or social and private surplus from subsidizing Bank $B$ exceeds its cost $\tau |S|$. Finally, we assume that the government does not have the ability to direct Bank $B$’s lending policy. It only can supply capital to Bank $B$ in period 1a.

3. **Analysis**
In this section, we solve for the subgame perfect Nash equilibrium of this game involving Banks $B$ and $C$ and the government. This is a sequential game of complete and perfect information and is easily solved using backwards induction. We first solve for Bank $B$’s lending decision in period 1b. It is found in proposition 1, for some parameter values, that Bank $B$ will suffer from debt overhang without government intervention if it holds toxic mortgages in period 1a. Then we solve for the government’s best response recapitalization in period 1a, given that debt overhang is a problem. It turns out the government is best off buying a large stake of common equity or toxic mortgage assets. There is no subsidy necessary to induce efficient lending, but the subsidy to induce Bank $B$ to participate in the recapitalization is strictly positive. This result is summarized in proposition 2. According to proposition 3, no subsidy is necessary for efficient lending if the recapitalization is forced. In proposition 4, we find the parameter values where it is a subgame perfect Nash equilibrium that a bailout will occur.

3.1 The lending decision without government intervention when mortgage securities are purchased in period 0

3.1.1 Debt overhang

Suppose that Bank $B$ buys the mortgage securities in period 0 and Bank $C$ agrees to act as Bank $B$’s counterparty. Let us denote the equity in Bank $B$ as “$E$.” Let us denote the fact that Bank $B$ has taken on the good loan by the superscript “$+$.”
\[
E^+ = p \max(M_H + N - F - D + \pi, 0) \\
+(1-p) \max(M_L + N - F - D + \pi, 0)
\]  

(4)

We know that \( E^+ = p(M_H + N - F - D + \pi) \) because \( M_H + N - F - D + \pi > M + N - F - D > 0 \), but \( M_L + N - F - D + \pi < 0 \). Therefore, the value of the bank is raised by good loan, but the debts of Bank B are still risky. The bank’s shareholders will raise new equity to finance the loan if the expected return to the positive NPV loan from the new equity issuance is positive. That is, if \( p(\pi + N) - N > 0 \).  

The bank suffers from the problem of debt overhang if

\[
\pi < \phi N, \text{ where } \frac{1-p}{p} \equiv \phi.
\]

(5)

The debt overhang problem is the largest when the probability that shareholders will be wiped out, \( 1 - p \), approaches unity and the probability of success, \( p \), approaches zero.

**Proposition 1**

*Bank B will not make the good loan due to debt overhang if \( \pi < \phi N \).*

This follows from equation (5) and the discussion in the paragraph immediately preceding equation (5). *Q.E.D.*
When debt overhang is a problem, the bank will not take on the good loan and the firm is only solvent when mortgage returns are high. Thus, the value of equity holders claims in period 1 without government intervention is the following:

\[ E^0 = p(M_H - F - D), \text{ when } \pi < \phi N. \]  

(6)

This is not an ideal situation for the bank’s shareholders or the government. We will focus on the situation where the government attempts to remedy the problem of debt overhang.

### 3.1.2 Bank B’s problem when it did not buy mortgage securities in period 0

Suppose that Bank B only bought risk-free securities. Its equity value is the following, where the subscript “F” denotes that the bank only has risk-free assets:

\[ E_F = \beta + \gamma > 0 \]  

(7)

Let us denote Bank B’s equity value if raises new equity to fund the good loan given it only has risk-free assets by the superscript “+.”

\[ E^+_F = N + \beta + \gamma + \pi \]  

(8)

The value of old equity investors claims has risen because
\[ E_P^+ - N - E_P = \pi > 0 \] (9)

The value of the original common equity claims will rise from \( \beta + \gamma \) to \( \beta + \gamma + \pi \) for a net increase of \( \pi > 0 \). Thus, given that Bank B invests in risk-free securities in period 0, it will strictly prefer to fund the good loan.

3.2 Regulatory Remedies

3.2.1 Government’s problem in period 1a when Bank B buys safe securities

The government’s problem in period 1a if Bank B does not buy speculative mortgages is trivial. In this case, there is no need for the government to intervene. Bank C is paid back. The good loan is undertaken and social welfare is first-best, \( \pi + \Pi \). Let us turn to the government’s problem when Bank B buys toxic mortgages.

3.2.2 Government’s problem in period 1a if when both \( \pi < \phi N \) and Bank B buys on toxic mortgages

In period 1a, the government will bail out Bank B if the \textit{ex post} benefits to society from the subsidy exceeds its cost. When \( \pi < \phi N \) and the Bank B has toxic mortgages, it will not take on the good loan in period 1b without government intervention. The expected social gains from government intervention from the perspective of period 1b are \( \pi + \Pi + K(1 - p) \). This is the total social gains from the good loan plus the expected loss
forgone from preventing a default on Bank B’s obligations to Bank C. This needs to be weighed against the cost of any subsidy to society. In this latter case, the government will bail out Bank B if $\pi + II + K(1 - p) > \tau |S|$.

3.2.2.1 Buying toxic mortgages

Suppose that the government offers to buy the toxic mortgages from Bank B for a price $M + S_M \equiv \tilde{M}$. Since $M$ is the fair market value of the mortgages, $S_M$ is the overpayment for the mortgages where $S_M \geq 0$. If Bank B sells its mortgage securities in period 1a and takes on the good loan in period 1b, then the value of its old common equity is the following:

$$E^{M+} \equiv \tilde{M} + N - F - D + \pi. \quad (10)$$

Because the cash received from the mortgage sales weakly exceeds $M$, we know that the bank is solvent, regardless of subsequent mortgage returns.

If the bank does not make the good loan after selling its toxic mortgages, its equity is worth the following:

$$E^{M0} \equiv \tilde{M} - F - D. \quad (11)$$

Bank B will make the good loan if its existing equity will rise with the good loan after it has sold its toxic mortgages. That is if $E^{M+} - N - E^{M0} \geq 0$. This is clearly the case
regardless of the magnitude of the subsidy. The efficient lending constraint below is slack, regardless of the subsidy.

\[(EL_M) \quad E^{M+} - N - E^{M0} = \pi > 0 \quad (12)\]

Thus the minimum non-negative efficient lending subsidy is \(\hat{S}_M = 0\).

Malaith and Mester (1994) observe that regulators have the power to take drastic steps that would be calamitous to bank’s shareholders such as pulling a bank’s charter. Yet, oftentimes these threats are not credible. Thus, regulators may not be able to credibly force recapitalizations. Instead, they may have to subsidize recapitalizations to induce participation.

The bank’s shareholders will only volunteer to make its cash flows risk-free if the following is true:

\[E^{M+} - N - E^0 = S_M + \pi + (1 - p)(M_L - D - F) \geq 0 \quad (13)\]

Equation (13) is obtained by subtracting \(N\) and \(E^0\) in equation (6) from \(E^{M+}\) in equation (10).

This implies that the minimum subsidy for voluntary participation is as follows:

\[(VP_M) \quad S_M \geq -p(\pi + \phi[M_L - D - F + \pi]) \equiv \hat{S}_M > 0 \quad (14)\]
From equation (2), we know that $M_L - D - F + \pi < -N$. Since we know that $\pi - \phi N < 0$ because there is a debt overhang problem, it must be the case that $\pi + \phi(M_L - D - F + \pi) < \pi - \phi N < 0$. Thus, the overall sign of (14) is positive. Merton (1974) argues that equity is a call option on the firm’s assets. A call option’s value is increasing in the volatility of its assets, all other things being equal. Thus, for Bank $B$ to voluntarily participate in the mortgage recapitalization, the government must compensate equity investors for the fact that their call option on the firm’s assets has become less volatile.

It is clear that the subsidy for voluntary participation, $\hat{S}_M > 0$, exceeds the subsidy for efficient lending, $\tilde{S}_M = 0$.

### 3.2.2.2 Preferred stock infusion

Suppose that the government offers to buy preferred stock for $\bar{R} \equiv R + S_{R} > 0$.

The true market value of the preferred stock that the government is buying is $R \in [0, \bar{R}]$. $R$ is the present value of the preferred stock dividends promised to the government. The promised payments to the government must be paid before common equity shareholders’ claims. Thus, the government is giving the troubled bank a subsidy, $S_{R} \in [0, \bar{R} - R]$. Let us denote the value of Bank $B$’s equity after this preferred stock infusion by the superscript “$R$”.

$$
E^R = p(M_H + S_{R} + N - D - F + \pi) + (1 - p) \max\{(M_L + S_{R} + N - D - F + \pi), 0\}
$$

(15)
Suppose that it is the case that preferred stock subsidy is not enough to keep Bank B solvent if the toxic mortgages have bad returns. That is,

$$M_L + S_R + N - D - F + \pi < 0.$$  \hfill (16)

If (16) holds and the bad bank does not take on the good loan after its preferred stock infusion, its equity will be worth:

$$E^{R_0} = p(M_H + S_R - D - F)$$  \hfill (17)

In equation (17), let us use the superscript “$R_0$” to denote that Bank B’s equity will be wiped out if its mortgage portfolio declines in value, despite the preferred stock investment. It is clear the bank will always prefer to not undertake the new loan if it is not solvent in both states of the world, even after the preferred stock recapitalization:

$$E^R - N - E^{R_0} = p(\pi - \phi N) < 0, \text{ when } M_L + S_R + N - D - F + \pi < 0.$$  \hfill (18)

When the bank is solvent in both states of the world, let us denote its equity value by $E^{R^+}$.

$$E^{R^+} = M + S_R + N - D - F + \pi$$  \hfill (19)
In this case, efficient lending requires that

\[ E^{R^+} - N - E^{R^0} = (1 - p)(M_L + S_R - D - F + \pi) \geq 0, \]
when \( M_L + S_R + N - D - F + \pi \geq 0. \]  \hspace{1cm} (20)

When subsidy is sufficient to make the bank solvent in both states of the world given that it takes on the loan, then some subsidy will make lending efficient. Rearranging equation (20) to solve for the minimum subsidy for efficient lending to take place, we are left with the following relationship:

\[ (EL_R) \quad S_R \geq -(M_L - D - F + \pi) \equiv \hat{S}_R > 0, \] \hspace{1cm} (21)

For the bank to agree to the recapitalization, the following inequality must be satisfied:

\[ (E^{R^+} - N - E^0) = (1 - p)(M_L - D - F) + S_R + \pi \geq 0. \] \hspace{1cm} (22)

In this case, the minimum voluntary participation subsidy is the following:

\[ (VP_R) \quad S_R \geq -p(\pi + \phi[M_L - D - F + \pi]) \equiv \hat{S}_R > 0 \] \hspace{1cm} (23)

This is the same as equation (14). Since the debt needs to become safe to spur efficient lending, the voluntary participation constraint requires that the existing common
stockholders must be compensated for the lost volatility of common stock. It is clear efficient lending requires a larger subsidy.

\[ \hat{S}_R - \tilde{S}_R = p(M_L - D - F) < 0 \]  

(24)

Therefore, the binding constraint is the efficient lending constraint for preferred stock recapitalizations.

3.2.2.3 Common stock infusion

In this subsection, the regulator buys a common equity stake worth \( E_G \) in the troubled bank for a price \( \bar{E} \equiv E_G + S_E \). The government is assumed to weakly overpay for its stake. The expected subsidy is \( S_E \geq 0 \). If the troubled bank makes the good loan, then the aggregate value of that banks’ common equity in period 1b is \( E^{E+} \) below.

\[
E^{E+} = p(M_H + N + E_G + S_E - D - F + \pi) \\
+ (1 - p) \max \{ (M_L + N + E_G + S_E - D - F + \pi), 0 \} 
\]

(25)

We know that \( M_H + N + E_G + S_E - D - F + \pi > 0 \), from our knowledge of equation (3) and equation (1). Let us denote the period 1b value of common equity as \( E^{E0} \) when \( M_L + N + E_G + S_E - D - F < 0 \).
Efficient lending is not feasible as long as the firm’s liabilities exceed its assets in the low mortgage returns state.

\[
E^{E_0} = p(M_H + E_G + S_E - D - F),
\]
when \( M_L + N + E_G + S_E - D - F < 0 \)  

(26)

\[
E^{E^+} - N - E^{E_0} = p[\pi - \phi N] < 0,
\]
when \( M_L + N + E_G + S_E - D - F < 0 \)  

(27)

Therefore, efficient lending requires that the bank be solvent in both states of the world. In that case,

\[
E^{E_0} = pM_H + (1 - p)M_L + E + S_E - D - F,
\]
when \( M_L + N + E + S_E - D - F \geq 0 \)  

(28)

We can find under what circumstance the original shareholders will choose the good loan by subtracting the equity promised to new investors, \( N \), and the value of the old shareholders’ equity without the good loan \( E^{E_0} \) in equation (28) from \( E^{E^+} \) in equation (27).  

\[
E^{E^+} - N - E^{E_0} = \pi > 0,
\]
when \( M_L + N + E_G + S_E - D - F \geq 0 \)  

(29)

Therefore, as long as Bank B is solvent in both states of the world after the equity recapitalization, then efficient lending will occur without a subsidy \( S_E = 0 \).
The amount of common stock necessary to induce efficient lending without subsidy is $E_G \geq \tilde{E}$, where

$$E_G \geq \tilde{E} \equiv -(M_L + N - D - F) > 0. \quad (30)$$

Given that lending is efficient, voluntary participation requires that the old equity investors’ stakes are no less valuable than $E_0$ after the equity sales of $N$ and $E_G$ to outside investors and the government, respectively. That is,

$$(E^e + N - E_G - E^0) \geq (1 - p)(M_L - D - F) + S_E + \pi \geq 0,$$

when $M_L + N + E_G + S_E - D - F \geq 0. \quad (31)$

Rearranging this relationship,

$$(VP_E) \quad S_E \geq -p \left( \pi + \phi[M_L - D - F + \pi] \right) \equiv \hat{S}_E > 0 \quad (32)$$

This is exactly the same magnitude as the voluntary participation constraint for buying up troubled mortgages in equation (14). The existing equity investors must be compensated for the fact that they are no longer benefiting from creditors’ misfortunes after the recapitalization. Thus, the binding constraint is the voluntary participation constraint when the common equity stake purchased by the government exceeds $\tilde{E}$ in equation (30). That is,
\[ \hat{S}_E - \tilde{S}_E = -p (\pi + \phi [M_L - D - F + \pi]) > 0, \text{ when } E_g \equiv E^*_g \geq \tilde{E}. \tag{33} \]

3.3 Comparing the costs

A summary of our findings in the previous discussions of various types of government capital injections are summarized in proposition 2 below:

**Proposition 2**

*Suppose that the troubled bank suffers from debt overhang problems, \( \pi < \phi N \), without a government recapitalization. The subsidy necessary for both efficient lending and voluntary participation are strictly lower when the government buys common stock or toxic mortgages.*

\[
\max \{ \tilde{S}_R, \hat{S}_R \} = \hat{S}_R > \max \{ \tilde{S}_M, \hat{S}_M \} = \hat{S}_M = \max \{ \tilde{S}_E, \hat{S}_E \} = \hat{S}_E = \hat{S}_R \tag{34} \]

Equation (34) and the proposition above follows from comparing equations (14), (23), and (30). *Q.E.D.*

**Proposition 3**

*If the government does not need to satisfy the voluntary participation constraint, then no subsidy is needed to satisfy efficient lending if the government buys the toxic mortgages or common equity stakes. Thus, the subgame perfect Nash equilibrium (SPE) is that Bank B will buy safe debt and there will be no bailout in equilibrium.*

This follows from the fact that \( \tilde{S}_E (E^*_G) = \hat{S}_M = 0 \). If Bank B expects there to be no bailout, then it will find it unprofitable to incur the transaction costs, \( \beta + \gamma \), of buying the toxic mortgages and borrowing Bank C’s deposits. *Q.E.D.*
3.4 Bank B’s investment decision in period 0

When debt overhang is a problem, the government’s marginal benefit from inducing efficient lending is \( \pi + \Pi + (1 - p)K \). The government will clearly prefer the \( \hat{S}_M = \hat{S}_E \) because this minimizes the deadweight losses from taxation. *Ex post* bailouts make sense to the government if

\[
\pi + \Pi + (1 - p)K > \tau \hat{S}_M = \tau \hat{S}_E > 0. \tag{35}
\]

Similarly, Bank B will only prefer to invest in the mortgage assets if the government’s subsidy exceeds the transaction costs of the toxic mortgages. That is,

\[
\hat{S}_M = \hat{S}_E > \beta + \gamma \tag{36}
\]

Therefore, the double coincidence of Bank B buying the toxic assets and the government being willing to bail out Bank B when it does so, occurs only when the following set of inequalities are met:

\[
\frac{1}{\tau} (\pi + \Pi + (1 - p)K) > \hat{S}_M = \hat{S}_E > \beta + \gamma \tag{37}
\]

**Proposition 4**

*If Bank B must voluntarily participate in any bailout and debt overhang is a problem, \( \pi < \phi N \), then the subgame perfect Nash equilibrium (SPE) is one of the following, depending on the magnitude of the parameter values:

i. If both \( \frac{1}{\tau} (\pi + \Pi + (1 - p)K) > \hat{S}_M = \hat{S}_E > \beta + \gamma \), then the subgame perfect Nash equilibrium (SPE) is that Bank B will buy toxic mortgages and there will be a
bailout, involving the government’s buying either common equity or toxic mortgages from Bank B.

ii. If either \( \pi + \Pi + (1 - p)K < \tau \hat{S}_M = \tau \hat{S}_E \) or \( \hat{S}_M = \hat{S}_E < \beta + \gamma \), then the subgame perfect Nash equilibrium (SPE) is that Bank B will buy safe assets and there will be no bailout.

This follows from equation (37). Q.E.D.

For many parameter values, when equation (35) holds then social welfare would be higher by \( \tau \hat{S}_M = \tau \hat{S}_E \) if the government could commit to not bailout Bank B.

Nevertheless, we have assumed that such a commitment is not possible.

It seems that the government cannot commit to bailout the banks in light of recent events. Fender and Gyntelberg (2008) and table 1 document that a large number of developed nations’ governments have committed to either purchase assets from their banks or to provide capital injections to their banks in the financial crisis of 2008.

4. Numerical example

Suppose that the following parameters describe the game:

\[
M = $1000, \quad M_H = $1050, \quad M_L = $800
\]
\[
p = 0.8 \Rightarrow \phi = 0.25
\]
\[
D = F = $500
\]
\[
N = $100
\]
\[
\beta = $20 \quad \text{and} \quad \gamma = $10
\]
\[
\pi = $5, \quad \Pi = $10, \quad \text{and} \quad K = $240
\]
\[
\tau = 0.2
\]
For these parameter values, debt overhang would be a problem if Bank $B$ bought mortgage securities because $\pi = $5 < $\phi N = $25. Let us assume that Bank $B$ does buy mortgages. In that case, in period 1a the government would bail out Bank $B$. If the government uses common equity, the government’s minimum best-response common equity infusion which is obtained by combining equations (30) and (38) would be $\tilde{E} = $100. This capital infusion is 10 percent of period 1a assets. The subsidy for common equity is $\max \{\tilde{S}_E, \tilde{S}_E^*\} = $35 according to equations (33) and (32). This is weakly the government’s best response. This weakly maximizes ex post social welfare.

Ex post social welfare in period 1a in this scenario rises by $\pi + \Pi + (1 - p)K - \tau \tilde{S}_E = $5 + $10 + 0.2*$240 – 0.2*$35 = $63.

Suppose that Bank $B$ buys preferred stock. What would be the minimum subsidy that satisfies efficient lending? The minimum subsidy to induce efficient lending and voluntary participation with a preferred stock recapitalization can be obtained by combining the parameters in equation (38) with the efficient lending and voluntary participation subsidies in equations (21) and (23). The subsidy that satisfies both constraints would need to be $\max \{\tilde{S}_g, \tilde{S}_g^*\} = \tilde{S}_g = $195 = $35 = $195, $35$. Thus any preferred stock infusion must exceed $195 or 19.5 percent of assets and include a subsidy of at least 19.5 percent of assets to be effective. Let us contrast this with the subsidy necessary for the strategy of the government of buying up toxic mortgages.

Finally, if the government buys up toxic mortgages, the minimum subsidies that satisfy Bank $B$’s voluntary participation constraint can be found by inserting the parameters in equation (38) into equation (14). The efficient lending constraint is
satisfied for all non-negative subsidies. The minimum subsidy that induces the bank to voluntarily participate is $\hat{S}_M = $35. The minimum mortgage subsidy is 3.5 percent of assets, but the outlay for the mortgage recapitalization is 100 percent of assets.

Not only does the equity recapitalization offer weakly the lowest subsidy, but it also requires the lowest dollar investment to induce the bank to lend properly in period 1b. Table 1 summarizes the minimum magnitude and the minimum subsidies needed for each type of capital infusions. In summary, the government will find it optimal to recapitalize the bank if Bank B buys toxic mortgages in period 0. Further, Bank B will find it optimal to buy mortgages in period 0 because it has to forfeit $\beta + \gamma = $30 of its equity value to receive a subsidy of $35. Thus, the subgame perfect Nash equilibrium is that Bank B will buy toxic assets and there will be a bailout involving the government’s buying either common stock or toxic mortgages. Social welfare is clearly worse than first-best of $\pi + \Pi = $15. If the government could commit to not bail out Bank B, then Bank B would not buy the toxic assets and there would be no need for the bailout. Yet, since the government is assumed to be unable to commit to that policy, social welfare is reduced by the deadweight losses associated with the subsidy to Bank B. Thus, social welfare is $\pi + \Pi - \tau \left| \hat{S}_E \right| = $8, which is less than first-best. Table 2 summarizes the results of this section. Common equity recapitalizations require the least investment and are tied with toxic asset purchases for requiring the lowest possible subsidy.
5. Conclusion

This paper has demonstrated that purchases of toxic mortgages and common stock in troubled banks are the most efficient way to bail out a bank suffering from debt overhang. Preferred stock recapitalizations are the least efficient. Preferred stock gives similar incentives to debt and exacerbates the problems of debt overhang. For common stock recapitalizations to be effective they must be accompanied by covenants prohibiting the payment of dividends, share repurchases, or cash acquisitions until the bank’s equity capital is rebuilt. The government need not exercise control rights beyond enforcing those covenants against increasing leverage. Further, the government could resell its common equity stakes to private investors, and leave intact the incentives that cure the problem of debt overhang.

Toxic mortgage purchases are as efficient as stock purchases. This is because toxic asset purchases require the same subsidy to encourage efficient lending (none) and voluntary participation (the voluntary participation subsidy could be large). Yet, they have two drawbacks relative to the government’s buying common stock, which are not explicitly modeled in this paper. First, mortgage securities are notoriously hard to value. Second, the government’s initial outlay is much larger for toxic asset purchases. Thus, if politicians are under pressure to engineer a bailout that does not involve a large initial outlay of funds, common stock recapitalizations will be strictly preferred to toxic asset purchases.
Finally, this paper shows that voluntary recapitalizations will be costly because they require the government to pay stockholders for reducing the variability of the stock price. Thus, the government must overpay for stock or toxic securities if the bank’s current shareholders have any say in the matter. The troubled bank’s shareholders’ love of variance also explains why it cannot find private investors to buy its stock and toxic assets. Since an over-levered firm derives most of its common stock value from the variability of its stock price, the government would be much better off if it could force the bank to accept the recapitalization. Forced recapitalizations also improve the bank’s ex ante incentives.

This paper considers the optimal securities for a government to buy from a distressed bank that is suffering from debt overhang. This analysis is most relevant for an institution which is so complex that a bankruptcy workout poses systematic risk. An institution that is “too-big-to-fail” imposes the extra social cost of subsidized recapitalizations in times of crisis. A longer term approach would involve requiring complex financial conglomerates to have a larger capital base as is argued in Freixas et al. (2007). With a larger capital base those institutions would have a harder time pursuing riskier activities that lead to costly bailouts. Further, extra capital requirements would provide a longer-term disincentive to grow to be “too-big-to-fail” in the first place.
References


Table 1: Selected government capital infusions into financial institutions from September 2008 through January 2009

<table>
<thead>
<tr>
<th>County Providing Capital</th>
<th>Recipient</th>
<th>Source</th>
<th>Date of the Article</th>
<th>Securities Purchased</th>
<th>Value in Billions of US dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>Erste</td>
<td>Forbes</td>
<td>10/30/2008</td>
<td>Preferred Stock</td>
<td>$3.50</td>
</tr>
<tr>
<td>Belgium</td>
<td>Fortis Bank NV/SA</td>
<td>Bloomberg</td>
<td>9/30/2008</td>
<td>Common Stock</td>
<td>$6.79</td>
</tr>
<tr>
<td>Belgium</td>
<td>KBC</td>
<td>Fidelity</td>
<td>1/22/2009</td>
<td>Preferred Stock</td>
<td>$7.10</td>
</tr>
<tr>
<td>Belgium &amp; non-state investors</td>
<td>Dexia</td>
<td>NYT &amp; FT</td>
<td>10/1/2008</td>
<td>NA</td>
<td>$4.26</td>
</tr>
<tr>
<td>France</td>
<td>Dexia</td>
<td>NYT &amp; FT</td>
<td>10/1/2008</td>
<td>Common Stock</td>
<td>$1.42</td>
</tr>
<tr>
<td>France</td>
<td>Various</td>
<td>FT</td>
<td>1/21/2008</td>
<td>Subordinated Debt and Preferred Shares</td>
<td>$13.40</td>
</tr>
<tr>
<td>Germany</td>
<td>Commerzbank</td>
<td>Reuters</td>
<td>1/8/2009</td>
<td>Common and Preferred Stock</td>
<td>$27.15</td>
</tr>
<tr>
<td>Germany</td>
<td>Hypo Real Estate</td>
<td>Bloomberg</td>
<td>1/30/2009</td>
<td>Common Stock</td>
<td>$13.08</td>
</tr>
<tr>
<td>Greece</td>
<td>Various</td>
<td>Reuters on Forbes.com</td>
<td>1/28/2008</td>
<td>Preferred Stock</td>
<td>$6.60</td>
</tr>
<tr>
<td>Ireland</td>
<td>Glitnir</td>
<td>Reuters</td>
<td>12/27/2009</td>
<td>Common Stock</td>
<td>$0.88</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>SNS Real</td>
<td>Reuters</td>
<td>11/13/2008</td>
<td>Preferred Stock</td>
<td>$1.60</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>Ageon NV</td>
<td>Reuters</td>
<td>10/28/2008</td>
<td>Preferred Stock</td>
<td>$3.70</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>ING</td>
<td>Guardian and Reuters</td>
<td>10/13/2008</td>
<td>Preferred Stock</td>
<td>$13.50</td>
</tr>
<tr>
<td>Neatherlands</td>
<td>SNS Real</td>
<td>Reuters</td>
<td>11/13/2008</td>
<td>Preferred Stock</td>
<td>$1.60</td>
</tr>
<tr>
<td>Neatherlands</td>
<td>Ageon NV</td>
<td>Reuters</td>
<td>10/28/2008</td>
<td>Preferred Stock</td>
<td>$3.70</td>
</tr>
<tr>
<td>Neatherlands</td>
<td>ING</td>
<td>Guardian and Reuters</td>
<td>10/13/2008</td>
<td>Preferred Stock</td>
<td>$13.50</td>
</tr>
<tr>
<td>Neatherlands</td>
<td>Fortis Bank Nederland</td>
<td>Bloomberg</td>
<td>9/30/2008</td>
<td>Common Stock</td>
<td>$5.78</td>
</tr>
<tr>
<td>Russia</td>
<td>Various</td>
<td>Reuters</td>
<td>1/27/2009</td>
<td>NA</td>
<td>$27.40</td>
</tr>
<tr>
<td>Switzerland</td>
<td>UBS</td>
<td>The Street.com and Guardian</td>
<td>10/16/2008</td>
<td>Convertible Bonds</td>
<td>$5.30</td>
</tr>
<tr>
<td>Switzerland</td>
<td>UBS</td>
<td>The Street.com and Guardian</td>
<td>10/16/2008</td>
<td>Asset Purchases</td>
<td>$60.00</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Various</td>
<td>S&amp;P</td>
<td>10/13/2008</td>
<td>Common and Preferred Stock</td>
<td>$37.00</td>
</tr>
<tr>
<td>United States</td>
<td>Various</td>
<td>NYT</td>
<td>10/14/2008</td>
<td>Preferred Stock and Warrents</td>
<td>$250.00</td>
</tr>
</tbody>
</table>

The dollar value of the capital injections announced were the amounts in US dollars stated in the article or they were converted to US dollars by the author. The author used the interbank rate quoted on Onanda.com on the date of the article for all conversions not in the main text of the article. The author thanks Luana Zhang for her excellent research assistance in identifying most of the articles referred to in this table. NA = not available; NYT = New York Times; FT = Financial Times; S&P refers to the report “Ratings Implications of U.K. Government Capital Injections into Certain Major U.K. Banks,” Standard & Poor’s.
Table 2: Comparing the various types of recapitalizations in the numerical example

<table>
<thead>
<tr>
<th>Type of Recapitalization</th>
<th>Toxic Asset Purchases</th>
<th>Preferred Stock</th>
<th>Common Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum capital for efficient lending required as a percent of assets</td>
<td>100%</td>
<td>19.5%</td>
<td>10%</td>
</tr>
<tr>
<td>Minimum subsidy required as a percent of assets with voluntary participation</td>
<td>3.5%</td>
<td>19.5%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Lowest subsidy required as a percent of assets with mandatory participation</td>
<td>0%</td>
<td>19.5%</td>
<td>0%</td>
</tr>
</tbody>
</table>