MEDICAL SURFACE RECONSTRUCTION FROM SPARSE DATA

RECONSTRUCTION DE SURFACES MEDICALES A PARTIR DE DONNEES LACUNAIRES

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ABSTRACT

Nous présentons dans cet article un algorithme pour reconstruire des surfaces lisses à partir de données lacunaires en utilisant un modèle déformable de type plaque mince. L'algorithme est une version améliorée de l'algorithme VSF (Variational Spline Fitting) développé par Richard Szeliski. Nous présentons une dérivation différente des équations discrètes pour l'énergie correspondant au modèle plaque mince. Les résultats obtenus sur des données simulées montrent que l'algorithme proposé converge plus rapidement que l'algorithme original. Ces travaux ont été utilisés pour reconstruire la surface du ventricule gauche du cœur à partir de données incomplètes.

INTRODUCTION

Surface reconstruction and smoothing methods are widely used in practice to best estimate the original surface represented by a scattered noisy point set arising in a number of medical imaging applications [1]. For instance, the number and the distribution of the initial samples of an organ surface lead to incomplete meshes. The problem of surface reconstruction can be solved using a variety of techniques: finite element methods such as a Delaunay triangulation [2], finite difference methods using deformable models [3] such as thin plate model [4], and Fourier-based interpolation methods [5], to mention a few. The former two approaches were selected for further study in this work as these are generally more appropriate for smooth surfaces such as surfaces of left ventricle of the heart.

ORIGINAL VSF ALGORITHM

In [6], Szeliski proposed to use a deformable model to estimate the missing points. In his VSF algorithm, the problem is formulated as an optimization one. The function to be minimized is written:

$$E(x) = E_d(x) + \lambda E_s(x) \qquad (1)$$

where $x_{i,j}$ (i = 0:N-1, j = 0:M-1) are the mesh regular points of the reconstructed surface, i and j indicate spatial positions. The previous formulation is usually expressed as an energy minimisation problem where an attracting force $E_d(x)$ draws the mesh towards the sparse data and a tension $E_s(x)$ in the mesh keeps the surface smooth. The regularization parameter λ ($\lambda>0$) is used to adjust the closeness of the fit between the surface and the sparse data set.

The expression for the discrete energy corresponding to the function above can be written as [6]:

$$E_d(x,d) = \frac{1}{2}\sum_{(i,j)} w_{i,j}(x_{i,j} - d_{i,j})^2 \qquad (2)$$

where $d_{i,j}$ ($d_{i,j} = 0$ at missing points) are the sparse samples of the original incomplete surface and the weights $w_{i,j}$ ($w_{i,j} = 0$ at missing points) are inversely related to the variance of the measurements.

Using the thin plate model, the energy function corresponding to the smoothness constraint can be written in continuous form as:

$$\varepsilon_s(f) = \frac{1}{2}\int\int\left(f_{uu}^2 + f_{vv}^2 + 2f_{uv}^2\right)dudv \qquad (3)$$

In the original VSF algorithm, the discrete form of the above energy function is derived using a nonconforming rectangular element

$$E_s(x) = \frac{h_u h_v}{2}\sum_{i,j}\left[\left(\frac{x_{i+1,j} - 2x_{i,j} + x_{i-1,j}}{h_u^2}\right)^2 + \left(\frac{x_{i,j+1} - 2x_{i,j} + x_{i,j-1}}{h_v^2}\right)^2 + 2\left(\frac{x_{i+1,j+1} - x_{i,j+1} - x_{i+1,j} + x_{i,j}}{h_u h_v}\right)^2\right]. \qquad (4)$$

IMPROVED VSF ALGORITHM

In this work, we propose a more accurate derivation process to get the discrete energy corresponding to the above continuous function (3) using Taylor formula. In particular, we propose to use the following approximation E_s for ε_s :

$$E_s(x) = \frac{h_u h_v}{2} \sum_{i,j} \left[\left(\frac{x_{i+1,j} - 2x_{i,j} + x_{i-1,j}}{h_u^2} \right)^2 + \left(\frac{x_{i,j+1} - 2x_{i,j} + x_{i,j-1}}{h_v^2} \right)^2 + \frac{1}{8} \left(\frac{x_{i+1,j+1} - x_{i+1,j-1} - x_{i-1,j+1} + x_{i-1,j-1}}{h_u h_v} \right)^2 \right] \quad (5)$$

The approximation of the Laplacian part of equation (3) is similar in both algorithms using the so-called 5-star points scheme. However, our approximation of the third part is more accurate as it uses the 4 diagonal points to approximate the crossed second-order derivative while Szeliski's approach uses the biased forward positions. Our approximation is found to lead to better performance compared to that used in original VSF algorithm.

RESULTS

Our first aim was to compare the convergence characteristics of the original and improved VSF algorithms for relatively smooth surfaces. The results displayed in Figure 1 show clearly that the proposed algorithm, IVSF, outperforms the original VSF algorithm, e.g., when the tolerance value of the iterative conjugate gradient algorithm is set to 10^{-4}, the modified algorithm converges about 100 iterations faster than the original algorithm.

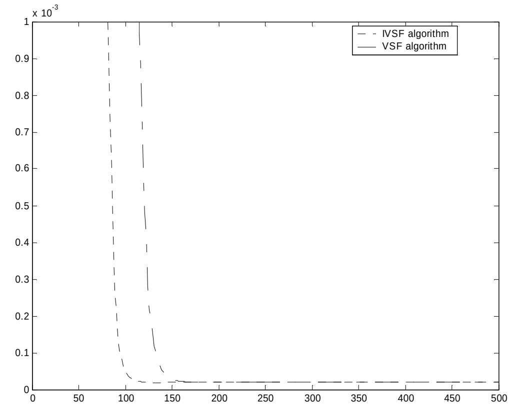
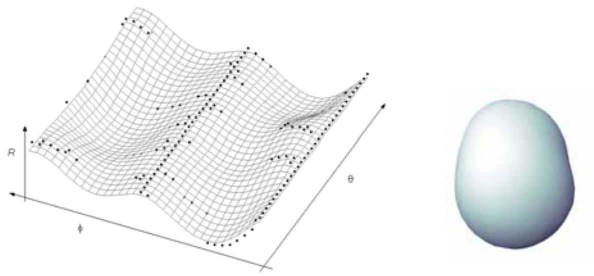


Figure 1. MSE comparison

The IVSF algorithm has been used to reconstruct the surface of the left ventricle of the heart (Figure 2.b) starting with data acquired along three contours (Figure 2.a). Even if results from both algorithms look visually similar, the computation time of the improved algorithm is reduced compared to the original VSF algorithm. This advantage can be used to accelerate medical surface reconstruction software.



(a) (b)

Figure 2. Reconstruction of the left ventricle :
(a) 2d developed surface from the original sparse data which are represented here by black dots,
(b) 3d reconstructed surface using IVSF algorithm

CONCLUSION

In this short paper, we presented an improved implementation of the Variational Spline Fitting Algorithm. The algorithm is based on a more general and precise approximation of the energy equations in discrete time. We have shown that the modified algorithm outperforms the original implementation discussed by Szeleski. We are currently investigating the potential of such algorithm in practical medical applications of surface reconstruction where very limited data are available.

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