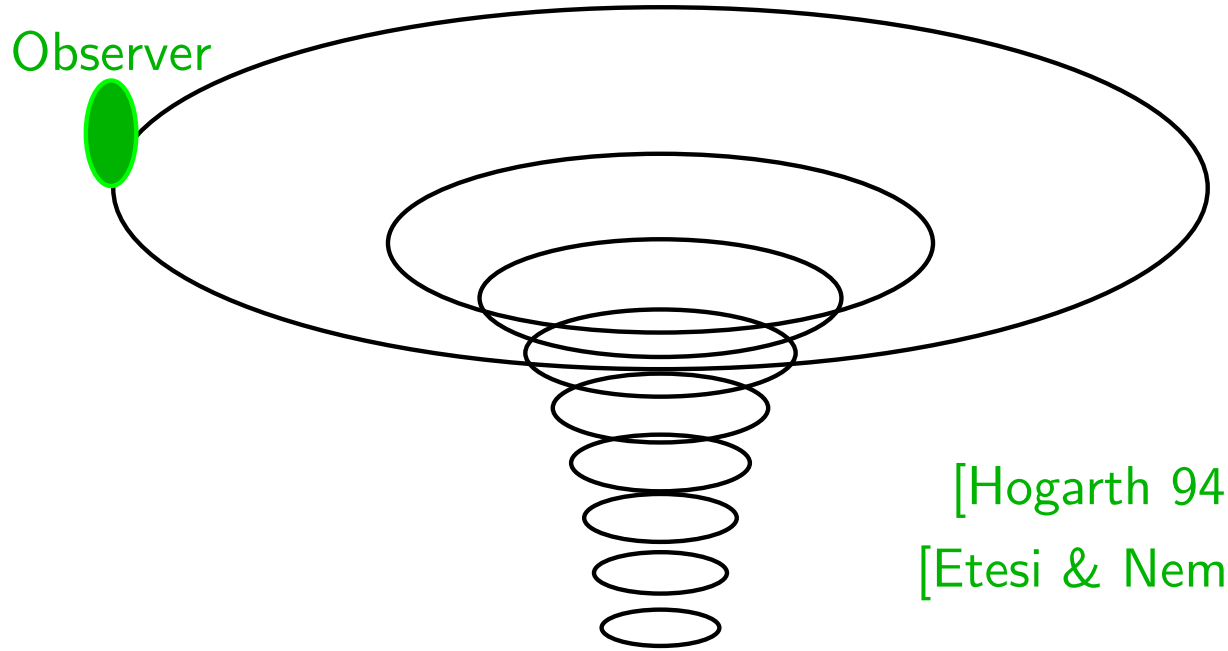
Abstract geometrical computation for Black hole computation

Jérôme Durand-Lose

Laboratoire d'Informatique Fondamentale d'Orléans,
Université d'Orléans, Orléans, FRANCE

Computations on the continuum
Lisboa, June 17th, 2005

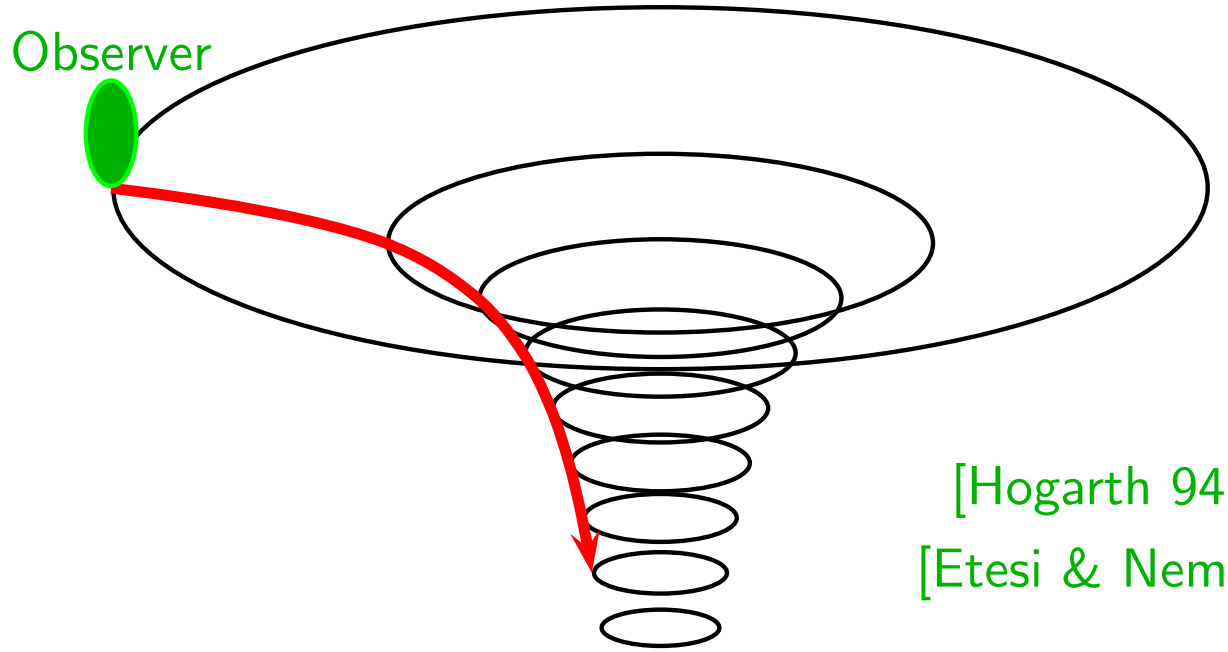
Black hole model



[Hogarth 94 & 00]
[Etesi & Nemeti 02]

1. Observer at the "edge"

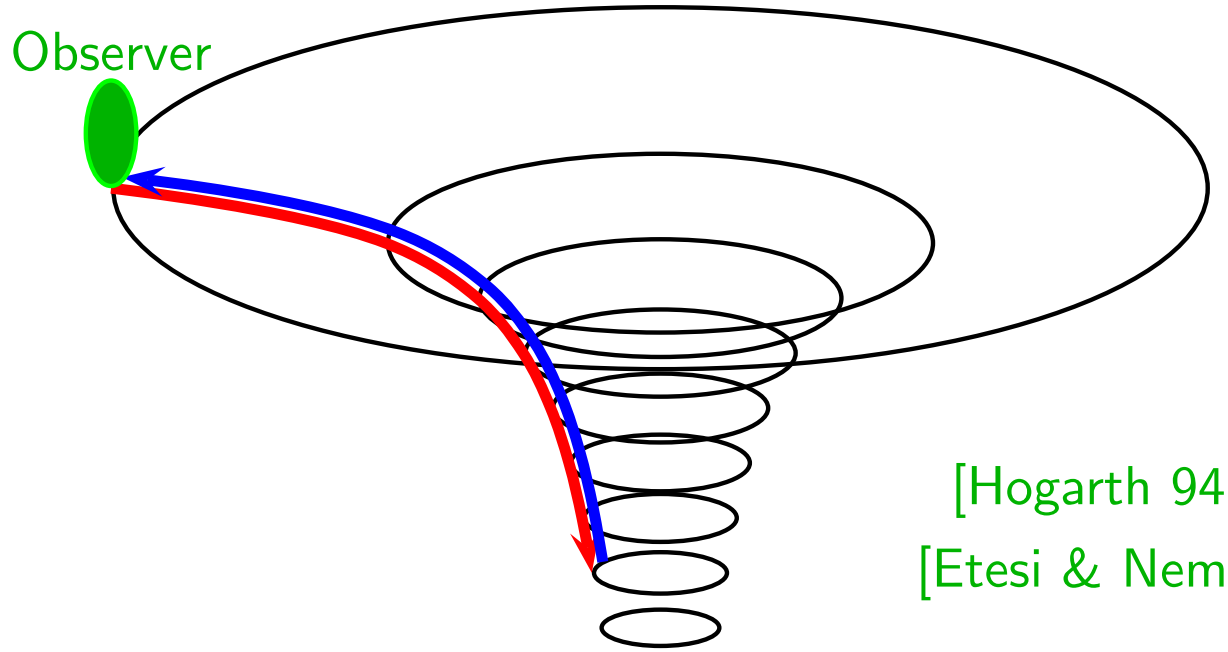
Black hole model



[Hogarth 94 & 00]
[Etesi & Nemeti 02]

1. Observer at the "edge"
2. Machine sent into the black hole *infinitely accelerated*

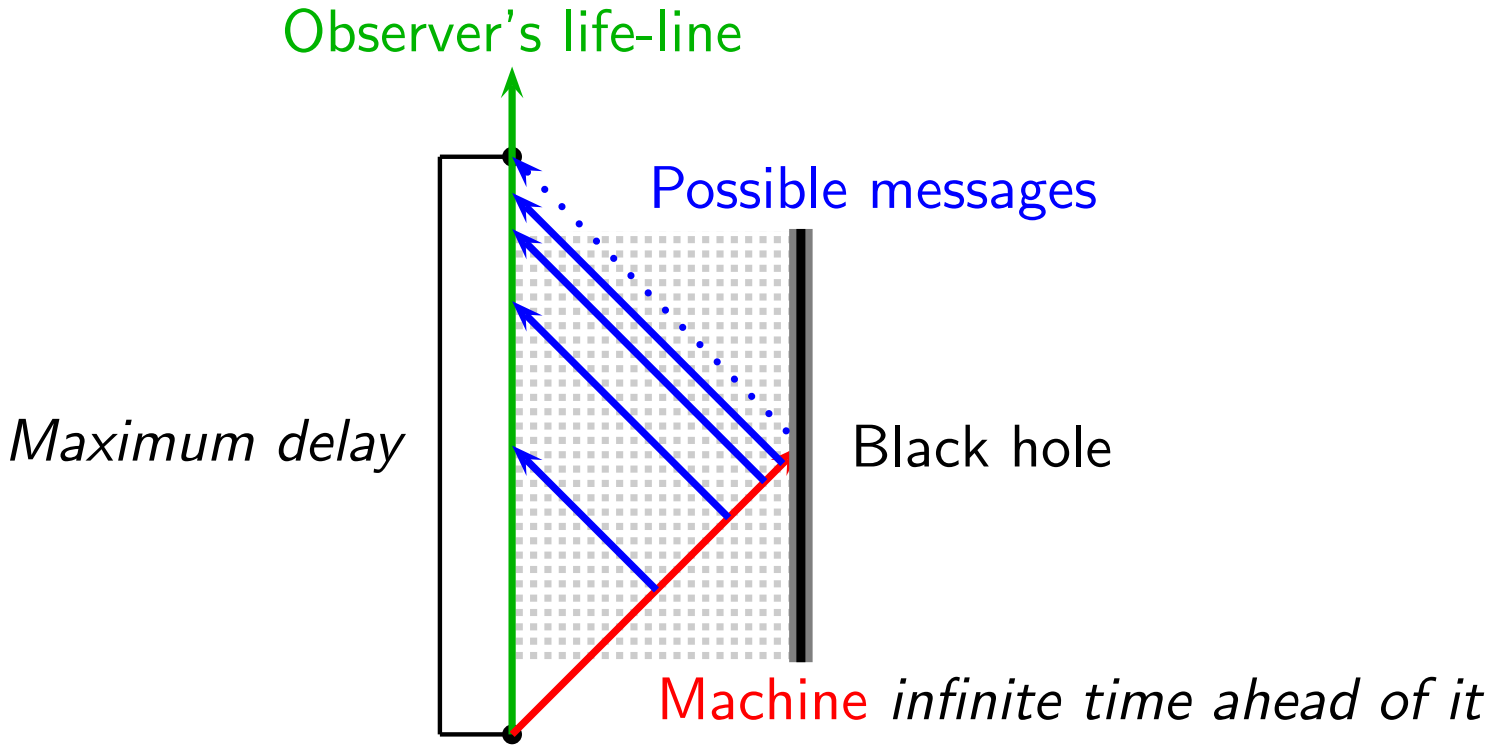
Black hole model



[Hogarth 94 & 00]
[Etesi & Nemeti 02]

1. Observer at the "edge"
2. Machine sent into the black hole *infinitely accelerated*
3. Message sent by the machine received by the observer *within a bounded delay*

Malament-Hogarth space-time



Message indicates the result of the computation

After the delay, the observer knows whether the computation stops

Any recursively enumerable problem can be decided!

Related models

Main idea: infinitely many "iterations" on a sub-time-scale

Can be achieved with a transfinite ordinal scale as in:

Infinite time Turing machines

[Hamkins 02]

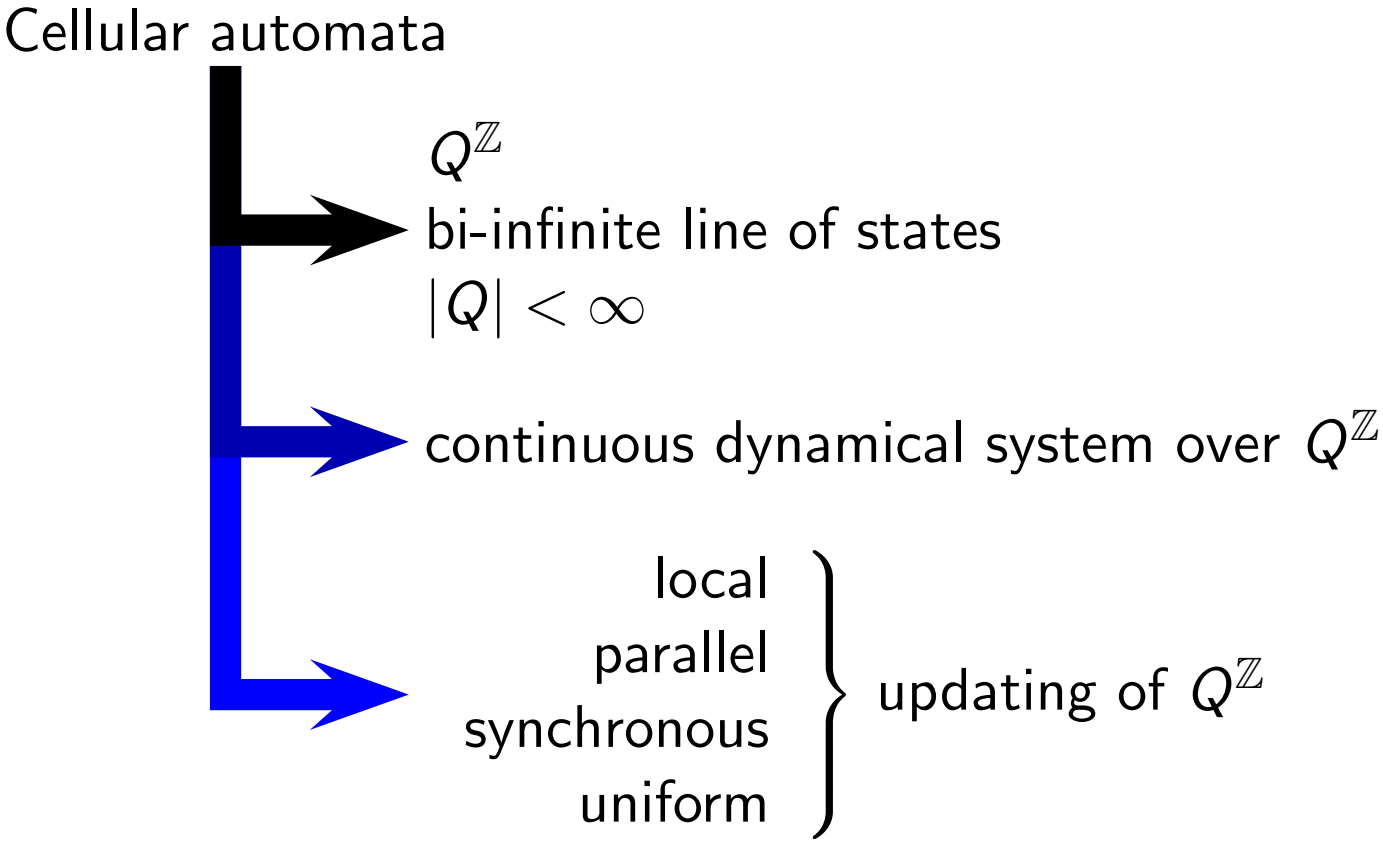
Or with a "Zeno" sub-scale as in:

Piecewise constant derivative systems

[Asarin & Maler 95, Bournez 99]

We use the last approach

Starting from discrete model...



. . . with discrete space-time diagrams

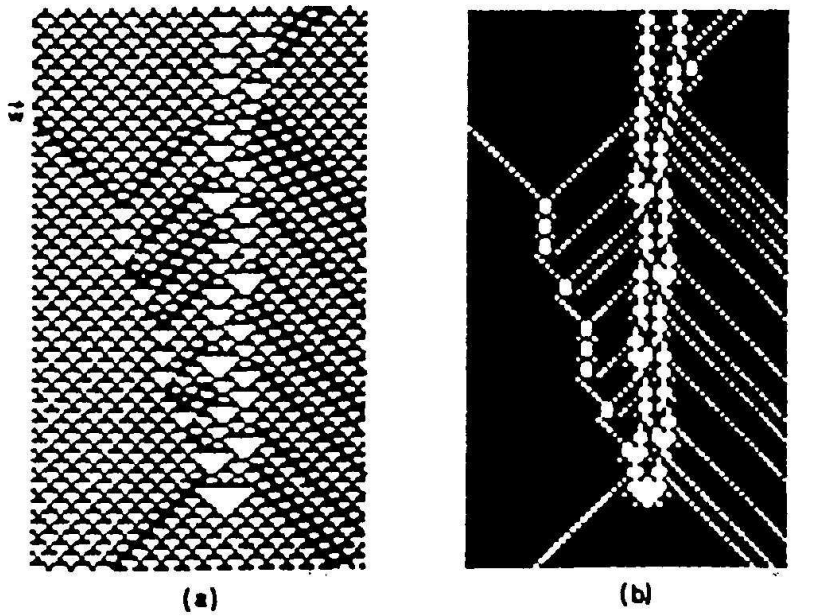


FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6.

[BNR91, Fig. 7]

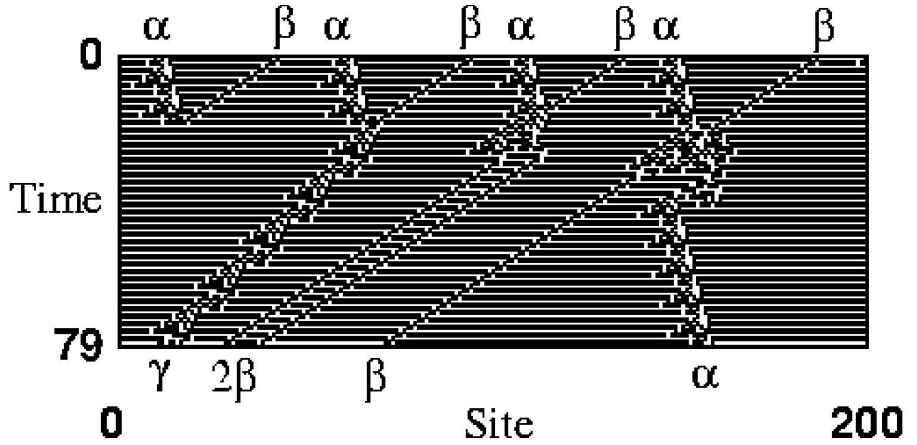


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction.

[HSC01, Fig. 7]

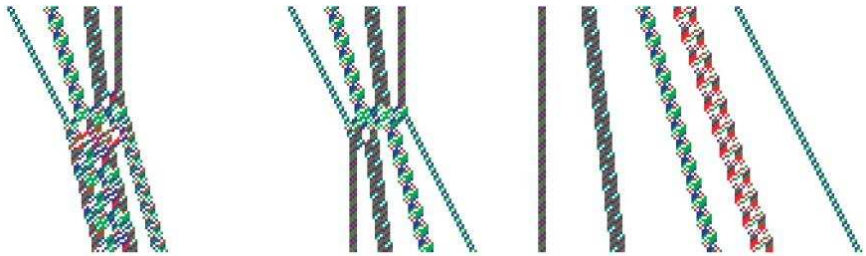


Figure 5. Two collisions of filtrons, and five free filtrons supported by the FPS model; ST diagram applies $q = 1$.

[Siw01, Fig. 5]

. . . with discrete space-time diagrams

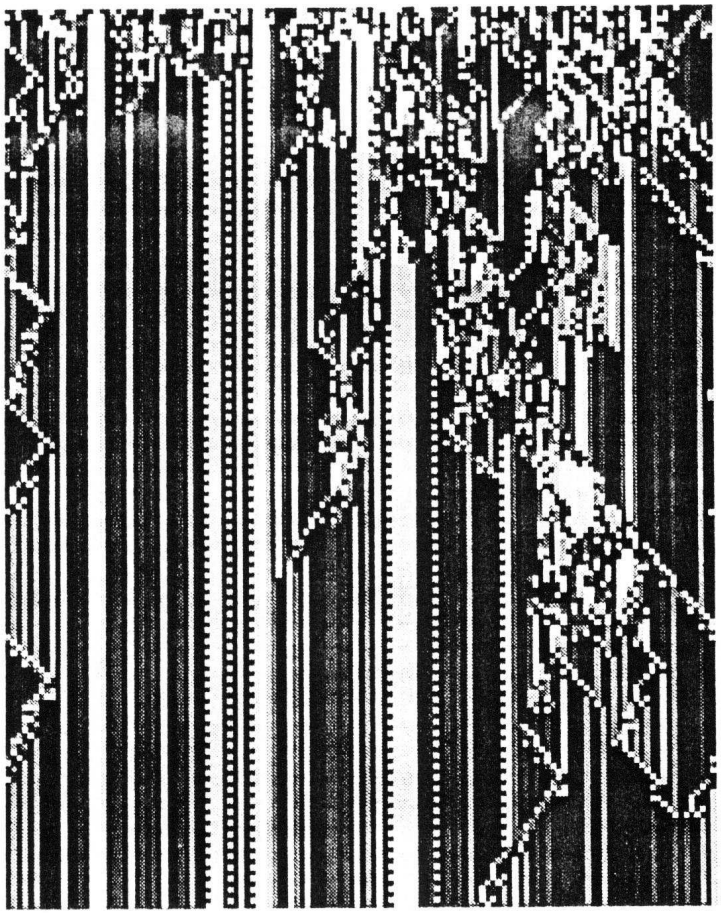


Figure 3: A simulation of the $k = 7$, $r = 1$ universal CA of table 3 for an uncorrelated initial state (with a density of blanks equal to 0.76). Symbols y, 0, 1, A, B, $\sqcup$, and T are represented by

[LN90, Fig. 4]

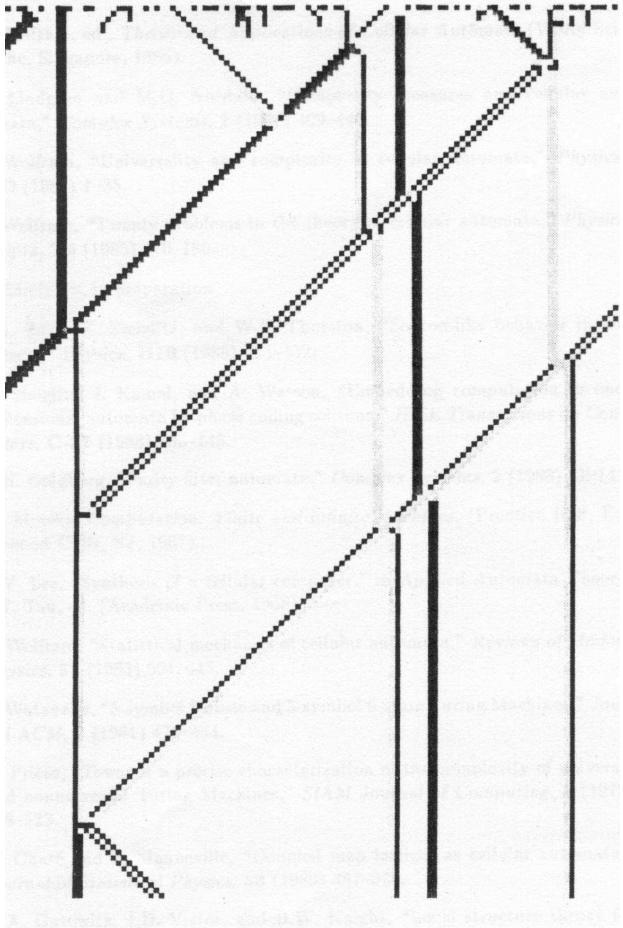


Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, $\sqcup$, and + are represented by

[LN90, Fig. 3]

. . . with discrete space-time diagrams

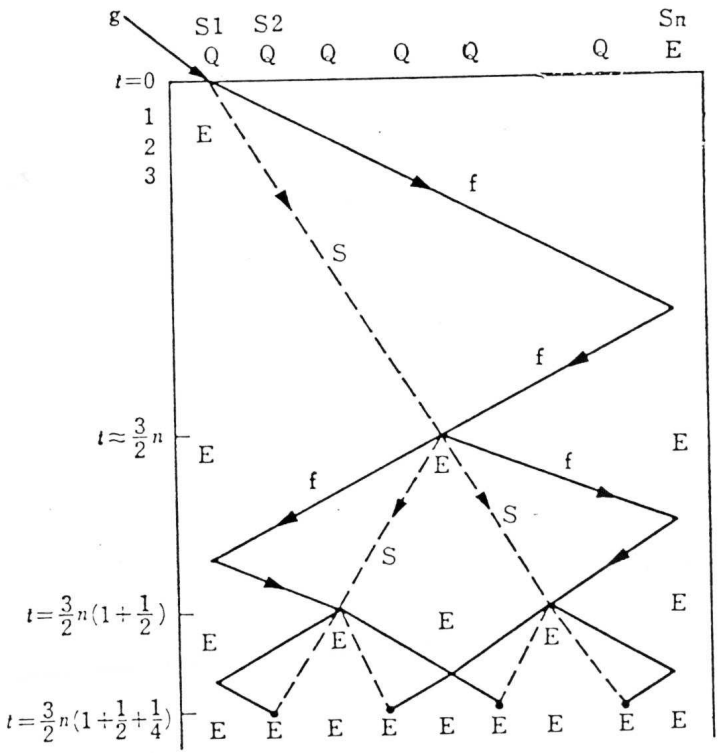


図 3·5 一斉射撃の問題（連続近似）

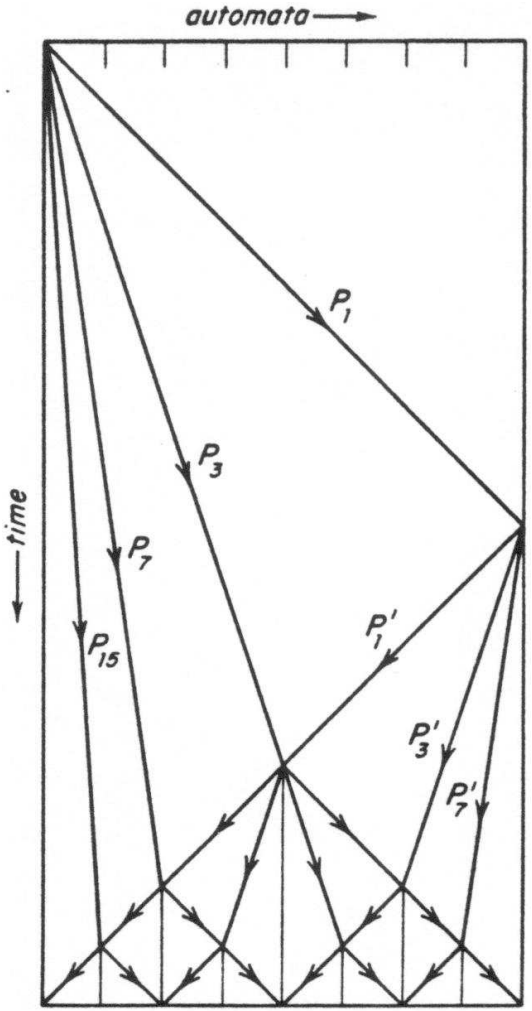
[Got66, Fig. 3]

G	s_1	s_2	s_3	s_4	s_5	s_6
g	Q	Q	Q	Q	Q	E
$t=0$	f's'Efs	Q	Q	Q	Q	E
1	E	Q2f	Q	Q	Q	E
2	E	Q1	Qf	Q	Q	E
3	E	Q&	Q	Qf	Q	E
4	E	Q	Q2	Q	Qf	E
5	E	Q	Q1	Q	Q	f'Ef
6	E	Q	QS	Q	f'Q	E
7	E	Q	Q	a'Q	Q	E
8	E	Q	f'S'ESf	f's'Esf	Q	E
9	E	f'2Q	E	E	Q2f	E
10	f'Ef	1Q	E	E	Q1	f'Ef
11	E	f'S'ESf	E	E	f's'Esf	E
12	a'Ea	E	a'Ea	a'Ea	E	a'Ea
13	F	F	F	F	F	F

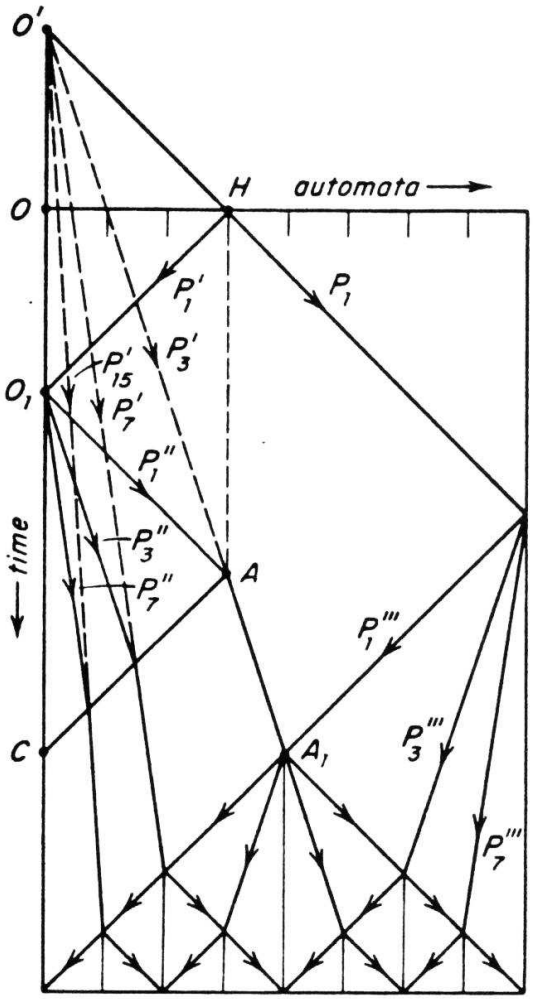
図 3·6 一斉射撃解 ($n=6$)

[Got66, Fig. 6]

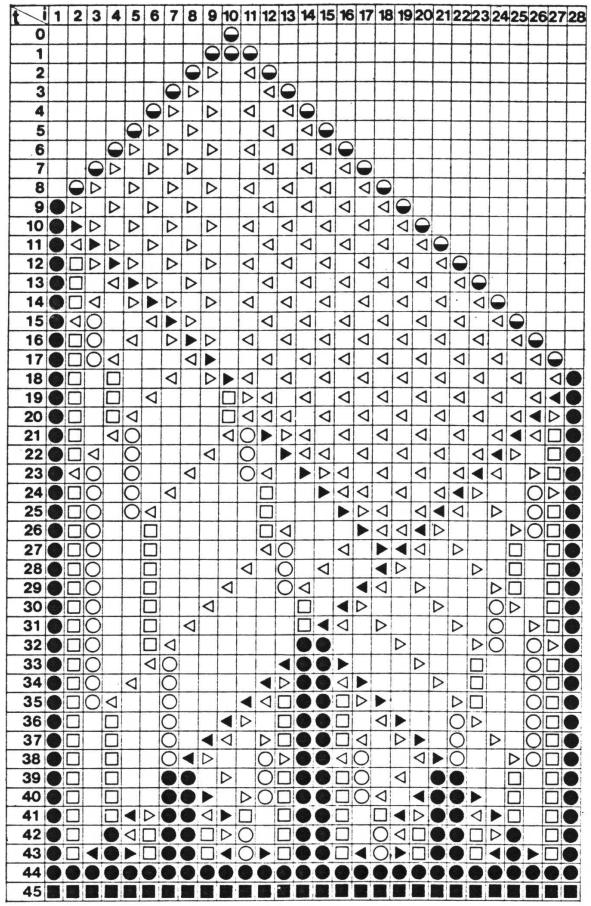
. . . with discrete space-time diagrams



[VMP70, Fig. 1]



[VMP70, Fig. 2]



[VMP70, Fig. 3]

. . . with discrete space-time diagrams

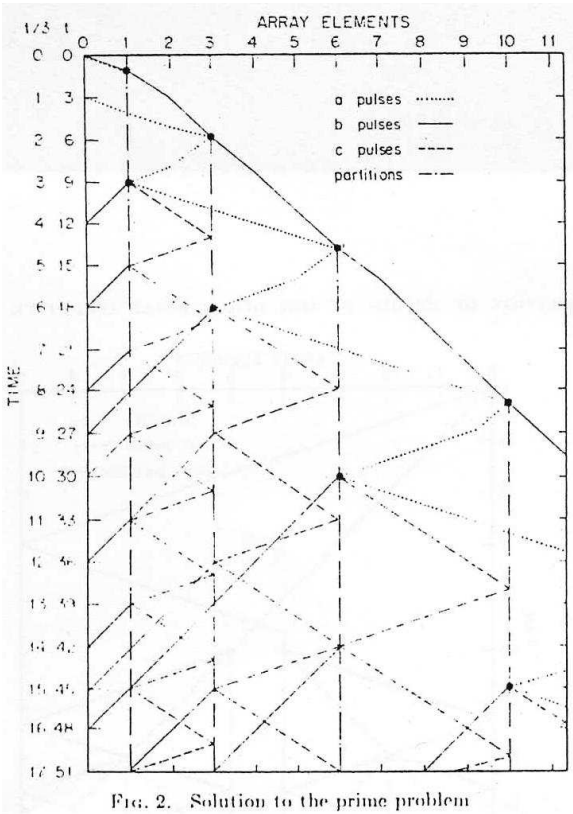


Fig. 2. Solution to the prime problem

[Fis65, Fig. 2]

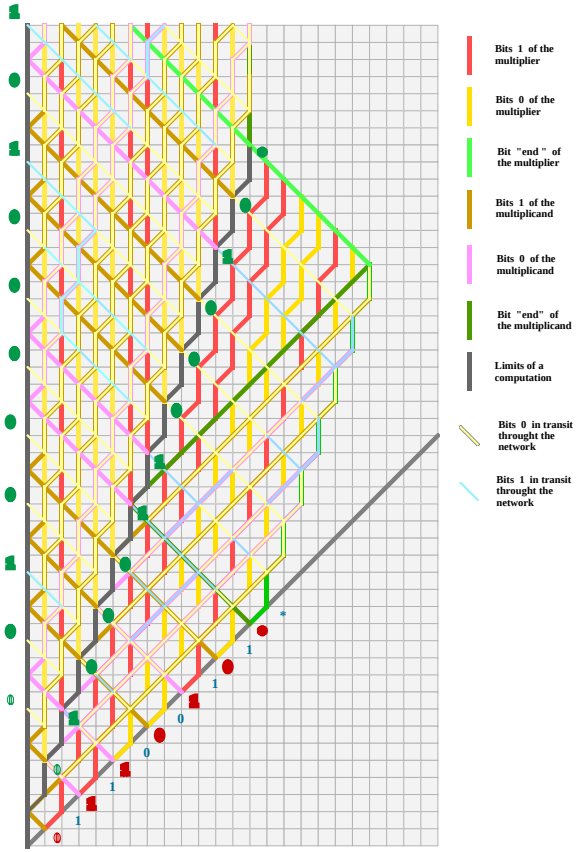


Figure 8: Computing $(ab)^2$.

[Maz96, Fig. 8]

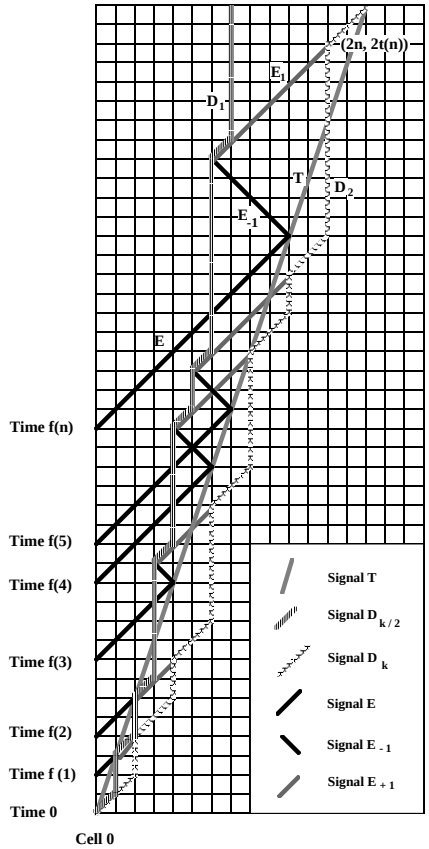
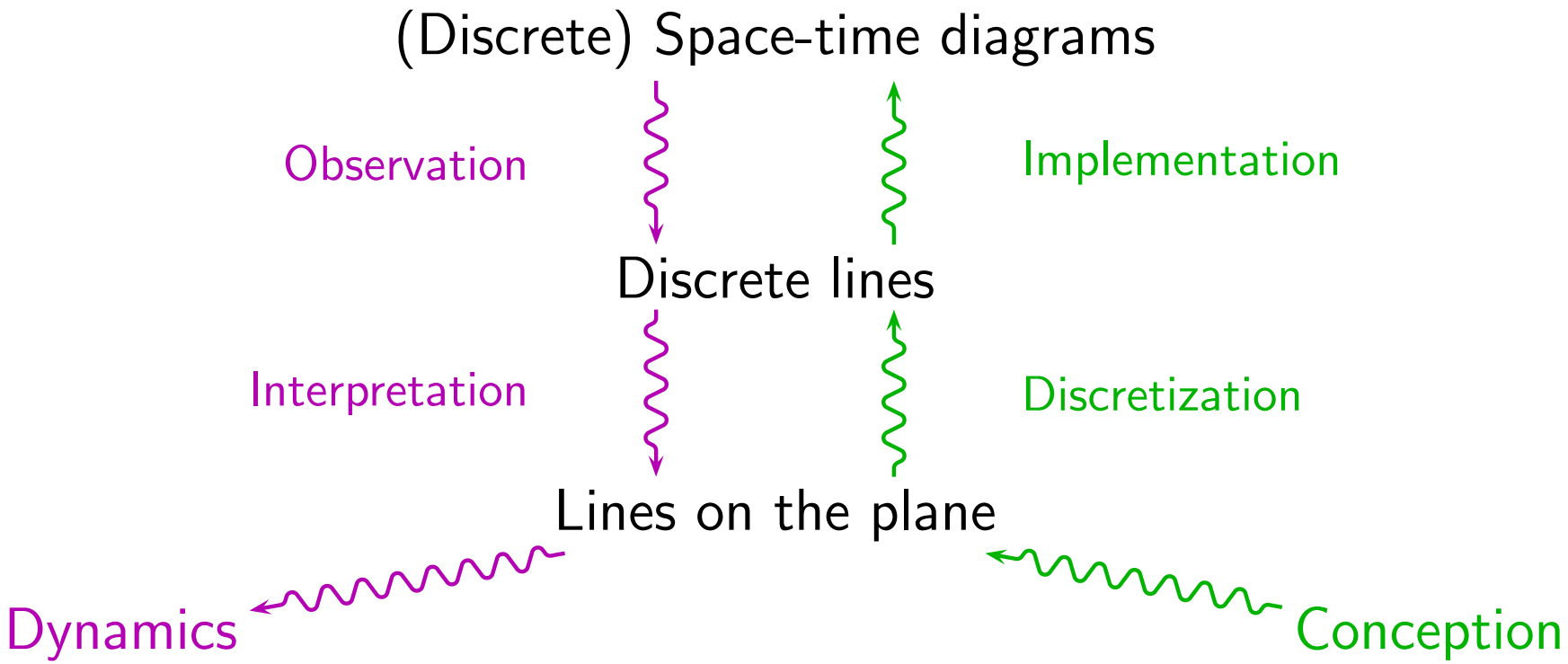
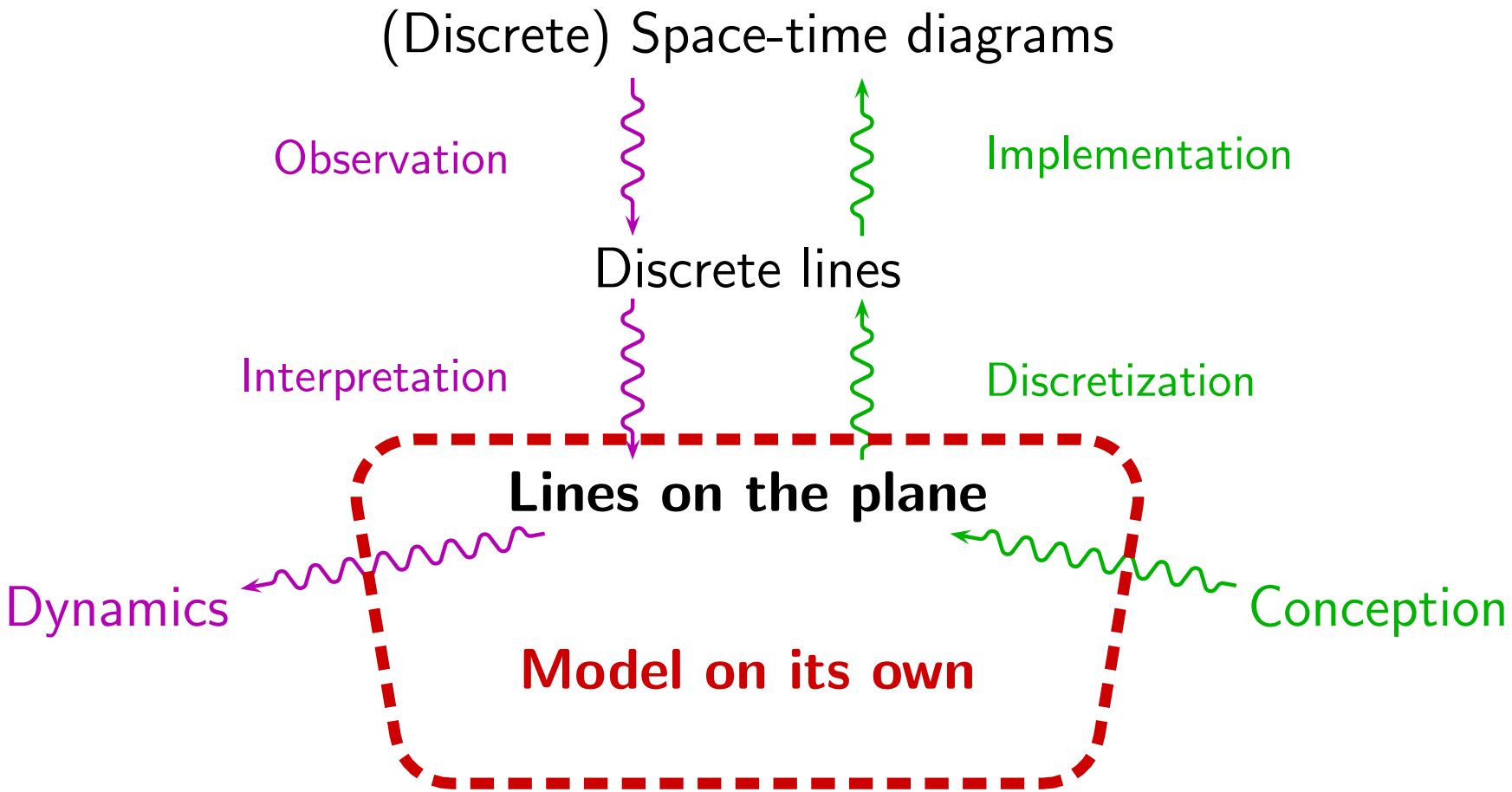


Figure 18: Characterization of the sites $(n, f(n))$.

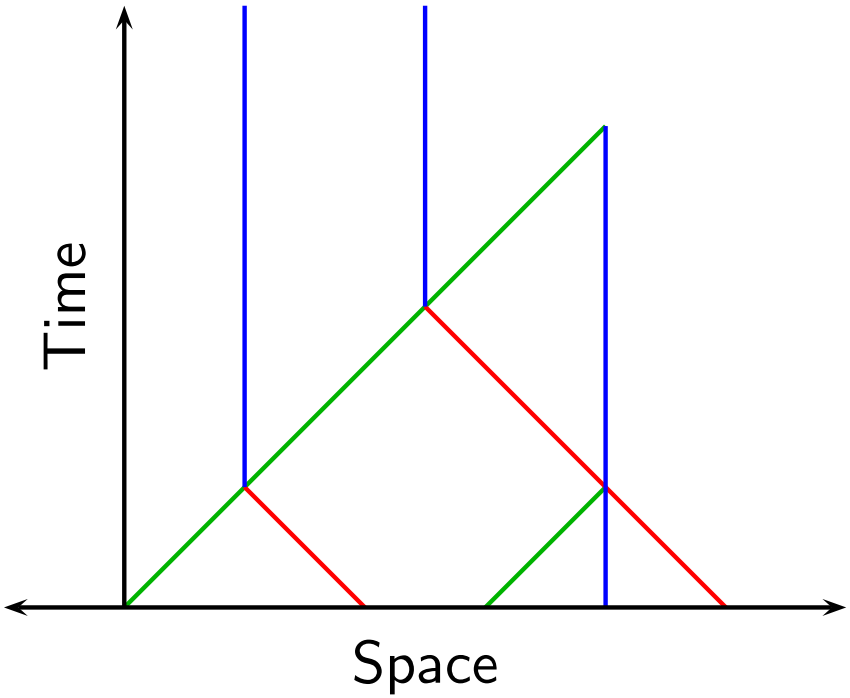
[MT99, Fig. 18]


(Discrete) Space-time diagrams
Observation
Implementation
Discrete lines
Interpretation
Discretization
Lines on the plane
Dynamics
Conception


(Discrete) Space-time diagrams
Observation
Implementation
Discrete lines
Interpretation
Discretization
Lines on the plane
Dynamics
Conception
Model on its own

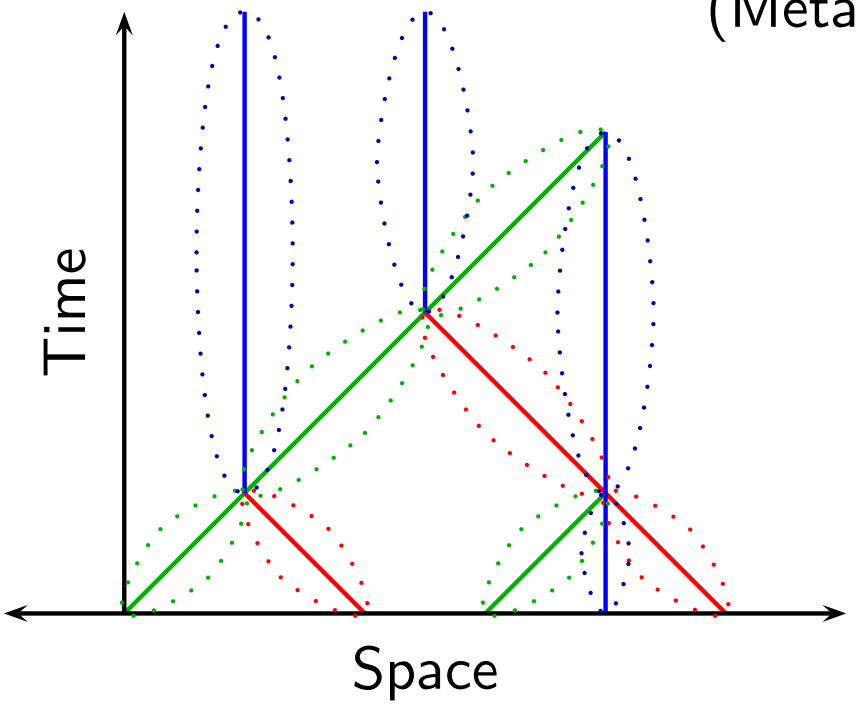
Continuous space-time,

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Continuous space-time, signals

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

(Meta-signal, position)

Position

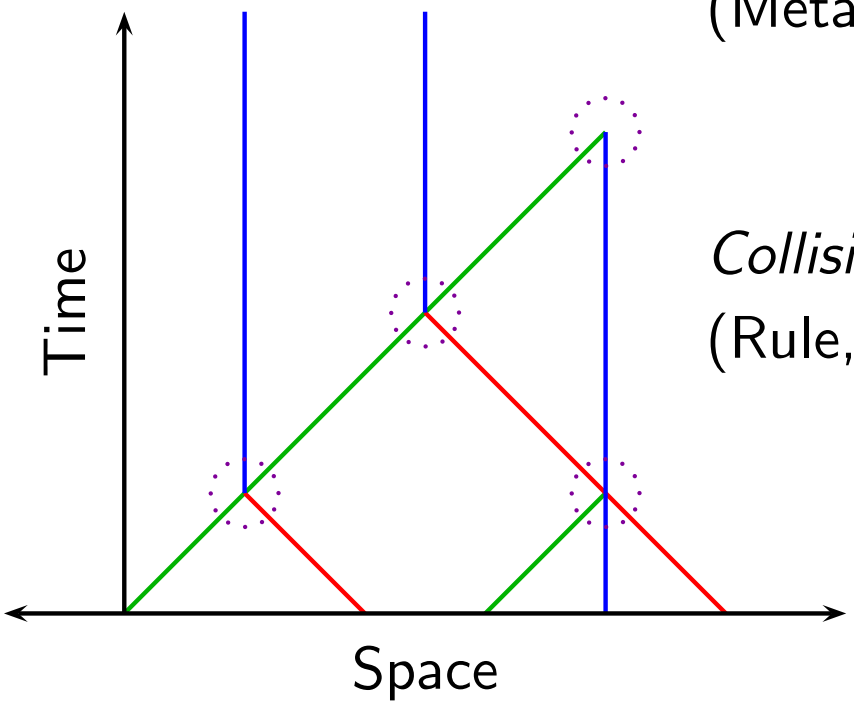
(x, t)

Meta-signal

$\mu = (\iota, \nu)$

Continuous space-time, signals and collisions

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

(Meta-signal, position)

Position

(x, t)

Meta-signal

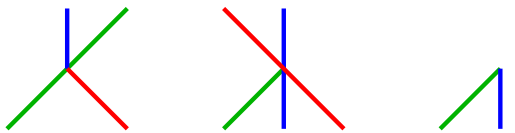
$\mu = (\iota, \nu)$

Collision

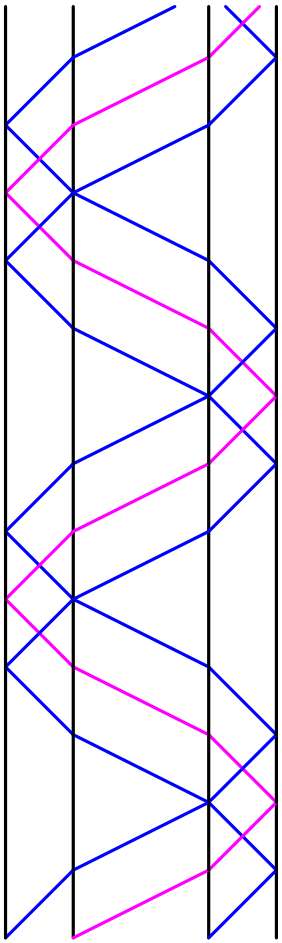
(Rule, position)

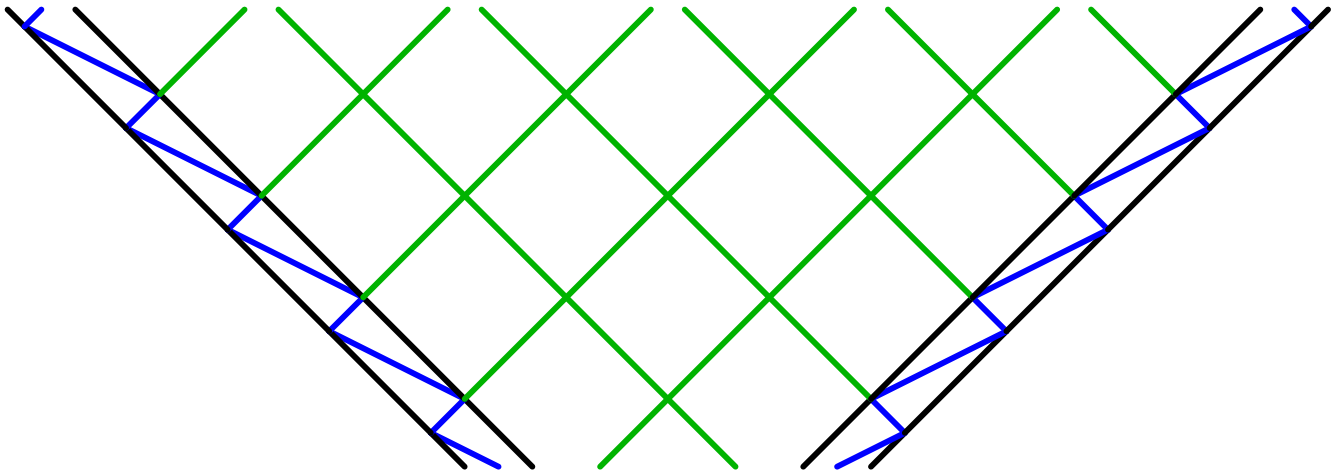
Rule

$\rho = \{\mu_i^-\}_i \to \{\mu_j^+\}_j$

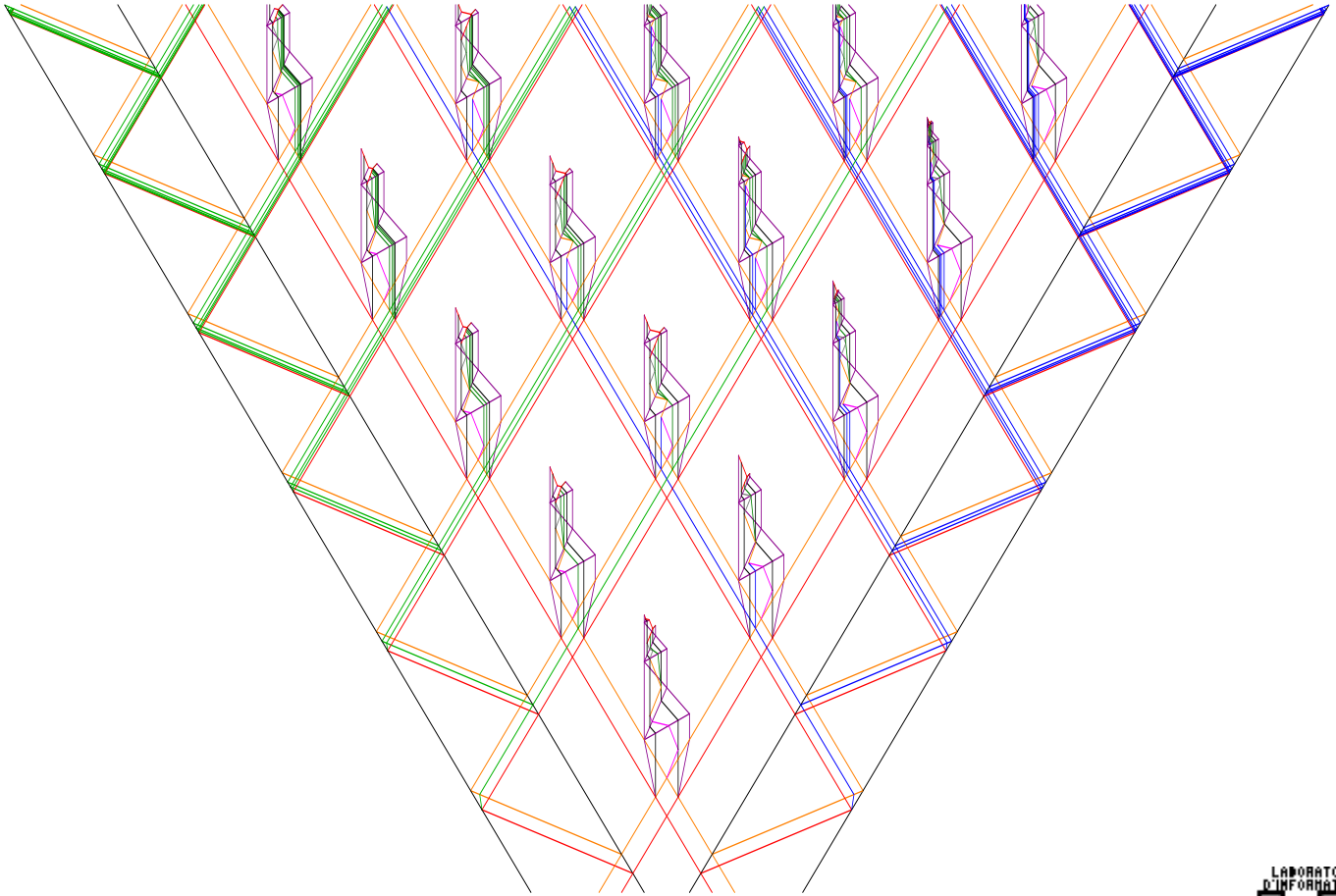


Continuous space-time diagrams



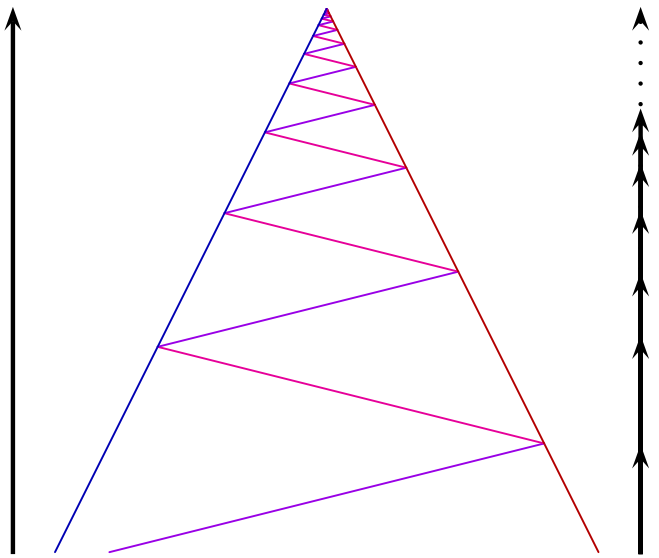


Continuous space-time diagrams



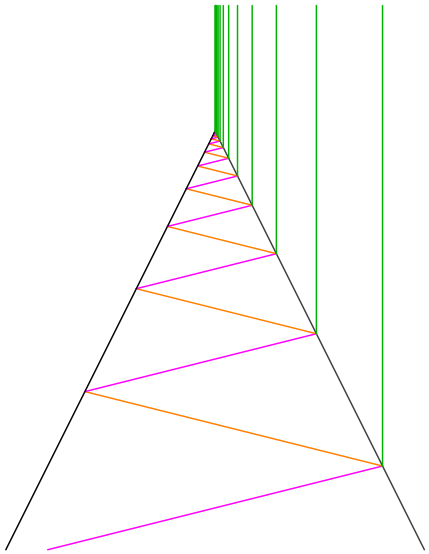
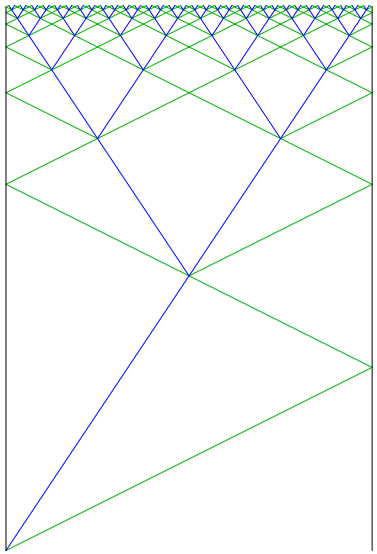
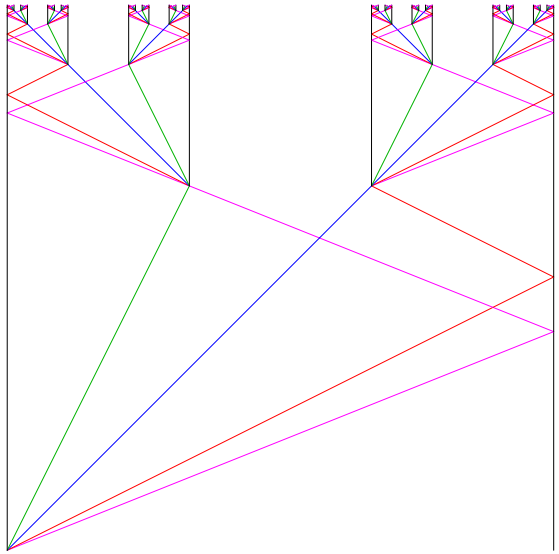
Zeno artifact for black hole implementation

Finite duration



infinitely many collisions

Unwanted space-time diagrams

Unwanted because

- The number of signals is bursting to infinity
  (free creation of mater/energy)
- Difficulty (if not impossibility) to define continuation there

Restriction

Idea:

- Associate to each meta-signal a minimal amount of energy
- Ensure that no energy is created

$\rightsquigarrow$ Collision must not create energy

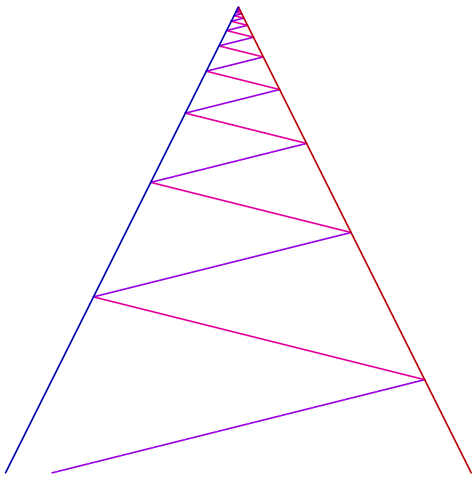
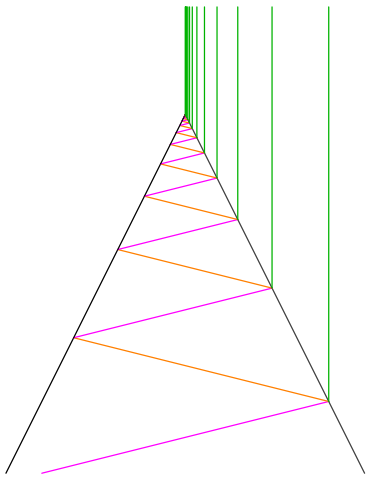
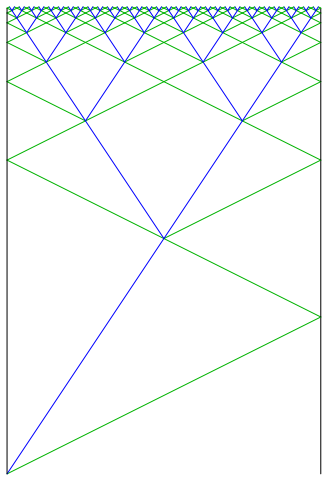
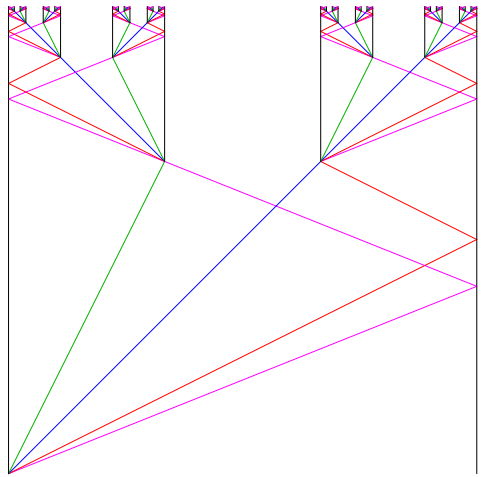
Restriction

- Energy: $\mu \longrightarrow E(\mu) \in \mathbb{N}^*$
- $\forall \rho = \{\mu_i^-\}_i \to \{\mu_j^+\}_j, \quad \sum E(\mu_i^-) \geq \sum E(\mu_i^+)$
- $E($ configuration $) = \sum E($ existing signals $)$
- Total energy quantified and bounded
- The total number of signals is bounded

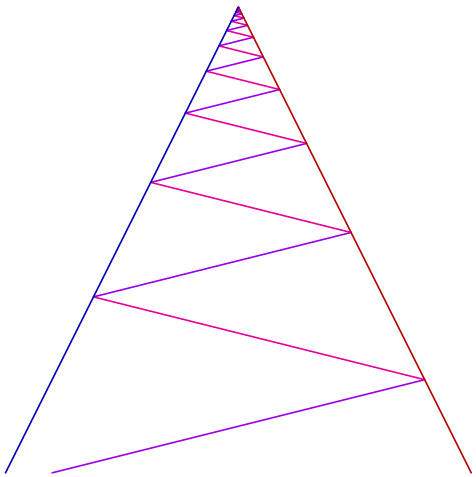
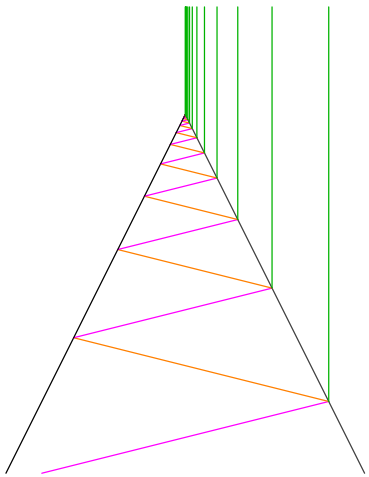
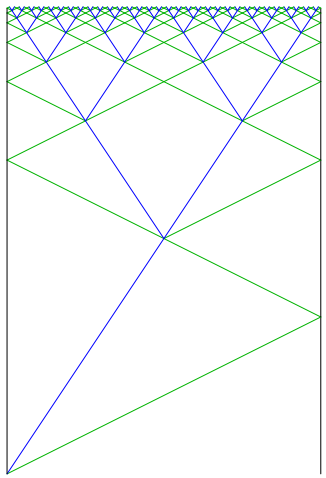
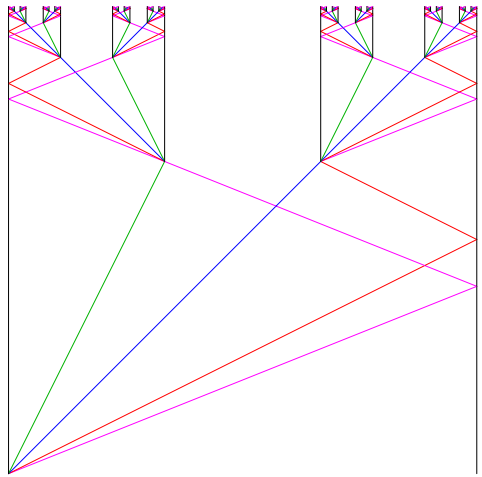
Special case

The number of signals is preserved by any collision

Back to the space-time diagrams

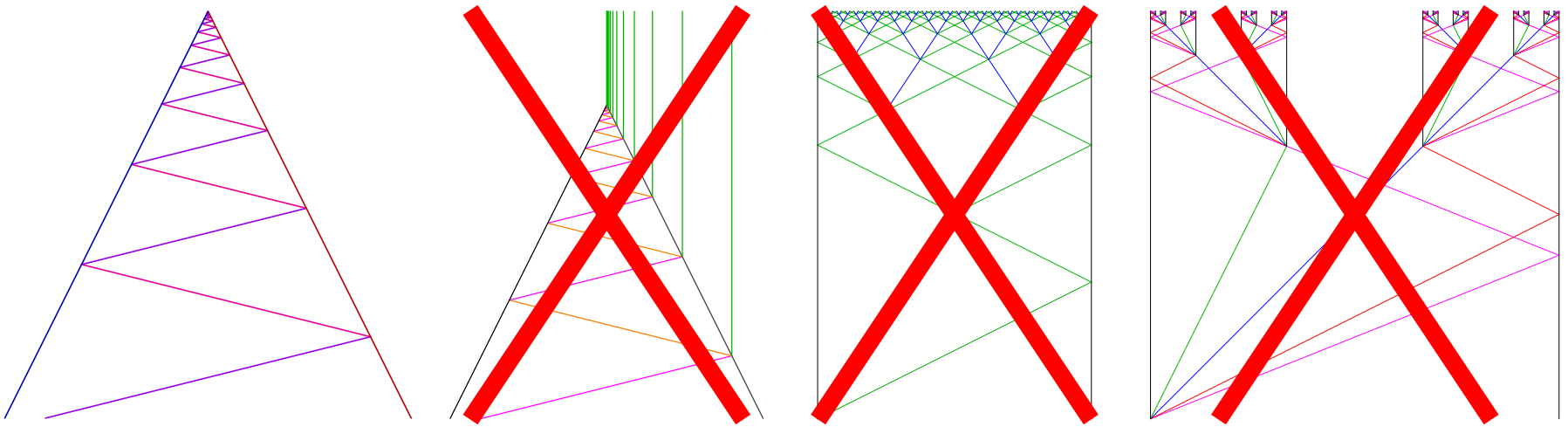
Back to the space-time diagrams

OK

Energy is lost in the accumulation

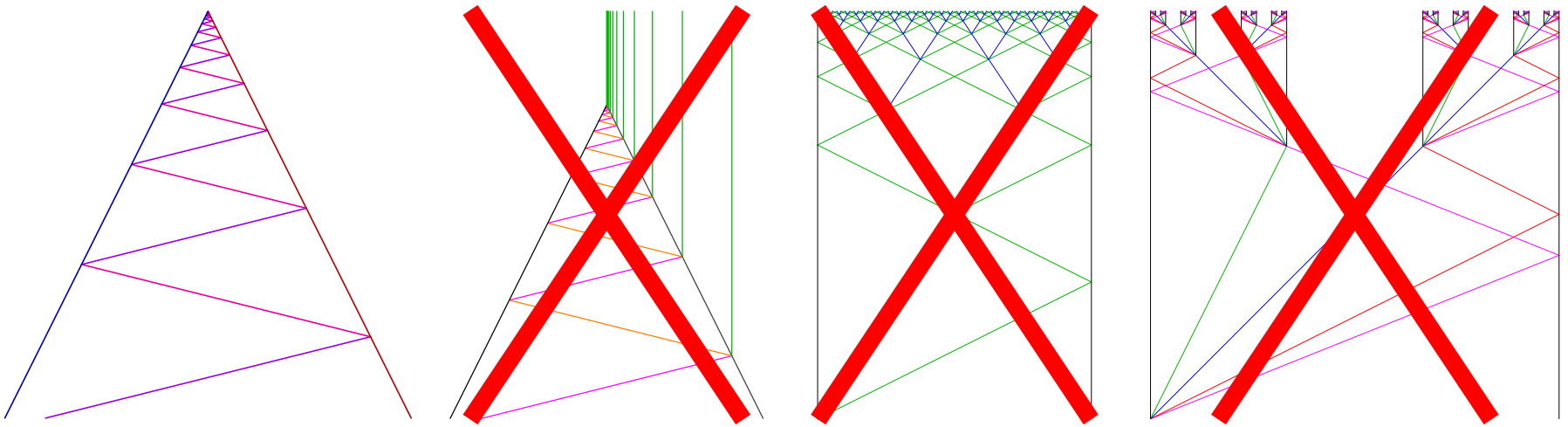
Back to the space-time diagrams



OK

Energy is lost in the accumulation

Back to the space-time diagrams



OK

Energy is lost in the accumulation

Can a Turing-computation be carried out with such a restriction?

2-counter automata (or 2-register machine)

```
 beg: B++
      A--
      IF A !=0 beg1
      IF B !=0 imp
beg1: A--
      IF A !=0 beg
 pair: B--
      A++
      IF B !=0 pair
      IF A !=0 beg
 imp: B--
      A++
      A++
      IF B !=0 imp1
      IF A !=0 beg
imp1: B--
      A++
      A++
      A++
      IF B !=0 imp1
      IF A !=0 beg
```

Turing-universal

A, B counters (values in $\mathbb{N}$)

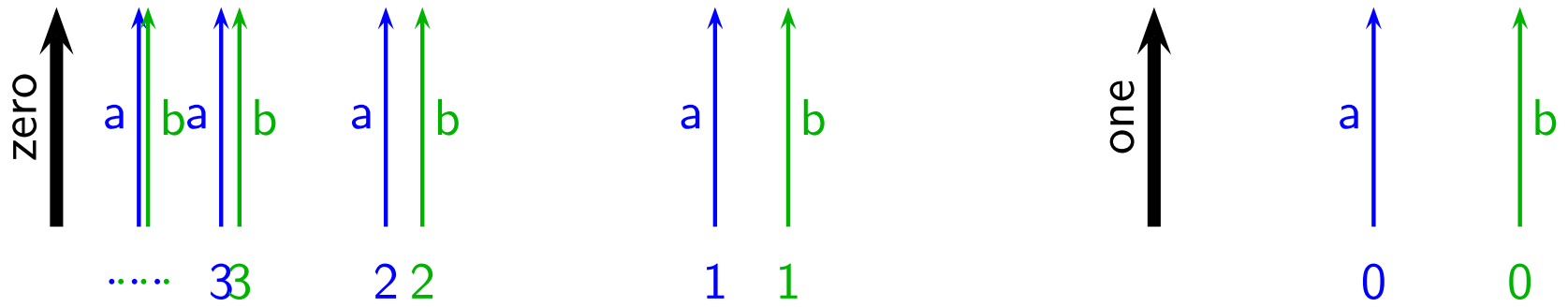
Operations

A++	B++
A--	B--
A !=0 m	B !=0 m

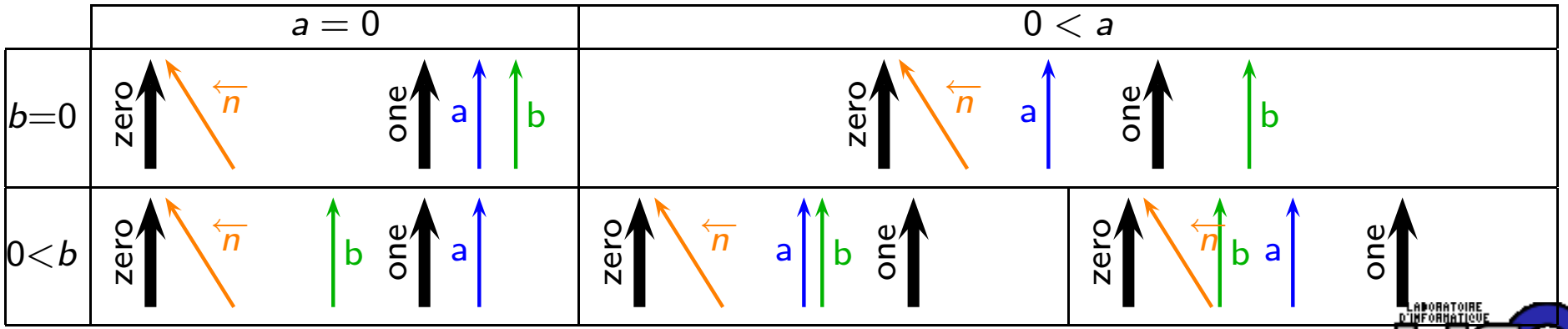
a configuration $\rightsquigarrow (n, a, b)$

Encoding (n, a, b) into a space-time diagram

Position encoding of a and b

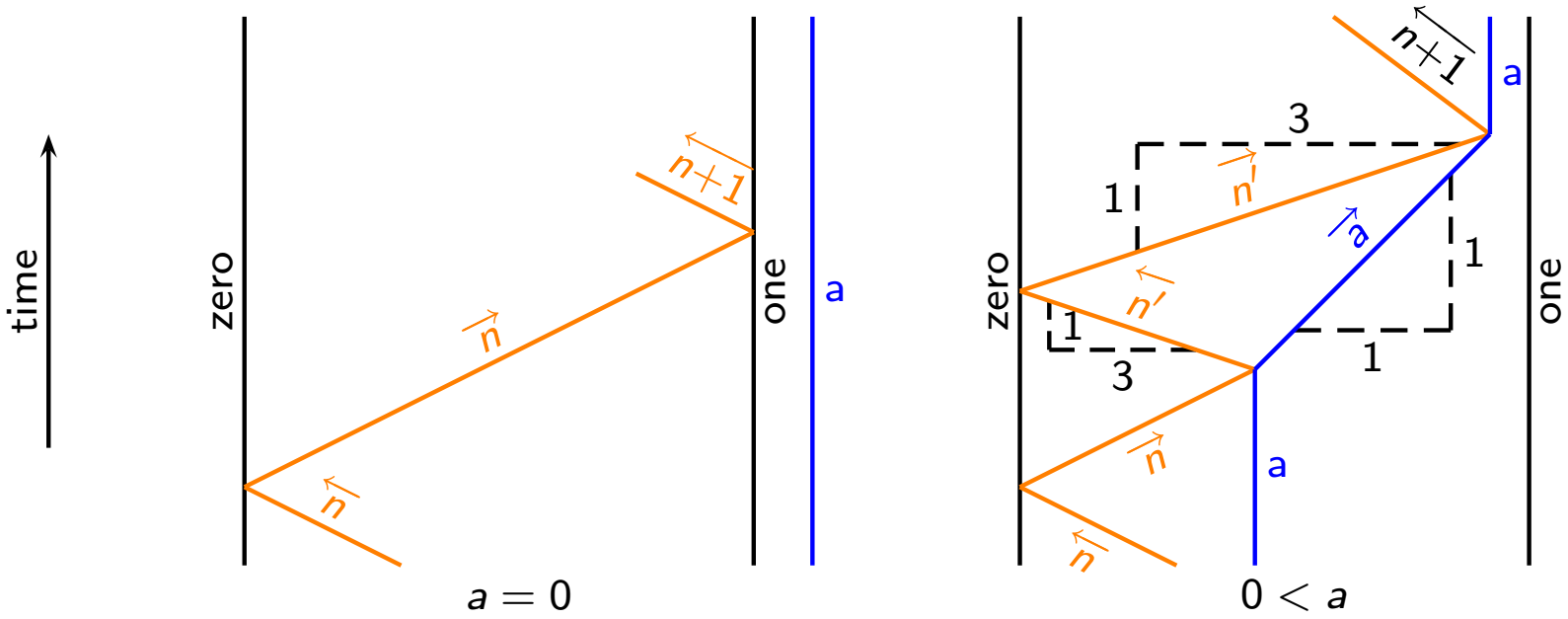


Encoding of configurations



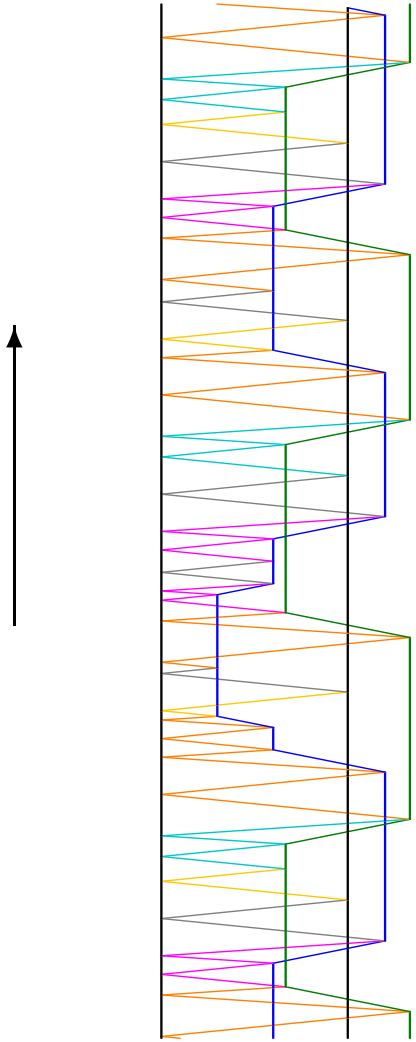
A set of signals for each line of instruction

Implementing "n A−−"

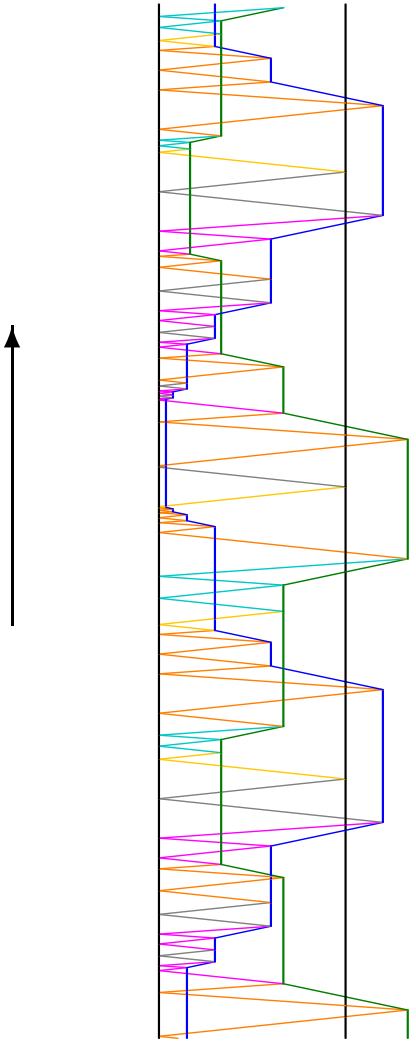


Other instructions are implemented similarly

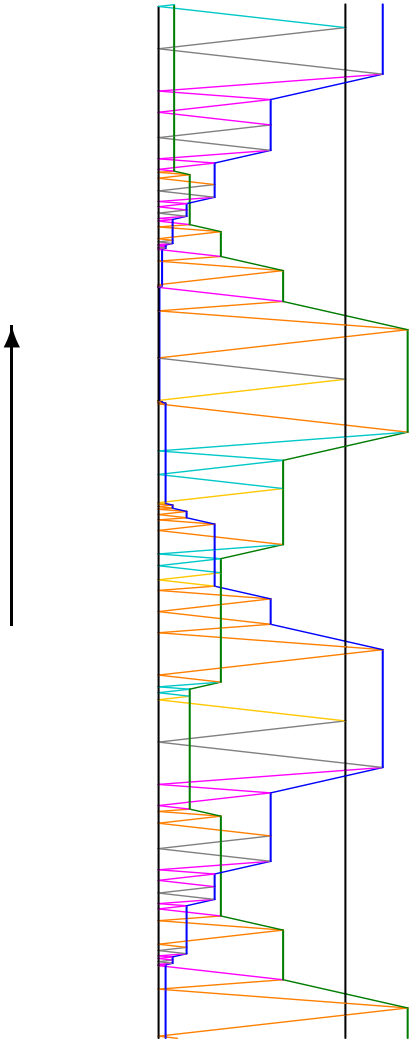
Examples



$a=1 \quad b=0$



$a=3 \quad b=0$



$a=5 \quad b=0$

Restriction and halting

Restriction is always satisfied but...

what about halting?

The instruction turns into a `yes/no` signal leaving on the left

Theorem

(Energy preserving) signal machines can simulate

any 2-counter automaton

Theorem

(Energy preserving) signal machines can carry out

any Turing computation

Turing-universal model of computation

Theorem

(Energy preserving) signal machines can simulate

any 2-counter automaton

Theorem

(Energy preserving) signal machines can carry out

any Turing computation

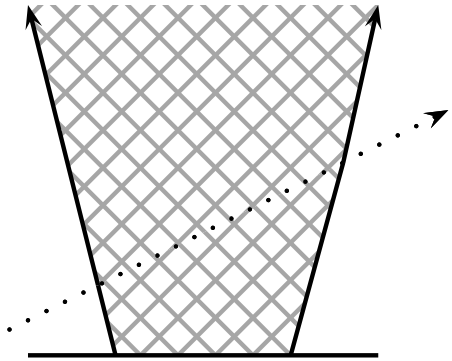
Turing-universal model of computation

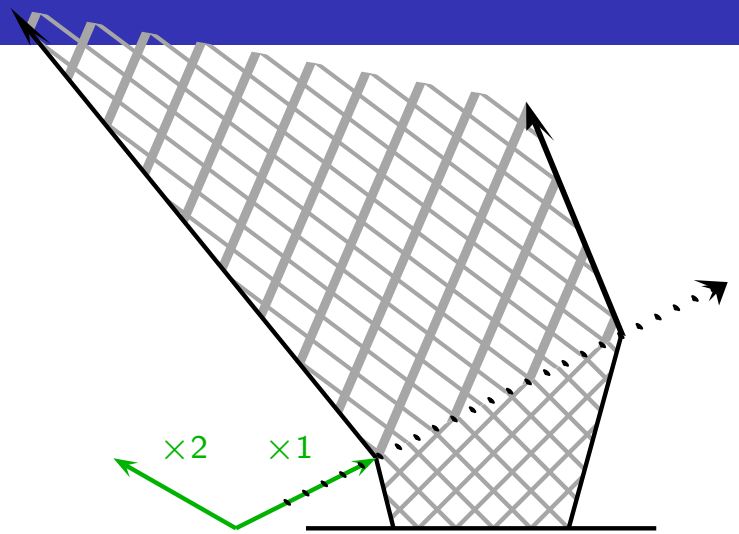
All is done with *rational* positions

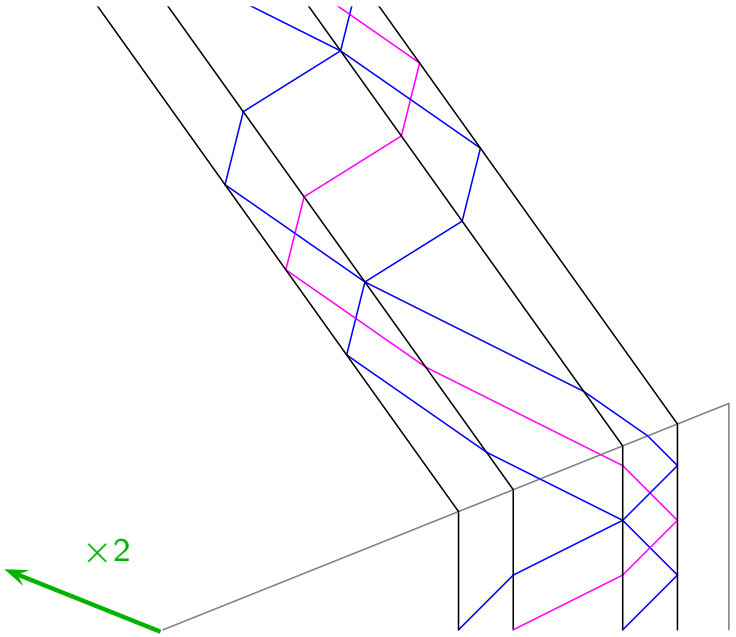
$\rightsquigarrow$ manipulable by classical Turing machines

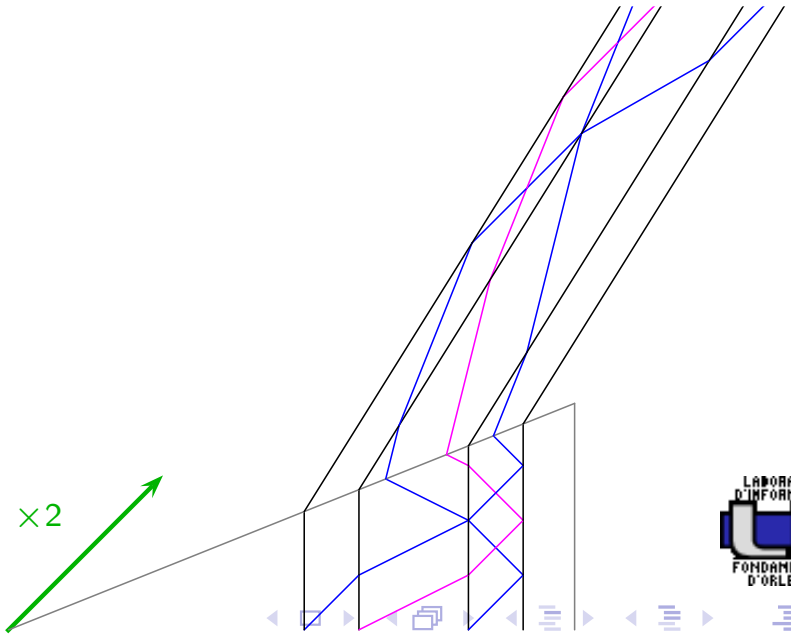
How to embed this into a Zeno artifact?

Providing a strain





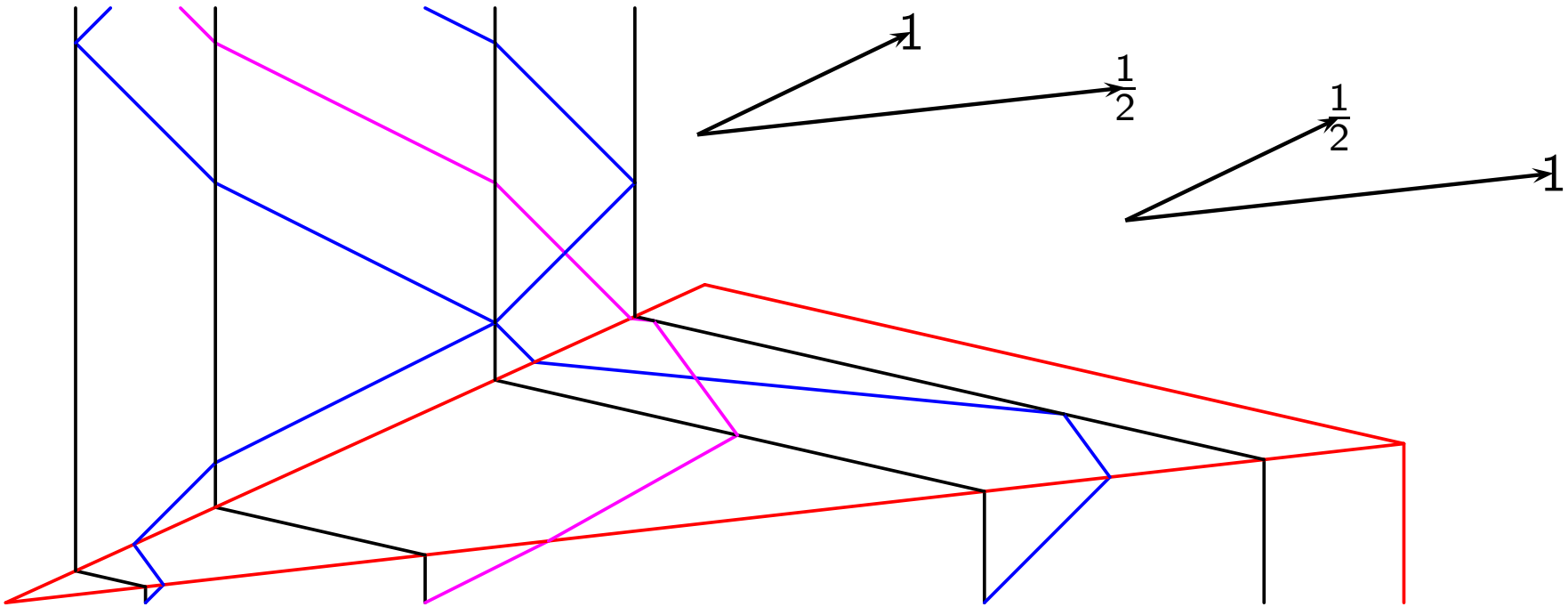




Providing a shrinking

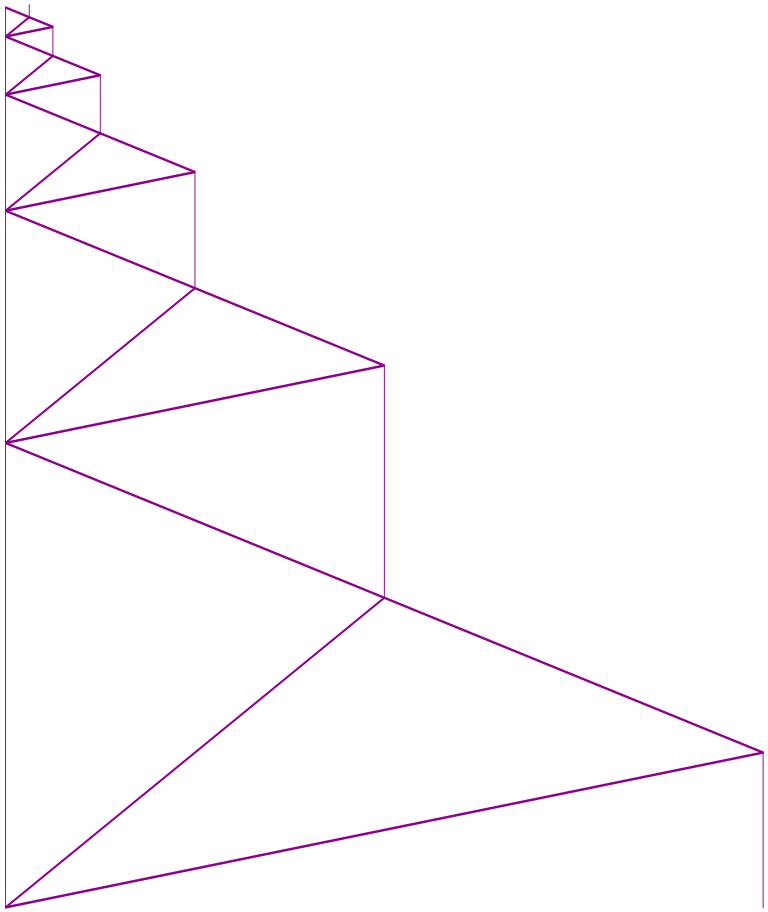
Two consecutive strains with the same directions

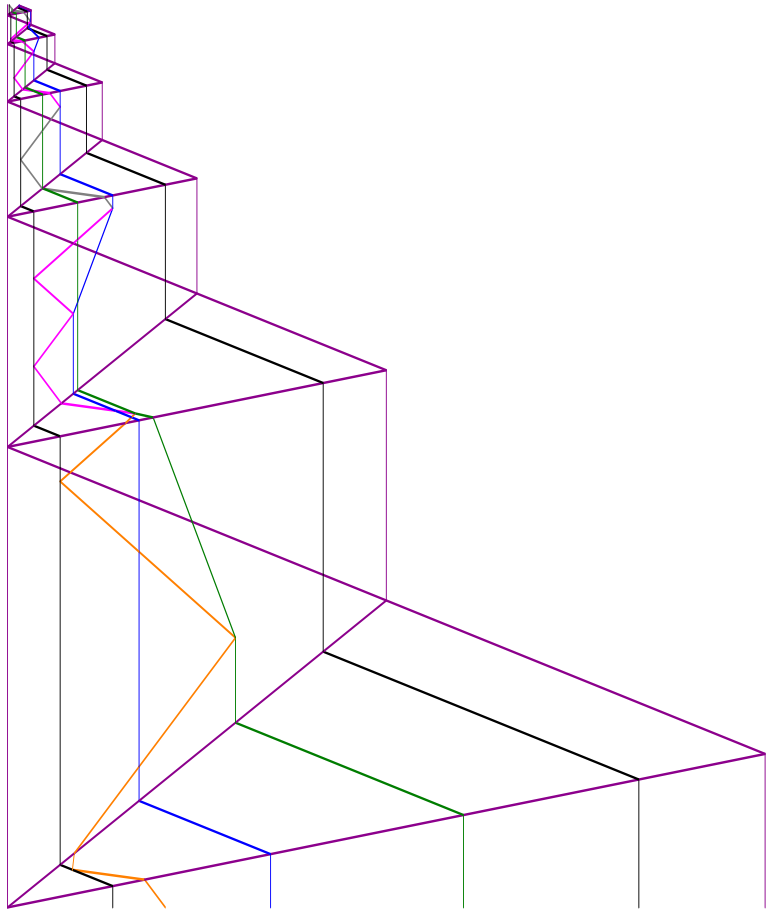
coefficient 1/2 on one direction then the other



Iterating possible if spatially bounded

Iterating shrinking

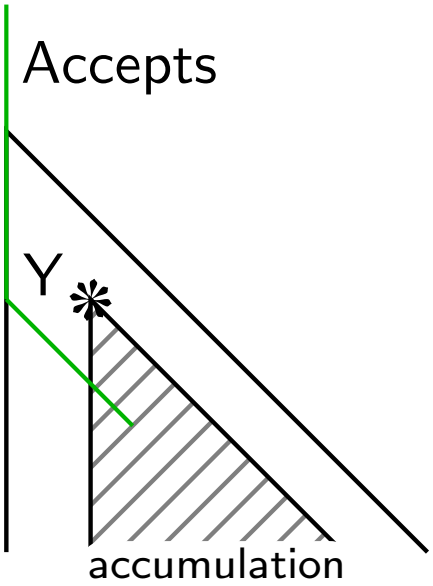


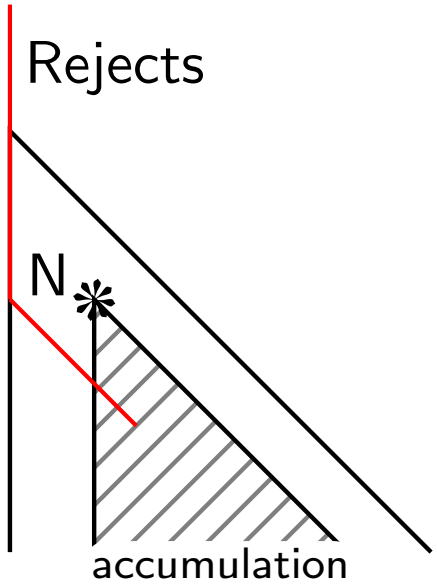


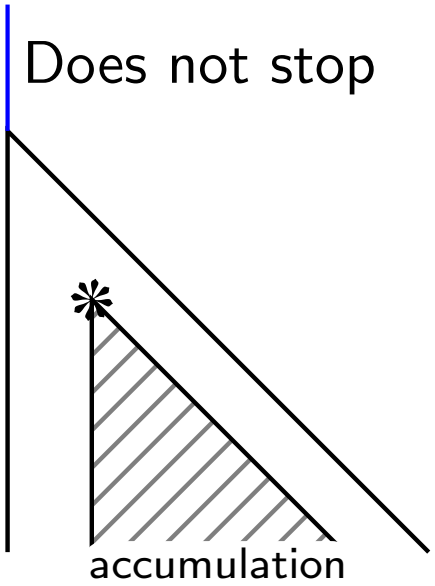
(for a spatially bounded computation)

Bounding delay

Simulation & iterated shrinking construction satisfy the restriction







Bounding signals indicate when it is too late to get any answer

Results

- New model of computation
- Turing computation power
- Computation on the continuum (space and time)
- Geometric model where geometric constructions allow Zeno effects
- Similarity with the Black hole model
- Rational numbers are enough to get all this (*i.e.* distinction lies in continuity and not in cardinality)

Work in progress

- Second (and higher) accumulation
  - infinite amount of energy at start
  - lifting the restriction (hierarchy climbing)
- Super Turing-computability
  - through real positions
- Analog computation
  - through real positions