La perspective du signal:
des automates cellulaires aux machines à signaux

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1 Introduction

2 Implicit use of signals

3 Discrete signals

4 Signal Machines

5 Conclusion
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Cellular Automata

Definition
- $Q$: finite set of states
- $f: Q^k \rightarrow Q$ local function

Dynamical system
Global function, $G : Q^\mathbb{Z} \rightarrow Q^\mathbb{Z}$

Orbit and space-time diagram
Value in $Q^{\mathbb{Z} \times \mathbb{N}}$
Image with big pixels

$Q = \{0, 1, 2, 3\}$

$f(x, y, z) = 3x + 2y + z + xy \mod 4$
Background and Signals

Background

(2-d) Pattern that may form a valid space-time diagram by bi-periodic repetition.

Signal

- Pattern that (legally) repeats 1-periodically on a background
- Pattern repeating 1-periodically and separating two backgrounds

Illustration by examples
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Understanding the dynamics

**FIG. 7.** Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6. [Boccara et al., 1991, Fig. 7]

**FIG. 7.** The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction. [Hordijk et al., 2001, Fig. 7]

Figure 5. Two collisions of filions, and five free filions supported by the FPS model; ST diagram applies if $\eta = 1$. [Siwak, 2001, Fig. 5]
Generating prime numbers

[Fischer, 1965, Fig. 2]
Implicit use of signals

Computing by simulating a Turing machine

Figure 4: The $k = 4, r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, ω, and + are represented by different colors.

[Lindgren and Nordahl, 1990, Fig. 4]
Firing Squad Synchronization

[Goto, 1966, Fig. 3+6]
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Firing Squad Synchronization (again)

[Varshavsky et al., 1970, Fig 1 and 3]
**Multiplication**

- strongest digit
- weakest digit

**end of the words**

- i-th digit of the multiplier
- i-th carry over of the partial sum
- i-th digit of the multiplicand
- i-th digit of the multiplier

The last partial sum is the result

Figure 1: A human multiplication.

Figure 3: Computations done on one cell out of two, one unit of time out of two.

[Figure 4: Multiplying 11001 by 10110.]

[Mazoyer, 1996, Fig. 1, 3 and 4]
A whole programming system

Figure 8: Computing $(ab)^2$.

Figure 9: Setting up an infinite family of regular safe grids (the darkness of the grid indicates its rank).

Figure 18: Characterization of the sites $(n, f(n))$.

[Mazoyer, 1996, Fig. 8 and 19] and [Mazoyer and Terrier, 1999, Fig. 18]
Introduction

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Conclusion
Moving to the continuum

Forget about discreteness

⇝ continuous
Signal Machines

Time ($\mathbb{N}$)
Space ($\mathbb{Z}$)

Time ($\mathbb{R}^+$)
Space ($\mathbb{R}$)

Vocabulary
- Signal (meta-signal)
- Collision (rule)
New kinds of *monsters*
Computability and undecidability [Durand-Lose, 2005]

Two-counter simulation
Turing-machine can also be simulated directly

Undecidable
- total erasing
- finite number of signal
- signal/collision apparition
Scaling down and bounding the duration
Computing inside bounded room
Accumulation forecasting is $\Sigma^2_0$-complete

[Durand-Lose, 2006b]
Link with the Black hole model [Durand-Lose, 2006a]

**Principe**

Two different timelike half-curves such that
- they have a point in common (used to set things and start)
- one is upward-infinite and fully contained in the casual past of a point of the other

**Solving recursively enumerable problems**

- **Accept**
  - Calcul
- **Refuse**
  - Calcul
- **Does not stop**
  - Calcul
Links with the Blum, Shub and Smale model

**Classical BSS model**

Variables hold real numbers in exact precision
- input / output
- test $0 < x$
- shift (to access other variables)
- compute a polynomial function

**Linear BSS [Durand-Lose, 2007]**

Restriction
- only linear function
- *i.e.* no inner multiplication
Encoding real numbers

- Common scale for all variables
- Sign test trivial
Encoding real numbers

scale + distance

- Common scale for all variables
- Sign test trivial
Encoding real numbers

- Common scale for all variables
- Sign test trivial
Copy and Addition
External multiplication

\[ \text{line}_{n+1} \]

\[ \text{accum} \]

\[ \text{val} \]

\[ \text{mul} \]

\[ \text{mul}^+ \]

\[ \text{mul}^+_a \]

\[ \text{mul}^+_b \]

\[ \text{mul}^+_c \]
Internal multiplication [Durand-Lose, 2008]

Computation

- Pre-treatment to ensure $0 < y < 1$
- Binary extension of $y$:
  \[ y = y_0 \cdot y_1 y_2 y_3 \cdots \]
- Computation
  \[ xy = \sum_{0 \leq i} y_i \left( \frac{x}{2^i} \right) \]

Principe

Computation on the margin
the margin is scaling down geometrically

Square rooting is also possible!
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Conclusion

- Natural filiation with CA
- Continuous time
  - Zeno effect

Links with other models

- Black hole model
- Blum, Shub and Smale model

Future work

- Relate with CA
- Characterize the analog computing power