Abstract geometrical computation: Turing-computing ability and undecidability

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Starting from discrete model...

Cellular automata

- $Q^{\mathbb{Z}}$
  bi-infinite line of states
  $|Q| < \infty$
- continuous dynamical system over $Q^{\mathbb{Z}}$
- local, parallel, synchronous, uniform } updating of $Q^{\mathbb{Z}}$

. . . with discrete space-time diagrams

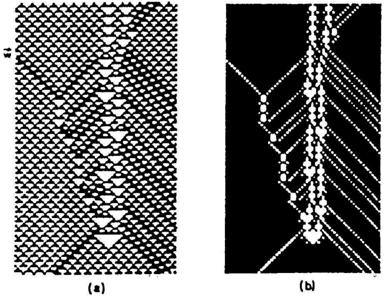


FIG. 7. Rule 54. (a) Annihilation of the radiating particle. (b) The same as (a) with the mapping defined in Fig. 6.

[BNR91, Fig. 7]

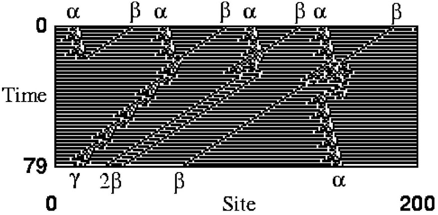


FIG. 7. The four different (out of 14 possible) interaction products for the $\alpha + \beta$ interaction.

[HSC01, Fig. 7]

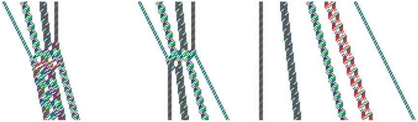


Figure 5. Two collisions of filtrons, and five free filtrons supported by the FPS model; ST diagram applies $q = 1$.

[Siw01, Fig. 5]

. . . with discrete space-time diagrams

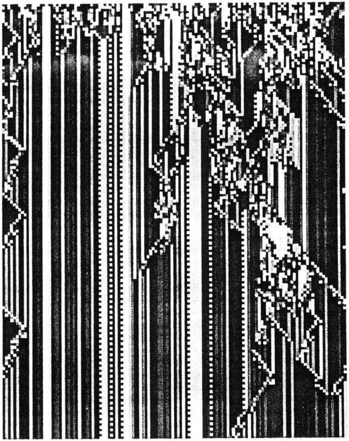


Figure 3: A simulation of the $k = 7$, $r = 1$ universal CA of table 3 for an uncorrelated initial state (with a density of blanks equal to 0.76). Symbols y, 0, 1, A, B, ␣, and T are represented by

[LN90, Fig. 4]

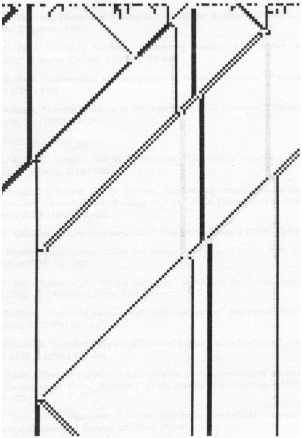


Figure 4: The $k = 4$, $r = 2$ universal cellular automaton of table 4 simulated starting from a random initial state. The symbols 0, 1, ␣, and + are represented by

[LN90, Fig. 3]

. . . with discrete space-time diagrams

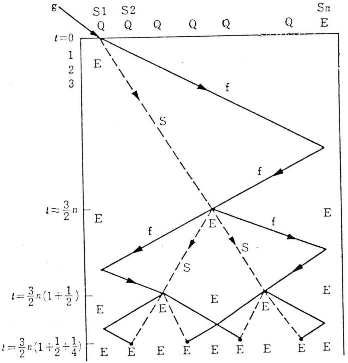
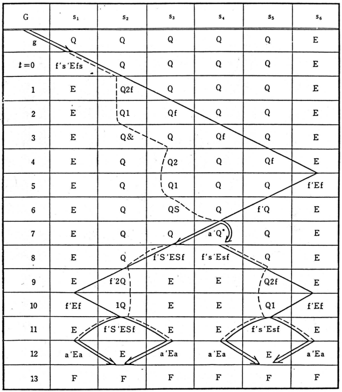


図 3·5 一斉射撃の問題（連続近似）

[Got66, Fig. 3]

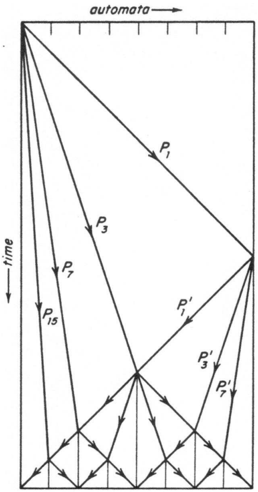


G	s_1	s_2	s_3	s_4	s_5	s_6
g	Q	Q	Q	Q	Q	E
t=0	f's'Efs	Q	Q	Q	Q	E
1	E	Q2f	Q	Q	Q	E
2	E	Q1	Qf	Q	Q	E
3	E	Q&	Q	Qf	Q	E
4	E	Q	Q2	Q	Qf	E
5	E	Q	Q1	Q	Q	f'Ef
6	E	Q	QS	Q	f'Q	E
7	E	Q	Q	a'Q'	Q	E
8	E	Q	f'S'ESf	f's'Esf	Q	E
9	E	f'2Q	E	E	Q2f	E
10	f'Ef	1Q	E	E	Q1	f'Ef
11	E	f'S'ESf	E	E	f's'Esf	E
12	a'Ea	E	a'Ea	a'Ea	E	a'Ea
13	F	F	F	F	F	F

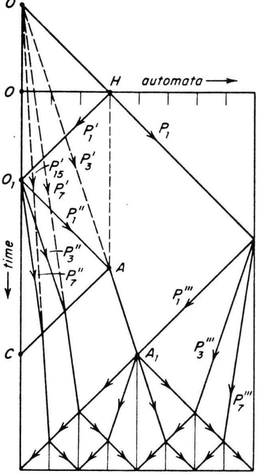
図 3·6 一斉射撃解 ($n=6$)

[Got66, Fig. 6]

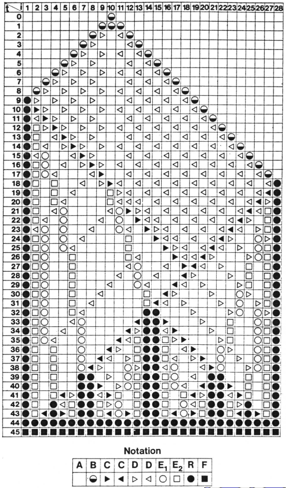
. . . with discrete space-time diagrams



[VMP70, Fig. 1]



[VMP70, Fig. 2]



[VMP70, Fig. 3]

. . . with discrete space-time diagrams

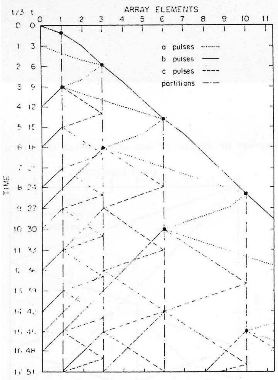


Fig. 2. Solution to the prime problem

[Fis65, Fig. 2]

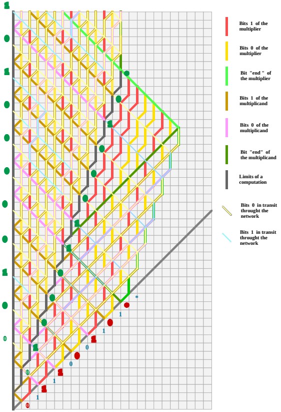


Figure 8: Computing $(ab)^2$.

[Maz96, Fig. 8]

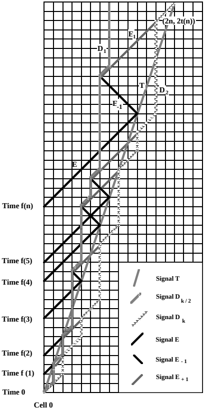
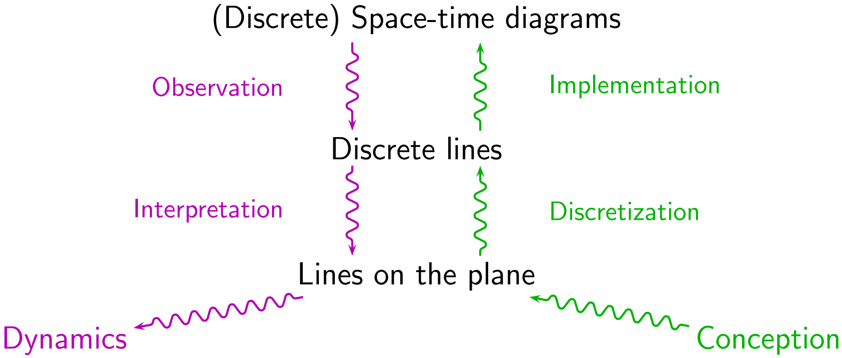
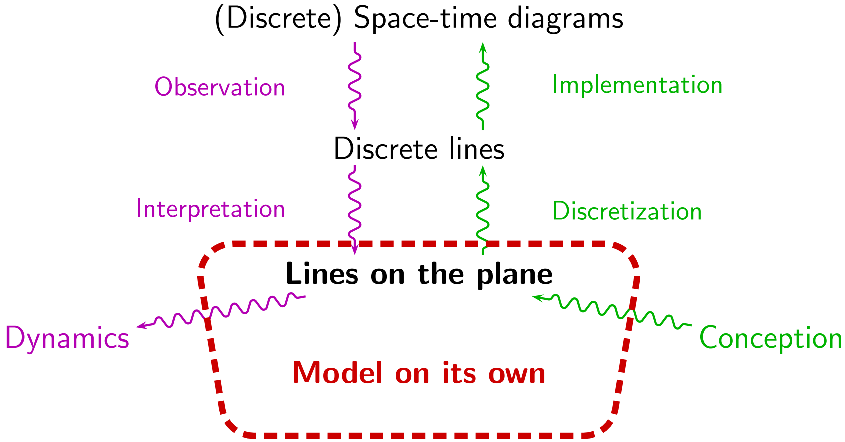


Figure 18: Characterization of the sites $(n, f(n))$.

[MT99, Fig. 18]


(Discrete) Space-time diagrams
Observation
Implementation
Discrete lines
Interpretation
Discretization
Lines on the plane
Dynamics
Conception


(Discrete) Space-time diagrams
Observation
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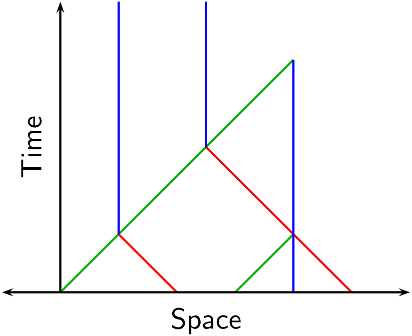
Conception
Model on its own

Continuous space-time,

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Continuous space-time, signals

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)

Signal

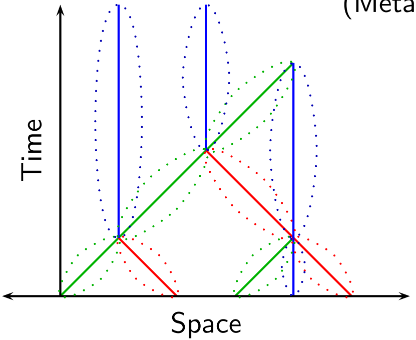
(Meta-signal, position)

Position

(x, t)

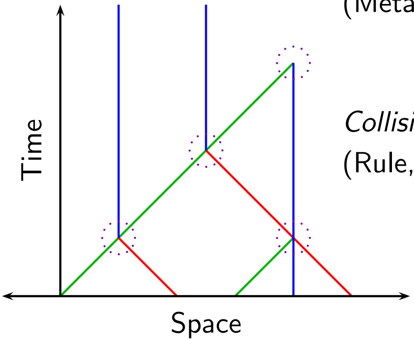
Meta-signal

$\mu = (\iota, \nu)$



Continuous space-time, signals and collisions

~~$\mathbb{Z} \times \mathbb{N}$~~ $\mathbb{R} \times \mathbb{R}^+$ (or $\mathbb{Q} \times \mathbb{Q}^+$)



Signal

(Meta-signal, position)

Position

(x, t)

Meta-signal

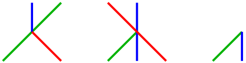
$\mu = (\iota, \nu)$

Collision

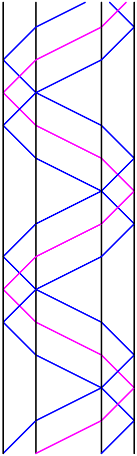
(Rule, position)

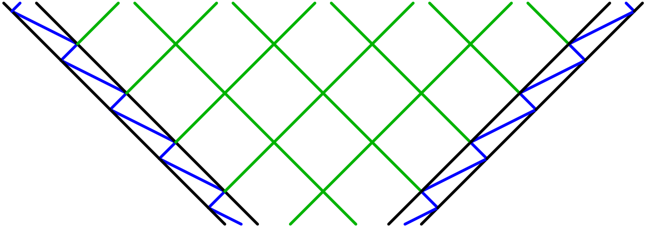
Rule

$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$

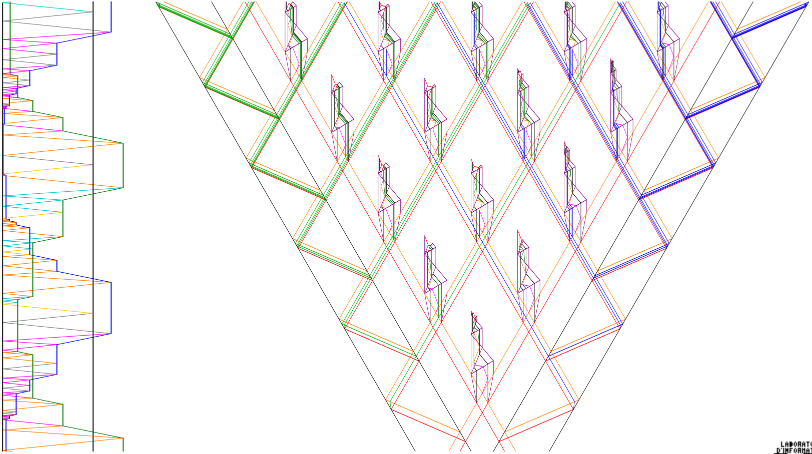


Continuous space-time diagrams





Continuous space-time diagrams



2-counter automata (or 2-register machine)

```
 beg: B++
      A--
      A != 0  beg1
      B != 0  imp
beg1: A--
      A != 0  beg
pair: B--
      A++
      B != 0  pair
      A != 0  beg
 imp: B--
      A++
      A++
      B != 0  imp1
      A != 0  beg
imp1: B--
      A++
      A++
      A++
      B != 0  imp1
      A != 0  beg
```

Turing-universal

A, B counters (values in $\mathbb{N}$)

Operations

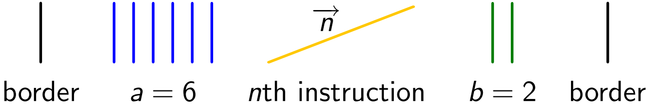
A++	B++
A--	B--
A != 0 m	B != 0 m

a configuration $\rightsquigarrow$ (n, a, b)

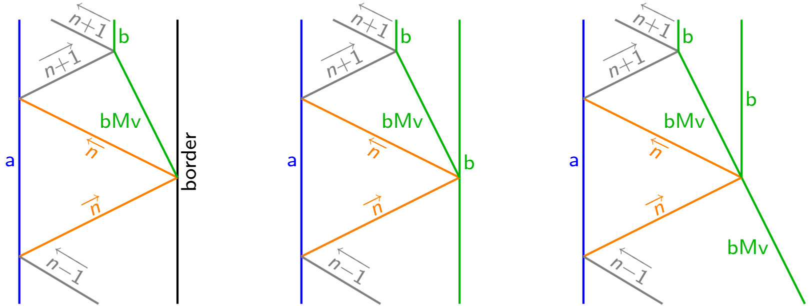
Encoding (n, a, b) into a space-time diagram

Unary encoding of a and b

A set of signals for each line of instruction

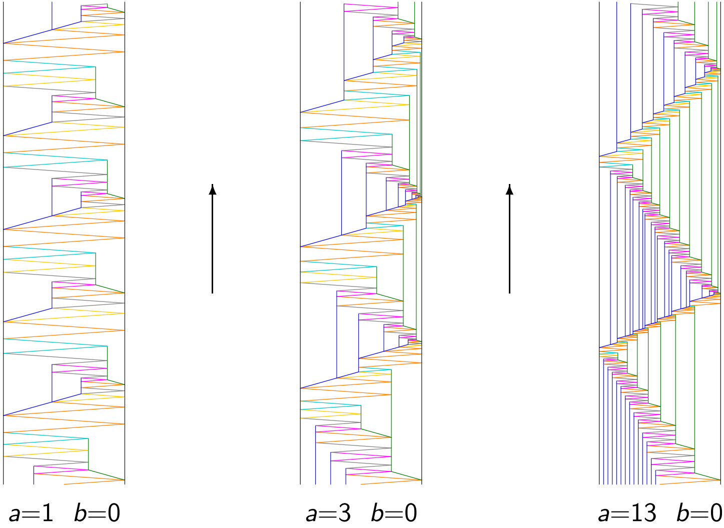


Implementing “n B++”



Other instructions are implemented similarly

Examples



Theorem

Signal machines can simulate any 2-counter automaton

Theorem

Signal machines can carry out any Turing computation

Turing-universal model of computation

Theorem

Signal machines can simulate any 2-counter automaton

Theorem

Signal machines can carry out any Turing computation

Turing-universal model of computation

All is done with *rational* positions

$\rightsquigarrow$ manipulable by classical Turing machines

Undecidable - 1

Instance **Finite number of collisions**
A rational signal machine, and an initial configuration

Question
Does the computation of the machine on the initial configuration stop?

Instance **Appearance of a given meta-signal**
A rational signal machine, an initial configuration, and a meta-signal

Question
Does the computation of the machine on the initial configuration ever generates a signal of this meta-signal?

Undecidable - 2

Instance **Collision with a given signal**
A rational signal machine, an initial configuration, and a signal in the initial configuration

Question
Is there any collision involving the given signal on the computation of the machine on the initial configuration?

Instance **Disappearance of all signals**
A rational signal machine, and an initial configuration

Question
Does the computation of the machine on the initial configuration stop on an empty configuration?

Results

- New model of computation
- Turing-universality
- Undecidable problems

Work in progress

- Super Turing-computability
  - through accumulation
- Super Turing-computability
  - through real positions
- Analog computation
  - through real positions