Abstract Geometrical Computation and the Linear Blum, Shub and Smale Model

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18-23 June 2007 – Siena, Italy

Context

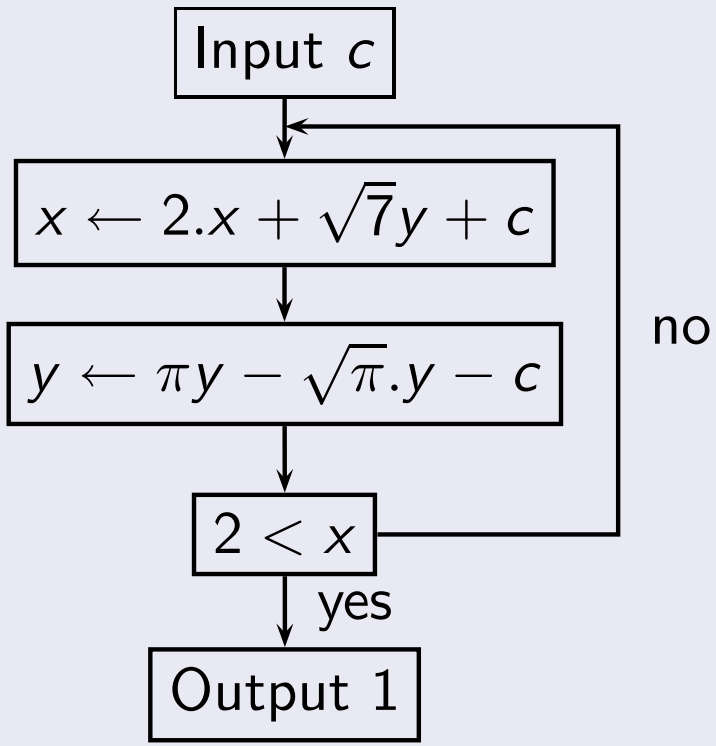
- Computation on the continuum
- Analog/continuous models

- No consensus on an *analog Turing thesis*
- Relating various models

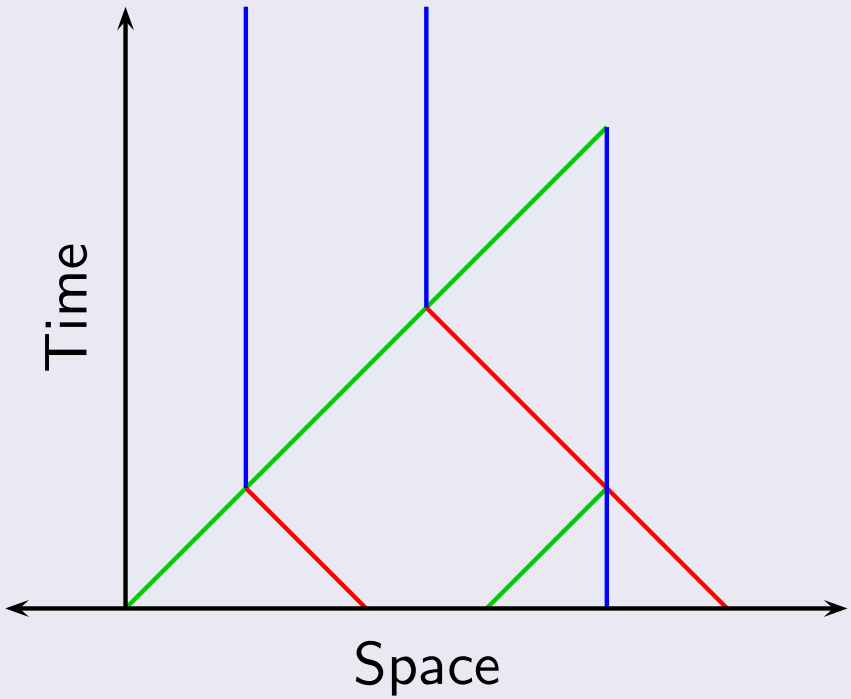
- Linear Blum, Shub and Smale model on $\mathbb{R}$ (lin-$\mathbb{R}$-BSS)
  [Blum, Shub, and Smale, 1989]
  [Blum, Cucker, Shub, and Smale, 1998]
- Abstract geometrical computation (AGC)
  [JDL: MCU 04, CiE 05]

Goal: to relate these

Lin-$\mathbb{R}$-BSS



Abstract Geometrical Computation



Definition: Linear-$\mathbb{R}$-BSS

- Variables hold real numbers
- Computing linear functions over the variables
- Branch with $0 \leq$ test

Definition: Linear-$\mathbb{R}$-BSS on unbounded sequences

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To handle unbounded number of variables

- Variables ordered in an infinite array
- `shift` operator

Definition: Linear-$\mathbb{R}$-BSS on unbounded sequences

- Variables hold real numbers
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To handle unbounded number of variables

- Variables ordered in an infinite array
- `shift` operator

Not the easiest to handle

- Switch for an equivalent model [Novak, 1995]
- $\rightsquigarrow$ *linear-$\mathbb{R}$-Unlimited Register Machines*

Definition: Linear-$\mathbb{R}$-Unlimited Register Machines

- An accumulator
- Infinite array of registers (holding real numbers)

Basic operations

- Done on the accumulator
- Store into / load from a register
- Addition of the value of a register
- Multiplication by a constant

Definition: Linear-$\mathbb{R}$-Unlimited Register Machines

- An accumulator
- Infinite array of registers (holding real numbers)
- Finitely many addresses (special registers)

Basic operations

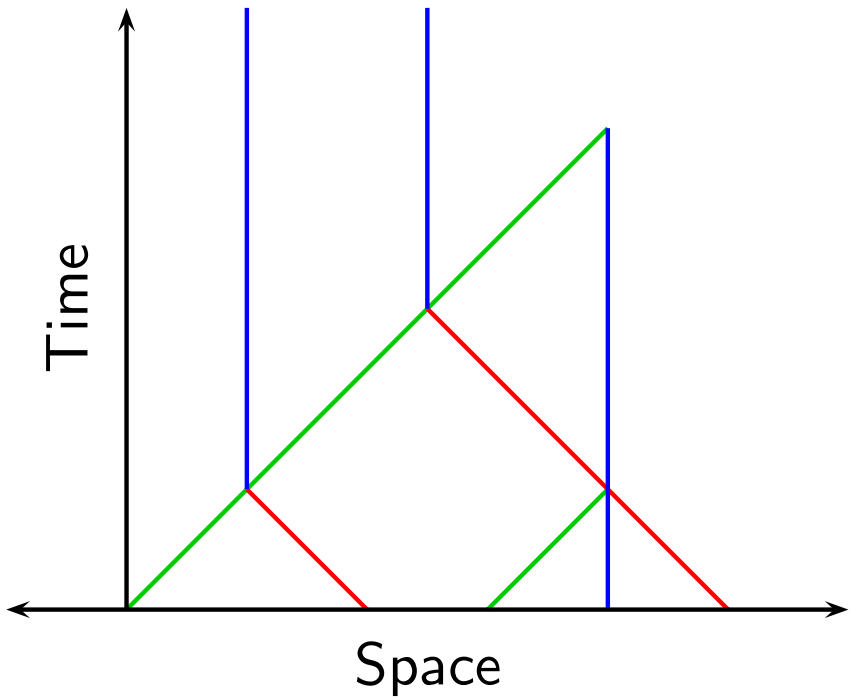
- Done on the accumulator
- Store into / load from a register
- Addition of the value of a register
- Multiplication by a constant

Indirect addressing

- Through address registers
- `Dec` and `Inc`

Definition: Abstract Geometrical Computation and Signal Machines

$$\mathbb{R} \times \mathbb{R}^+$$



Definition: Abstract Geometrical Computation and Signal Machines

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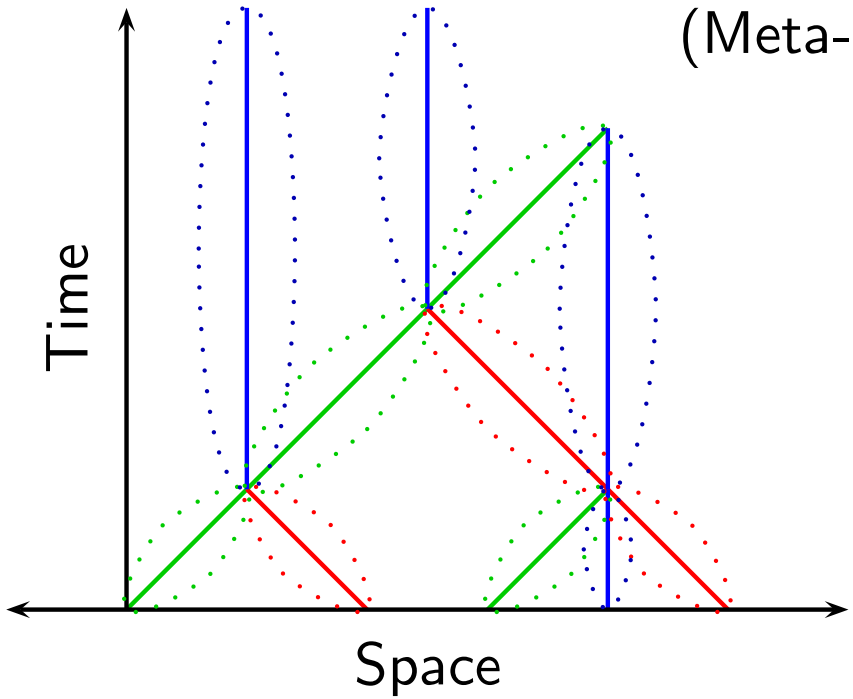


Signal

(Meta-signal, position)

Position

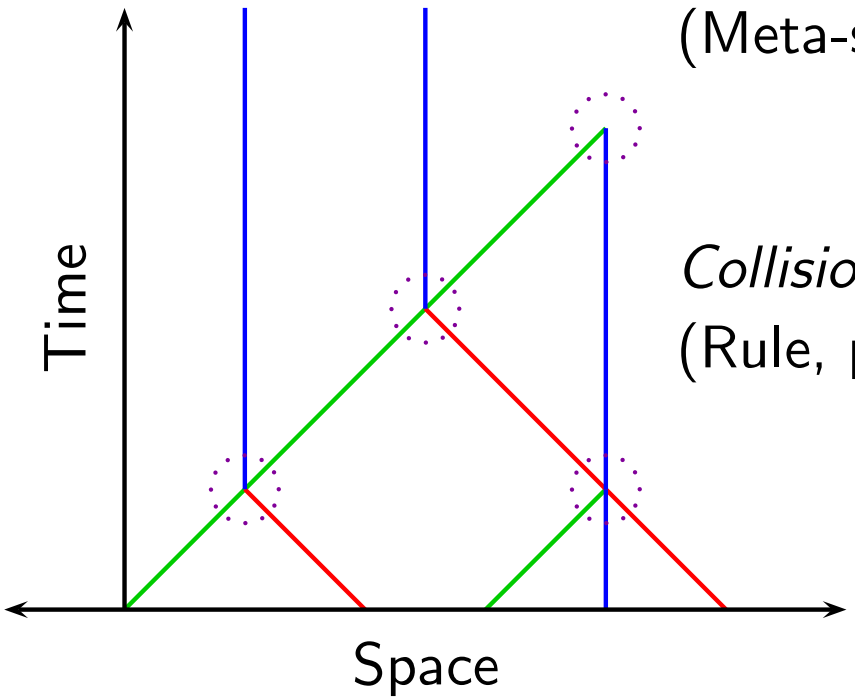
(x, t)

Meta-signal

$\mu = (\iota, \nu)$

Definition: Abstract Geometrical Computation and Signal Machines

$\mathbb{R} \times \mathbb{R}^+$



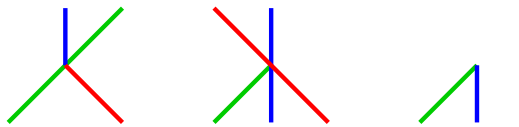
Signal
(Meta-signal, position)

Position
(x, t)

Meta-signal
$\mu = (\iota, \nu)$

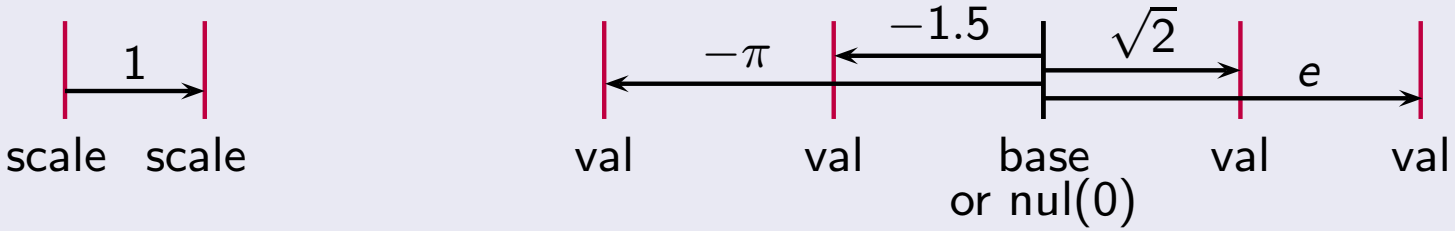
Collision
(Rule, position)

Rule
$\rho = \{\mu_i^-\}_i \to \{\mu_j^+\}_j$



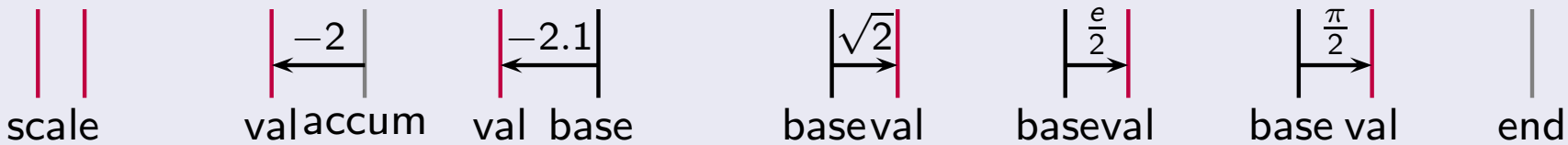
Encoding of Lin-$\mathbb{R}$-URM configuration - 1

Real values



Scale and positions of val for values $-\pi$, -1.5, 0, $\sqrt{2}$ and e

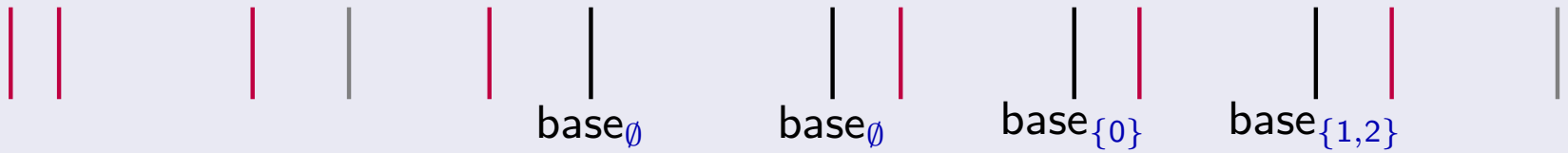
Accumulator and registers



Accumulator (-2) and registers $(-2.1, \sqrt{2}, \frac{e}{2}, \frac{\pi}{2})$

Encoding of Lin-$\mathbb{R}$-URM configuration - 2

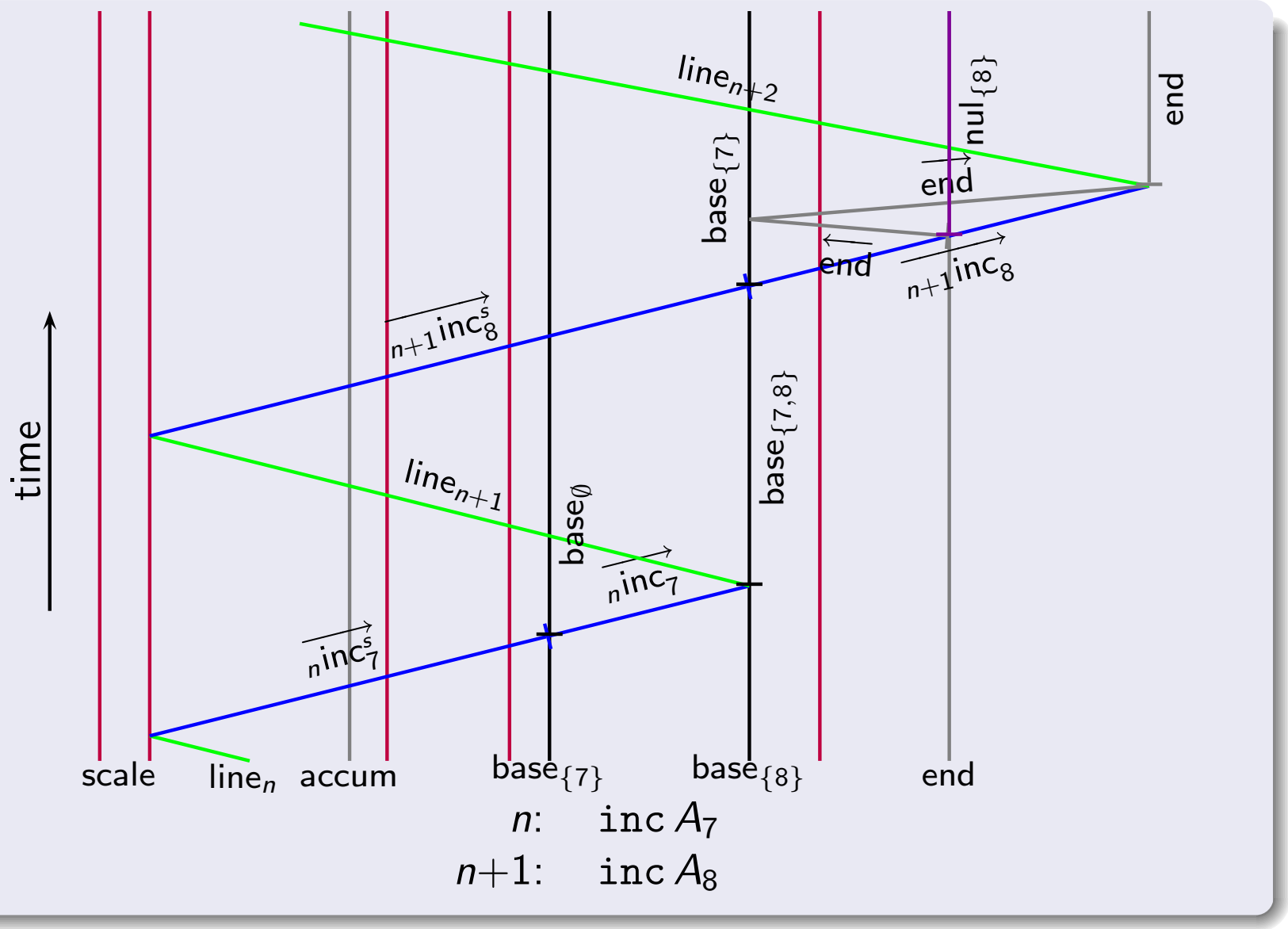
Addresses



$$A_0 = 2,\ A_1 = 3 \text{ and } A_2 = 3$$

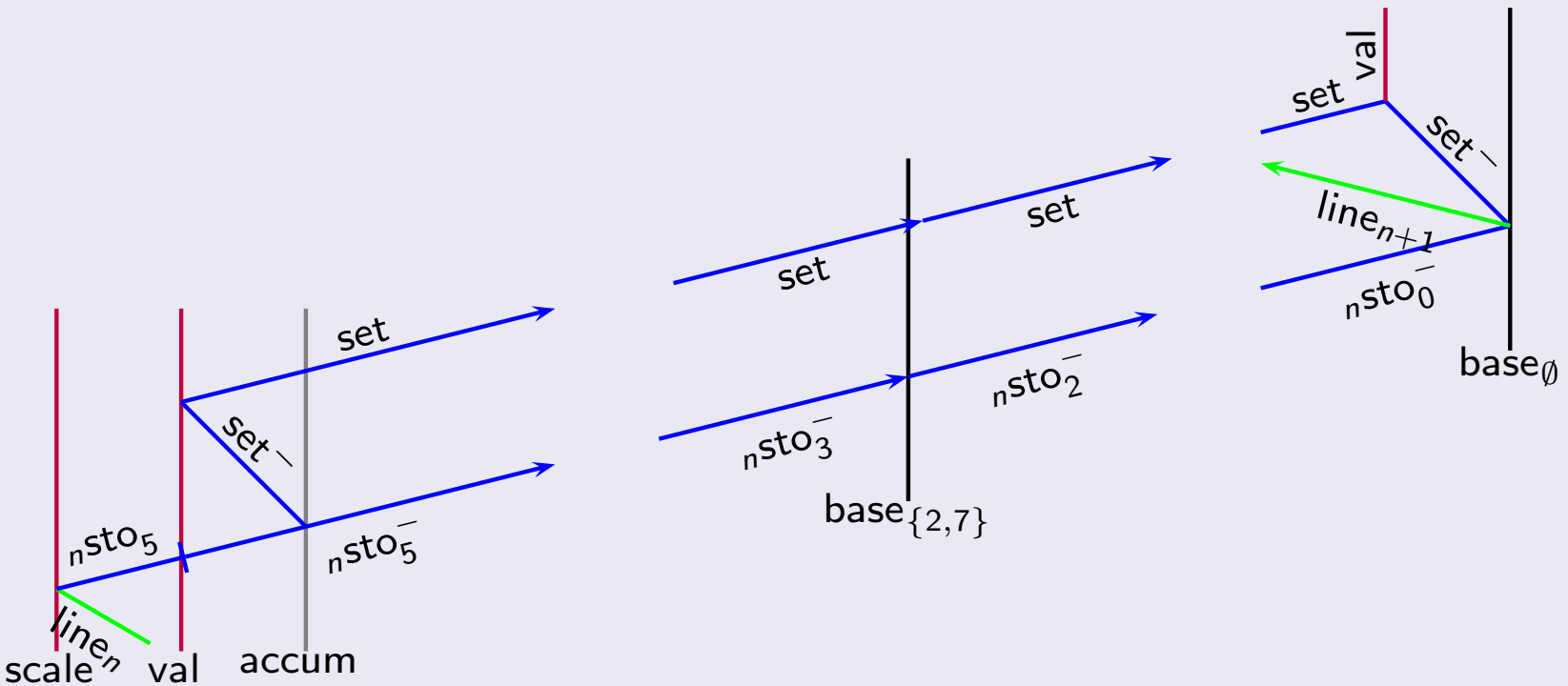
State / line number

Updating the addresses



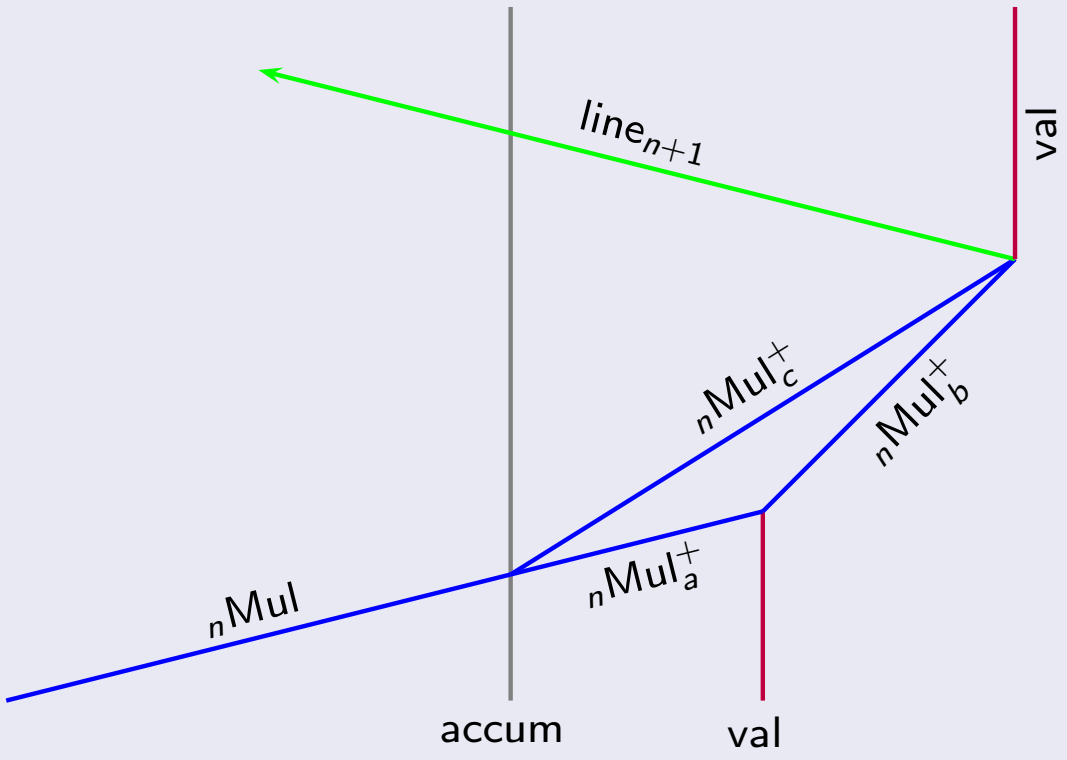
Load and store

Example of `store` R_5



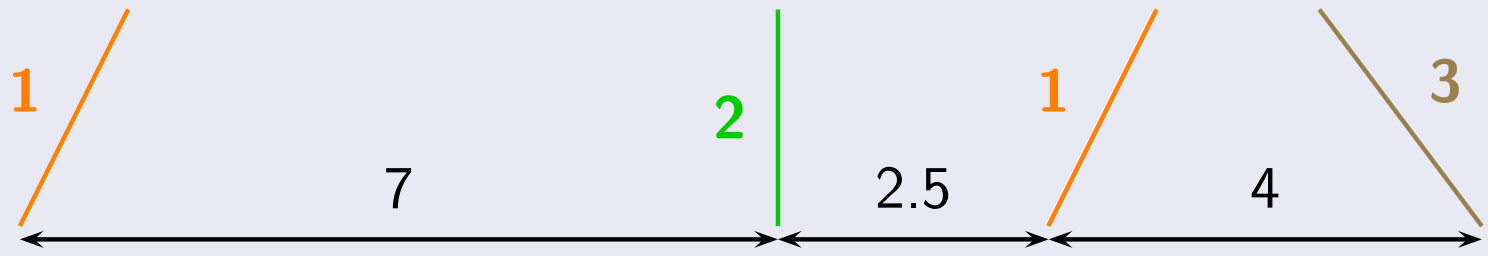
Multiplication

By 2



Encoding of Signal machine configurations

Alternatively signal id and distance to the next



1	7	2	2.5	1	4	3	-1	0	0	···

Updating

Main loop

- Find next collision date
  - Go through the configuration
  - Find minimal time to zero a distance
- Update the distances (Go through the configuration again)
- Treat the collision (Go through the configuration again)
  - Find maximal sequences of zero distances
  - Replace the signals and shift the rest if necessary

Linear URM because...

- Finite number of signals (and collision rules)
- $\rightsquigarrow$ Bounded number of signals involved in a collision
- $\rightsquigarrow$ Switch (nested `if`) to get to the right case
- Constant speeds $\rightsquigarrow$ Constants of the lin-$\mathbb{R}$-URM

Conclusion

Result: Equivalence

- Lin-$\mathbb{R}$-URM / Lin-$\mathbb{R}$-BSS
- Abstract geometrical computation / Signal machines

Future work: natural (extension) of the models

- Multiplication inside AGC for full $\mathbb{R}$-BSS
- BBM counterpart of accumulations in AGC (Zeno effect)