Abstract geometrical computation with accumulations: beyond the Blum, Shub and Smale model

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Context

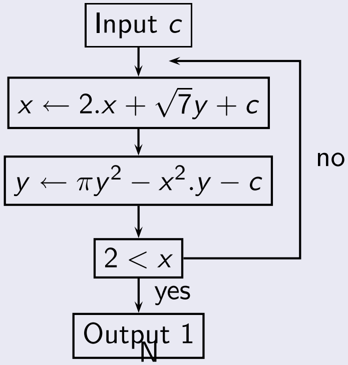
- Computation on the continuum
- Analog models

- No consensus on an *analog Turing thesis*
- Relating various models

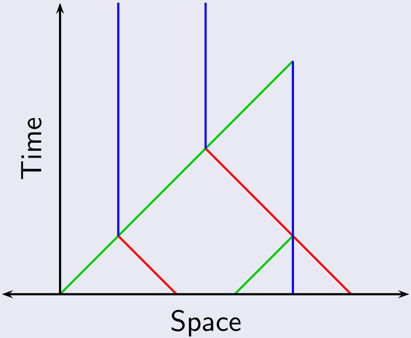
- Blum, Shub and Smale model on $\mathbb{R}$
  [Blum, Shub, and Smale, 1989]
  [Blum, Cucker, Shub, and Smale, 1998]
- Abstract geometrical computation (AGC)
  [JDL: MCU 04, CiE 05]

Goal: to relate these

$\mathbb{R}$-BSS



Abstract Geometrical Computation



Definition: $\mathbb{R}$-BSS

- Variables hold real numbers
- Computing polynomial functions over the variables
- Branch with $0 \leq$ test

Definition: $\mathbb{R}$-BSS on unbounded sequences

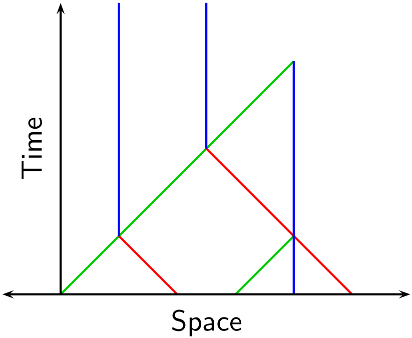
- Variables hold real numbers
- Computing polynomial functions over the variables
- Branch with $0 \leq$ test

To handle unbounded number of variables

- Variables ordered in an infinite array
- `shift` operator

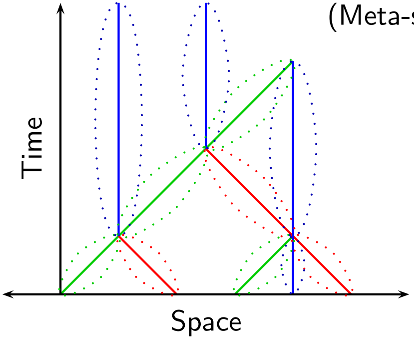
Definition: Abstract Geometrical Computation and Signal Machines

$$\mathbb{R} \times \mathbb{R}^+$$



Definition: Abstract Geometrical Computation and Signal Machines

$\mathbb{R} \times \mathbb{R}^+$



Signal

(Meta-signal, position)

Position

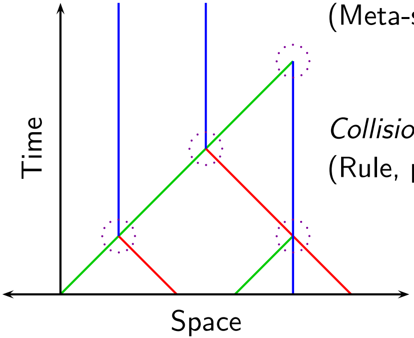
(x, t)

Meta-signal

$\mu = (\iota, \nu)$

Definition: Abstract Geometrical Computation and Signal Machines

$\mathbb{R} \times \mathbb{R}^+$



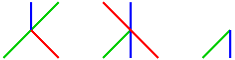
Signal
(Meta-signal, position)

Position
(x, t)

Meta-signal
$\mu = (\iota, \nu)$

Collision
(Rule, position)

Rule
$\rho = \{\mu_i^-\}_i \rightarrow \{\mu_j^+\}_j$



Previous results

[JDL CiE '07]

AGC (without accumulation)

$\Updownarrow$ *simulations in both way*

BSS without inner multiplication

Linear BSS

i.e. multiplication only by constant

CiE '08 Result

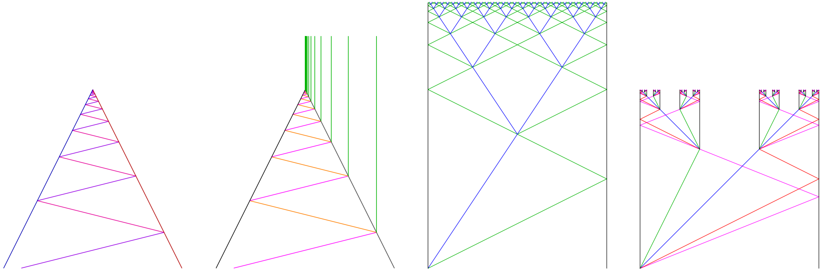
One step ahead

AGC with accumulations

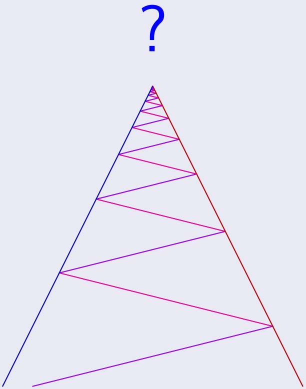
simulates

(full) BSS

What can they be?



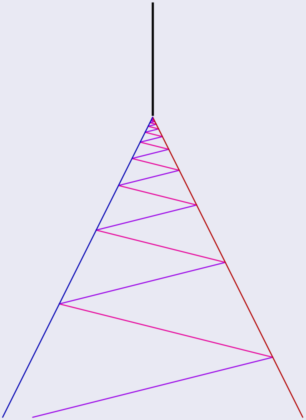
What is addressed?



Simple

Isolated

What is addressed?



Simple

Isolated

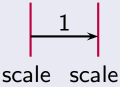
$\rightsquigarrow$ replaced by a signal

Principle

Data	$\longleftarrow$	same as in CiE '07 (recalled)
Operators	$\longleftarrow$	add, mul by cte same as in CiE '07 *inner multiplication*
Control	$\longleftarrow$	same as in CiE '07

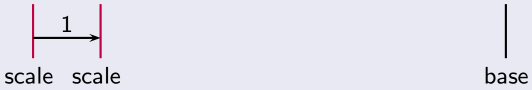
Real encoding

Continuous space with *no scale*



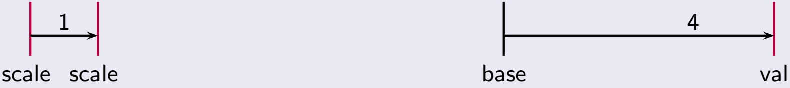
Real encoding

Continuous space with *no scale*



Real encoding

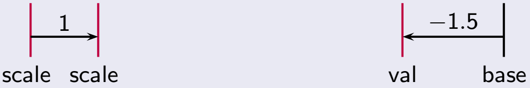
Continuous space with *no scale*



Value: 4

Real encoding

Continuous space with *no scale*

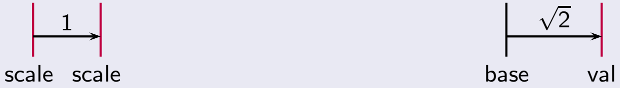


Value: -1.5

All real numbers can be exactly represented

Real encoding

Continuous space with *no scale*

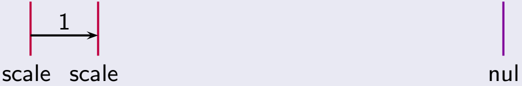


Value: $\sqrt{2}$

All real numbers can be exactly represented

Real encoding

Continuous space with *no scale*



Value: 0

All real numbers can be exactly represented

Multiplication scheme

$$0 < y < 1 \quad \begin{cases} y = 0.\mathrm{y}_1\mathrm{y}_2\mathrm{y}_3\ldots \\ xy = \sum_{0<i} \mathrm{y}_i \left(\frac{x}{2^i}\right) \end{cases}$$

$$1 \leq y \qquad yx = \left(x2^k\right)\left(\frac{y}{2^k}\right) \text{ with } 0 < \frac{y}{2^k} < 1$$

Zero and signs are treated easily

Recurrence

$$p_n = \sum_{0<i\leq n} \mathtt{y}_i \left(\frac{x}{2^i}\right)$$

$$x_n = \frac{x}{2^n}$$

$$y_n = 0.00\ldots 0\mathtt{y}_n\mathtt{y}_{n+1}\mathtt{y}_{n+2}\cdots$$

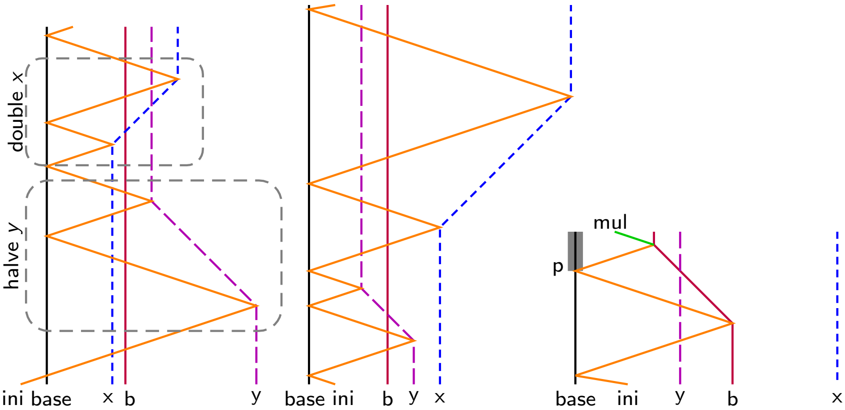
$$b_n = \frac{1}{2^n}$$

Next values computed with test, sum, and division by 2

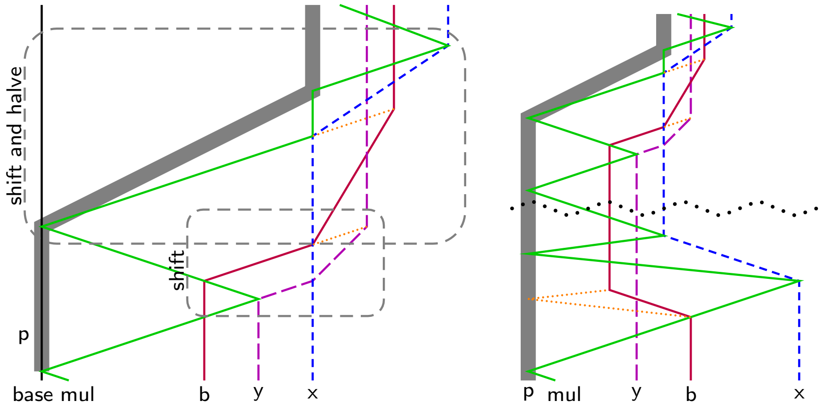
$p_n \longrightarrow xy$ so that accumulation should be on its limit

All other values $\longrightarrow 0$ $\rightsquigarrow$ accumulation is possible

Normalisation (to ensure $0 < y < 1$)



Infinite loop



Result and limit

AGC with accumulations can simulate the full BSS

With the same kind of construction $\sqrt{\ }$ can be computed

So that there is no hope of simulation in the other way

Future work

Identify which functions can be computed

Relate to other models of analog computation