Geometrical accumulations and computably enumerable real numbers

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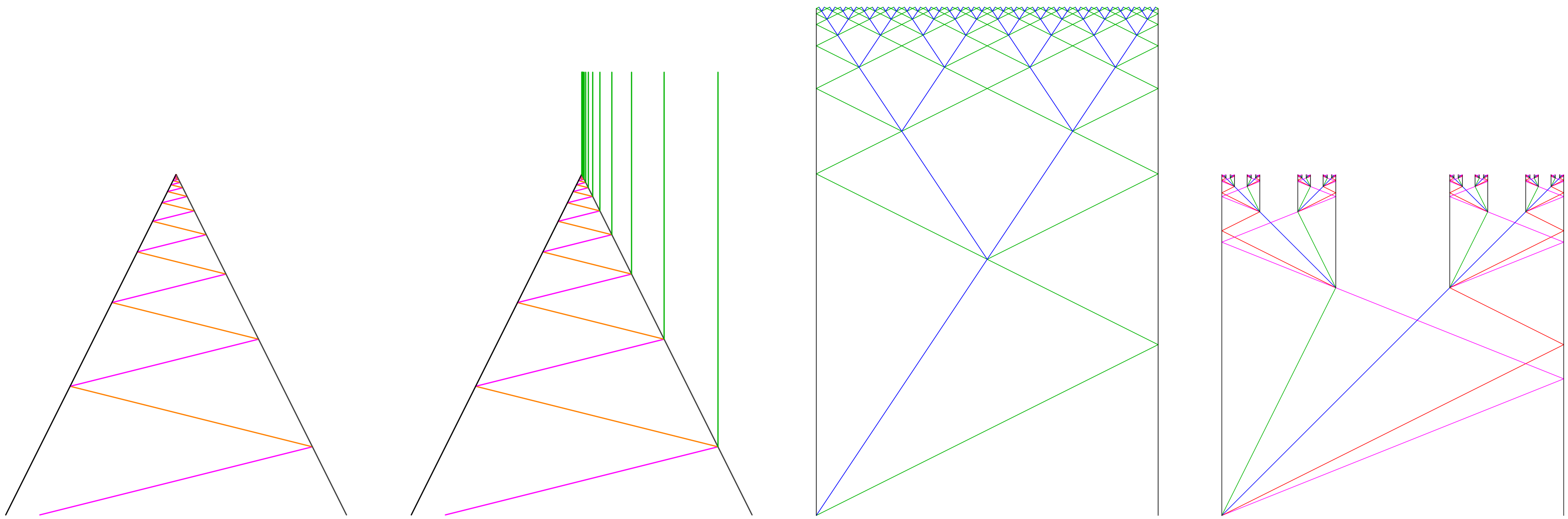
"Nice regular drawings"



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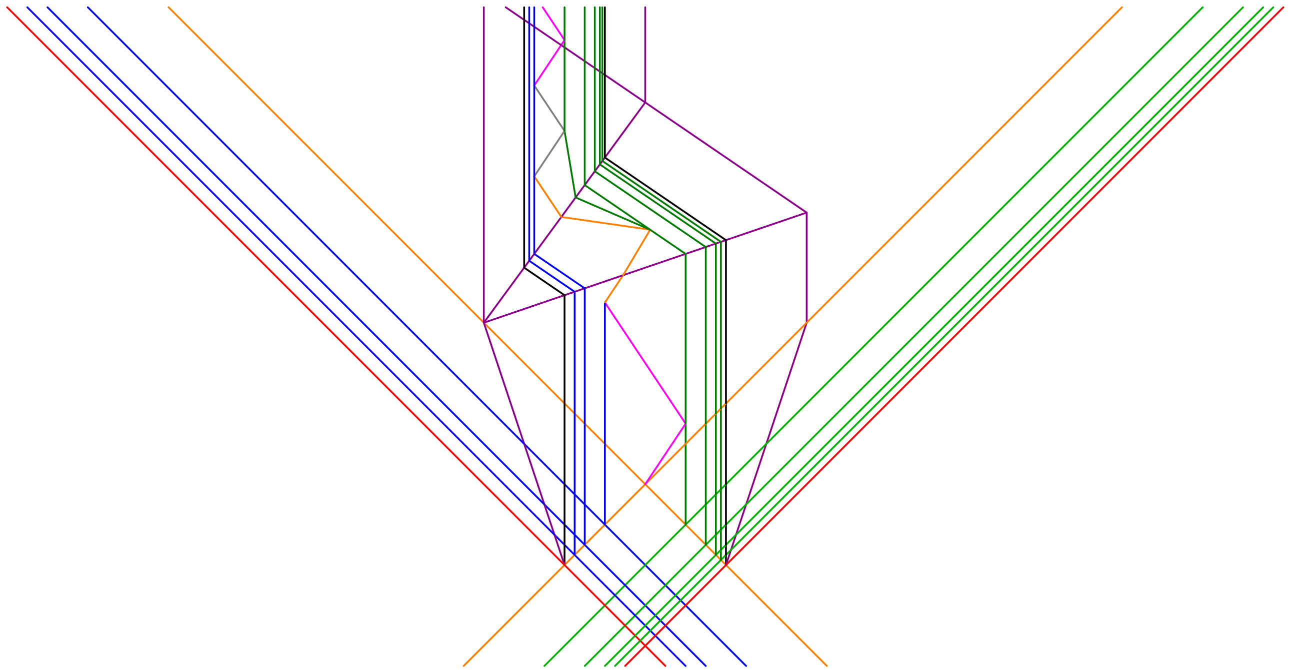

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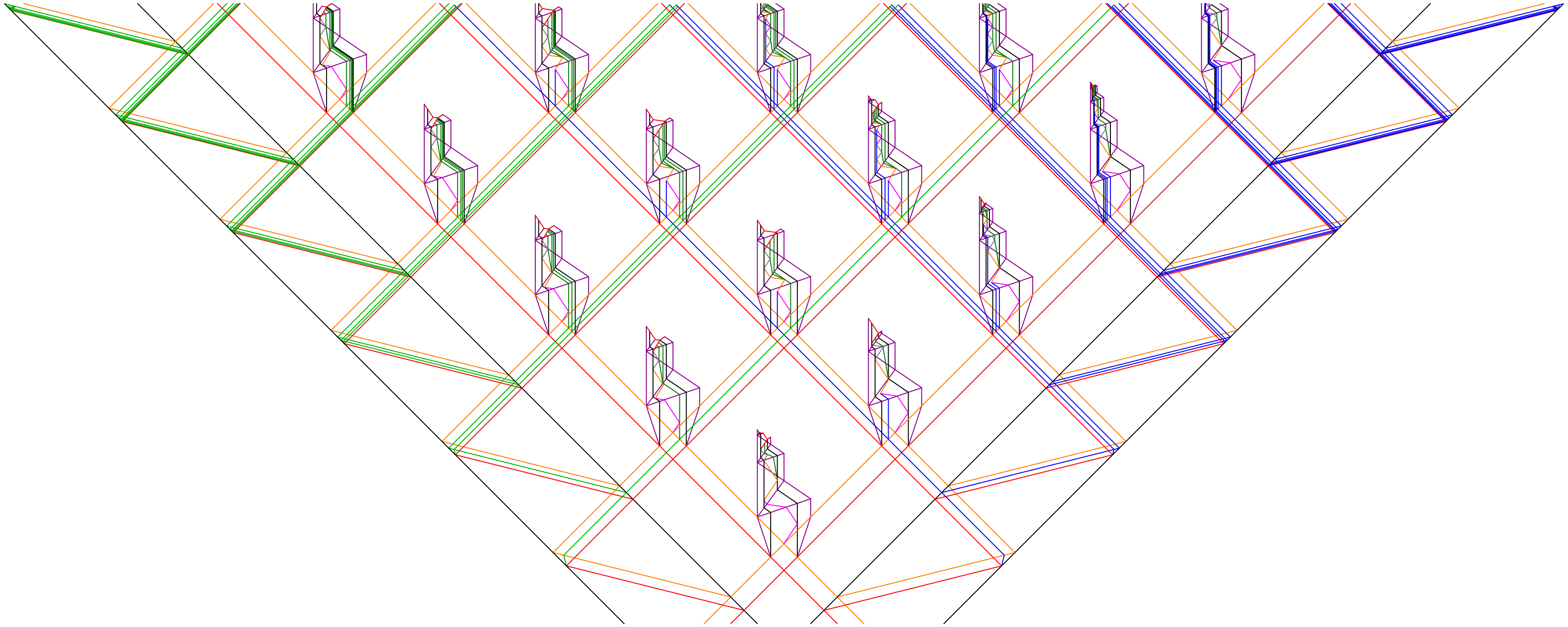

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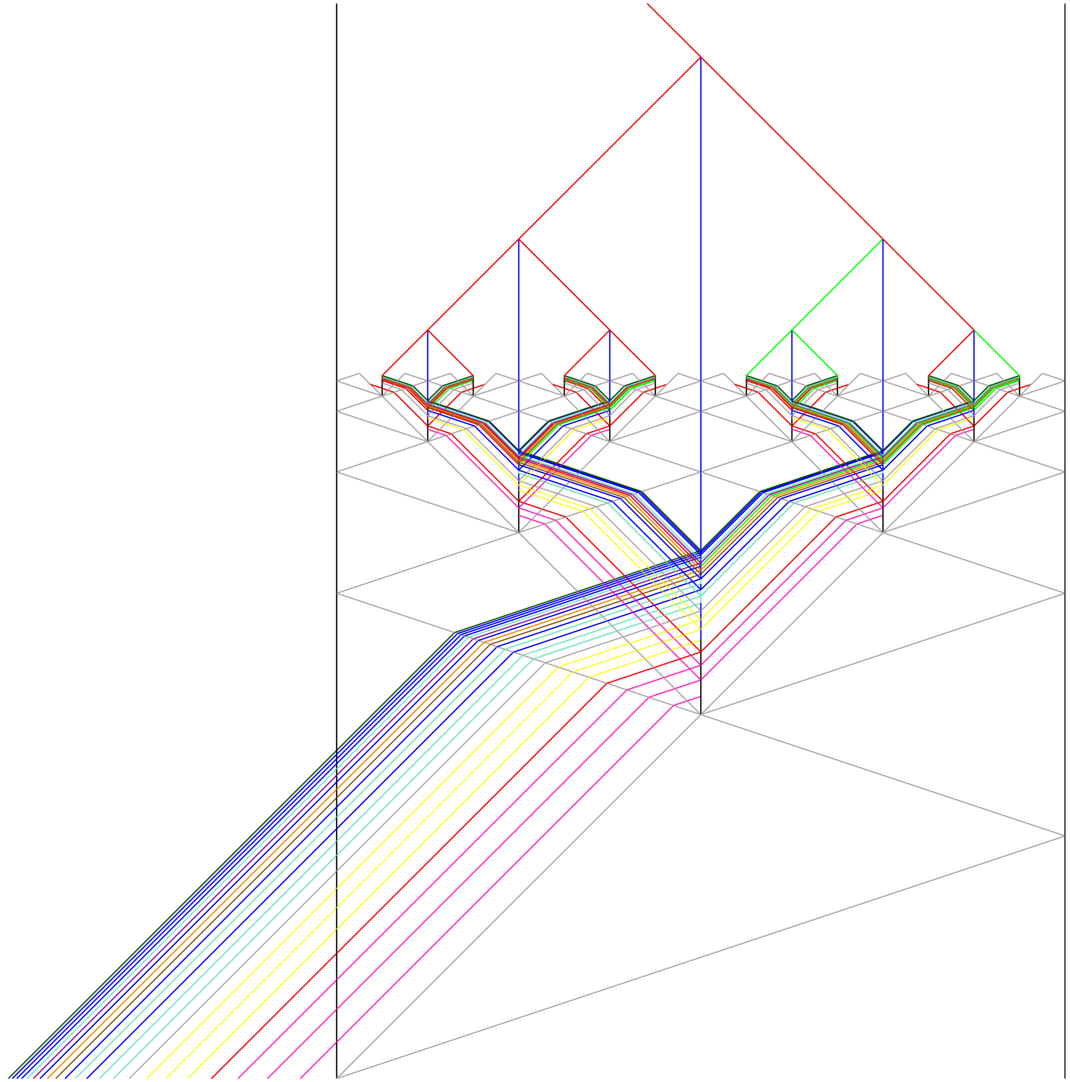
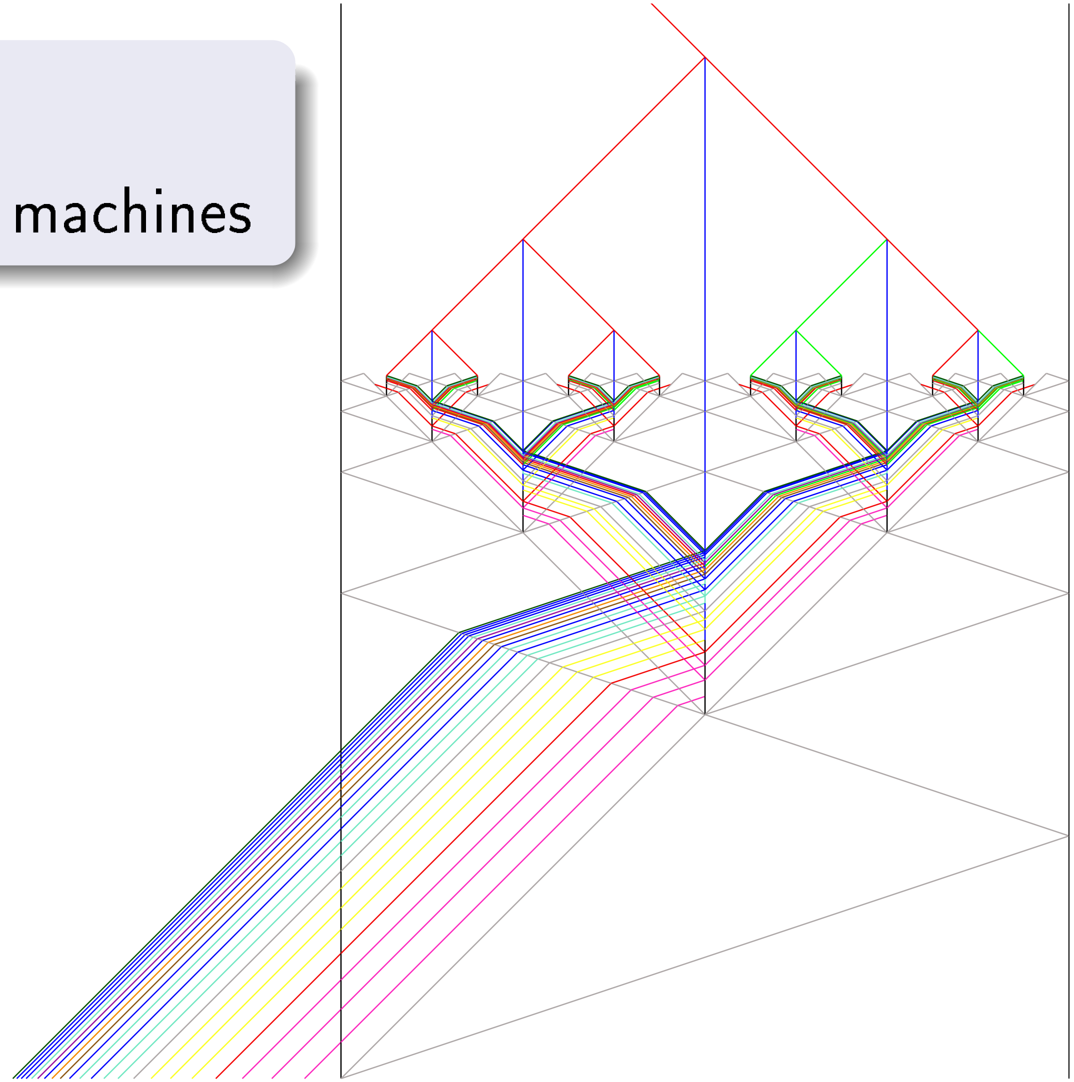


“Nice regular drawings”

Lines: traces of signals

Space-time diagrams of signal machines



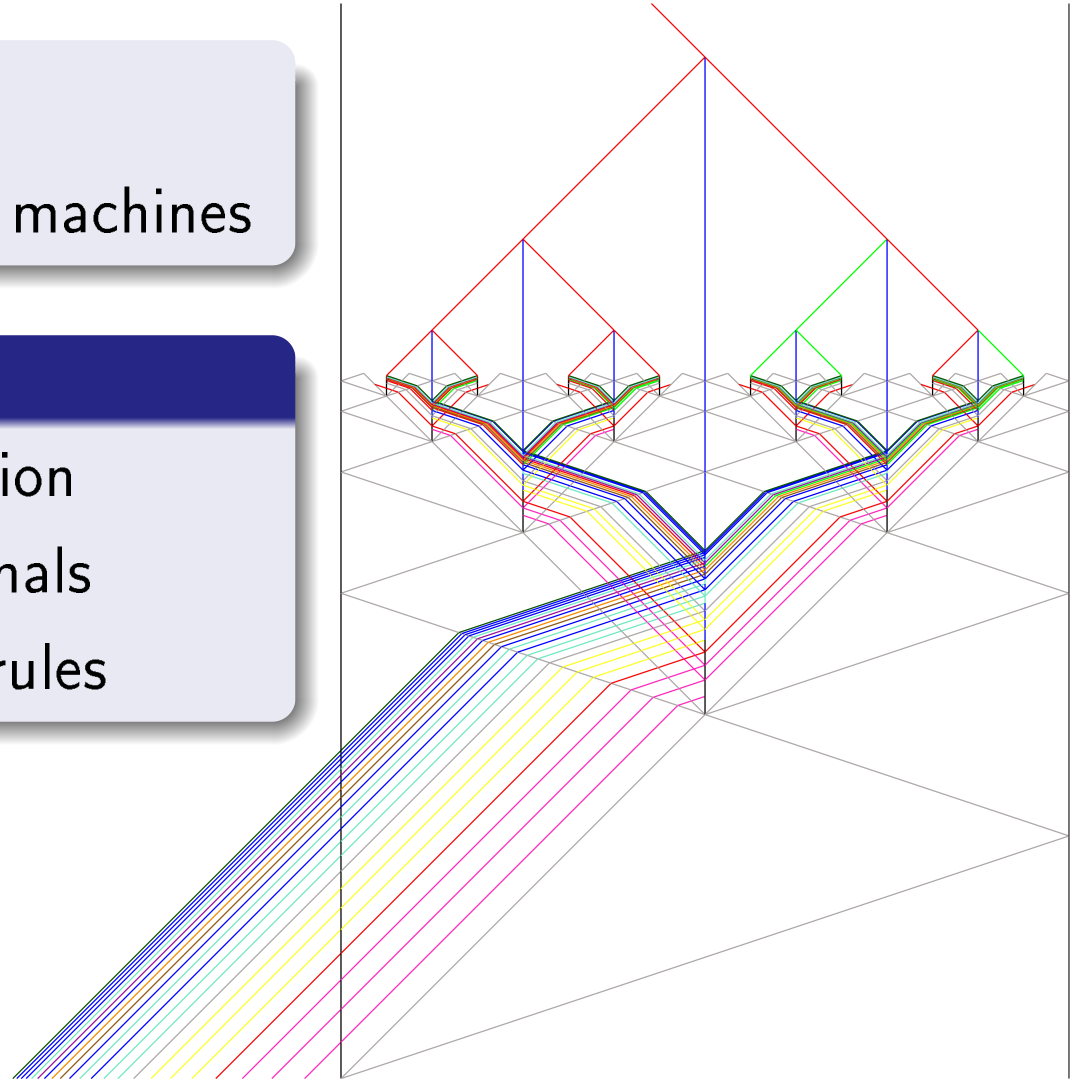
"Nice regular drawings"

Lines: traces of signals

Space-time diagrams of signal machines

Defined by

- bottom: initial configuration
- lines: signals ⇝ meta-signals
- end-points: collisions ⇝ rules



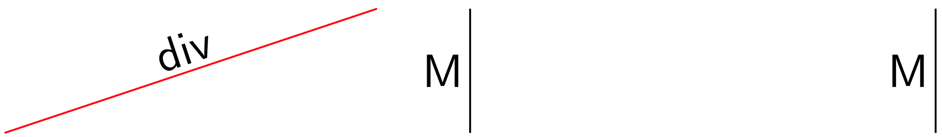
Example: find the middle

Meta-signals (speed)

M (0)

Collision rules

Example: find the middle

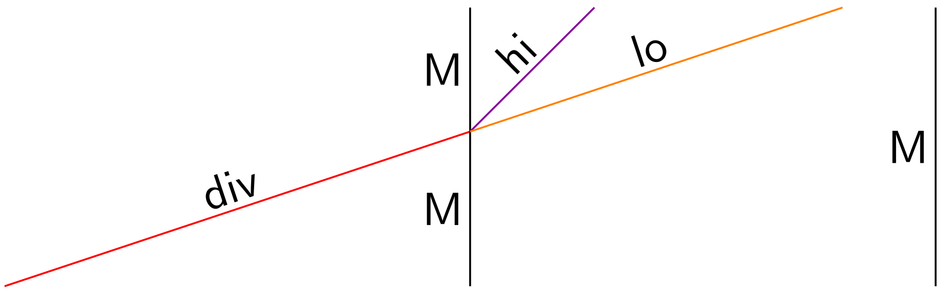


Meta-signals (speed)

M (0)
div (3)

Collision rules

Example: find the middle



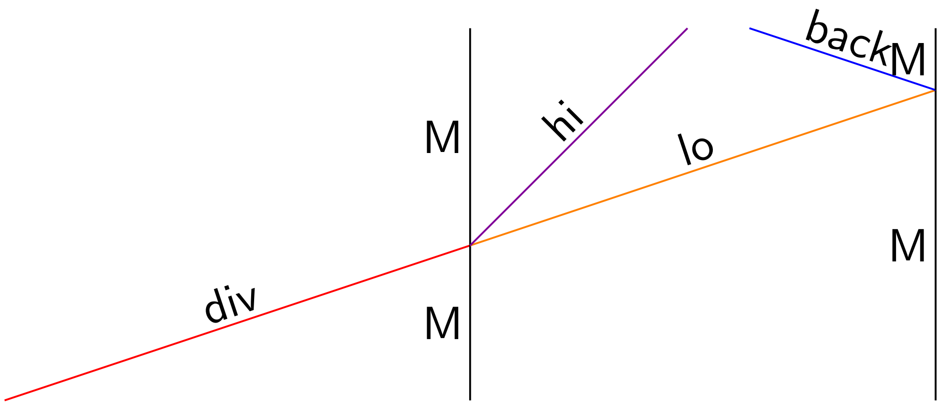
Meta-signals (speed)

M (0)
div (3)
hi (1)
lo (3)

Collision rules

$\{$ div, M $\} \rightarrow \{$ M, hi, lo $\}$

Example: find the middle



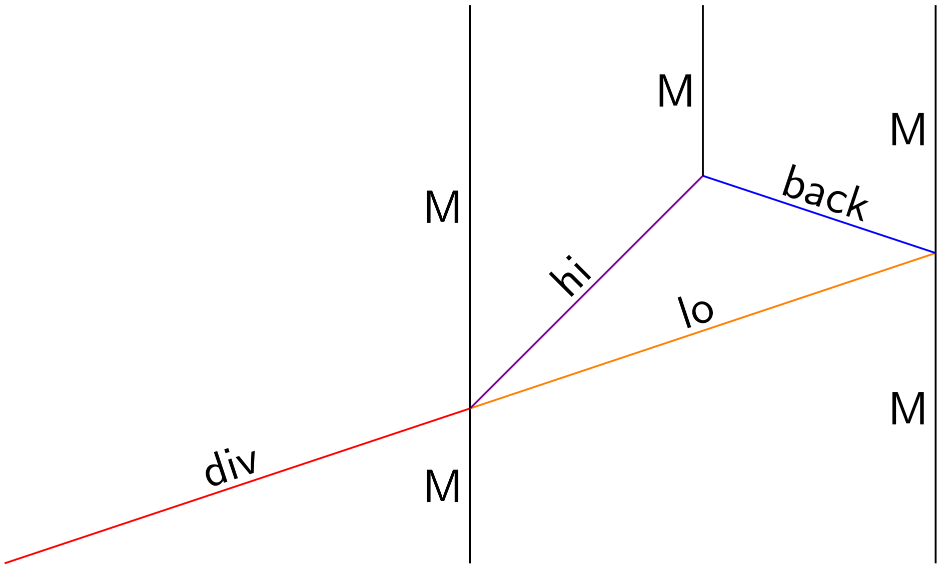
Meta-signals (speed)

M (0)
div (3)
hi (1)
lo (3)
back (-3)

Collision rules

{ div, M } → { M, hi, lo }
{ lo, M } → { back, M }

Example: find the middle



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Collision rules

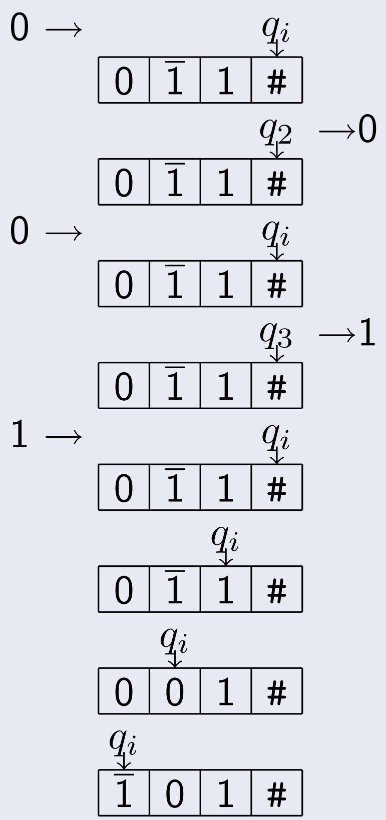
{ div, M } → { M, hi, lo }
{ lo, M } → { back, M }
{ hi, back } → { M }

Known results

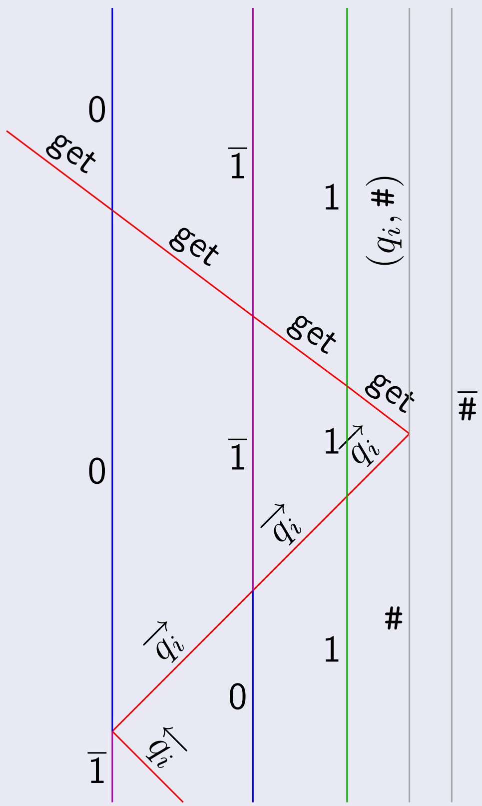
Turing computations

- [Durand-Lose, 2011]

TM run



Simulation



Known results

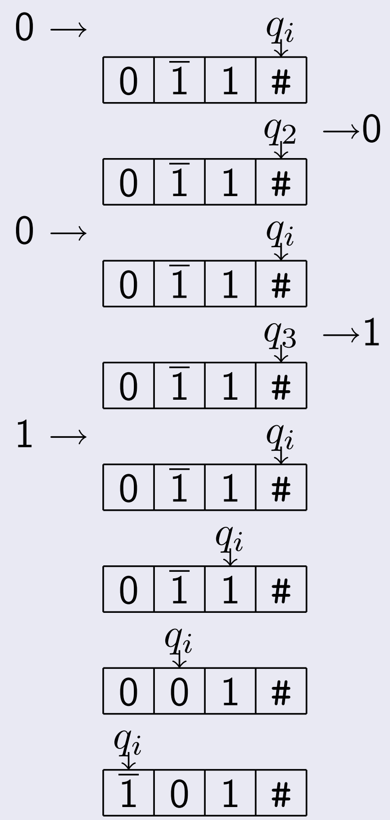
Turing computations

- [Durand-Lose, 2011]

Analog computations

- Computable analysis [Weihrauch, 2000] [Durand-Lose, 2010a]
- Blum, Shub and Smale model [Blum et al., 1989] [Durand-Lose, 2008]

TM run



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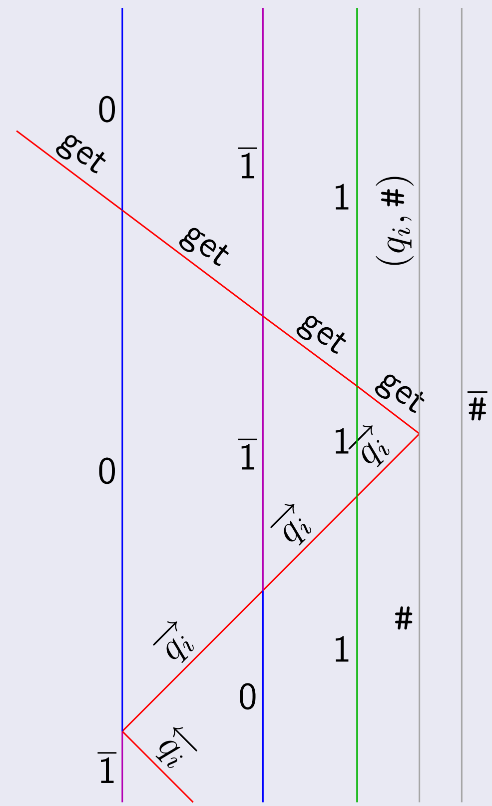

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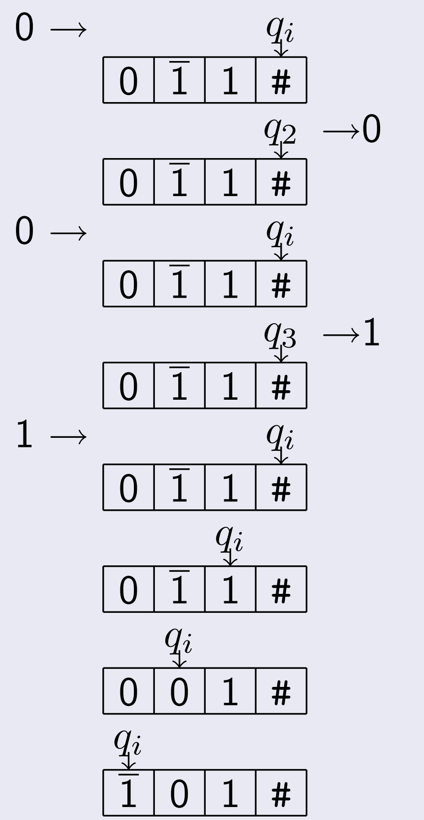
"Black hole" implementation

- [Durand-Lose, 2009]

TM run



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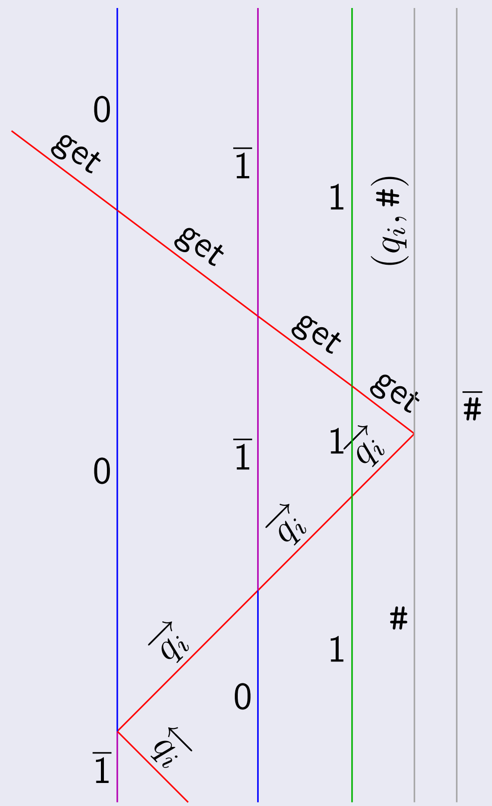
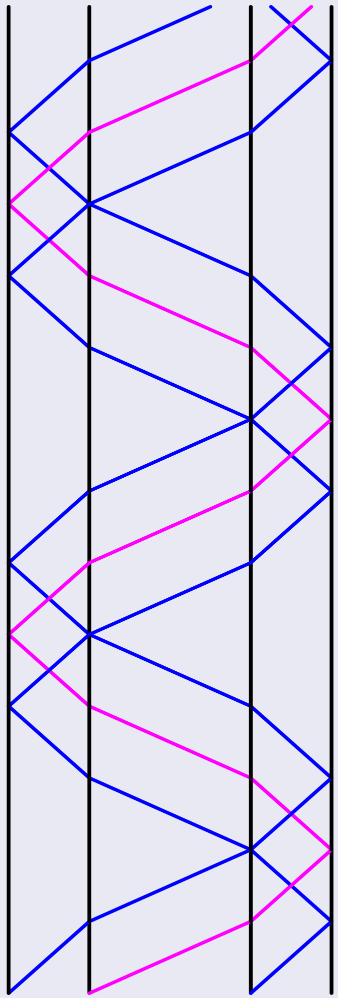
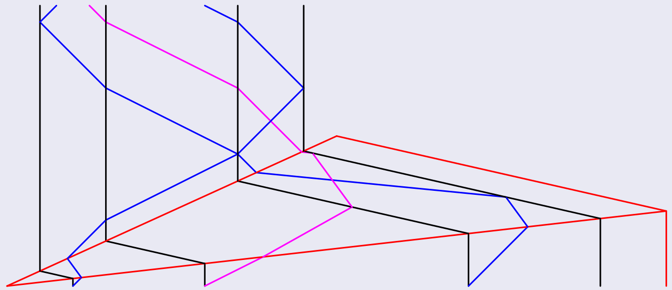


Geometric primitives: accelerating and bounding time

Normal



Shrunk



Geometric primitives: accelerating and bounding time

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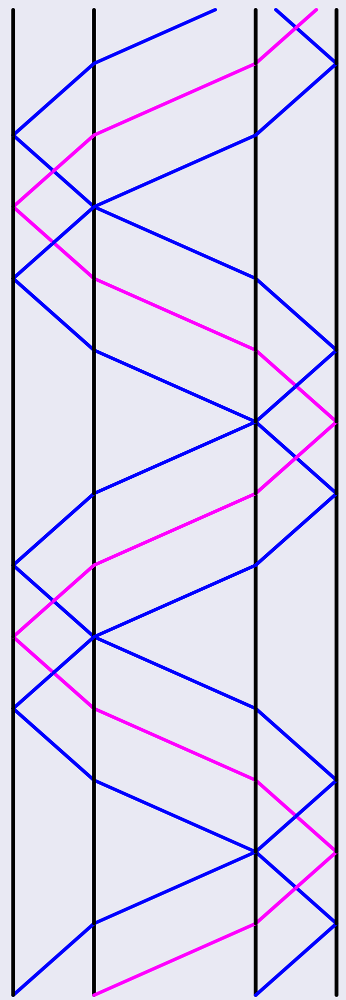

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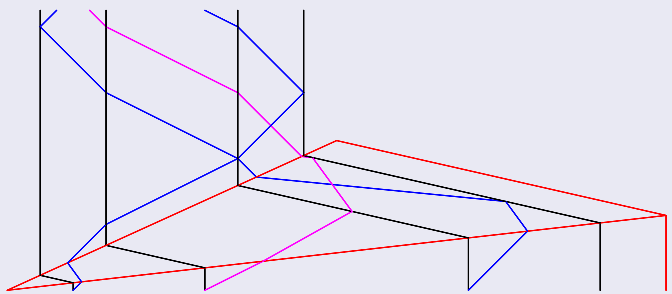
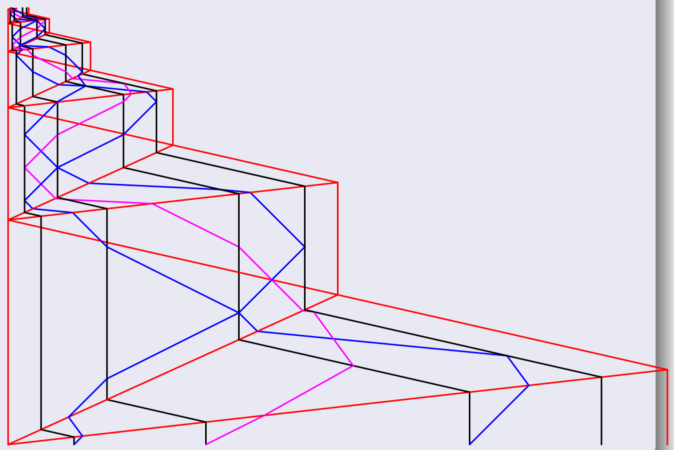


Iterated



Rational signal machines and isolated accumulations

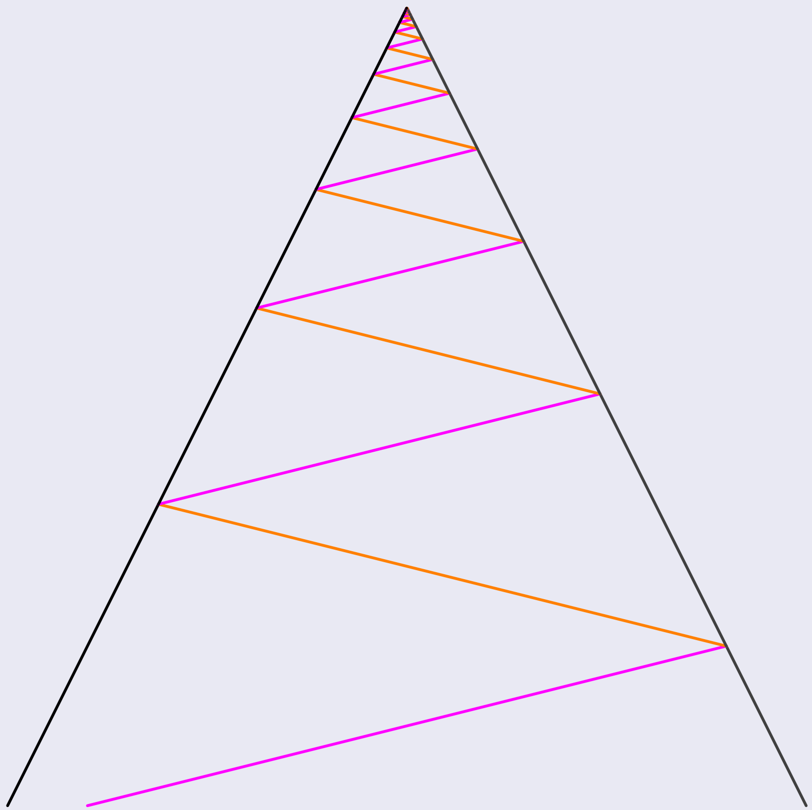
$\mathbb{Q}$ signal machine

- all speed are in $\mathbb{Q}$
- all initial positions are in $\mathbb{Q}$
- $\Rightarrow$ all location remains in $\mathbb{Q}$

Space and time location

- Easy to compute

Simplest accumulation



Rational signal machines and isolated accumulations

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- all speed are in $\mathbb{Q}$
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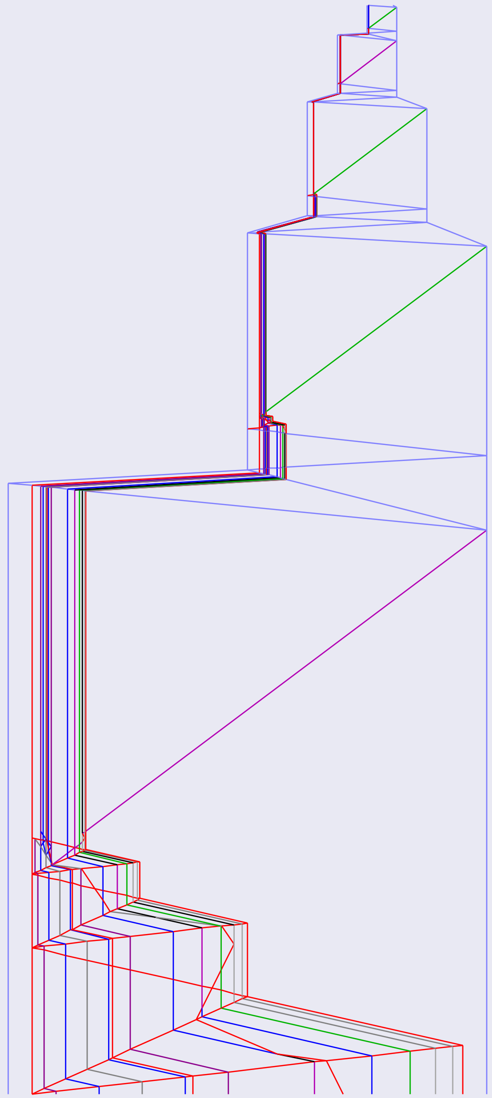
Space and time location

- Easy to compute
- Not so easy to guess

Accumulation?



Rational signal machines and isolated accumulations

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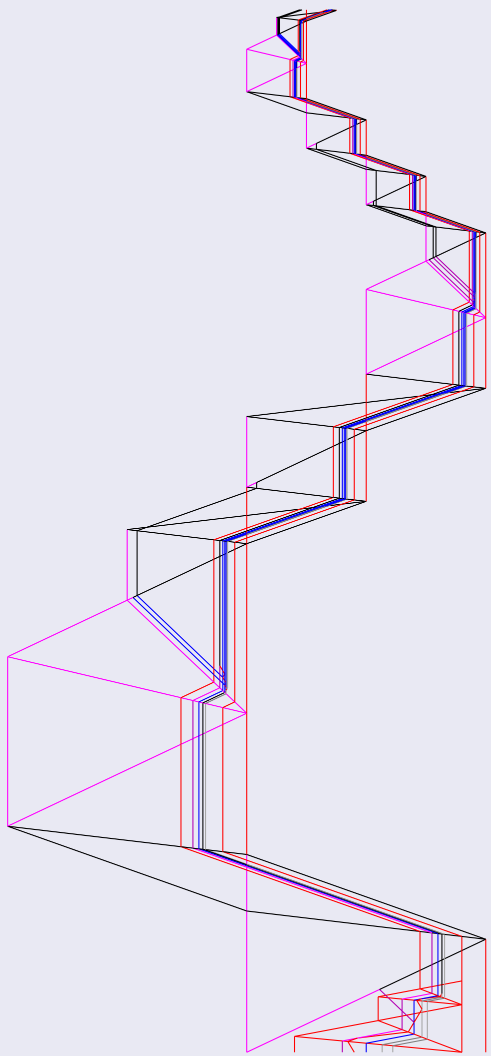
Forecasting any accumulation

Highly undecidable
(Σ_2^0 in the arithmetic hierarchy)
[Durand-Lose, 2006]

Accumulation?



Temporal coordinate

$\mathbb{Q}$-signal machine

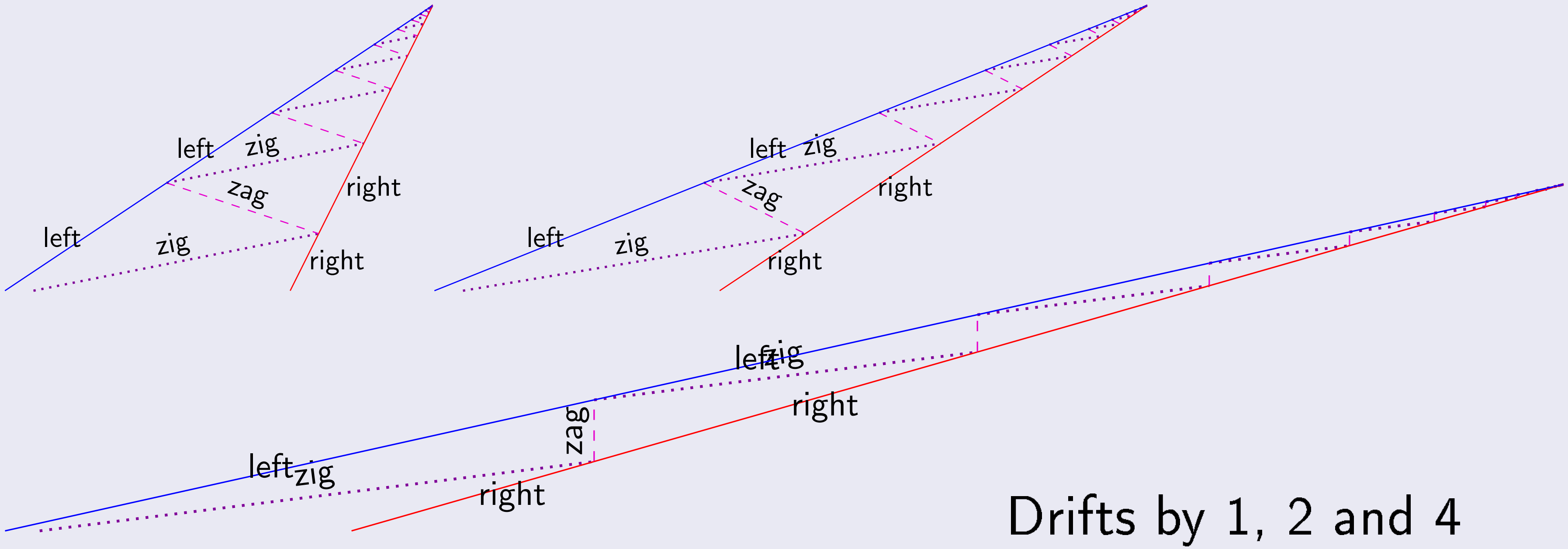
- $\mathbb{Q}$ on computers/Turing machine
  - exact representation
  - exact operations
- exact computations by TM (and implanted in Java)

Simulation near an isolated accumulation

- on each collision, print the date
- $\leadsto$ increasing computable sequence of rational numbers (converges iff there is an accumulation)

Spacial coordinate

Static deformation by adding a constant to each speed



Drifts by 1, 2 and 4

With all speeds positive

- the left most coordinate is increasing (and computable) converges iff there is an accumulation
- correction by subtracting the date times the drift

c.e. real number

- limit of a convergent increasing computable sequence of rational numbers
- no bound on the convergence rate
- represents a *c.e.* set (of natural numbers)
- stable by positive integer multiplication but not by subtraction

d-c.e. real number

- difference of two *c.e.* real number
- form a field
- [Ambos-Spies et al., 2000]
  these are exactly the limits of a computable sequence of rational numbers that converges *weakly effectively*, *i.e.*,
  $$\sum_{n \in N} |x_{n+1} - x_n| \text{ converges}$$

Encoding

For *d-c.e.* real numbers

$$x = \sum_{i \in \mathbb{N}} \frac{z_i}{2^i} \text{ , } z_i \in \mathbb{Z}$$

the sequence $i \to z_i$ is computable and

$$\sum_{i \in \mathbb{N}} \left| \frac{z_i}{2^i} \right| \text{ converges}$$

For *c.e.* real numbers

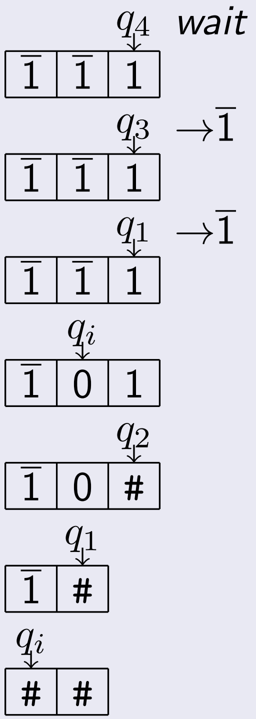
- identical but $z_i \in \mathbb{N}$

- z_i in signed unary representation

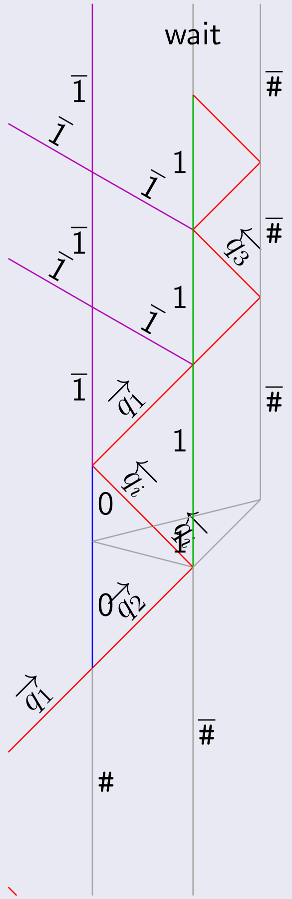
TM outputting the infinite sequence

Run

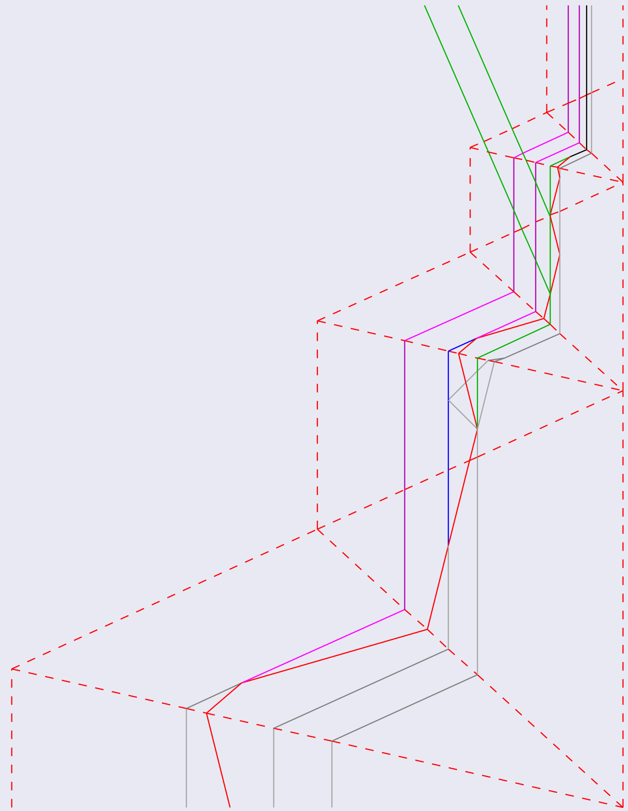
wait between each z_i



Simulation



Shrunk to output in bounded time

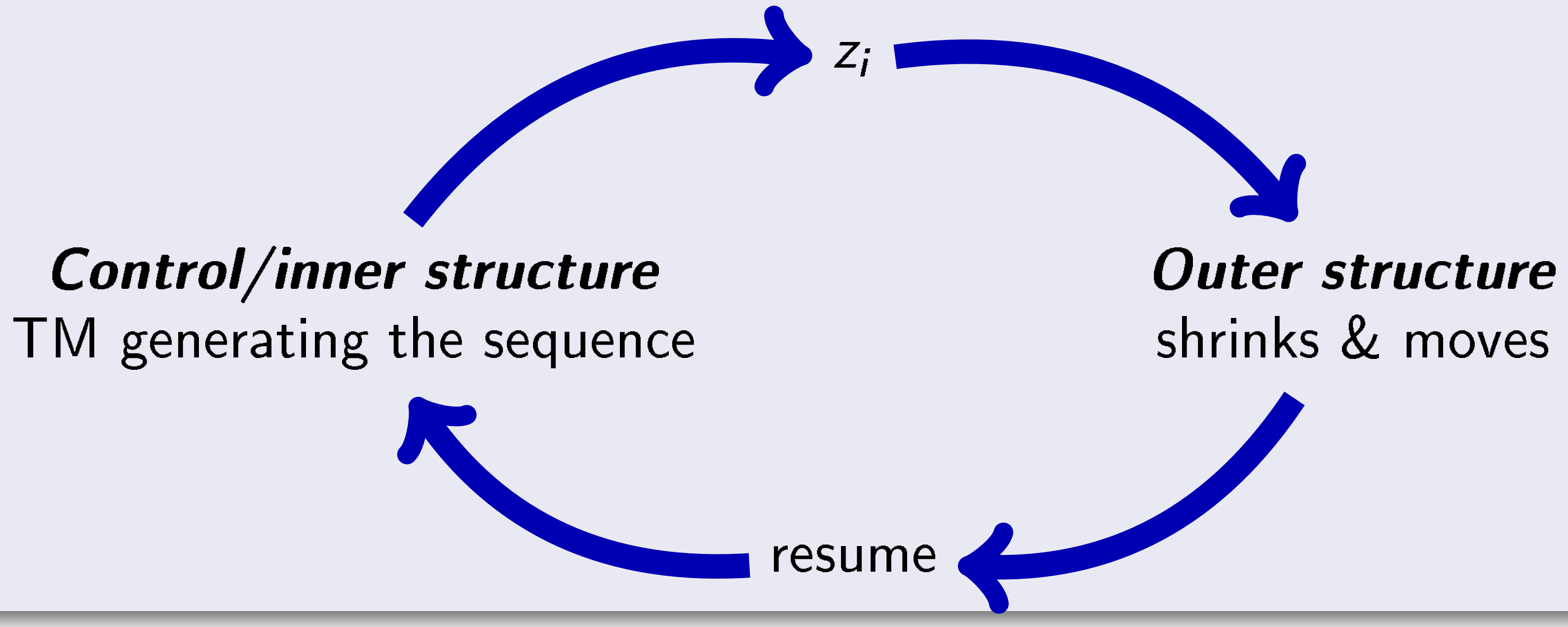


Simulation and shrinking structure stop after each value

Two-level scheme

Control/inner structure

- Provide the data for accumulating

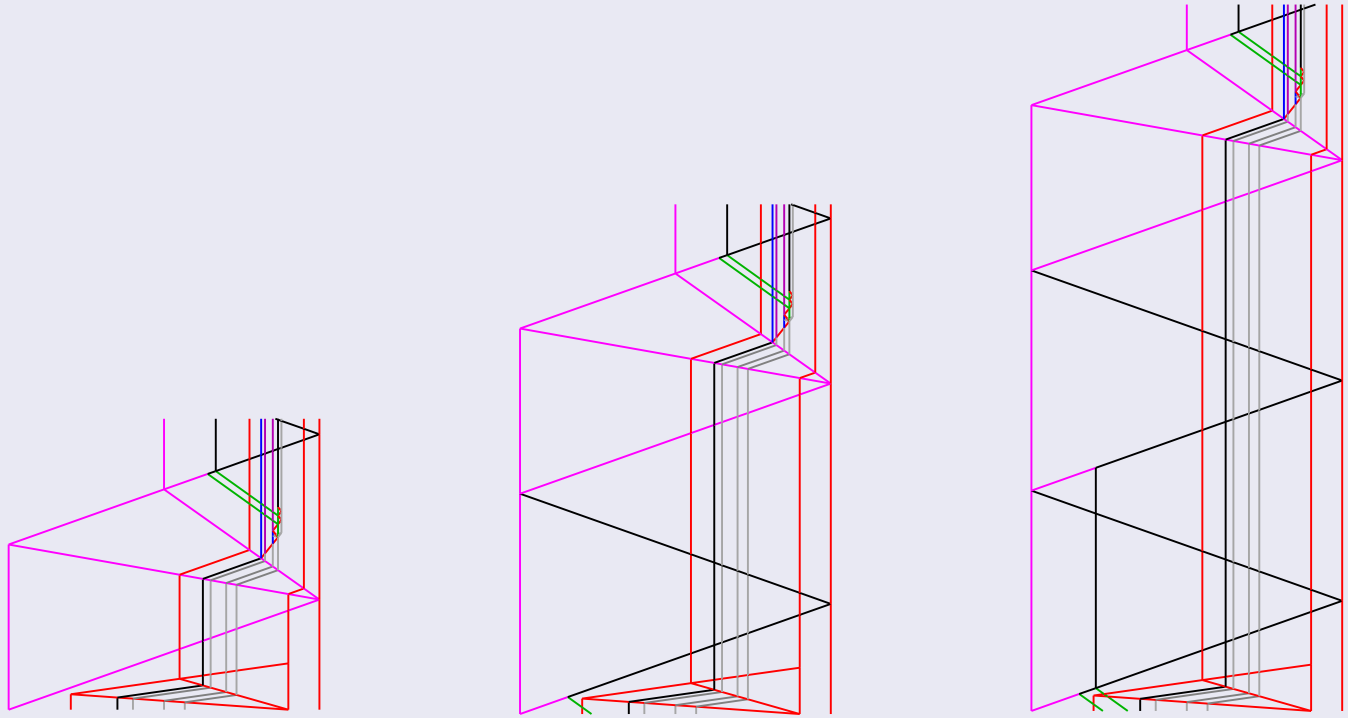


Outer structure

- shrink and move the whole structure $\rightsquigarrow$ accumulation

Temporal coordinate

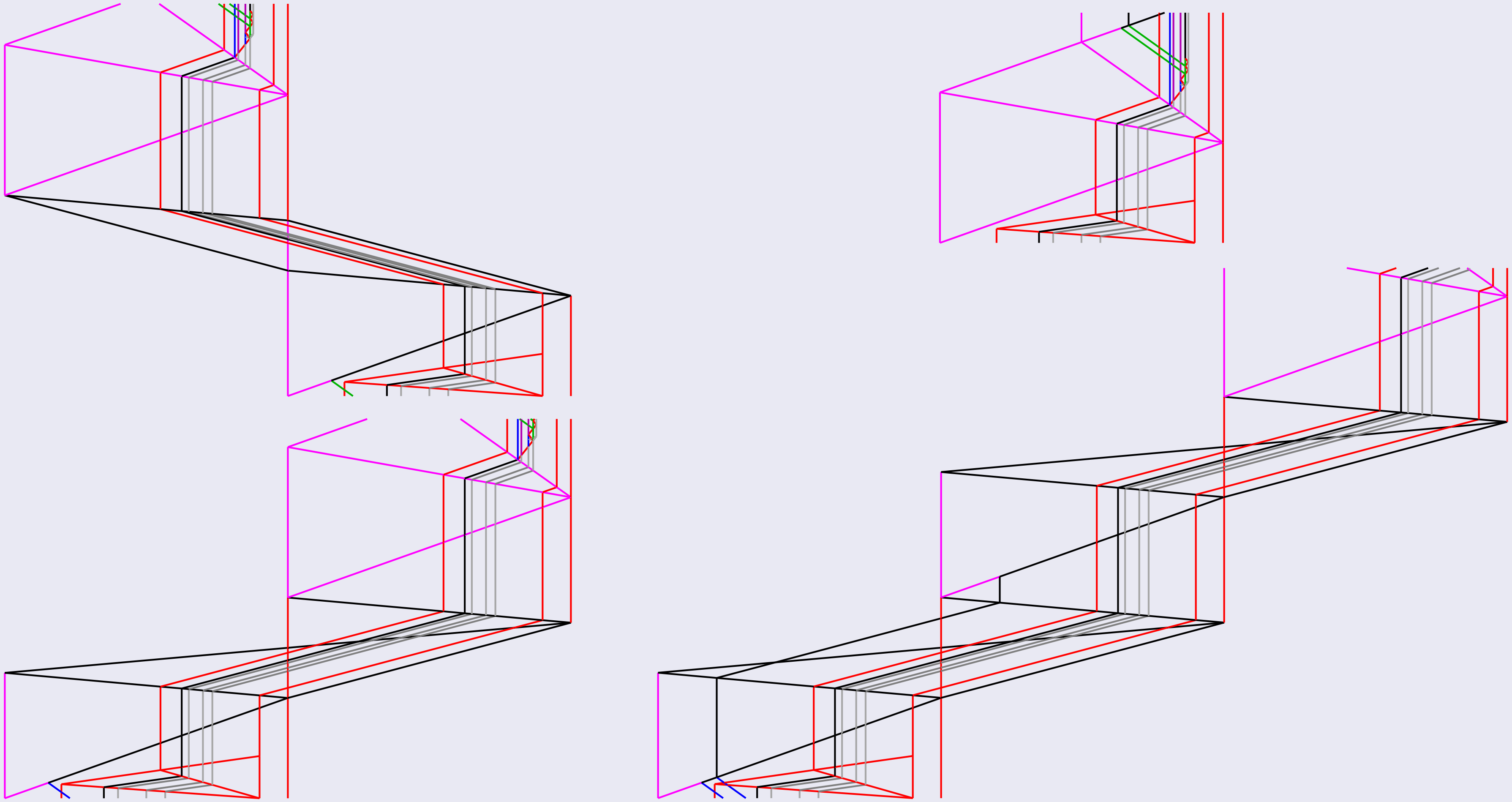
Wait the corresponding time



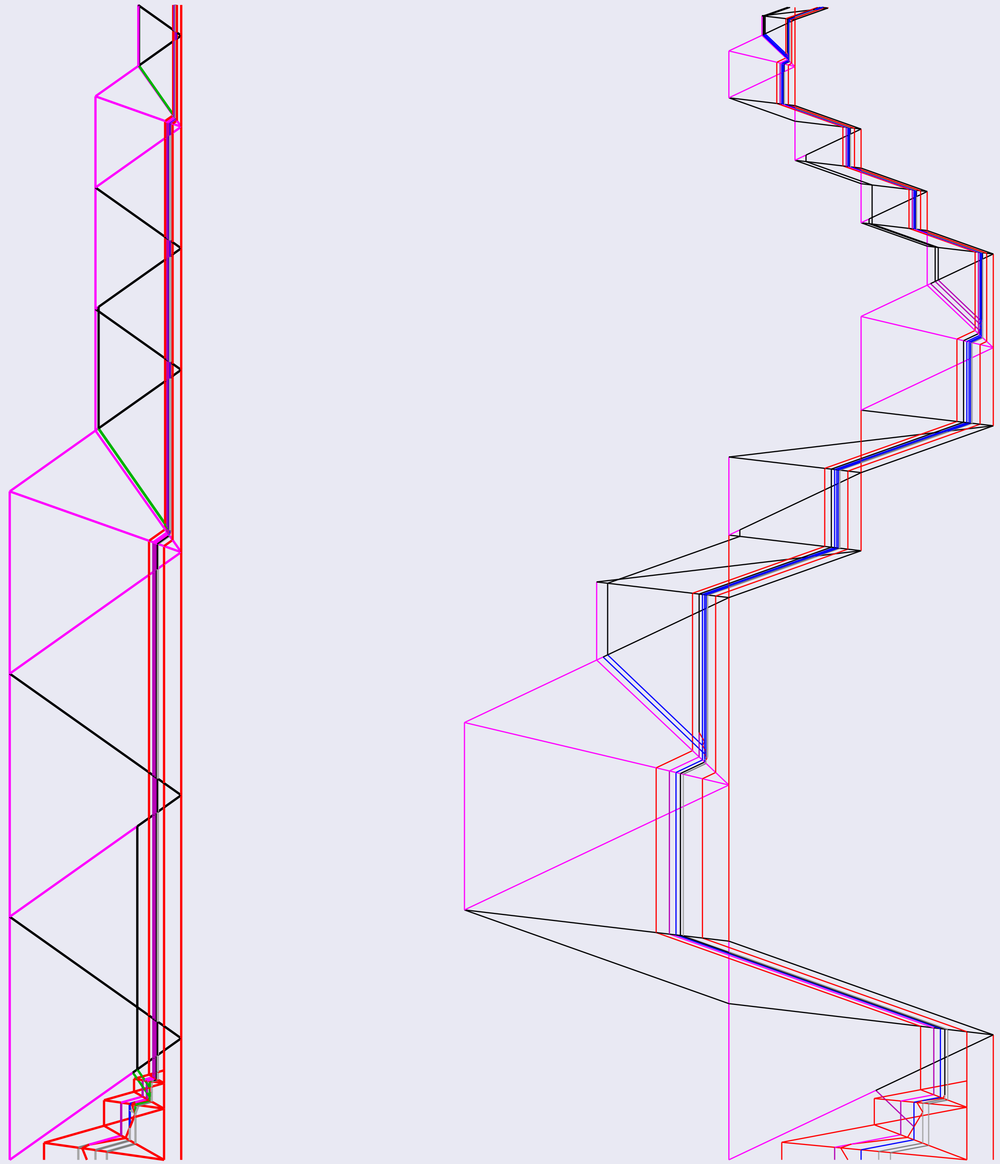
- Constant (up to scale) delay before outer structure action
- total delay is rational and should be previously subtracted

Spatial coordinate

Move left or right, more or less



Examples



Results

- Isolated accumulations happen at *d-c.e.* spacial and *c.e.* temporal coordinates
- Accumulation at any *c.e.* temporal coordinate is possible
- Accumulation at any *d-c.e.* spacial coordinate is possible

Perspectives

- Uncorrelate space and time coordinate
  it is possible for *computable* coordinates [Durand-Lose, 2010b]
- Higher order isolated accumulations
- Non isolated accumulations

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