Irrationality is needed to compute with signal machines with only three speeds

CiE 2013 — Special session on Computation in Nature

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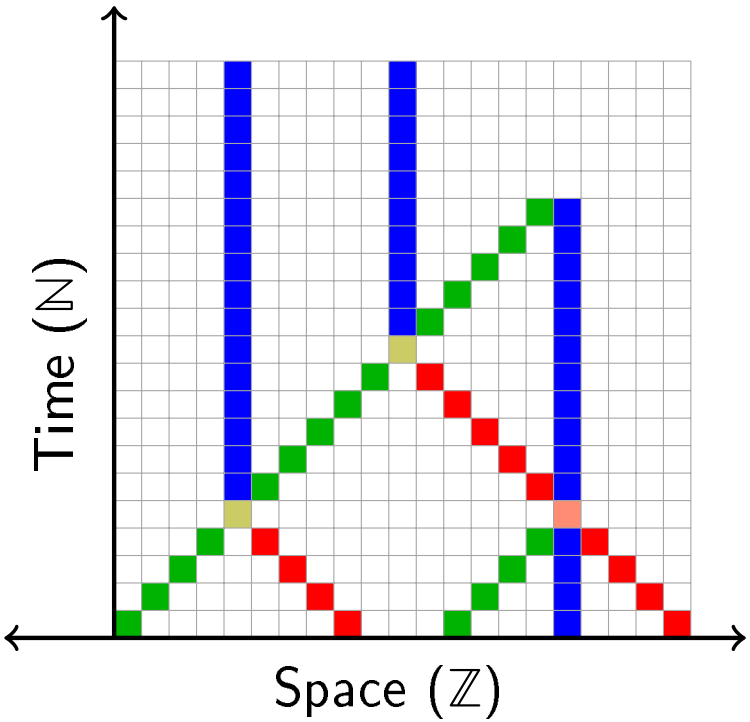
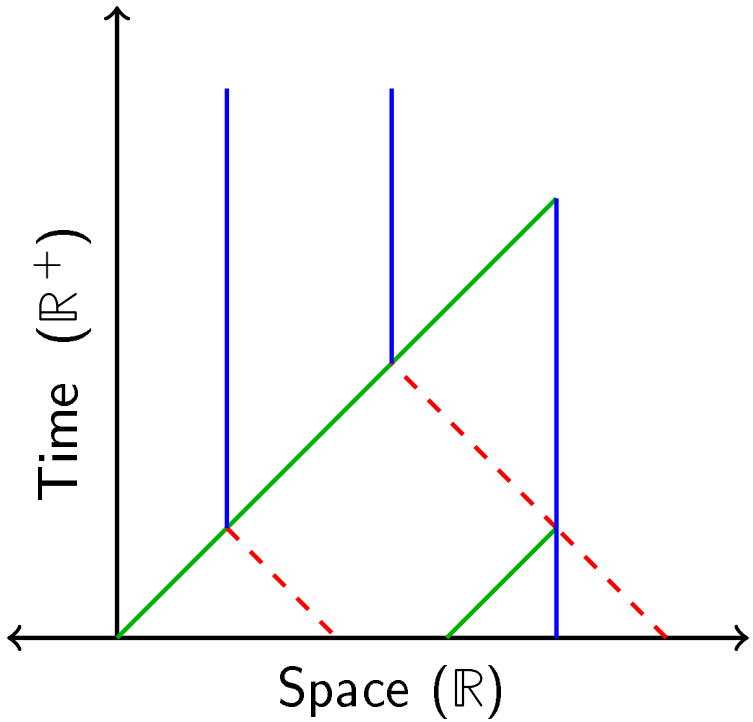
Signals in cellular automata





- Signal (meta-signal)
- Collision (rule)

Vocabulary and example: find the middle

Meta-signals (speed)

M (0)

Collision rules

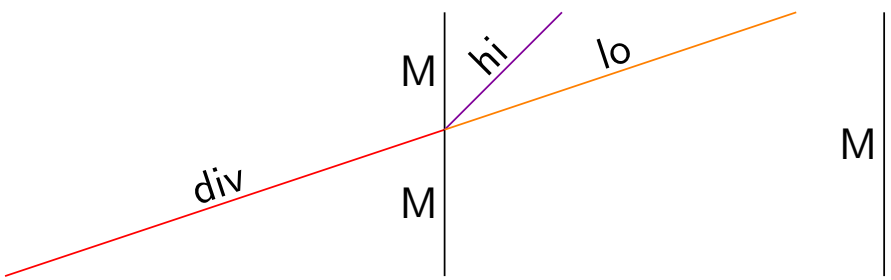
Vocabulary and example: find the middle

Meta-signals (speed)

M	(0)
div	(3)

Collision rules

Vocabulary and example: find the middle



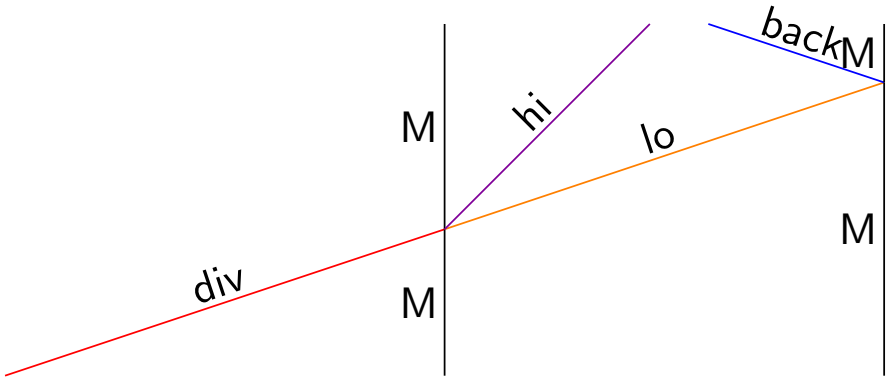
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)

Collision rules

$$\{ \text{div}, \text{M} \} \rightarrow \{ \text{M}, \text{hi}, \text{lo} \}$$

Vocabulary and example: find the middle



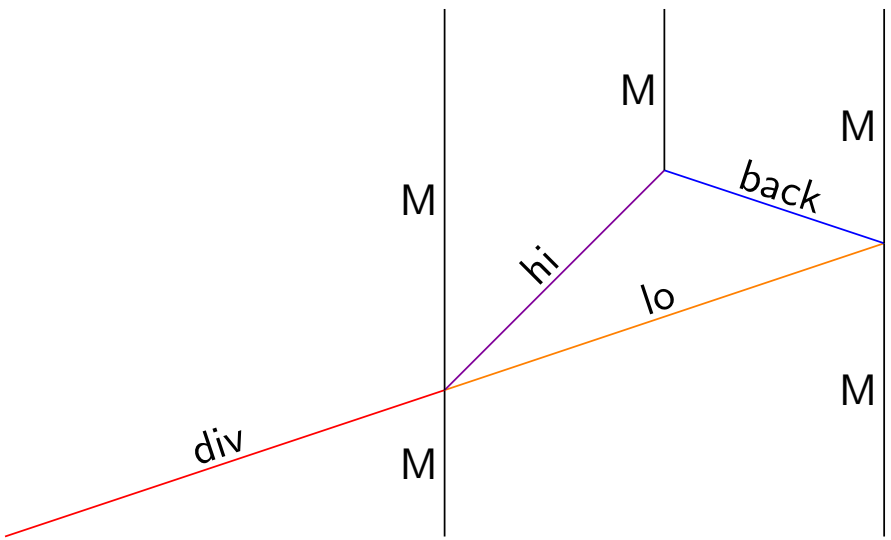
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)
back	(-3)

Collision rules

$$\{ \text{div}, \text{M} \} \rightarrow \{ \text{M}, \text{hi}, \text{lo} \}$$
$$\{ \text{lo}, \text{M} \} \rightarrow \{ \text{back}, \text{M} \}$$

Vocabulary and example: find the middle



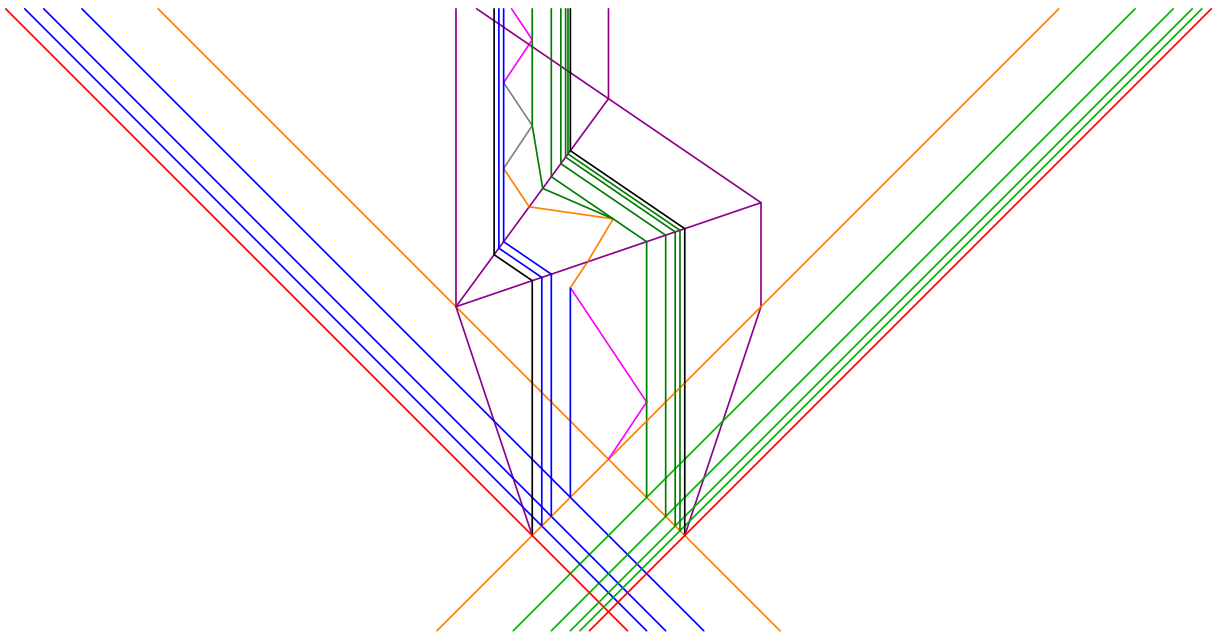
Meta-signals (speed)

M	(0)
div	(3)
hi	(1)
lo	(3)
back	(-3)

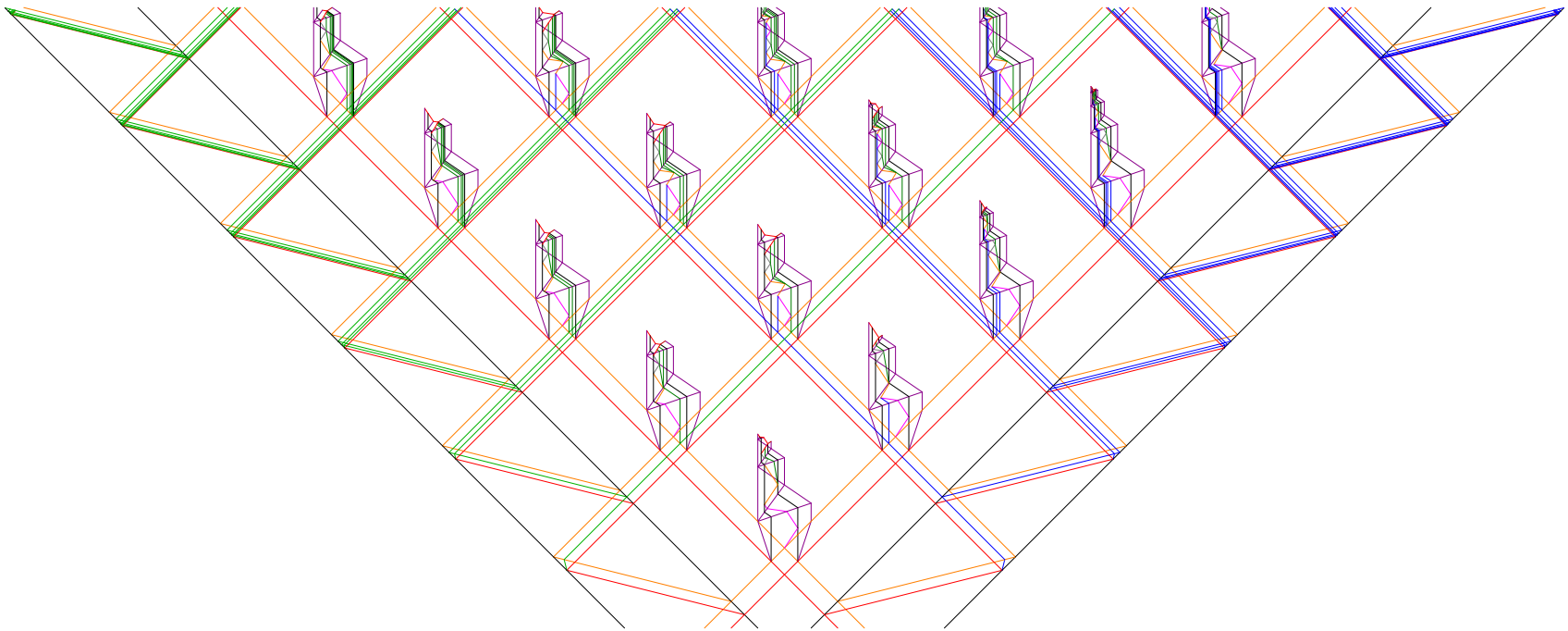
Collision rules

$$\{ \text{div}, \text{M} \} \to \{ \text{M}, \text{hi}, \text{lo} \}$$
$$\{ \text{lo}, \text{M} \} \to \{ \text{back}, \text{M} \}$$
$$\{ \text{hi}, \text{back} \} \to \{ \text{M} \}$$

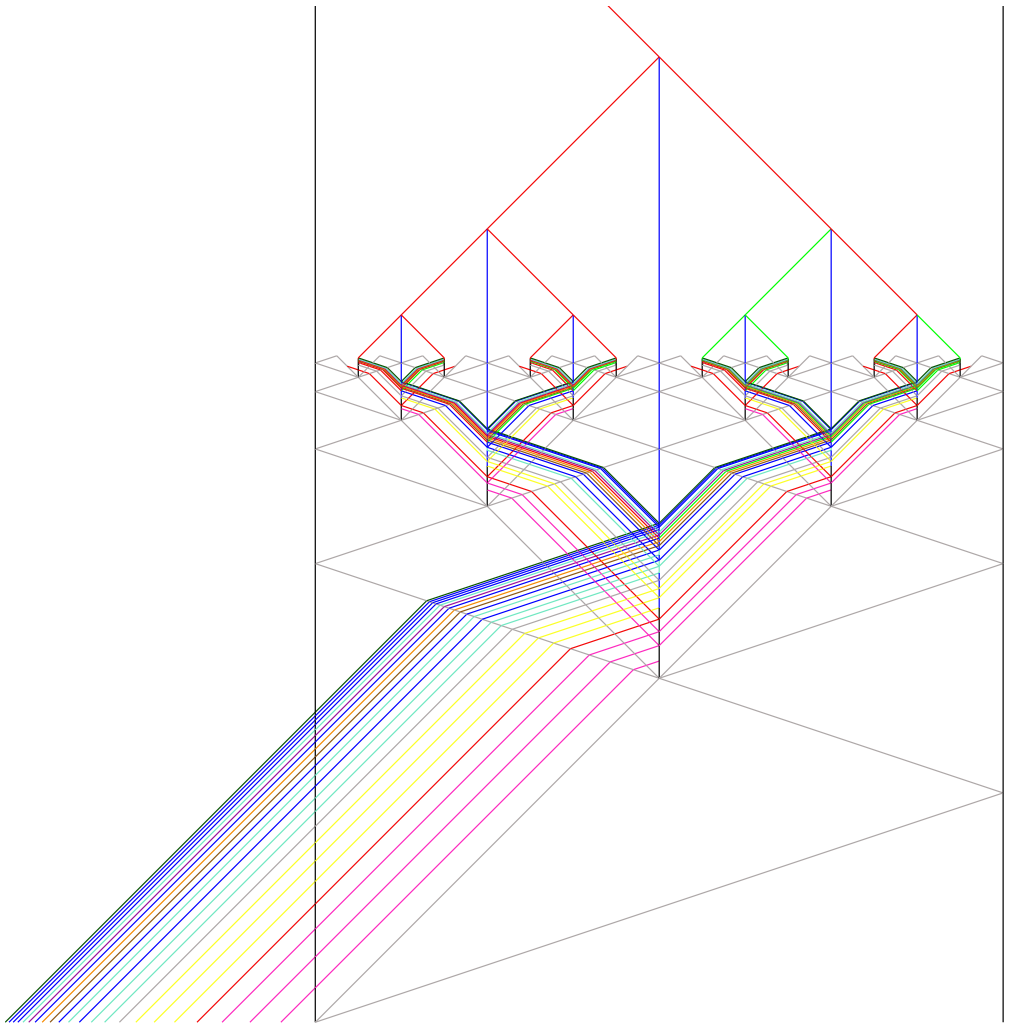
Complex behavior



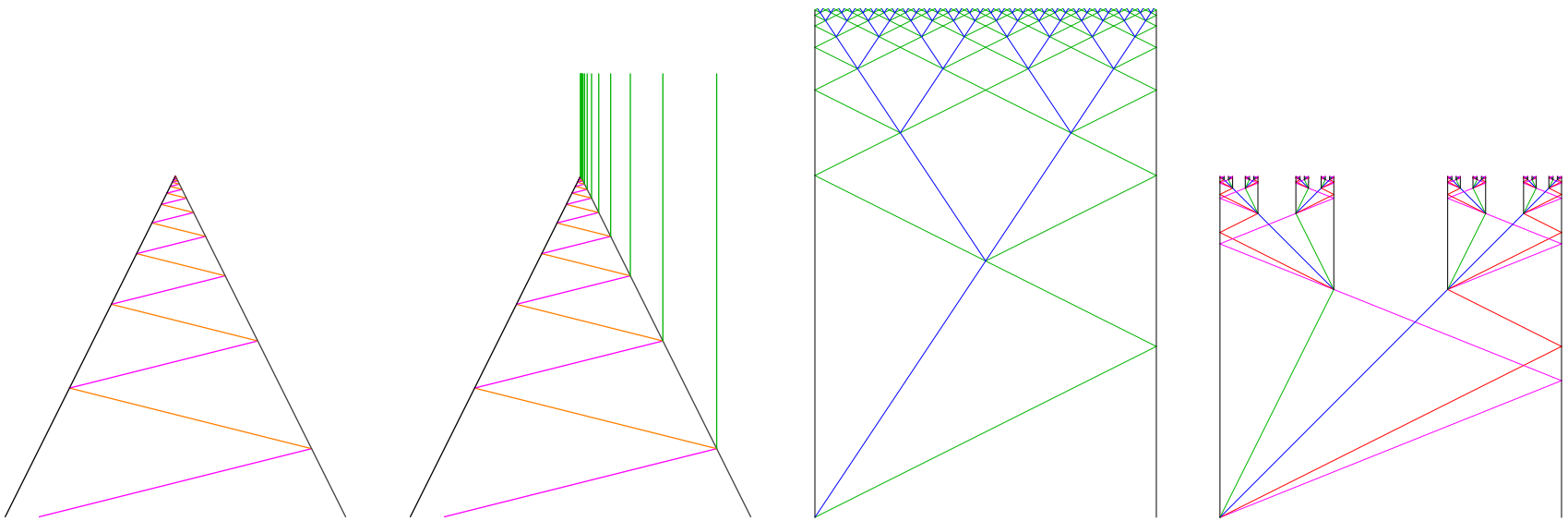
Complex behavior



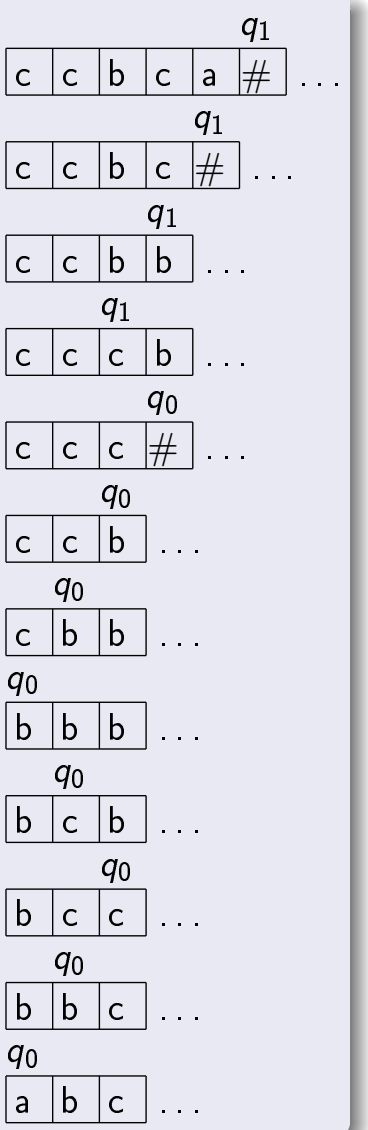
Complex behavior

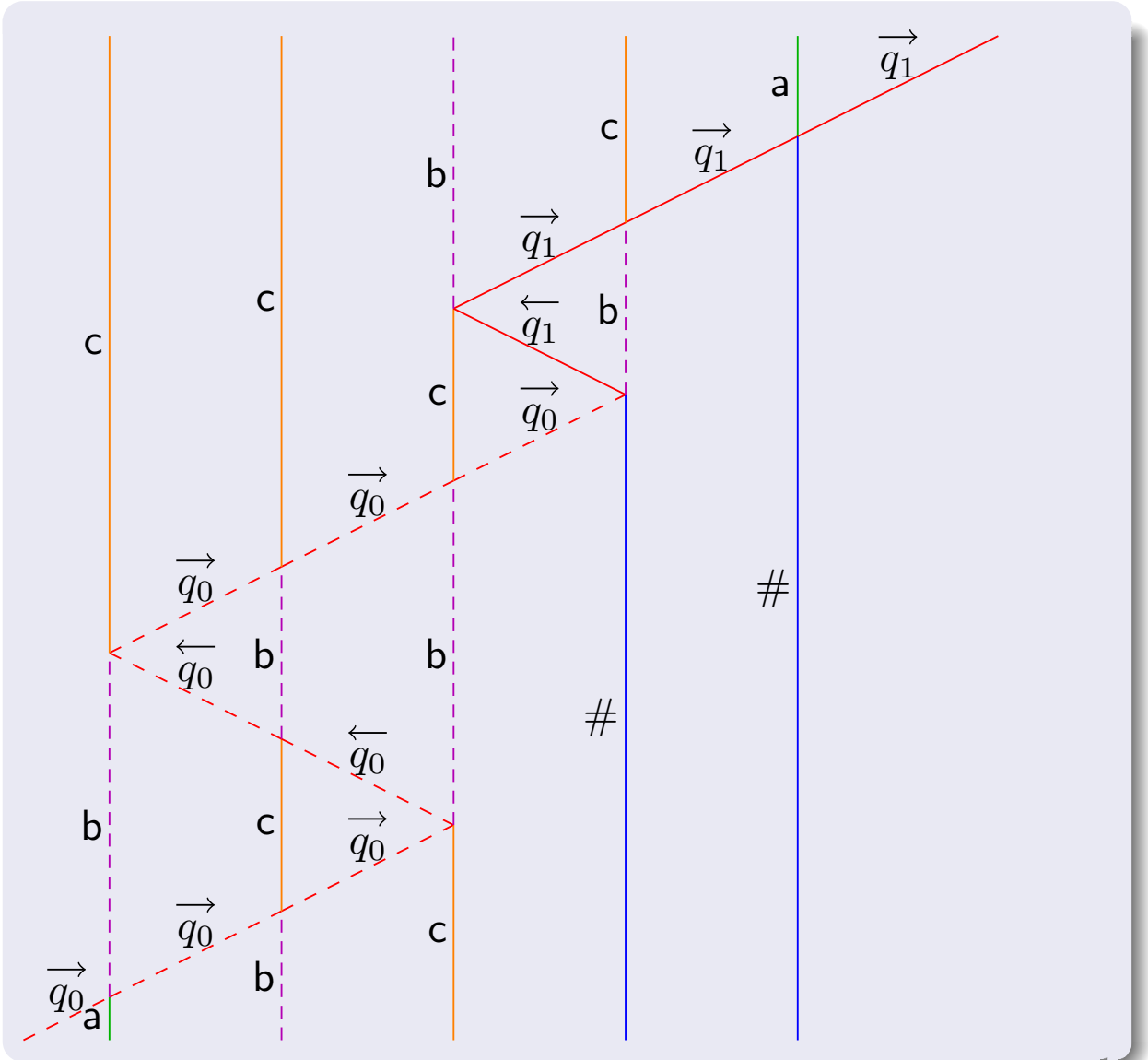


Accumulations are quite common



Simulating a Turing machine (on a finite tape)

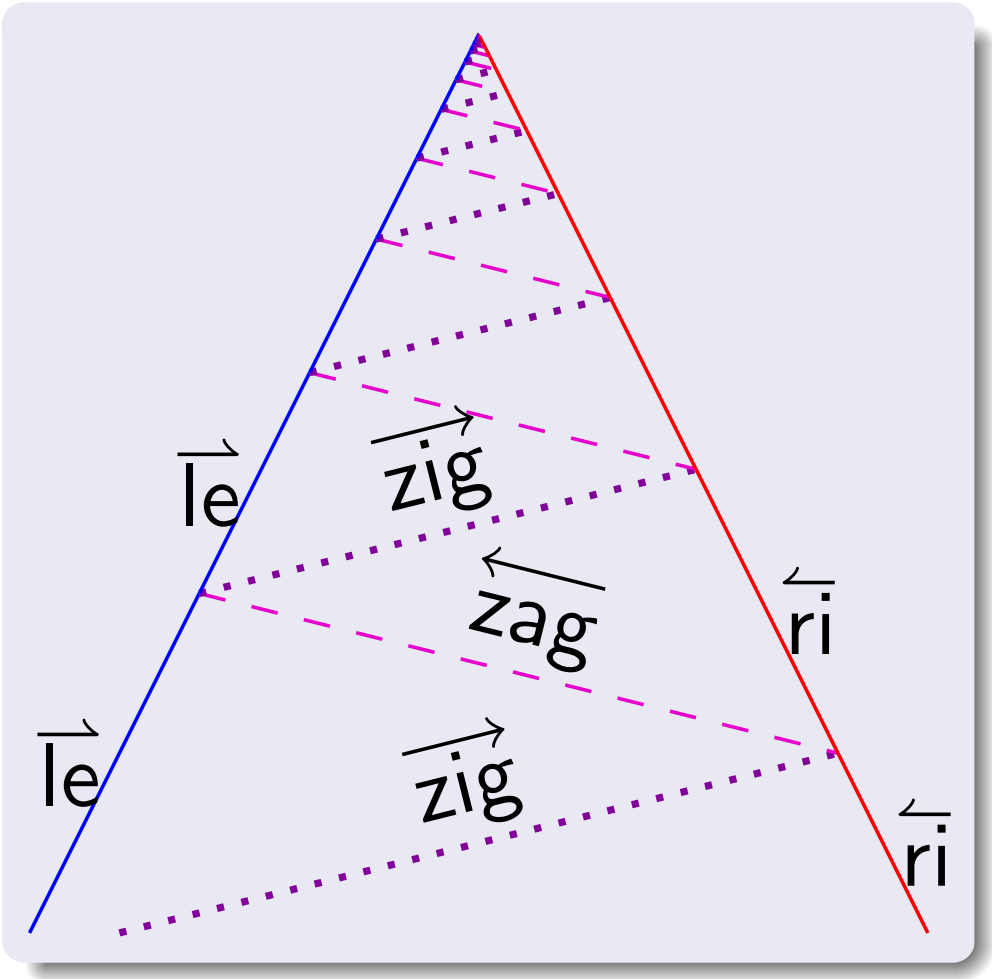




Minimality — bounds on the number of...

...*meta-signals* to...

Accumulate 4

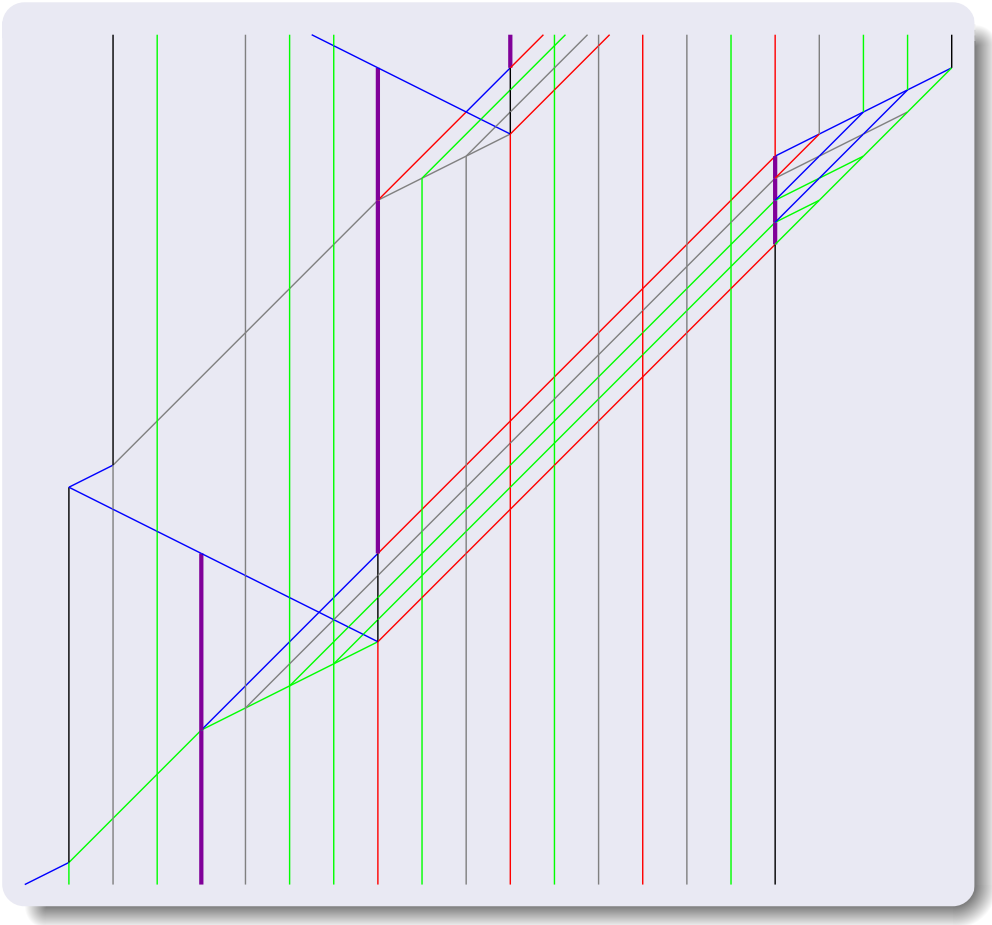


Minimality — bounds on the number of...

...*meta-signals* to...

Accumulate 4

Compute 13 meta-signals
(21 collision rules)
Cyclic tag system
[Durand-Lose, 2011]



Minimality — bounds on the number of...

...*meta-signals* to...

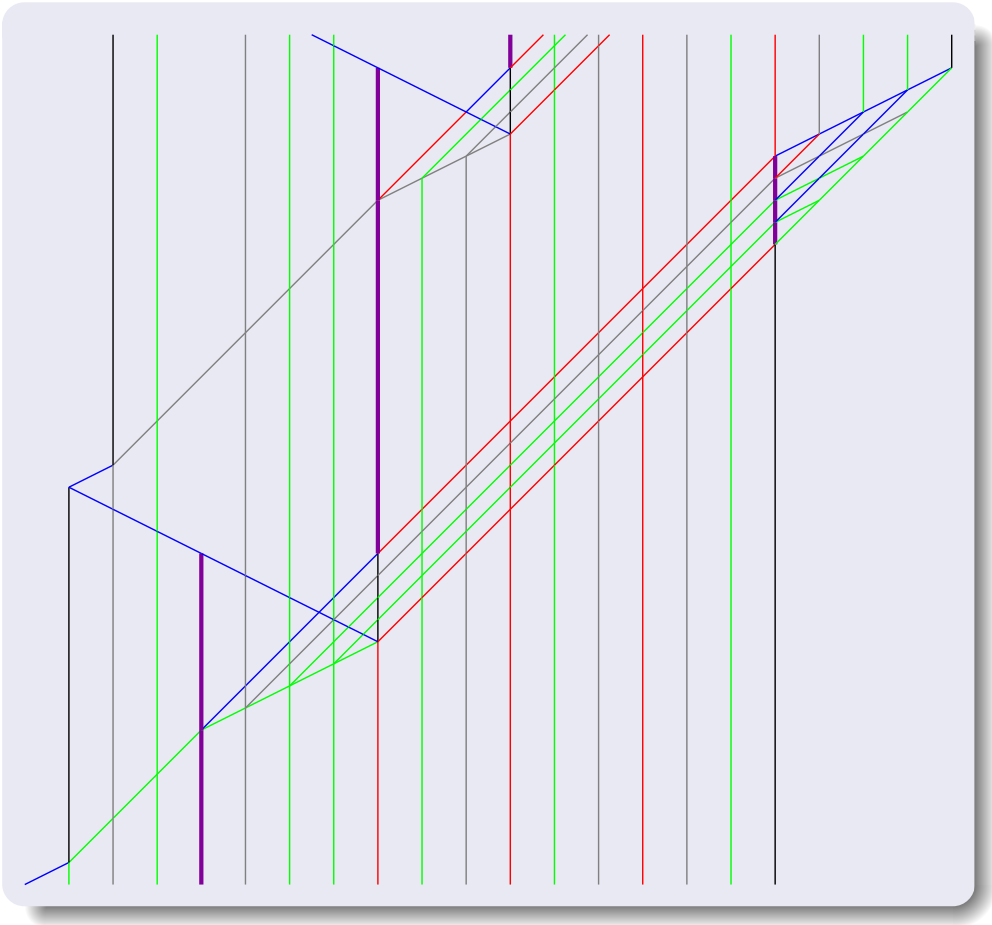
Accumulate 4

Compute 13 meta-signals
(21 collision rules)
Cyclic tag system
[Durand-Lose, 2011]

...*speeds* to...

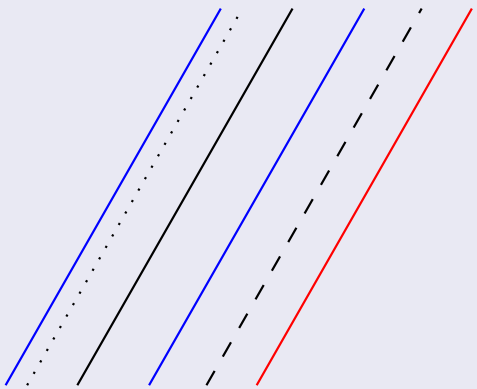
Accumulate *this talk*
[Becker et al., 2013]

Compute *this talk*



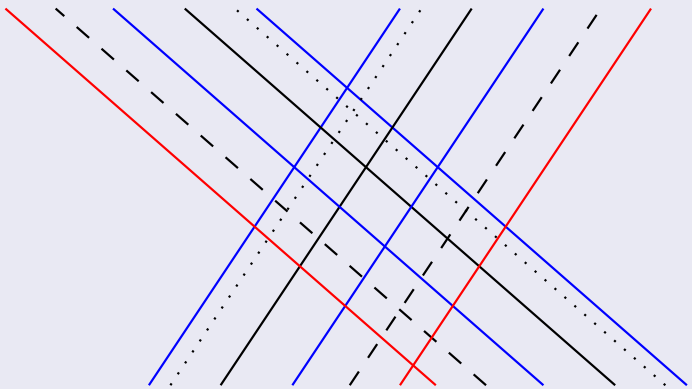
1 or 2 speeds

1 speed



No collision at all

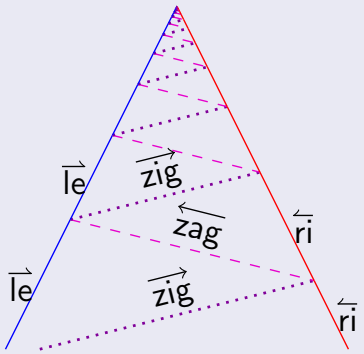
2 speeds



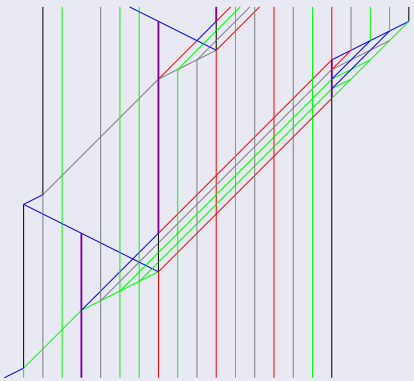
Bounded number of collisions

- No accumulation
- Not Turing-universal

Accumulation

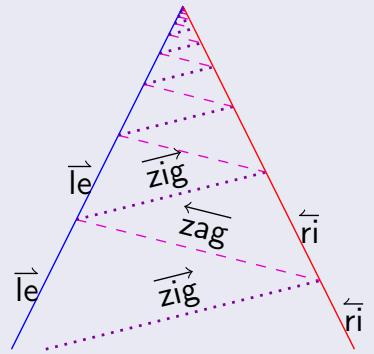


Computation

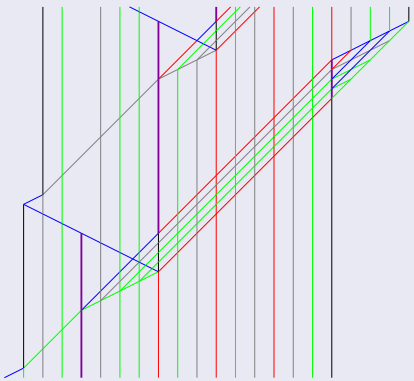


- Accumulation
- Turing-universal

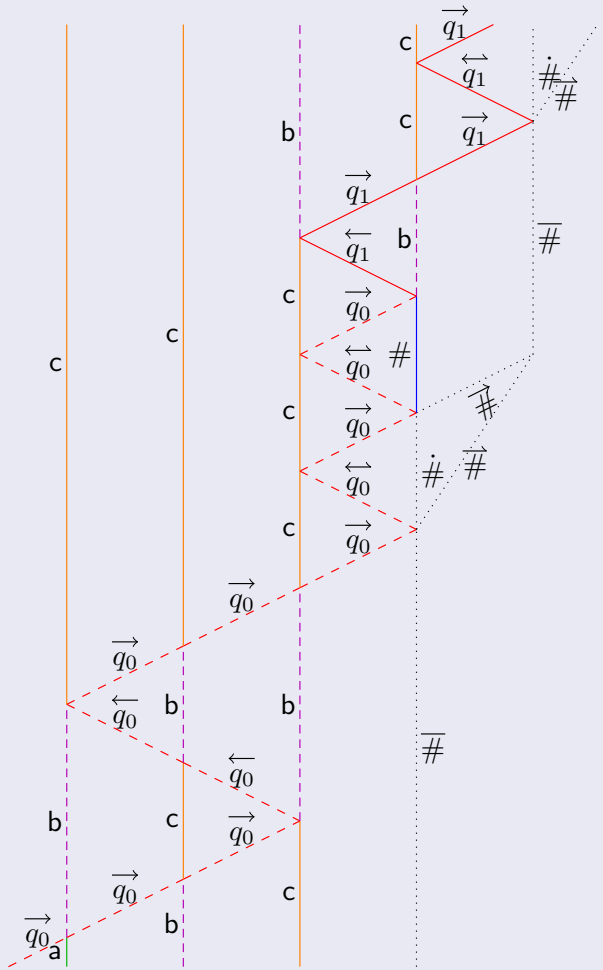
Accumulation



Computation



Computation by TM simulation



- Accumulation
- Turing-universal

Rational case ($\mathbb{Q}$)

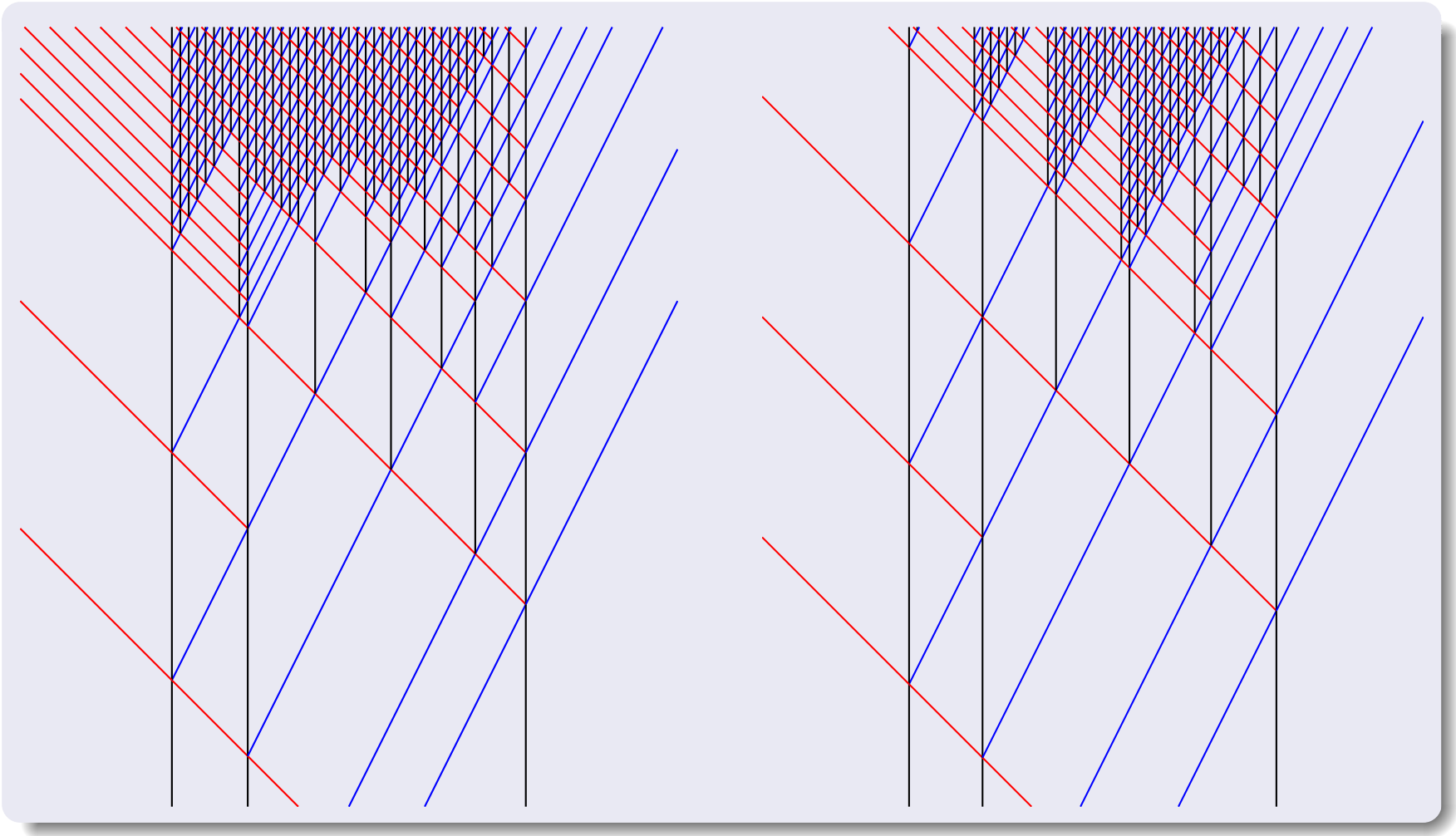
- Rational speeds
- Rational initial positions

$\rightsquigarrow$ Collisions at rational positions
as the solution of systems of two rational linear equations

Implemented in Java

- Exact precision (on $\mathbb{Q}$)
- Tons of space-time diagrams

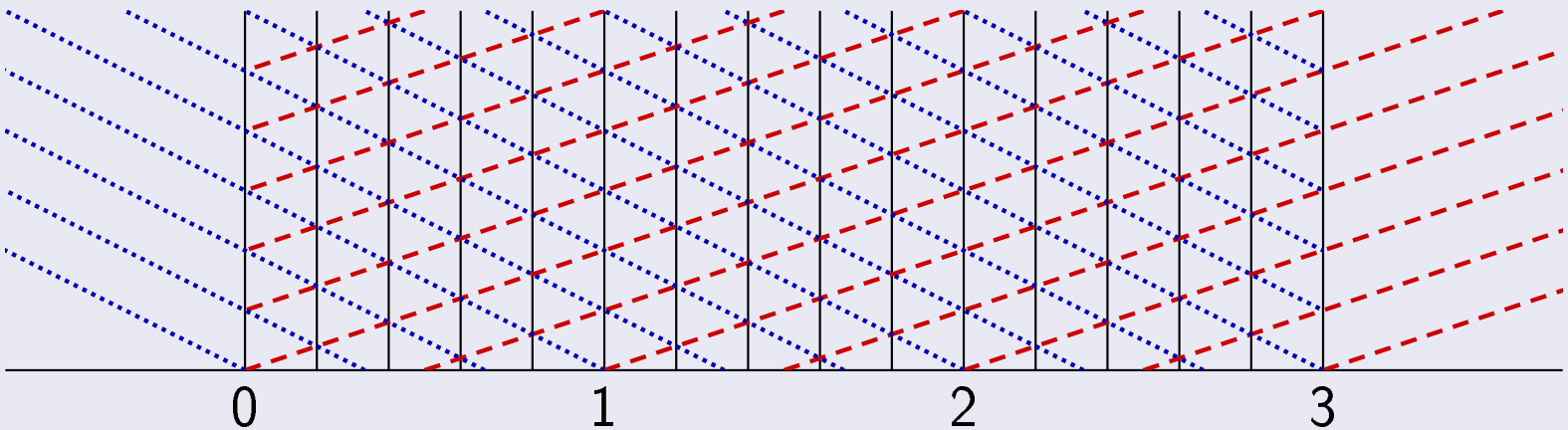
Rational space-time diagrams



Embedded in a mesh

- Some gcd computations [Becker et al., 2013]
- Embedded in a mesh [Becker et al., 2013]

Mesh

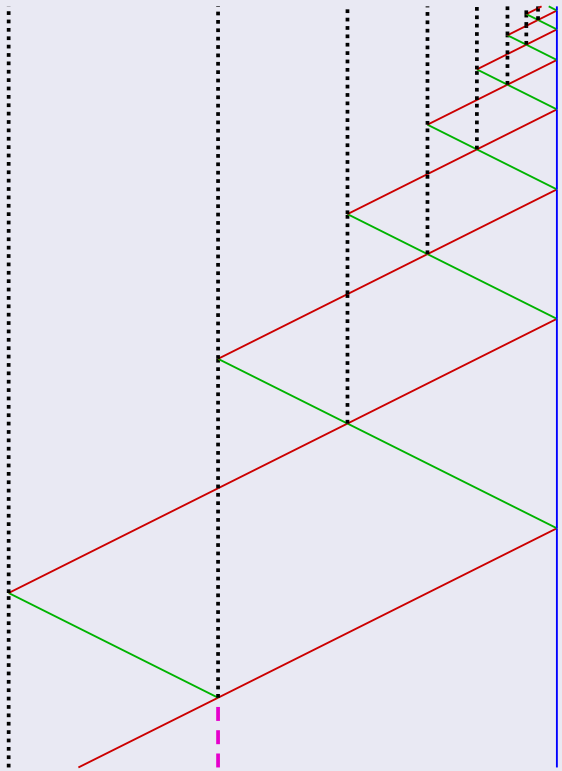


Results

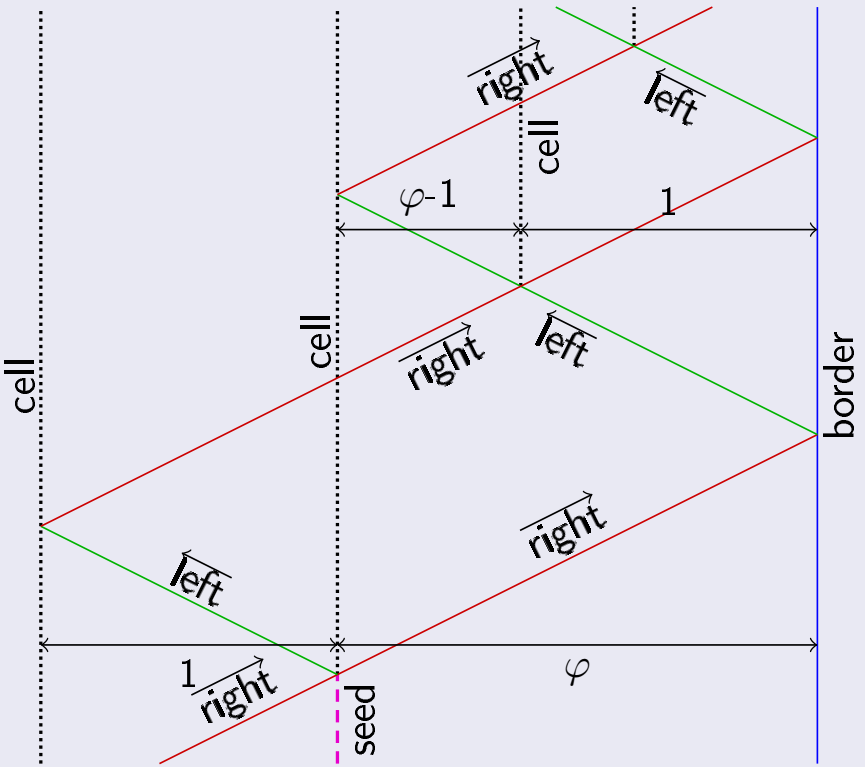
- No accumulation [Becker et al., 2013]
- No computation

Simple fractal construction [Becker et al., 2013]

Fractal



Fractal construction

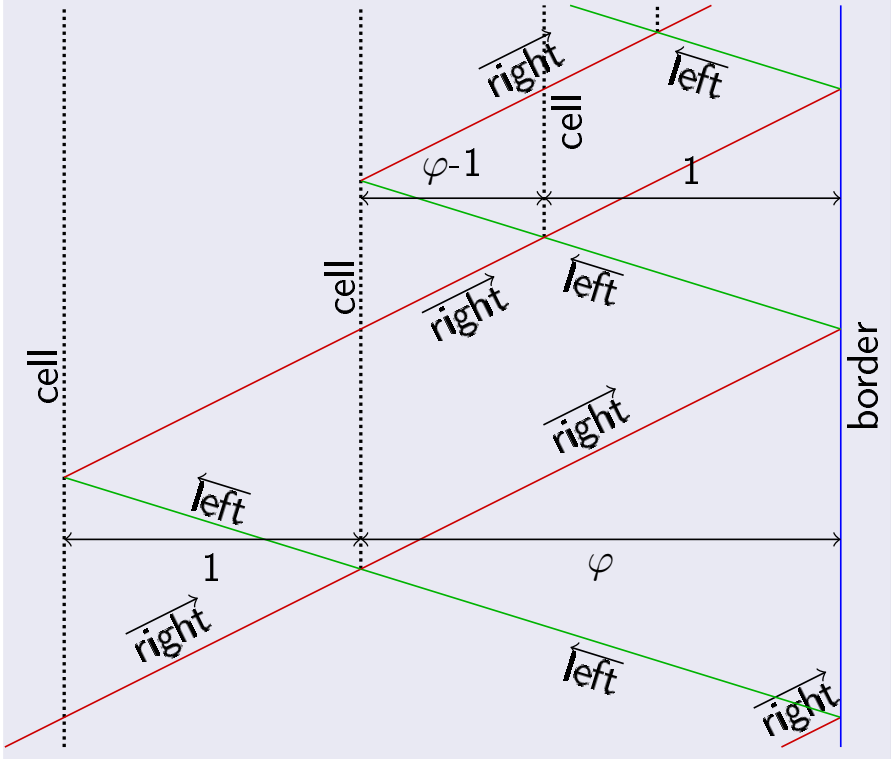


Irrational initial positions $(0, 0.6, 1, \varphi)$, *rational* speeds $(-2, 0, 2)$

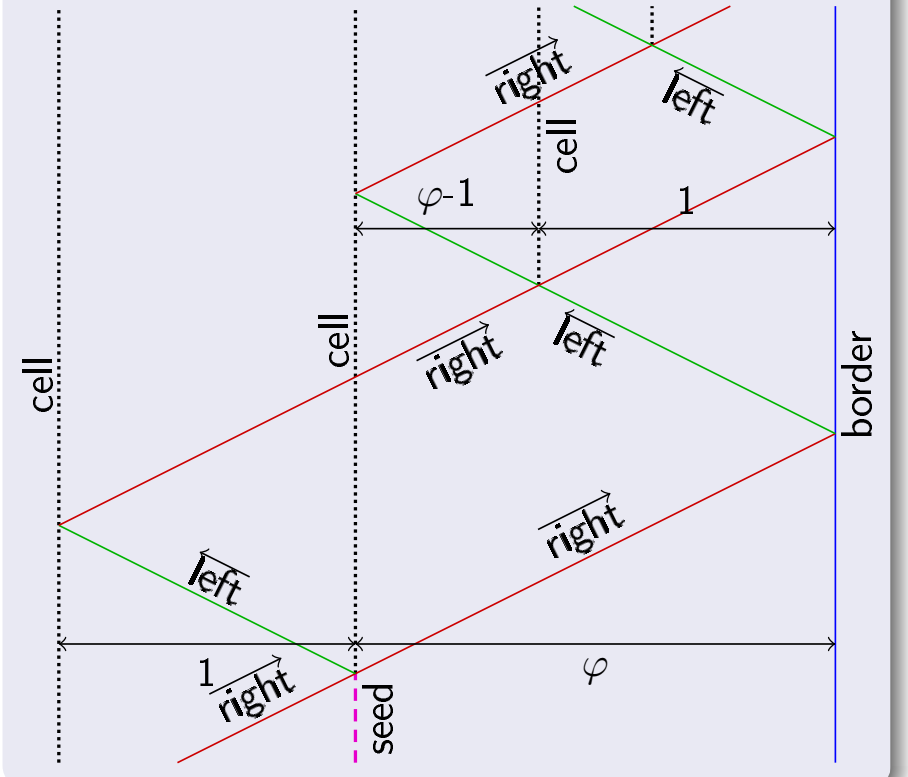
φ must satisfy $\frac{\varphi}{1} = \frac{1}{\varphi - 1}$ φ is the *Golden ratio*

Simple fractal construction [Becker et al., 2013]

Fractal construction



Fractal construction

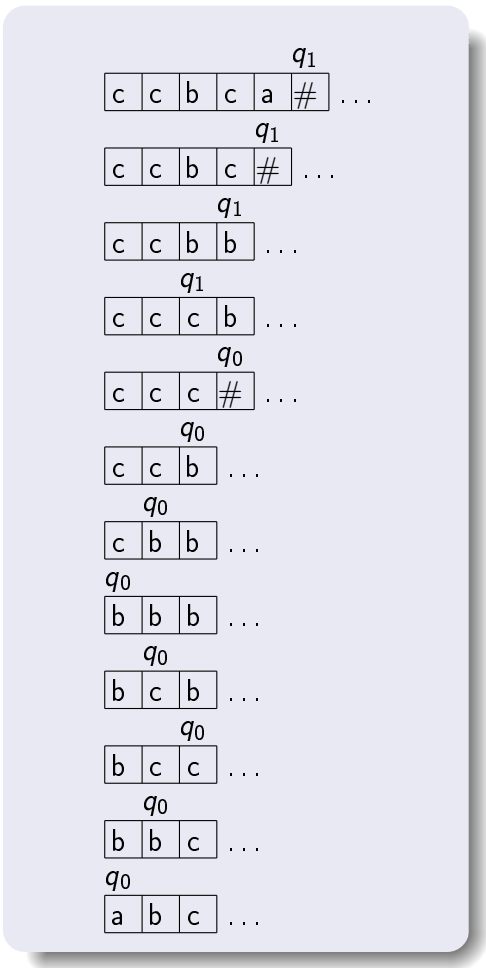


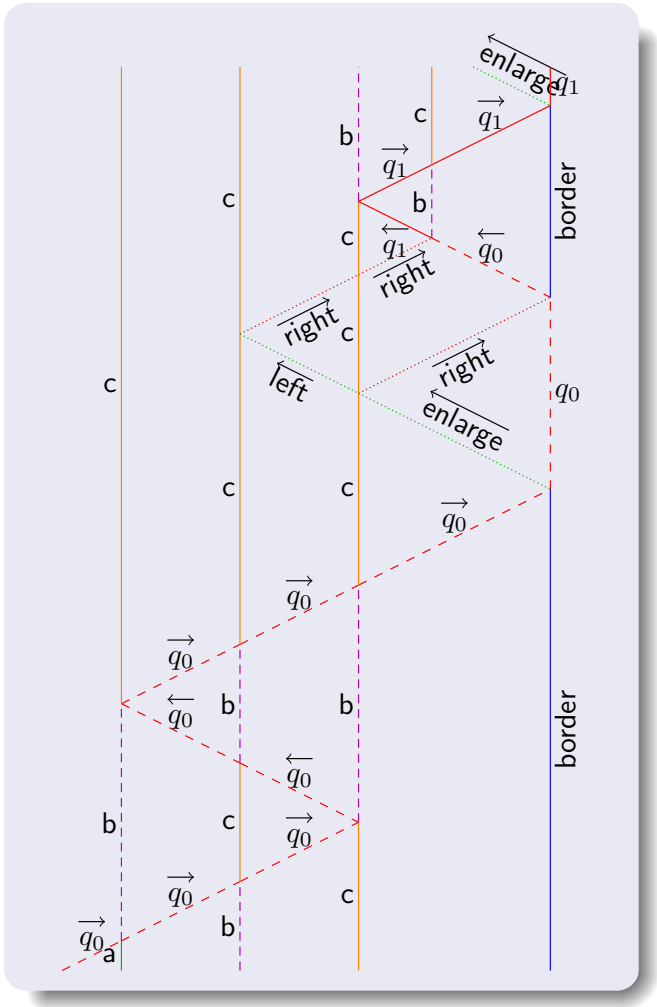
Rational initial positions $(-.1, 0, .9, 1)$, *irrational* speeds $(-2\varphi, 0, 2)$

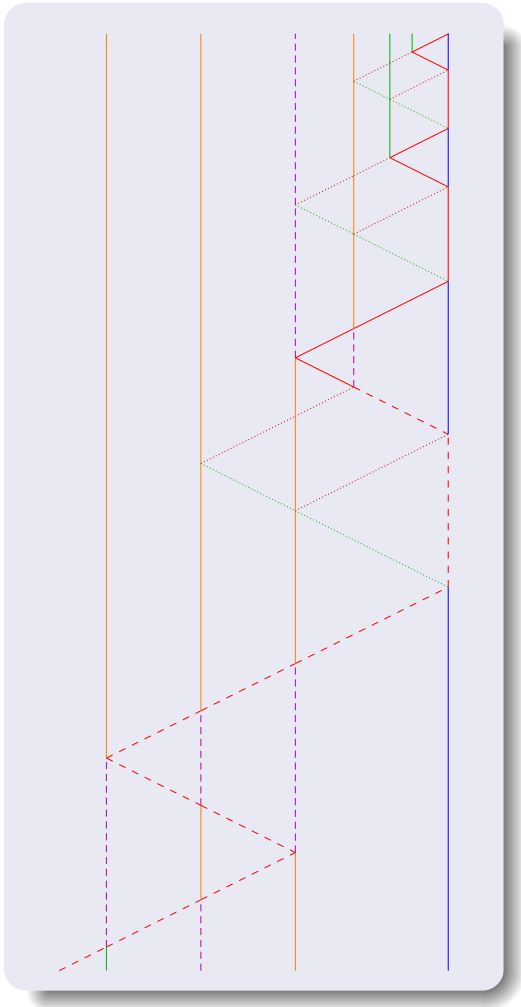
φ must satisfy $\frac{\varphi}{1} = \frac{1}{\varphi - 1}$ φ is the *Golden ratio*

How to enlarge the tape?

- Use the fractal... without generating it!







Results

Rational signal machines

- Up to *normalization* ($\rightsquigarrow$ rational ratios)
- At least 4 speeds to accumulate or compute

With an irrational ratio between initial distances

- 3 (rational) speeds are enough to accumulate and compute
- Turing-universal 25-meta-signal 3-speed signal machine with the Golden ratio

With an irrational ratio between speeds

- Can be used to get an irrational ratio between distances
- 3 (rational) speeds are enough both to accumulate and compute

Future work

- Use irrational values as oracle
- Black hole (hyper-)computation
- Analog computation?

Becker, F., Chapelle, M., Durand-Lose, J., Levorato, V., and Senot, M. (2013).

Abstract geometrical computation 8: Small machines, accumulations & rationality.

Submitted.

Durand-Lose, J. (2011).

Abstract geometrical computation 4: small Turing universal signal machines.

Theoret. Comp. Sci., 412:57–67.