Sort and Search: Exact algorithms for generalized domination

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Abstract

In 1994, Telle introduced the following notion of domination, which generalizes many domination-type graph invariants. Let $\sigma$ and $\varrho$ be two sets of non negative integers. A vertex subset $S \subseteq V$ of an undirected graph $G = (V, E)$ is called a $(\sigma, \varrho)$-dominating set of $G$ if $|N(v) \cap S| \in \sigma$ for all $v \in S$ and $|N(v) \cap S| \in \varrho$ for all $v \in V \setminus S$. In this paper, we prove that decision, optimization, and counting variants of $(\{p\}, \{q\})$-domination are solvable in time $2^{\frac{|V|}{2}} \cdot |V|^{O(1)}$. We also show how to extend these results for infinite $\sigma = \{p + m \cdot \ell : \ell \in \mathbb{N}_0\}$ and $\varrho = \{q + m \cdot \ell : \ell \in \mathbb{N}_0\}$. For the case $|\sigma| + |\varrho| = 3$, these problems can be solved in time $3^{\frac{|V|}{2}} \cdot |V|^{O(1)}$, and similarly to the case $|\sigma| = |\varrho| = 1$ it is possible to extend the algorithm for some infinite sets.

1 Introduction

Let $G = (V, E)$ be a finite undirected graph without loops or multiple edges. Here $V$ is the set of vertices and $E$ the set of edges. Throughout the paper we reserve $n = |V|$. We call two vertices $u, v$ adjacent if they form an edge, i.e., if $uv \in E$. The open neighborhood of a vertex $u \in V$ is the set of the vertices adjacent to it, denoted by $N(u) = \{x : xu \in E\}$. A set of vertices $S \subseteq V$ is dominating if every vertex of $G$ is either in $S$ or adjacent to a vertex in $S$. Finding a dominating set of the smallest possible size is one of the basic optimization problems on graphs. This problem is also known to be notoriously hard. The problem is NP-hard even for chordal graphs (cf. [6]), and the parameterized version is W[2]-complete [2].

Many generalizations have been studied, such as independent dominating set, connected dominating set, efficient dominating set, etc. (cf. [6]). In [10], Telle introduced the following framework of domination-type graph invariants. Let $\sigma$ and $\varrho$ be two non empty

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sets of non negative integers. A vertex subset $S \subseteq V$ of an undirected graph $G = (V, E)$ is called a $(\sigma, \varrho)$-dominating set of $G$ if $|N(v) \cap S| \in \sigma$ for all $v \in S$ and $|N(v) \cap S| \in \varrho$ for all $v \in V \setminus S$. The following table shows a sample of previously defined and studied graph invariants which can be expressed in this framework.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\varrho$</th>
<th>$(\sigma, \varrho)$-dominating set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{N}_0$</td>
<td>$\mathbb{N}$</td>
<td>dominating set</td>
</tr>
<tr>
<td>${0}$</td>
<td>$\mathbb{N}_0$</td>
<td>independent set</td>
</tr>
<tr>
<td>$\mathbb{N}_0$</td>
<td>${1}$</td>
<td>efficient dominating set</td>
</tr>
<tr>
<td>${0}$</td>
<td>${1}$</td>
<td>1-perfect code</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>${0, 1}$</td>
<td>strong stable set</td>
</tr>
<tr>
<td>${0}$</td>
<td>$\mathbb{N}$</td>
<td>independent dominating set</td>
</tr>
<tr>
<td>${1}$</td>
<td>${0}$</td>
<td>total perfect dominating set</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>$\mathbb{N}$</td>
<td>total dominating set</td>
</tr>
<tr>
<td>${1}$</td>
<td>$\mathbb{N}_0$</td>
<td>induced matching</td>
</tr>
<tr>
<td>${r}$</td>
<td>$\mathbb{N}_0$</td>
<td>$r$-regular induced subgraph</td>
</tr>
</tbody>
</table>

Table 1: Examples of $(\sigma, \varrho)$-dominating sets, $\mathbb{N}$ is the set of positive integers, $\mathbb{N}_0$ is the set of non negative integers.

We are interested in the computational complexity of decision, search and counting problems related to $(\sigma, \varrho)$-domination. Explicitly, we consider the following problems for some special sets $\sigma$ and $\varrho$.

$\exists (\sigma, \varrho)$-DS: Does an input graph $G$ contain a $(\sigma, \varrho)$-dominating set?

$\#-(\sigma, \varrho)$-DS: Given a graph $G$, determine the number of $(\sigma, \varrho)$-dominating sets of $G$.

MAX-$(\sigma, \varrho)$-DS: Given a graph $G$, find a $(\sigma, \varrho)$-dominating set of maximum size.

MIN-$(\sigma, \varrho)$-DS: Given a graph $G$, find a $(\sigma, \varrho)$-dominating set of minimum size.

It is interesting to note that already the existence problem is NP-complete for many parameter pairs $\sigma$ and $\varrho$, including some of those listed in Table 1 (1-perfect code and total perfect dominating set). In fact, Telle [10] proves that $\exists (\sigma, \varrho)$-DS is NP-complete for every two finite non empty sets $\sigma, \varrho$ such that $0 \not\in \varrho$.

In this paper we show that for a sufficiently large set of decision, optimization, and even counting $(\sigma, \varrho)$-dominating problems there are exact algorithms of running time $O^*(2^{n/2})$.\footnote{As has recently become standard, we write $f(n) = O^*(g(n))$ if $f(n) \leq p(n) \cdot g(n)$ for some polynomial $p(n)$.}

Our approach is built on a classical technique of Horowitz & Sahni [7], and Schroeppel & Shamir [9] (see also the survey of Woeginger [11]). The basic idea is a clever use of sorting and searching, and thus we call it Sort & Search.

Let us briefly recall the main ideas of this paradigm. The original problem of size $n$, say an input graph $G$ on $n$ vertices, is divided into two subproblems, say two disjoint vertex subsets $V_1$ and $V_2$ of size $n/2$. For each subset $S \subseteq V_i$ ($i \in \{1, 2\}$) a vector of length $n$ is assigned and stored in a table $T_i$. The definition of the vectors is of course problem dependent. Now $T_1$ and $T_2$ contain each at most $2^{n/2}$ different vectors. Then each solution of the problem corresponds to a vector $\vec{a}$ of the first subproblem and a vector
Sort & Search algorithms for the case $|\sigma| = |\varrho| = 1$

Recall that even for one element sets $\sigma = \{p\}$ and $\varrho = \{q\}$, $\exists(\sigma, \varrho)$-DS remains NP-complete [10] if $q \neq 0$. It is known that Perfect Code (or $\exists(\{0\}, \{1\})$-DS) can be solved in time $O(1.1730^n)$ by reduction to the exact satisfiability problem (called XSAT) [1]. Our use of Sort & Search is inspired by the aforementioned algorithms.

**Theorem 1.** $\exists(\{p\}, \{q\})$-DS, $\#-(\{p\}, \{q\})$-DS, MAX-(\{p\}, \{q\})-DS and MIN-(\{p\}, \{q\})-DS are solvable in time $O^*(2^n/2)$.

**Proof.** Let $p, q \in \mathbb{N}_0$. Let $G = (V, E)$ be the input graph and let $k = \lfloor n/2 \rfloor$. As explained in the introduction, the algorithm partitions the set of vertices into $V_1 = \{v_1, v_2, \ldots, v_k\}$ and $V_2 = \{v_{k+1}, \ldots, v_n\}$. Then for each subset $S_1 \subseteq V_1$, it computes the vector $s_1 = (x_1, \ldots, x_k, x_{k+1}, \ldots, x_n)$ where

$$x_i = \begin{cases} 
p - |N(v_i) \cap S_1| & \text{if } 1 \leq i \leq k \text{ and } v_i \in S_1 
p - |N(v_i) \cap S_1| & \text{if } 1 \leq i \leq k \text{ and } v_i \notin S_1 \
|N(v_i) \cap S_1| & \text{if } 1 \leq i \leq k \text{ and } v_i \notin S_1 \end{cases}$$

and for each subset $S_2 \subseteq V_2$, it computes the corresponding vector $s_2 = (y_1, \ldots, x_y, y_{k+1}, \ldots, y_n)$ where

$$y_i = \begin{cases} 
|N(v_i) \cap S_2| & \text{if } 1 \leq i \leq k \n p - |N(v_i) \cap S_2| & \text{if } k + 1 \leq i \leq n \text{ and } v_i \in S_2 
q - |N(v_i) \cap S_2| & \text{if } k + 1 \leq i \leq n \text{ and } v_i \notin S_2. \end{cases}$$

After computing all these vectors (the total number of vectors is at most $2^{k+1}$), we sort vectors corresponding to $V_2$ lexicographically. Then for each vector $s_1$ representing $S_1 \subseteq V_1$, we use binary search to test whether there exists a vector $s_2$ representing $S_2 \subseteq V_2$, such that $s_2 = s_1$. Note that the choice of the vectors guarantees that $s_2 = s_1$ if and only if $S_1 \cup S_2$ is a $(\{p\}, \{q\})$-dominating set. We use here the fact that for $i \leq k$, the vertex $v_i$ has $p - x_i$ neighbors in $S_1$ if $v_i \in S_1$ ($q - x_i$ neighbors if $v_i \notin S_1$ respectively), and it has $y_i$ neighbors in $S_2$. Symmetrically, for $i \geq k + 1$, $v_i$ has $x_i$ neighbors in $S_1$, and it has $p - y_i$ neighbors in $S_2$ if $v_i \in S_2$ ($q - y_i$ neighbors if $v_i \notin S_2$ respectively).
can be found in time $n \log 2^{n/2}$ among the lexicographically ordered vectors of $V_2$. Thus $\exists\{\{p\}, \{q\}\}$-DS is solvable in time $O^*(2^{n/2})$. the overall running time is $O^*(2^{n/2})$.

Now we consider $\#\{(p), \{q\}\}$-DS. The algorithm of the previous theorem only needs to be modified as follows: Instead of storing all vectors corresponding to $V_1$ and $V_2$ multiple copies are removed and each vector is stored with an entry indicating its number of occurrences. Denote by $X_1$ the set of all different vectors corresponding to subsets of $V_1$, and by $X_2$ the set of vectors corresponding to subsets of $V_2$. Let $#_1(s_i)$ be the number of subsets of $V_1$ which correspond to $s_i \in X_1$, and let $#_2(s_2)$ be the number of subsets of $V_2$ corresponding to $s_2$. As for $\exists\{\{p\}, \{q\}\}$-DS, for every $s \in X_1$, we check whether $s$ is included to $X_2$ as well. Then the number of different $(\sigma, \varrho)$-dominating sets is

$$\sum_{s \in X_1 \cap X_2} #_1(s) \cdot #_2(s)$$

if $X_1 \cap X_2 \neq \emptyset$, and this number is 0 otherwise.

Furthermore, for $\text{MAX-}\{\{p\}, \{q\}\}$-DS, with each vector $s \in X_1$ we store the subset $S_i(s) \subseteq V_i$ of maximum cardinality that generates this vector. It can be easily seen that a $(\sigma, \varrho)$-dominating set of maximum size (if it exists) is the set $S = S_1(s^*) \cup S_2(s^*)$ such that $s^*$ is a vector of $X_1 \cap X_2$ with $|S_1(s^*)| + |S_2(s^*)| = \max_{s \in X_1 \cap X_2} |S_1(s)| + |S_2(s)|$. It is not hard to see that $\text{MIN-}\{\{p\}, \{q\}\}$-DS can be solved in the same way by replacing maximum by minimum.

Now we extend our approach to certain infinite $\sigma$ and $\varrho$. Let $m \geq 2$ be a fixed integer and $k \in \{0, 1, \ldots, m-1\}$. We denote by $k + m \mathbb{N}_0$ the set $\{k + m \cdot \ell : \ell \in \mathbb{N}_0\}$. Particularly, for $m = 2$ and $k = 0, 1$, the set $k + m \mathbb{N}_0$ is either the set of all even nonnegative integers or the set of all odd positive integers respectively. The cases when $\sigma$ and $\varrho$ are such sets are of importance in the coding theory, and they were considered in [3, 5].

**Theorem 2.** Let $m \geq 2$ and $p, q \in \mathbb{N}_0$. The problems $\exists(p + m \mathbb{N}_0, q + m \mathbb{N}_0)$-DS, $\#\{(p + m \mathbb{N}_0, q + m \mathbb{N}_0)\}$-DS, $\text{MAX-}(p + m \mathbb{N}_0, q + m \mathbb{N}_0)$-DS, and $\text{MIN-}(p + m \mathbb{N}_0, q + m \mathbb{N}_0)$-DS are solvable in time $O^*(2^{n/2})$.

**Proof.** For $\#\{(p + m \mathbb{N}_0, q + m \mathbb{N}_0)\}$-DS, the algorithm in Theorem 1 is modified such that for each subset $S_1 \subseteq V_1$ and each $S_2 \subseteq V_2$, we compute the vectors $s^*_1$ and $s^*_2$ respectively in the same way as before with only difference that all component are taken modulo $m$.

For $\#\{(p + m \mathbb{N}_0, q + m \mathbb{N}_0)\}$-DS, $\text{MAX-}(p + m \mathbb{N}_0, q + m \mathbb{N}_0)$-DS and $\text{MIN-}(p + m \mathbb{N}_0, q + m \mathbb{N}_0)$-DS, the modification of the algorithm is similar to the one from Theorem 1, and we omit it here.

Let $\text{EVEN}$ be the set of all even non negative integers and $\text{ODD}$ be the set of odd positive integers. It was shown in [5] that $\exists(\text{EVEN}, \text{EVEN})$-DS, $\exists(\text{EVEN}, \text{ODD})$-DS, $\exists(\text{ODD}, \text{EVEN})$-DS and $\exists(\text{ODD}, \text{ODD})$-DS can be solved in polynomial time while maximization and minimization problems are NP-hard. It follows immediately from Theorem 2 that for $\sigma, \varrho \in \{\text{EVEN}, \text{ODD}\}$, the problems $\#\{\sigma, \varrho\}$-DS, $\text{MAX-}\{\sigma, \varrho\}$-DS and $\text{MIN-}\{\sigma, \varrho\}$-DS are solvable in time $O^*(2^{n/2})$.

Special variants of these problems for bipartite graphs were considered in [3]. Suppose that $G = (R, B, E)$ is a bipartite graph with $R, B$ a bipartition of the vertex set. To distinguish different sets, vertices of $R$ are called red and vertices of $B$ are blue. Let $S \subseteq R$ be a non empty set of red vertices. It is said that $S$ is an even set if for every vertex $v \in B$,
\(|N(v)| \in \text{EVEN}\), and \(S\) is an odd set if for every vertex \(v \in B\), \(|N(v)| \in \text{ODD}\). The proof of the following theorem is based on combining the Sort & Search approach with dynamic programming.

**Theorem 3.** Let \(G = (R, B, E)\) be a red/blue bipartite graph. All even or odd sets can be counted, and maximum or minimum even or odd sets can be found in time \(O^*(2^{\min\{|R|, |B|}) = O^*(2^n/3))\).

**Proof.** We prove this claim for the counting problem for even sets. (All other problems can be solved similarly.) Let \(R = \{u_1, \ldots, u_k\}\) and \(B = \{v_1, \ldots, v_r\}\).

If \(k/2 \leq r\), then we apply the following Sort & Search algorithm. Let \(s = \lfloor k/2 \rfloor\). We partition the set of vertices \(R\) into \(R_1 = \{u_1, \ldots, u_s\}\) and \(R_2 = \{u_{s+1}, \ldots, u_k\}\). For each subset \(S_1 \subseteq R_1\), we compute its corresponding vector \(s_1^2 = (x_1, \ldots, x_r)\), where

\[
x_i = \begin{cases} 0 & \text{if } |N(v_i) \cap R_1| \in \text{EVEN} \\ 1 & \text{if } |N(v_i) \cap R_1| \in \text{ODD.} \end{cases}
\]

Similarly for each subset \(S_2 \subseteq R_2\), we compute the corresponding vector \(s_2^2 = (x_1, \ldots, x_r)\), such that

\[
x_i = \begin{cases} 0 & \text{if } |N(v_i) \cap R_2| \in \text{EVEN} \\ 1 & \text{if } |N(v_i) \cap R_2| \in \text{ODD.} \end{cases}
\]

Denote by \(X_1\) the set of all different vectors corresponding to subsets of \(V_1\), and by \(X_2\) the set of vectors corresponding to subsets of \(V_2\). Let \(#_1(s^1)\) be the number of subsets of \(R_1\) corresponding to \(s^1 \in X_1\), and let \(#_2(s^2)\) be the number of subsets of \(R_2\) corresponding to \(s^2\). After vectors are computed, we sort the vectors of \(X_2\) lexicographically. For each vector \(s^1 \in X_1\) we search for a vector \(s^2 \in X_2\) such that \(s^2 = s^1\). The total number of non empty even sets is

\[
\sum_{s \in X_1 \cap X_2} \#_1(s) \cdot \#_2(s) - 1,
\]

and then the running time of this procedure is \(O^*(2^{|R|/2})\).

For the case \(k/2 > r\), we use dynamic programming approach across the subsets. For every subset \(S \subseteq R\), let \(\tilde{s}(S) = (x_1, \ldots, x_r)\), where

\[
x_i = \begin{cases} 0 & \text{if } |N(v_i) \cap S| \in \text{EVEN} \\ 1 & \text{if } |N(v_i) \cap S| \in \text{ODD.} \end{cases}
\]

For every \(i \in \{1, \ldots, k\}\), and every vector \(\tilde{s} \in Z_2^r\), we put

\[
\#(i, \tilde{s}) = |\{S \subseteq \{u_1, \ldots, u_i\} : \tilde{s}(S) = \tilde{s}\}|.
\]

We also put \(#(0, 0) = 0\) for all non zero vectors \(\tilde{s}\), and \(#(0, 0) = 1\). For \(i \in \{1, \ldots, k\}\), we denote by \(\tilde{z}_i\) the vector \((y_1, \ldots, y_r)\), where

\[
y_j = \begin{cases} 1 & \text{if } v_j \in N(u_i) \\ 0 & \text{if } v_j \notin N(u_i). \end{cases}
\]

Since \(#(i, \tilde{s}) = #(i-1, \tilde{s}) + #(i-1, \tilde{s} + \tilde{z}_i)\), we have that all values \(#(i, \tilde{s})\) can be computed in time \(O^*(2^{|B|/2})\) by a dynamic programming approach considering the values \(i\) by increasing order. It remains to note that the number of non empty even sets is \(#(k, 0) - 1\).
3 Extending the Sort & Search approach

It is possible to extend (albeit with a worse running time) our results for single-element sets for the case when one set contains two elements and the other set is a singleton.

**Theorem 4.** The problems \(\exists(\sigma, \varrho)\)-DS, \(\#(\sigma, \varrho)\)-DS, \(\text{Max}-(\sigma, \varrho)\)-DS, and \(\text{Min}-(\sigma, \varrho)\)-DS are solvable in time \(O^*(3^n/2)\) if \(|\sigma| + |\varrho| = 3\).

**Proof.** We prove the theorem for \(\exists(\sigma, \varrho)\)-DS and \(\sigma = \{p_1, p_2\}, \varrho = \{\varrho\}\). Let \(G = (V, E)\) be a graph and \(k = \lfloor n/2 \rfloor\). As in all algorithms above, we partition the set of vertices \(V\) into three sets \(\{v_1, v_2, \ldots, v_k\}\) and \(V_2 = \{v_{k+1}, \ldots, v_n\}\). Now for every partition of \(V_1\) into three sets \(\{S^{(1)}_1, S^{(1)}_2, S^{(1)}_3\}\) (some of these sets can be empty), we compute the vector \(s_1 = (x_1, x_2, x_3, \ldots, x_n)\) where

\[
x_i = \begin{cases} 
    p_1 - |N(v_i) \cap S^{(1)}_i| & \text{if } 1 \leq i \leq k \text{ and } v_i \in S^{(1)}_i \\
    p_2 - |N(v_i) \cap S^{(1)}_2| & \text{if } 1 \leq i \leq k \text{ and } v_i \in S^{(1)}_2 \\
    q - |N(v_i) \cap S^{(1)}_3| & \text{if } 1 \leq i \leq k \text{ and } v_i \in S^{(1)}_3 \\
    |N(v_i) \cap (S^{(1)}_1 \cup S^{(1)}_2)| & \text{if } k + 1 \leq i \leq n.
\end{cases}
\]

Symmetrically, for each partition of \(V_2\) into three sets \(\{S^{(2)}_1, S^{(2)}_2, S^{(2)}_3\}\), we compute the corresponding vector \(s_2 = (x_1, x_2, x_3, \ldots, x_n)\), where

\[
x_i = \begin{cases} 
    |N(v_i) \cap (S^{(2)}_1 \cup S^{(2)}_2)| & \text{if } 1 \leq i \leq k \\
    p_1 - |N(v_i) \cap S^{(2)}_1| & \text{if } k + 1 \leq i \leq n \text{ and } v_i \in S^{(2)}_1 \\
    p_2 - |N(v_i) \cap S^{(2)}_2| & \text{if } k + 1 \leq i \leq n \text{ and } v_i \in S^{(2)}_2 \\
    q - |N(v_i) \cap S^{(2)}_3| & \text{if } k + 1 \leq i \leq n \text{ and } v_i \in S^{(2)}_3.
\end{cases}
\]

After computing these \(3^k+1\) vectors, the algorithm sorts vectors of \(V_2\) lexicographically, and for each vector \(s_1\) (corresponding to a partition of \(V_1\), search for a vector \(s_2\) from \(V_2\), such that \(s_2 = s_1\). Note that \(s_2 = s_1\) if and only if \((S^{(1)}_1 \cup S^{(1)}_2) \cup (S^{(2)}_1 \cup S^{(2)}_2)\) is a \((\{\sigma\}, \{\varrho\})\)-dominating set. Since the search of \(s_2\) can be done in time \(n \log 3^n/n\), we have that the overall running time of the algorithm is \(O^*(3^n/2)\).

The problems \(\exists(\sigma, \varrho)\)-DS with \(\sigma = \{p\}\) and \(\varrho = \{\varrho_1, \varrho_2\}\) are solved similarly. Moreover the algorithm can easily be extended to solve the counting, maximization and minimization version of the problem as it was done in Theorem 1 for single-element sets.

The algorithms of Theorem 4 can be modified to handle some infinite sets as it was done in Theorem 2. In that case, all components of vectors are taken modulo \(m\) and the addition and/or subtraction of vector components is taken modulo \(m\).

**Corollary 5.** Let \(m \geq 2\) and \(p_1, p_2, q_1, q_2 \in \mathbb{N}_0\). The problems \(\exists(\sigma, \varrho)\)-DS, \(\#(\sigma, \varrho)\)-DS, \(\text{Max}-(\sigma, \varrho)\)-DS and \(\text{Min}-(\sigma, \varrho)\)-DS are solvable in time \(O^*(3^n/2)\) for pairs of sets \(\sigma = (p_1 + m\mathbb{N}_0) \cup (p_2 + m\mathbb{N}_0), \varrho = q_1 + m\mathbb{N}_0\) and \(\sigma = p_1 + m\mathbb{N}_0, \varrho = (q_1 + m\mathbb{N}_0) \cup (q_2 + m\mathbb{N}_0)\).
4 Conclusion

We considered exact algorithms for \((\sigma, \varrho)\)-dominating set problems for some special sets \(\sigma\) and \(\varrho\), assuming they are the same for all vertices. However it is possible to define a more general problem. Let \(G\) be a graph such that for any vertex \(v \in V\), two non empty sets of non negative integers \(\sigma(v)\) and \(\rho(v)\) are given. A vertex subset \(S \subseteq V\) of the graph \(G\) is called now a \((\sigma, \varrho)\)-dominating set of \(G\) if \(|N(v) \cap S| \in \sigma(v)\) for all \(v \in S\) and \(|N(v) \cap S| \in \varrho(v)\) for all \(v \in V \setminus S\). It should be noted that all of our algorithms can be adopted to solve these problems too.

A natural open question is whether \((\sigma, \varrho)\)-dominating set problem can be solved in time \((2 - \varepsilon)^n\) for some \(\varepsilon > 0\) for any choice of sets \(\sigma\) and \(\varrho\). It does not seem that Sort & Search can be used to settle this question. In [4], we suggested a different approach for obtaining \((2 - \varepsilon)^n\) algorithms for various choices of \(\sigma\) and \(\varrho\), but we are still far from the complete answer.

References


