

Debugging Systems for Constraint Programming

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SIMPLIFICATION OF FINITE DOMAIN CONSTRAINTS

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This paper is an extended abstract of the DEA dissertation (in French) in appendix.

Abstract

An important issue in Constraint Logic Programming (CLP) systems is how to output constraints in a usable form. Typically, only a small subset \tilde{x} of the variables in constraints is of interest, and so an informal statement of the problem at hand is: given a conjunction $C(\tilde{x}, \tilde{y})$ of constraints, express $\exists \tilde{y} C(\tilde{x}, \tilde{y})$ in the simplest form. [13] showed how a set of constraints over the real domain can be simplified. We present here how we can simplify a set of constraints over finite domains. First, we start by a brief sight on the simplification of the constraints over real domain, then we show how we can extend those techniques of simplification over real domain to the finite domain.

This work takes part in the project: ESPRIT DiSCiPl (Debugging Systems for Constraints Programming).

The work is done in the LOCO project (common to INRIA and University of Orléans).

In several parts of the project, the problem of the display of a set of constraints was stated. This problem is not new, indeed it appears yet in the realisation of CLP systems for the display of solutions to a goal: It's the *presentation problem*. For that matter, in the DiSCiPl project, several solutions were suggested: Such as graphics tools, approximations,...etc. Here, we chose the transformation into an equivalent set of constraints, more legible.

One of the LOCO task in the DiSCiPl ESPRIT project is the declarative error diagnosis (task T.WP2.1, declarative debugging).

During a declarative error diagnosis session, the tool can be led to ask questions to the user, for example: of the form:

$$\exists \tilde{y} C \longrightarrow a \quad ?$$

where

- C is a conjunction of constraints;
- a is an atom;
- and \tilde{y} are the free variables of C which are not free in a .

The aim is to simplify (meaning making it easier to understand, to read), the formula $\exists \tilde{y} C$. For that, we can eliminate existential variables, eliminate redundant constraints and make symbolic transformations (more precisely, substitution of the formula by another one straightforward and equivalent, using for example techniques of automatic demonstration).

1 Introduction

The unit came out from the CLP(\mathbb{R}) was described with details in [13]. The wording of the problem of the constraint simplification over finite domains is the same as the wording on the real domain or more precisely:

Given a conjunction $C(\tilde{x}, \tilde{y})$ of constraints, express $\exists \tilde{y} C(\tilde{x}, \tilde{y})$ in the simplest form.

Example. x and y are the target variables:

- (a) the constraints $x = z+1, z = y+1$ can be output as $x = y+2$;
- (b) the constraints $x < z, z \leq y, z \leq y+2$ can be output as $x < y$;
- (c) the constraints $x = z * y$, can be output as x is multiple of y .

As in the case of the real domain, we can classify the simplification of those constraints in three directions:

- (I) Elimination of auxiliary variables (as in (a), (b));
- (II) Elimination of redundant constraints (as in (b));
- (III) Replacement of expressions par simpler equivalent ones (as in (c)).

The problem of elimination of the redundant constraints (II) on the finite domain seems to be not difficult since it can be treated in parts by methods similar to those on the domaines real. Without any doubt, the elimination of auxiliary variables (I) is the most intricate part for the simplification of constraints over finite domains, indeed, the following example show exactly the difficulty of the problem:

Example. On the real domain, the formula $\exists x (y = 2x)$ can be output as y *real*.

On the finite domain, the formula $\exists x (y = 2x)$ can be output as y *even*.

So the elimination of Fourier become wrong.

The problem of equivalent expression research is also a difficult problem.

We are going to see how we can simplify this wording on finite domains, but before we present a brief sight in the simplification of constraints over the real domain.

2 Output in CLP(\mathbb{R})

In [13], the unit output from the CLP(\mathbb{R}) is described in details. The point is settled on the well known problem of the projection in the linear arithmetic constraints. It starts with the Fourier's classic algorithm and increases it thanks to the elimination process of the redundant constraints generated by this algorithm. Then, it studies the other kinds of constraints, equations over trees and nonlinear constraints and it shows how they can be simplified with the linear constraints. For any kind of details, the reader can refer to [13].

Unfortunately, extension of those simplifications over the real domain to finite domains is not easy. As in the linear programming of constraints case, the simplification of constraints over finite domain is more intricate than over real domain.

3 Output in finite domain

The aim of this paragraph is to present a set of techniques in order to simplify a set of constraints over finite domain. By simplification, we mean reducing as much as possible a set of constraints into another more simple and readable (legible). However, we must make the difference between simplification and resolution which does not mean the same thing (which does not have the same signification).

As it was already evoqued, the simplification of constraints over finite domains can be classified into 3 three directions.

3.1 Elimination of redundant constraints

The problem of elimination is in general a problem which is very difficult, as it is similar to the entailment problem of constraints. We remind that the different categories of redundancy defined over the real domain are: Tautology, Syntactic redundancy, Quasi-syntactic redundancy, Hull redundancy, Facet redundancy, Quasi facet redundancy, Independent redundancy, Implicit redundancy.

Our first approach is to extend all these classes of redundancies which were defined over real domain to finite domains. This time, we remind that our polyhedral defined in the real case by $P = \{Ax \leq b, x \text{ real}\}$, will be integer that is to say:

- all the variables of matrix A will be integer;
- all the variables of vector x will be integer;
- all the variables of vector b will be integer.

The definitions will be the same as those for continuous domains, but this time, every time we must add the above conditions.

However the problem of the detection of this class of redundancy is stated, indeed, we saw that in order to detect the redundancy in the continuous case, we had to resolve the maximisation problem: $\max \{a_i x \mid a_j x \leq \beta_j, j \neq i\}$, which indicates that the constraint $a_j x \leq \beta_j$ is redundant independent if the value returned by the linear program is lower or equal to β_i .

Another problem is that of the detection of implicit equalities, indeed, the constraint $a_i x \leq \beta_i$ is an implicit equality if β_i is returned as optimal solution of the linear program $\min \{a_i x \mid Ax \leq b\}$.

In the real case, those problem of optimisation with constraints are not difficult to solve (by using for example simplexe method) which is not the case for finite domain.

We notice that for the Tautology, Syntactic redundancy, Quasi-syntactic redundancy, there is no problem of detection since their detection in the case of finite domains is the same for real domain.

For the other kinds of redundances: Hull redundancy, Facet redundancy, Quasi facet redundancy, independent redundancy, Implicit redundancy, their detection is not easy at all, but as in the continuous case, these constraints will be transformed by the hull transformation respectively in Tautology, Syntactic redundancy, Quasi-syntactic redundancy, equalities redundancy.

Remark. We remind that our objective is not to solve a constraints system on finite domains, but it is to simplify as much as possible this system into an easier and more legible one. \diamond

In [2], Bockmayr has suggested a method to solve the linear constraint system of Pseudo-Boolean constraints, using approximations for the convex hull of the integer solutions set, by the cutting plane. One of our approach is to extend our techniques for a linear constraints system on the finite domain. Indeed:

Noticing that, any variable x just can take $k+1$ integer values $0, 1, 2, \dots, k$, so we can substitute it by a linear combination:

$$x = y_0 + 2y_1 + 4y_2 + \dots + 2^p y_p$$

of $p+1$ variable: y_0, \dots, y_p , each of them being obliged to take just two value 0 or 1 (bivalents variables) (p is the smallest integer such as $k \leq 2^{p+1} - 1$).

So, we can always came back to the case where the linear system on the finite domain variable is a linear system on the bivalent variables.

(NB) The precedented transformation, which is always possible, is not necessarily always worthwhile in practice).

3.2 Elimination of auxiliary variables

As in the case of continuous domains, the traditional approach for the simplification of constraints is to use the “canonical form” equipped with an efficient algorithm for its calculation.

For linear equation, it is well known that the canonical form is called where equations are represented in the parametric form where the equations are represented in the form $\tilde{x} = t(\tilde{y})$ where the \tilde{y} are distincts of \tilde{x} and t is a tuple of linear expressions.

In this paragraph, we describe an algorithm for the output of $\exists \tilde{y} C(\tilde{x}, \tilde{y})$, where C is linear, only according to target variables, treating first equations and then inequalities.

The algorithm must produce this time an output containing the particular target variable x which appears in C , for example if we eliminate y from $\exists y \mid x = 2y$ (else the information x is even will be lost). So in the contrary of the case of continuous domain, the algorithm will only modify the constraints of the form: $\tilde{x} = t'(\tilde{y})$ where t' is a tuple of linear expressions formed by coefficients equal to 1 or to -1 i.e: all variable x_i is in the form: $x_i = \sum_i \alpha_i y_i$ where $\alpha_i \in \{1, -1\}$.

3.2.1 Linear equations

The equations are always maintained in the *parametric form*, $\tilde{u} = t(\tilde{v})$ where variables of \tilde{u} are called objects variables and are distinct from variables of \tilde{v} , called, parametres.

The algorithm has the same form as that of continuous domains, but this time, it will only modify the equation shown above. the other kinds of linear equations will be simplified with other constraints.

Remark. We can think to turn the constraints $x = 2y$ into the form $x = y + y$, which become the kind of the constraints seen above and then apply the algorithm described above, unfortunately those constraints must be maintained under the parametric form. \diamond

3.2.2 Linear Inequalities

In this paragraph, we adopt the same approach than that of continuous domains i.e: Fourier's algorithm + Cernikov + elimination of strict redundant constraints. Since our aim is to simplify a system of linear inequalities and not to solve it, we can extend the algorithm given in the continuous case to the case of the finite domain.

Example. We want to simplify the formula $\exists y \mid x + y \leq 1$ et $x - y \leq 2$

If we apply the Fourier's algorithm (adding both inequalities), we obtain the similar formula $x \leq 3/2$.

Geometrically, the constraint $x \leq 3/2$ is just the projection of the polyhedron $P = \{x + y \leq 1, x - y \leq 2\}$ in the abscissa axis. (refer to figure 1)

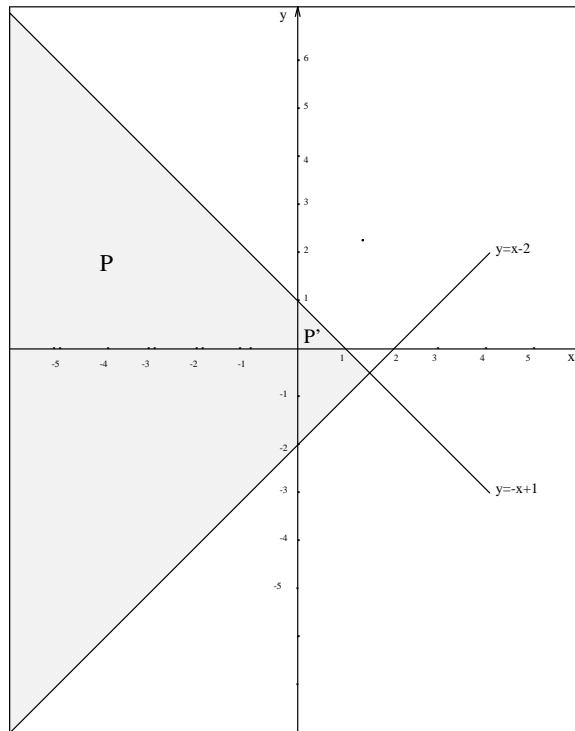


Figure 1: Simplification of linear inequalities

In the finite domain case, the projection of the integer polyhedron $P' = \{x + y \leq 1, -x - y \leq 2, x \text{ integer} \geq 0, y \text{ integer} \geq 0\}$ on the abscissa axis is the constraint $x \leq 1$. (refer to figure 1)

The simplification of linear inequalities over finite domains is the same as over the real domain, except that an inequalities of the form $x \leq 3/2$ may be substitute by $x \leq E(3/2)$, where $E(x)$ refers to the integer part of x .

3.3 Nonlinear Constraints

In the section 3.2, we had to face the problem of the simplification of the constraint $\exists y \mid x = 2y$. We choose to keep this constraints as it is, that is to say that those kind of constraints are not going to be simplified with the lineair constraints. They will be simplified with the constraints of the form $y \times z$, which are non linear constraints.

4 Conclusion

In this work, we were led to study the solutions put foward for the simplification of the constraint over real domain and suggest methods to solve the problem of simplification of the constraints over finite domain.

This task started with a synthesis of works on the output (Simplification, introduction) of constraints over the real domain.

We reminded the main classes of redundancy and the methods of elimination of auxiliary variables as well as tecniques to identify them.

As for finite domains, the techniques suggested for continous domains constitute a part of the solution. Indeed, a redundant constraints over continous polyhedron has also effects over integer polyhedron. Only the problem of the identification of some redundant constraints is stated. For that purpose, we put foward a method to solve this problem which becomes a problem of optimisation.

The main difficulty seem to be the elimination of auxiliary variables where Fourier's method can't be used. We suggest modification to adapt the elimination to finite domains.

This work should enable an introduction of a unit of presentation. (simplification, display) of constraints over finite domain in the DiSCiPl ESPRIT project.

The other kinds of constraints such as disequalities and strict inequalities were not evoked. This work could be the objective of an other work.

Another point should be to find ad hoc method for variable elimination in finite domain constraints.

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