A Theoretical and Empirical Comparison of Systemic Risk Measures: MES versus ΔCoVaR

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Macroprudential Regulation and Systemic Risk

- If financial regulation and supervision were historically focused on banks' risk in isolation, with the 2008 crisis it became clear that macro-prudential rules need to be established to limit systemic risk.
- Basel III proposes that capital surcharges need to be imposed for **systemically important financial institutions** (SIFI).
- Recently (12/20/2011), the FED has introduced such a surcharge for eight banks (Bank of America, Bank of New York Mellon, Citigroup, Goldman Sachs, JPMorgan, Morgan Stanley, State Street et Wells Fargo) that will be implement between 2016 and 2019.

1. Market risk
2. Credit risk
3. Liquidity risk
4. Operational risk
5. **Systemic risk**

La Fed s'attaque au risque systémique des banques américaines

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Se saisissant des recommandations formulées par le cadre de réglementation international dit de Bâle 3, la banque centrale des Etats-Unis, la Fed, a lancé mardi 20 décembre le processus de mise en œuvre de la surcharge financière pour les banques systémiques mondiales.

Ce lancement s'inscrit dans un projet durcissant la réglementation financière s'appliquant aux groupes financiers géants américains, dont l'application devrait prendre plusieurs années. Ce projet de réglementation est destiné à *"toutes les holdings bancaires américaines dont l'actif consolidé est supérieur ou égal à 50 milliards de dollars"* et à tous les groupes financiers non bancaires que les autorités américaines jugeront d'importance systémique, a indiqué la Réserve fédérale dans un communiqué. La Fed propose notamment un durcissement des normes de fonds propres et de trésorerie pour ces groupes.

1. **How to modifiy the reglementary capital requirement for the financial institutions that contribute the more to the systemic risk?**

2. **How to identify these financial institutions?**

   1. Balance sheet approach: total asset, cross positions, etc.
      => **Econometrics is useless...**

   2. Approach based on publicly available data (financial returns, leverage)
      => **Econometrics is essential**

   3. Both approaches give the same results (Engle and Brownlees, 2012): BIS versus SRisK
      So, let do some econometrics

How financial econometricians measure the systemic risk?

- Brownless, T. C. and R. Engle (2012), Volatility, correlation and tails for systemic risk measurement, forthcoming in Review of Financial Studies.
- Adrian, T., and M. K. Brunnermeier (2011), CoVaR, Technical report, Federal Reserve Bank of New York. Staff report No. 348.
- Acharya, V. V., Pedersen L. H., Philippon T., and R. Richardson (2010), Measuring Systemic Risk, Technical report, NYU-Stern.
- White, H., Kim, T.-H. and S. Manganelli (2010), VAR for VaR: Measuring Systemic Risk Using Multivariate Regression Quantiles, Working Paper, ECB.

SRISK and Marginal Expected Shortfall (MES)

Brownlees and Engle (2012), build a Systemic Risk index (SRISK) that captures the **expected capital shortage** of a firm given its degree of leverage and MES.

Definition

The MES is defined as the expected equity loss per dollar invested in a particular financial institution if the overall market declines by a certain amount.

CoVaR and ΔCoVaR

The second popular systemic risk measure is the **CoVaR**, introduced by Adrian and Brunnermeier (2011).

Definition

The CoVaR corresponds to the VaR of the market returns obtained given the effect of a specific event on the firm's returns. In this framework, it is possible to define the contribution of the institution to systemic risk, termed ΔCoVaR, as the difference between its CoVaR and the CoVaR calculated in the median state.

Objectives of the paper

- In this paper, we propose an unified and theoretical framework similar to the one used by Brownlees and Engle (2011) to compare both measures.
- This paper aims to determine whether they are convergent - going in the same direction - or whether they are complementary - capturing different components of systemic risk.
- This paper does not aim to determine which of the two measure is superior.

If you forgot what are the VaR and ES...

Definition (Value-at-Risk)

VaR is an estimate of how much a certain portfolio can lose within a given time period, for a given confidence level (Engle et Manganelli, 2004).

Engle, R. F., and Manganelli, S. (2004), "CAViaR: Conditional Autoregressive Value-at-Risk by regression quantiles", Journal of Business and Economic Statistics, 22, pp. 367-381.

$$\Pr\left[r_t < VaR_t\left(\alpha\right)\right] = \alpha$$

$$VaR_t\left(\alpha\right) = F_t^{-1}\left(\alpha\right)$$

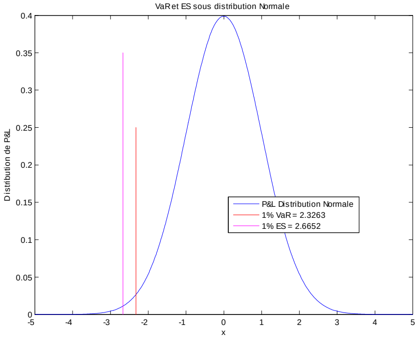
where $F_t\left(.\right)$ denotes the cdf of the returns r_t at time t.

If you forgot what are the VaR and ES...

Definition

L'**Expected Shortfall** (ES) associée à un taux de couverture de $\alpha\%$ correspond à la moyenne des $\alpha\%$ pires pertes attendues telle que :

$$ES_t(\alpha) = E\left(r_t \mid r_t < VaR_t(\alpha)\right) = -\frac{1}{\alpha}\int_0^{\alpha} F_t^{-1}(p)\, dp$$



Marginal Expected Shortfall

Definition

MES measures the *marginal contribution* of an institution i to systemic risk, measured by the ES of the system.

- We consider N firms and denote as r_{it} the return of each firm i at time t.
- The market return, r_{mt}, is defined as the value-weighted average of all firms $r_{mt} = \sum_{i=1}^{N} w_i r_{it}$, where w_i denotes the market size weight of each firm i.

Marginal Expected Shortfall (MES)

Strictly, the ES at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of the cases, but it can be extended to the general case, in which the returns are beyond a given threshold C.

Formally, the **conditional ES** of the system is defined as follows:

$$ES_{m,t-1}(C) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C) = \sum_{i=1}^{N} w_i \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C).$$

Marginal Expected Shortfall (MES)

Definition (Brownlees and Engle, 2012)

The MES is then defined as the partial derivative of the system's ES with respect to the weight of firm i in the economy.

$$MES_{it}\left(C\right) = \frac{\partial ES_{m,t-1}\left(C\right)}{\partial w_i} = \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C).$$

CoVaR and ΔCoVaR

Definition

The CoVaR corresponds to the α%-VaR of the market returns obtained conditionally on the financial stress for the firm i:

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|\mathbb{C}(r_{it})} \;\middle|\; r_{it} = VaR_{it}(\alpha)\right) = \alpha.$$

The ΔCoVaR is then defined as the difference between the VaR of the financial system conditional on the distress of a particular financial institution i and the VaR of the financial system conditional on the median state of the institution i.

$$\Delta CoVaR_{it}(\alpha) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}.$$

The Framework

Thus, we consider the following bivariate process of firm and market returns:

$$r_{mt} = \sigma_{mt}\varepsilon_{mt},$$

$$r_{it} = \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\xi_{it},$$

$$(\varepsilon_{mt}, \xi_{it}) \sim D,$$

where $\nu_t = (\varepsilon_{mt}, \xi_{it})'$ satisfies $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t\nu_t') = I_2$, and D denotes the bivariate distribution of the standardized innovations.

Marginal Expected Shortfall

The conditional MES can be expressed as a function of the firm's equity price volatility, its correlation with the market return and the comovement of the tails of the distribution:

$$\begin{aligned} MES_{it}(C) &= \sigma_{it}\rho_{it}\mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \\ &\quad + \sigma_{it}\sqrt{1-\rho_{it}^2}\mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right). \end{aligned}$$

An Empirical Comparison of Systemic Risk Measures

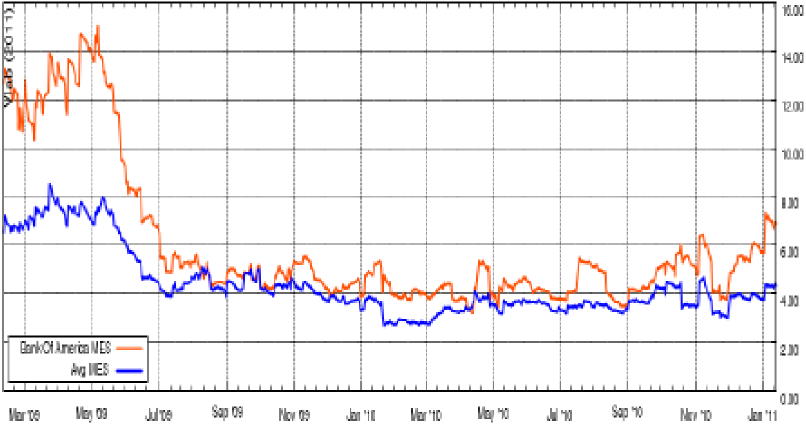
If the standardized innovations ε_{mt} and ζ_{it} are *i.i.d.* over time, then the nonparametric estimates of the tail expectations are given by

$$\widehat{\mathbb{E}}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \kappa\right) = \frac{\sum\limits_{t=1}^{T} \varepsilon_{mt} \; K\left(\frac{\kappa - \epsilon_{mt}}{h}\right)}{\sum\limits_{t=1}^{T} K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right)},$$

$$\widehat{\mathbb{E}}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \kappa\right) = \frac{\sum\limits_{t=1}^{T} \xi_{mt} \; K\left(\frac{\kappa - \epsilon_{mt}}{h}\right)}{\sum\limits_{t=1}^{T} K\left(\frac{\kappa - \varepsilon_{mt}}{h}\right)},$$

where $\kappa = VaR_m\left(\alpha\right) / \sigma_{mt}$, $K\left(x\right) = \int_{-\infty}^{x/h} k\left(u\right) \, du$, $k\left(u\right)$ is a kernel function and h is a positive bandwidth.

An Empirical Comparison of Systemic Risk Measures



Definition

The SRISK is simply given by the capital shortfall which tells us how much capital does the firm need to add if an other crisis were to happen.

$$SRISK_{it}^{SR} = k \; D_{it} - (1 - k) \;\; W_{it} \;\; MES_{it},$$

where k is the prudential capital ratio of equity to asset equal to 8%, D_{it} is the quarterly book value of total liabilities, W_{it} is the daily market value and MES_{it} the short term marginal expected shortfall of institution i

An Empirical Comparison of Systemic Risk Measures

Systemic Risk Top Ten

TOP 10	SRISK%	MES	MV
Bank Of America	18%	5.93	126770.3
Citigroup	17.79%	5.83	128401.6
JP Morgan Chase	12.08%	4.75	158790.8
Morgan Stanley	11.04%	5.05	40166.86
Wells Fargo	9.66%	6.07	150009.2
Goldman Sachs	6.85%	3.61	85490.06
Prudential Financial	5.49%	4.02	25937.7
MetLife	4.33%	4.86	40316.61
Hartford Financial Services Group	3.09%	5.16	11540.28
Genworth Financial	1.9%	8.59	5904.52

An Empirical Comparison of Systemic Risk Measures

TABLE 4.1 Systemic Risk Rankings during the Financial Crisis of 2007 to 2009

	July 1, 2007 Risk % (Rank)			March 1, 2008 Risk % (Rank)			September 12, 2008 Risk % (Rank)			March 31, 2009 Risk % (Rank)		
	SRISK%		MES	SRISK%		MES	SRISK%		MES	SRISK%		MES
Citigroup	14.3	#1	3.27	12.9	#1	4.00	11.6	#1	6.17	8.8	#4	12.55
Merrill Lynch	13.5	#2	4.28	7.8	#3	5.36	5.7	#5	6.86		—	—
Morgan Stanley	11.8	#3	3.25	6.7	#6	3.98	5.2	#7	4.87	2.8	#7	9.16
JPMorgan Chase	9.8	#4	3.44	8.5	#2	4.30	8.6	#4	5.2	12.1	#2	10.55
Goldman Sachs	8.8	#5	3.6	5.3	#9	3.14	4.2	#9	3.58	3.7	#5	6.61
Freddie Mac	8.6	#6	2.35	5.9	#7	4.60		—	—		—	—
Lehman Brothers	7.2	#7	3.91	5.0	#9	4.88	4.6	#8	15.07		—	—
Fannie Mae	6.7	#8	2.47	7.1	#4	5.88		—	—		—	—
Bear Stearns	5.9	#9	4.4	2.9	#12	4.16		—	—		—	—
MetLife	3.6	#10	2.57	2.2	#15	2.93	1.9	#12	3.20	3.2	#6	11.93
Bank of America	0	#44	2.06	6.7	#5	3.60	9.6	#2	6.33	12.7	#1	13.41
AIG	0	#45	1.51	5.5	#8	4.63	9.6	#3	10.86		—	—
Wells Fargo	0	#48	2.38	1.9	#16	4.14	3.0	#10	5.40	10.4	#3	12.15
Wachovia	0	#51	2.2	4.6	#11	4.64	5.7	#6	9.61		—	—
Prudential Fin.	3.3	#11	3.09	2.6	#13	3.94	2.1	#11	4.17	2.6	#8	15.89
U.S. Bancorp	0	#40	1.62	0	#54	2.41	1.1	#15	5.20	2.6	#9	10.4
PNC Financial	0	#49	2.46	0	#43	2.84	0.3	#32	3.78	1.6	#10	10.03

CoVaR and ΔCoVaR

This unconditional CoVaR can be estimated using a standard quantile regression (Koenker and Bassett (1978)).

$$r_{mt} = \mu_{\alpha}^{i} + \gamma_{\alpha}^{i} r_{it}.$$

Then, the estimated conditional CoVaR is defined as

$$CoVaR_{t}^{m|VaR_{i}(\alpha)} = \widehat{\mu}_{\alpha}^{i} + \widehat{\gamma}_{\alpha}^{i} \widehat{VaR}_{it}(\alpha)$$

By definition, the quantile-regression-based ΔCoVaR is equal to

$$\Delta CoVaR_{it}(\alpha) = \widehat{\gamma}_{\alpha}^{i} \left[\widehat{VaR}_{it}(\alpha) - \widehat{VaR}_{it}(0.5) \right].$$

According to proposition 2, the estimated DCC-ΔCoVaR is defined as

$$\Delta CoVaR_{it}\left(\alpha\right)=\widehat{\gamma}_{it}\left[\widehat{VaR}_{it}\left(\alpha\right)-\widehat{VaR}_{it}\left(0.5\right)\right],$$

where $\widehat{\gamma}_{it}=\widehat{\rho}_{it}\widehat{\sigma}_{mt}/\widehat{\sigma}_{it}$

An Empirical Comparison of Systemic Risk Measures