Journée "Risque"

Modèles mathématiques et risques naturels

Etat de l'art, verrous scientifiques, prospectives

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Outline & Main ideas

I - Various complex phenomena

II - Need for models & schemes

III - Modelling hazardous flows

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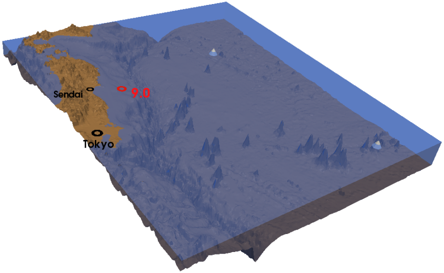
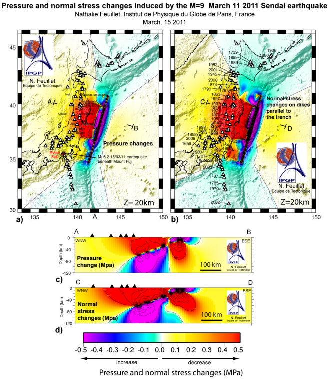
- Need for mathematical modelling
  - non smooth solutions, few dissipation
  - complex phenomena
  - not only mathematicians
- Modelling vs. prediction
  - from deterministic to statistical description

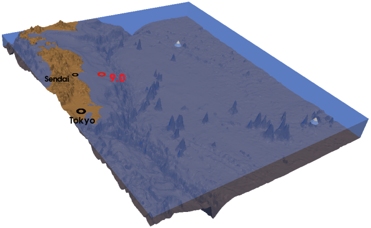
Seism : Japan, march 2011

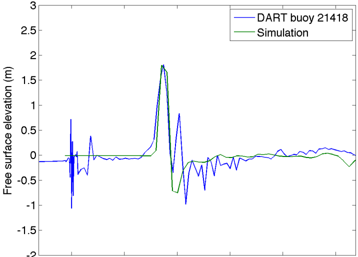


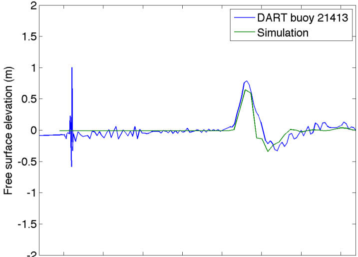


source IPGP (A. Mangeney)

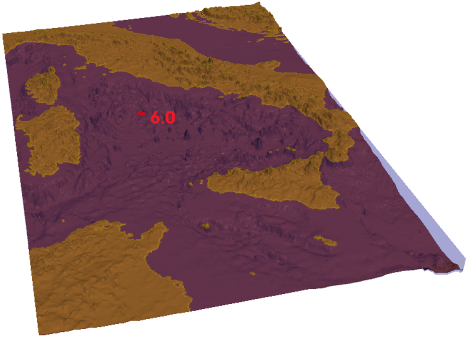
Comparison with DART buoys





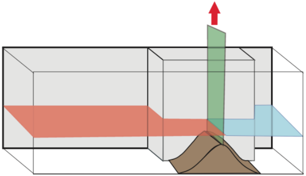


The next tsunami !

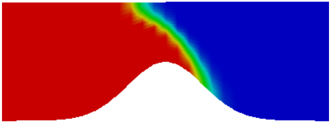


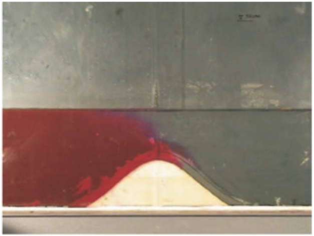
Archimedes law (with C. Pares)

- Experimental set up (simulation)



- Velocity field (6 s) (Gibraltar strait)





Fluid mechanics : Navier-Stokes & Euler equations

3d Navier-Stokes $H, \mathbf{u} = (u, v, w)$

$$\begin{cases} \operatorname{div} \underline{\mathbf{u}} = 0, \\ \dot{\underline{\mathbf{u}}} + (\underline{\mathbf{u}}.\nabla)\underline{\mathbf{u}} + \nabla p = \mathbf{G} + \operatorname{div} \underline{\underline{\Sigma}}, \end{cases}$$

- Many applications
- Very complex to analyse/simulate
- L. Euler in *Principes généraux du mouvement des fluides* (1757)

> Or nous voyons par là suffisamment, combien nous sommes encore éloignés de la connoissance complette du mouvement des fluides, & que ce que je viens d'expliquer, n'en contient qu'un foible commencement. Cependant tout ce que la Théorie des fluides renferme, est contenu dans les deux équations rapportées cy-dessus (§. XXXIV.), de sorte que ce ne sont pas les principes de Méchanique qui nous manquent dans la poursuite de ces recherches, mais uniquement l'Analyse, qui n'est pas encore assés cultivée, pour ce dessein : & partant on voit clairement, quelles découvertes nous restent encore à faire dans cette Science, avant que nous puissions arriver à une Théorie plus parfaite du mouvement des fluides.

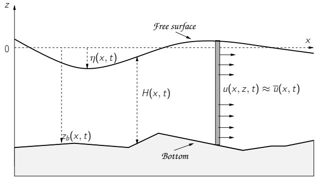
Shallow Water approximation of Navier-Stokes

$$(NS)\left\{\begin{array}{l}\operatorname{div}\underline{\mathbf{u}}=0,\\ \underline{\dot{\mathbf{u}}}+(\underline{\mathbf{u}}.\nabla)\underline{\mathbf{u}}+\nabla p=\mathbf{G}+\operatorname{div}\underline{\underline{\Sigma}},\end{array}\right.$$

- small parameter $\varepsilon=\frac{H_0}{L_0}$, expansion in $\mathcal{O}(\varepsilon^2)$
- Saint-Venant 1872, Gerbeau 2001, Saleri 2004, Marche 2007

$$(SV)\left\{\begin{array}{l}\frac{\partial H}{\partial t}+\frac{\partial(H\bar{u})}{\partial x}=0,\\ \frac{\partial(H\bar{u})}{\partial t}+\frac{\partial}{\partial x}\left(H\bar{u}^2+\frac{g}{2}H^2\right)=-gH\frac{\partial z_b}{\partial x}\end{array}\right.$$

- Reduced complexity
- Many applications
- Hyperbolic CL



NOTATIONS: free surface η, bottom z_b, water height $H=\eta-z_b$, velocities $\underline{\mathbf{u}}=(u_x,u_z)$, $u(x,t)=\frac{1}{H}\int_{z_b}^{\eta}u_z(x,z,t)dz$

Need for efficient num. methods

$$(SV)\begin{cases} \frac{\partial H}{\partial t} + \frac{\partial (H\bar{u})}{\partial x} = 0, \\ \frac{\partial (H\bar{u})}{\partial t} + \frac{\partial}{\partial x}\left(H\bar{u}^2 + \frac{g}{2}H^2\right) = -gH\frac{\partial z_b}{\partial x} \end{cases}$$

- The system is hyperbolic (nonlinear, resonant)
- The water depth satisfies

$$H \geq 0, \qquad \frac{d}{dt}\int H = 0$$

- Static equilibrium, "lake at rest"

$$u = 0, \qquad H + z_b = Cst$$

- It admits a convex entropy (the energy)

$$\frac{\partial}{\partial t}\left(H\frac{\bar{u}^2}{2} + \frac{g}{2}H^2 + gHz_b\right) + \frac{\partial}{\partial x}\bar{u}\left(H\frac{\bar{u}^2}{2} + gH^2 + gHz_b\right) \leq 0$$

$\Rightarrow$ Positivity, well-balancing, (consistency), discrete entropy,. . .

Num. methods for the Saint-Venant system

- Finite volume scheme [Bouchut'04]
- Various solvers (relaxation, Roe, HLL, kinetic,. . .)

$$\frac{\partial H}{\partial t} + \frac{\partial (H\bar{u})}{\partial x} = 0, \quad \Rightarrow H_i^{n+1} = H_i^n - \frac{\Delta t}{\Delta x}(\mathcal{F}^n_{i+1/2} - \mathcal{F}^n_{i-1/2})$$

with e.g.

$$\mathcal{F}^n_{i+1/2} = \frac{H_i u_i + H_{i+1} u_{i+1}}{2} + \frac{\max(\sqrt{gH_i}, \sqrt{gH_{i+1}})}{2}(H_i - H_{i+1})$$

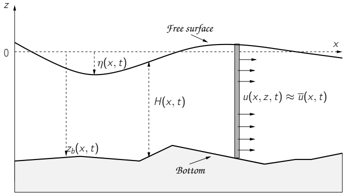
- Hydrodynamic limit of Boltzmann equation
  - Basis : adopt a microscopic description

$$Cont.\ model \quad \Leftrightarrow \quad \frac{\partial M}{\partial t} + \xi \frac{\partial M}{\partial x} = Q(x, t, \xi)$$

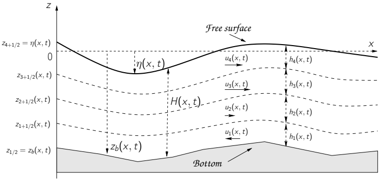
  - $M(x, t, \xi)$ particle density, $Q(x, t, \xi)$ collision term ($= 0$ a.e.)
  - $\int_{\mathbb{R}} \xi^p M \, d\xi$ gives the macroscopic variables
  - linear transport equation

Beyond the Saint-Venant system

Objective



$\Rightarrow$



- Many contributions (Audusse, Bouchut, LeVeque, Pares, . . .)
- Valid for non-miscible fluids
- Pb. with underlying physics,. . .

Key idea

Saint-Venant
$u(x,z,t) \approx \overline{u}(x,t)$

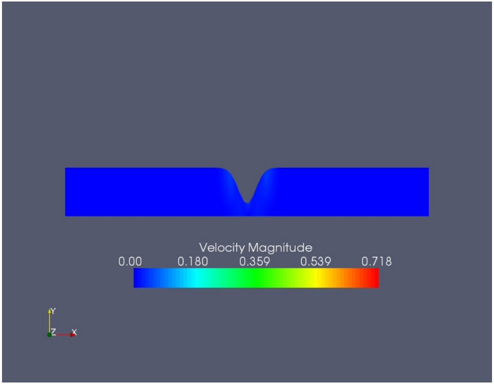
$\Rightarrow$

Multilayer Saint-Venant
$u(x,z,t) \approx \sum_{\alpha=1}^{N} \mathbb{I}_{z\in L_\alpha}(z) u_\alpha(x,t)$

Gravity waves & non-hydrostatic systems

- Dispersive models

$$\begin{cases} \frac{\partial H}{\partial t} + \frac{\partial (H\bar{u})}{\partial x} = 0, \\ \frac{\partial (H\bar{u})}{\partial t} + \frac{\partial}{\partial x}\left(H\bar{u}^2 + \frac{g}{2}H^2\right) + \mathcal{D}(H)\frac{\partial^3 \bar{u}}{\partial^2 x \partial t} = -H\frac{\partial z_b}{\partial x} - \kappa(\bar{u}) \end{cases}$$



- Difficult to simulate (costly in 3d)

Scientific challenges (for math)

⇒ GdR EGRIN

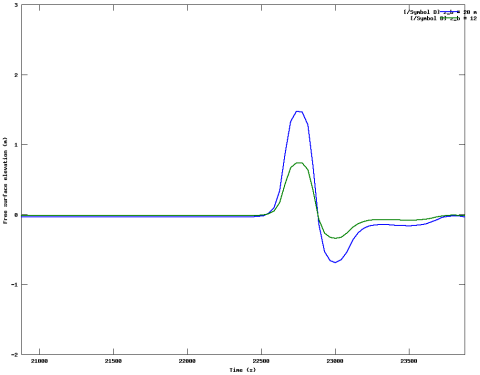
- Interaction with other scientists
- Non-hydrostatic models (large vertical acceleration)
  - large slopes (ANR METHODE - MAPMO), tsunami
  - complex models, costly simulations
- Complex fluids
  - multiphysics, landslides, pyroclastic flows
  - not only water, complex rheology
- What to do with models
  - simulation of past disasters (not enough !)
  - Risk management

Modelling & uncertainties

- Realistic/useful predictions required
- Ingredients needed for modelling of hazardous flows
  - a set of equations, a discrete model (scheme+geom. model)
  - initial (BC) conditions, forcing e.g. flooding : rain, dam break
  - probability of occurence + intensity
- Modelling the uncertainties
  - initial condition e.g. dam break configuration
  - propagation of uncertainties in deterministic models
  - mean value + confidence
- But often very complex

Small variations of the I.C.

- Japan seism
  - $\Delta z_b = 20$ m and $\Delta z_b = 12$ m



- disaster ≠ mean of events

Conclusion & outlook

- Interesting/challenging problems
  - large scales (space & time)
  - coupled phenomena
  - a lot of uncertainties (init. cond.)
- Very complex phenomena
  - a lot of bad models (math/mech/physics)
  - efficient techniques when they exist
  - need for collaborative works
- Interest of efficient numerical methods
  - in fluid mechanics, geophysics
  - non smooth solutions, few dissipation
  - useful in practice: for scientists, industrials
- Modelling vs. prediction ?