Query Completeness of Skolem Machine Computations

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GL as a fragment of FOL

Geometric formula: $C \Rightarrow D$, where

$C = A_1 \wedge \ldots \wedge A_n$ ($n \geq 0$, A_i atoms) and

$D = E_1 \vee \ldots \vee E_m$ ($m \geq 0$), where each

$E_j = (\sum x_1 \ldots x_k)\, C_j$ ($k \geq 0$ may vary with j,

each C_j a conjunction of atoms, $\sum$ for `exist´).

Geometric theory = set of geometric formulas

Examples

- Skolem (1920): lattices and projective geometry
- Horn clauses and CNF (resolution)
- Generating natural numbers:

  true => nat(0)

  nat(x) => ($\sum$ y) (nat(y) $\wedge$ s(x,y))
- General form: $A_1 \wedge \ldots \wedge A_n$ =>
  $((\sum \mathbf{x})\ A_{11} \wedge \ldots \wedge A_{1i}) \vee \ldots \vee ((\sum \mathbf{y})\ A_{k1} \wedge \ldots \wedge A_{kj})$

Machine Model

- Older than Turing Machine (not the only one …)
- Skolem's `Erzeugungsprinzipien´ (1920), production rules, geometric formulas as instructions of a `Skolem Machine´
- State: set (of sets) of closed atoms
- Inference procedure as computation: forward chaining + case distinction + introduction of `witnesses´ (*new?)*
- Essentially non-deterministic (not by $\vee$, but since different axioms may be applied)

Universality

- Horn Clause Logic: reg2horn.gl
- Geometric Logic: reg2gl.gl
- Geometric Logic, only constants: reg2gl0.gl

Geometric Logic for Automated Reasoning in First-Order Logic

- More expressive than CNF
- FOL to GL: no Skolemization needed
- Good for Interactive Theorem Proving
- Some success at CASC

Query completeness

true => $p \vee (\sum x)\ q(x)$

p => r

r => false

q(y) => false

Is $r \vee (\sum x)\ q(x)$ a logical consequence?

Tape1: p,r,false   Tape2: q(a),false

Yes!

Finite-model completeness

true => p $\vee$ ($\sum$ x) q(x)

p => r

r => false

r $\wedge$ q(y) => false

Is r $\vee$ ($\sum$ x) (r $\wedge$ q(x)) a logical consequence?

Tape1: p,r,false   Tape2: q(a) saturated!

A *finite* countermodel {q(a)} is found.

Infinite models are not found

true => s(0,1)

s(x,y) => (∑ z) s(y,z)

s(x,x) => false

Is (∑ x) s(x,x) a logical consequene?

The infinite countermodel {s(0,1),s(1,2),…}
is not found.

The End