Four small time universal Turing machines

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Introduction

- Shannon first posed the question of finding the smallest possible universal Turing machine (UTM). (The size of a Turing Machine TM is given by the number of states and symbols).

- In 1962 Minsky created a 7-state, 4-symbol UTM that simulates TMs via 2-tag systems.

- Minsky's technique was more recently used by Rogozhin et al. to create the smallest known UTMs.

- For this talk we consider deterministic single tape TMs with a single tape head.

New universal Turing machines

- Here we present new small polynomial time universal Turing machines with state-symbol pairs of $(5, 5)$, $(6, 4)$, $(9, 3)$ and $(18, 2)$.
- These machines simulate our new variant of tag system, the bi-tag system and are the smallest known universal Turing machines with 5, 4, 3, and 2 symbols respectively.

Bi-tag systems

A bi-tag systems is defined by the tuple (A, E, e_h, P). Each production in P is of one of the following three forms:

$$P(a) = a, \quad P(e, a) = AE, \quad P(e, a) = AAE$$

where $a \in A$ and $e \in E$.

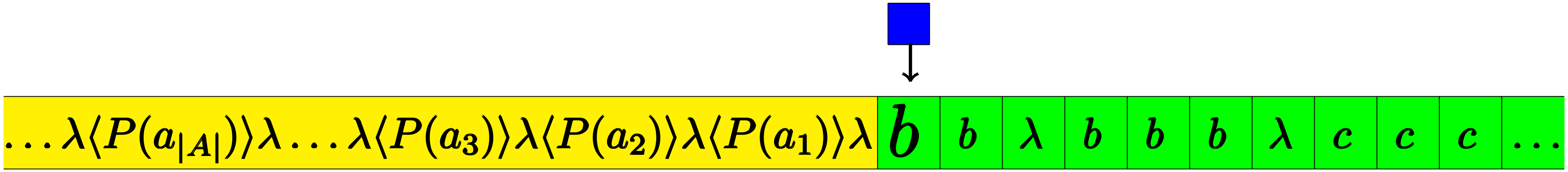
A configuration of a bi-tag system is a word of the form $s = A^*(AE \cup EA)A^*$ called the data word. If a configuration s_2 is obtained from a configuration s_1 via the application of a single production we write $s_1 \vdash s_2$.

A bi-tag systems computation step executes in one of two ways:

(i) If $s = as'$ then $as' \vdash s'P(a)$.

(ii) If $s = eas'$ then $eas' \vdash s'P(e, a)$.

Simulation of a bi-tag system computation step by UTM$(9,3)$



Encoded bi-tag system

Encoded data word

UTM tape head

$\langle X\rangle$ represents the encoding of object X

$P(a), P(e,a) \in P$, $a \in A$, $e \in E$

$\langle a\rangle, \langle e\rangle \in b^*$

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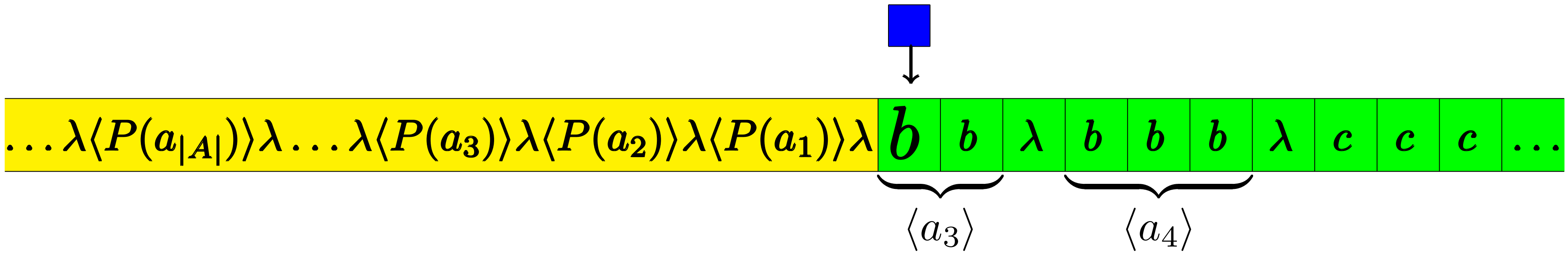

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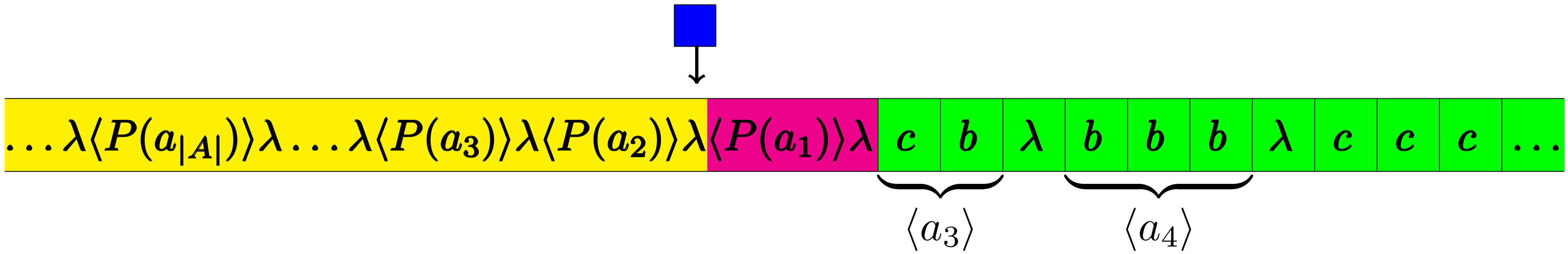

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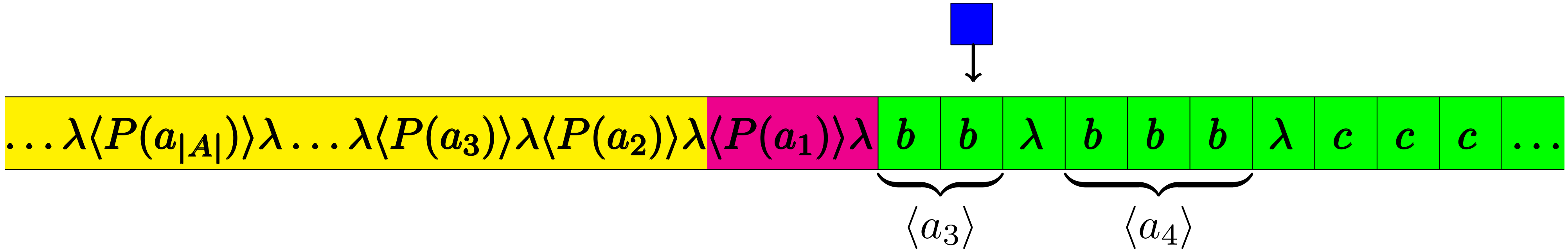

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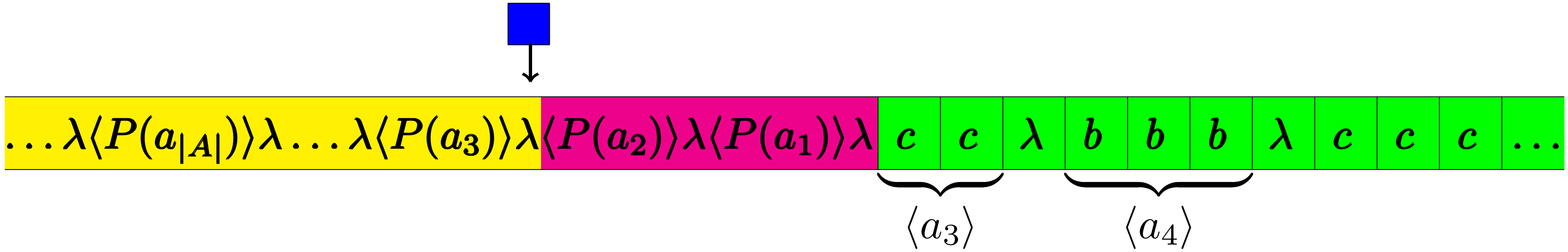

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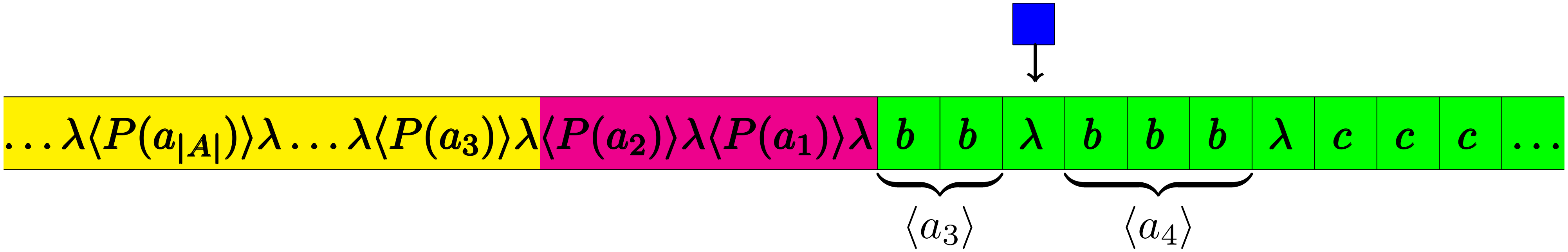

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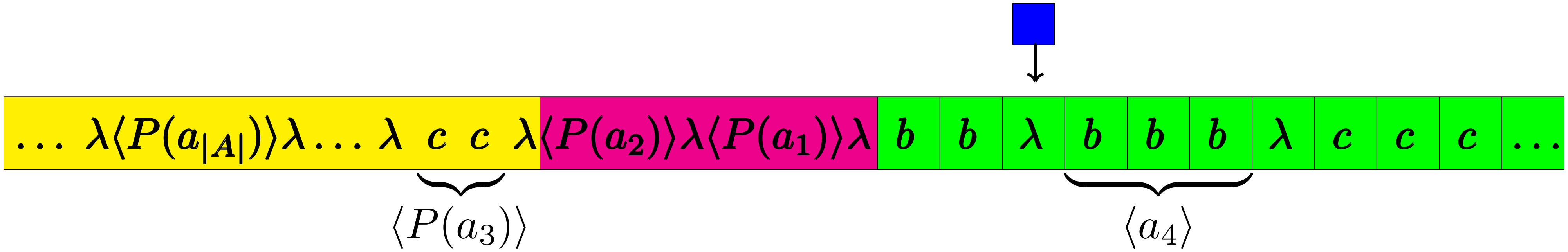

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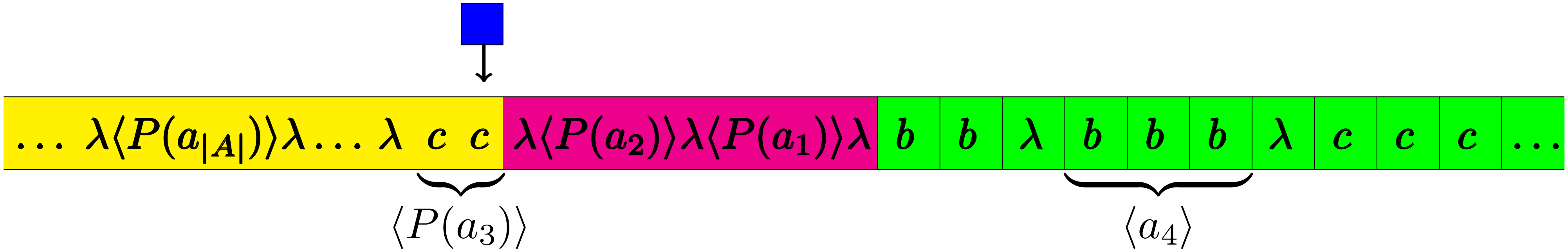

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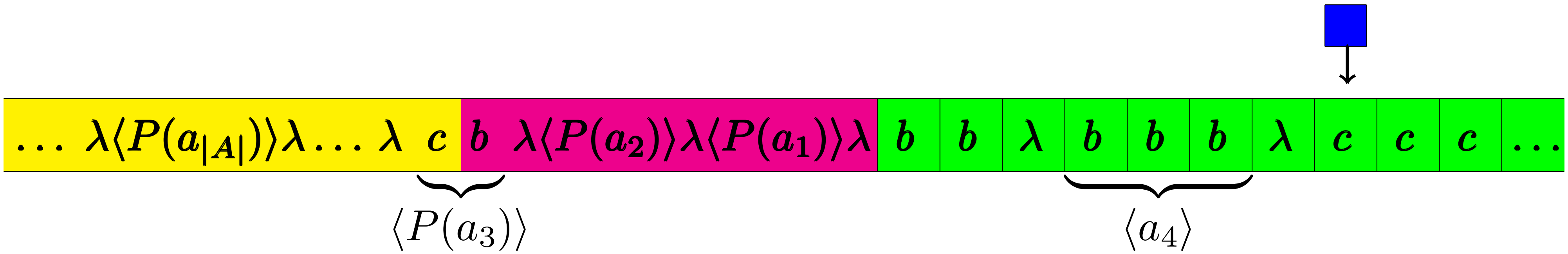

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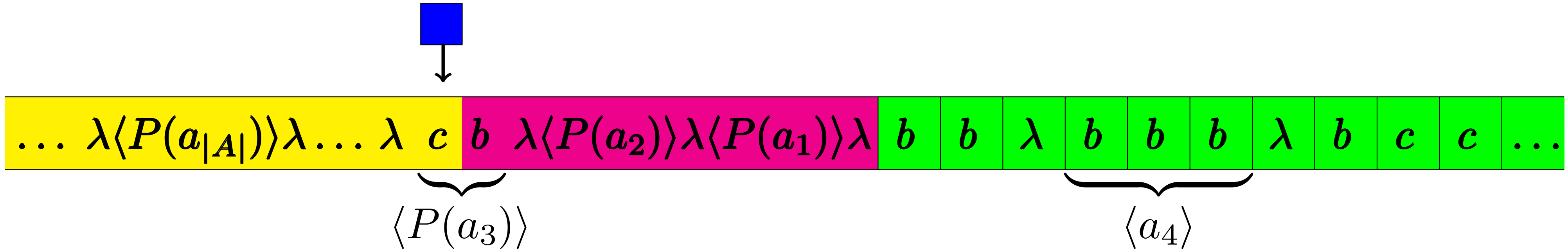

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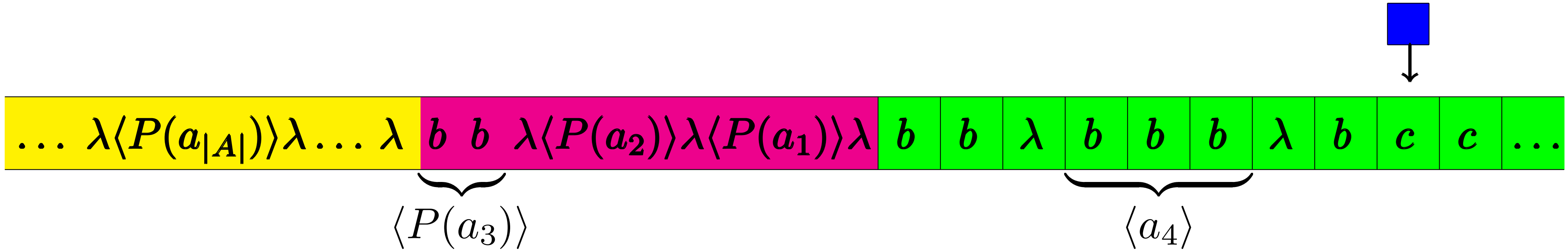

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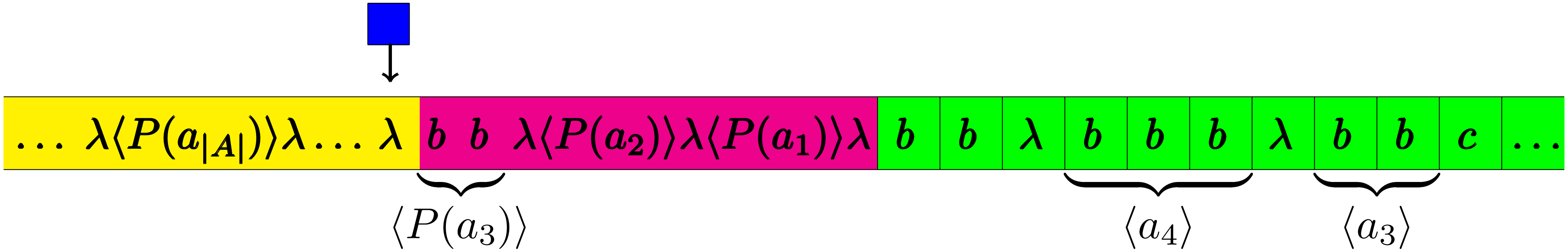

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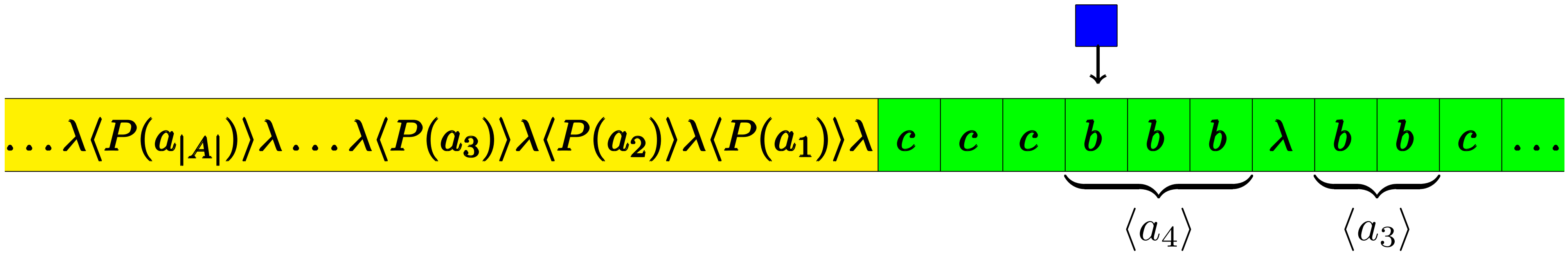

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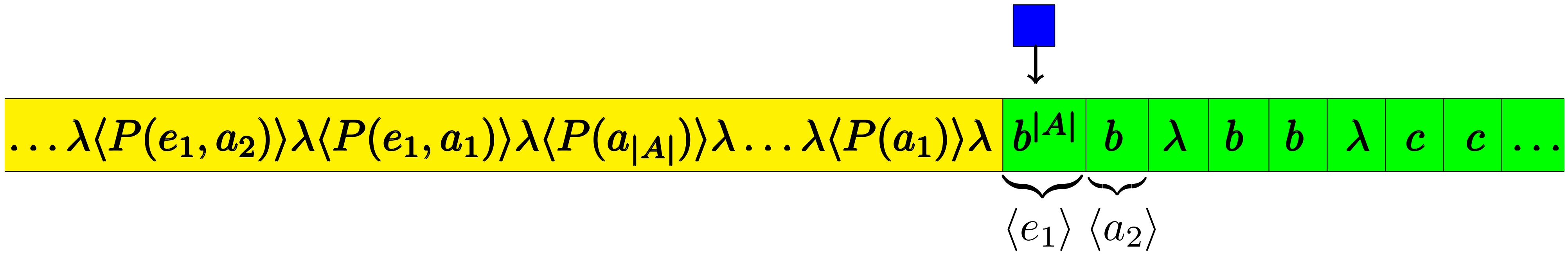

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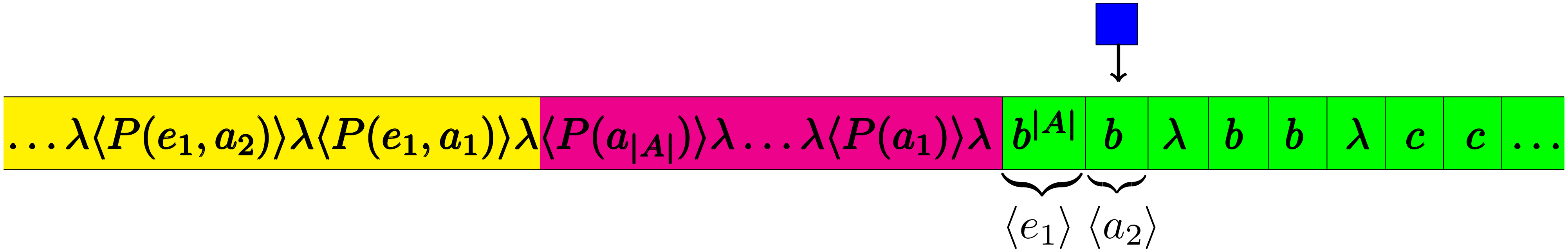

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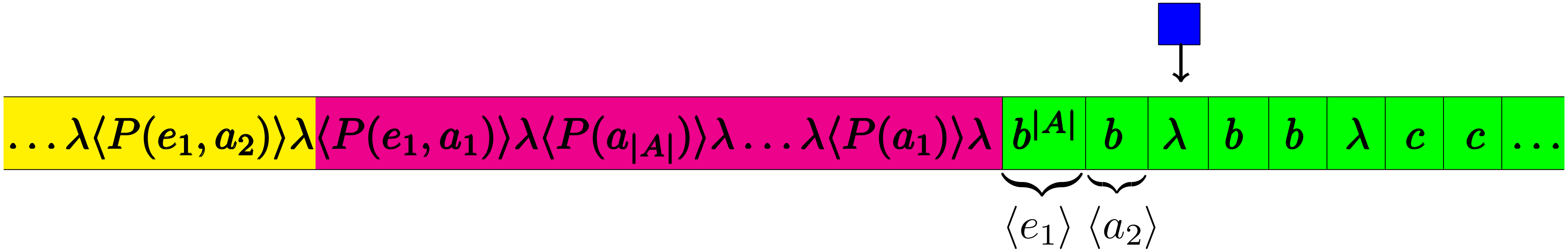

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Non-halting TMs

- Different definitions of UTM have been given by a number of different authors (Priese 1979, Davis 1956, Davis 1958).

- Non-halting UTM: Instead of the computation ending with a single halting configuration, the computation ends with a finite configuration sequence called the terminal configuration sequence.

- The output of the simulated TM is retrieved by applying the recursive decoding function to this terminal configuration sequence.

Lower bounds for non-halting UTM

- There are many ways to define a terminal configuration sequence. Some examples of possible terminal configurations sequence are:
  - The configurations containing a given sequence of states.
  - A repeated sequence configurations.
  - A single configuration containing a constant word.

- Lower bounds: Given a definition of a terminal configuration sequence we may prove that the terminal sequence problem (will a machine ever execute a terminal configuration sequence) is decidable.

- More general approach is to prove that the terminal sequence problem for all possible terminal sequences, of a machine or set of machines, is decidable.

Future work

- The universality question remains open for 42 state-symbols pair.
- The following are some approaches to find smaller UTMs.
  - Look for other universal systems that require less instructions to simulate.
  - Simplify some existing universal model in order to make it easier to simulate.
  - Find an encoding that allows many different operations to be carried out by the same group of instructions.