Changing the Neighborhood of Cellular Automata

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A new definition of CA by neighborhood function

induces countably many CA,

which have the same local function and different neighborhoods.

By this, we begin the research of CA from a new point of view.

Cellular Automaton (S, Q, f_n, ν)

1. S: discrete cellular space such as $\mathbb{Z}$, $\mathbb{Z}^2$, hyperbolic grid ...
2. Q: set of states of a cell. $Q = GF(q)$ where $q = p^k$.
3. $f_n(x_0, x_1, ..., x_{n-1})$: local function in n variables.
4. ν: injection from $\{0, 1, ..., n-1\}$ to S, called the neighborhood function.

A neighborhood function defines connection between variables of f_n and neighbors for CA: x_i is connected to $\nu(i)$, $0 \leq i \leq n-1$.

$\mathrm{range}(\nu) \equiv (\nu(0), \nu(1), ..., \nu(n-1))$ is the neighborhood N in the usual definition of CA (S, Q, f, N).

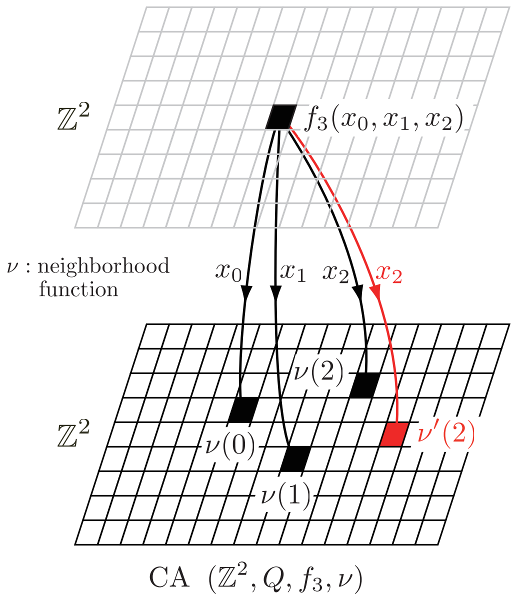


Figure 1: Neighborhood function

The global map $F_\nu : C \to C$ where $C = Q^S$ is defined as usual: for any $c \in C$ and $j \in S$, $c(j)$ is the state of cell j in c and we have

$$F_\nu(c)(j) = f_n(c(j+\nu(0)), c(j+\nu(1)), ..., c(j+\nu(n-1))). \quad (1)$$

The local function f_n is expressed by a polynomial over Q in n variables $(x_0, x_1, x_2, ..., x_{n-1})$, see [3].

In case of tertiary function,

$$f_3(x,y,z) = u_0 + u_1 x + u_2 y + \cdots + u_i x^h y^j z^k + \cdots + u_{q^3-2} x^{q-1} y^{q-1} z^{q-2} + u_{q^3-1} x^{q-1} y^{q-1} z^{q-1}, \quad where\ u_i \in Q,\ 0 \le i \le q^3 - 1. \quad (2)$$

In case of binary states $Q = GF(2) = \{0,1\}$,

$$f_3(x,y,z) = u_0 + u_1 x + u_2 y + u_3 z + u_4 xy + u_5 xz + u_6 yz + u_7 xyz, \quad where\ u_i \in \{0,1\},\ 0 \le i \le 7. \quad (3)$$

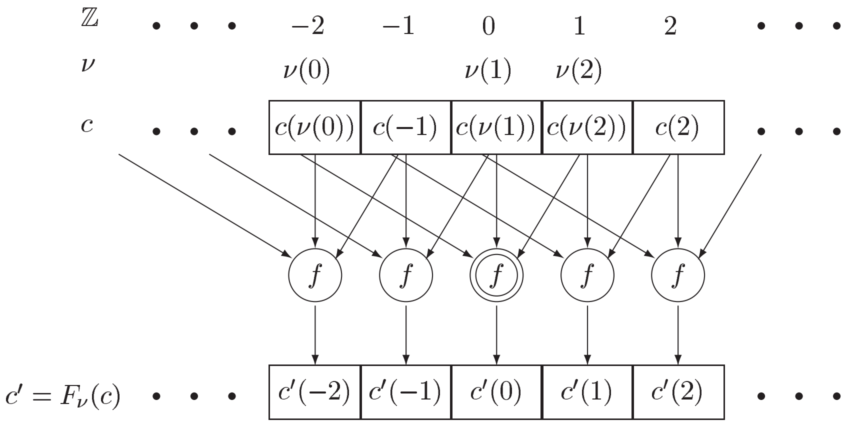


Figure 2: 1-d CA with $\boldsymbol{\nu(0) = -2, \nu(1) = 0, \nu(2) = 1}$

Countably many CA induced by changing the neighborhood

Theorem 1 *By changing the neighborhood function* ν*, infinitely many different global CA functions* F_ν *are induced from any single local function* $f_3(x, y, z)$ *which is not constant.*

Proof:
It is clear that to each non-constant function f_3 at least one of the following three cases applies.

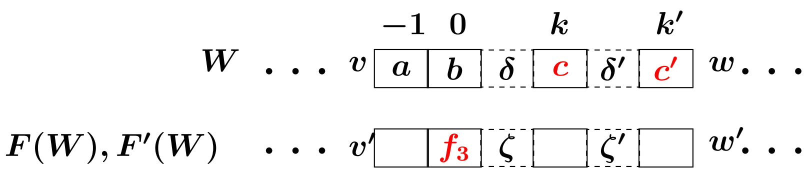
Case 1) $f_3(a, b, c) \neq f_3(a, b, c')$ for $a, b, c \neq c' \in Q$.

Case 2) $f_3(a, b, c) \neq f_3(a, b', c)$ for $a, b \neq b', c \in Q$.

Case 3) $f_3(a, b, c) \neq f_3(a', b, c)$ for $a \neq a', b, c \in Q$.

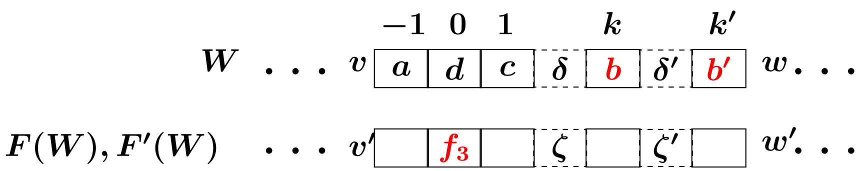
Case 1)

Consider CA and CA' which have the same local function $f_3(x, y, z)$ and different neighborhoods $(-1, 0, 1 + k)$ and $(-1, 0, 1 + k')$ where $0 \leq k < k'$. Then, for configuration $W = vab\delta c\delta' c' w$, where $W(0) = b$, δ and δ' are words of lengths $k - 1$ and $k' - k - 1$ and v, w are semi-infinite words over Q, we have $F(W)(0) = f_3(a, b, c) \neq f_3(a, b, c') = F'(W)(0)$. That is $F(W) \neq F'(W)$. In this way, countably many CA $\{(\mathbb{Z}, Q, f_3, (-1, 0, 1+k)),\ k \geq 1\}$ are induced from a single local function f_3.



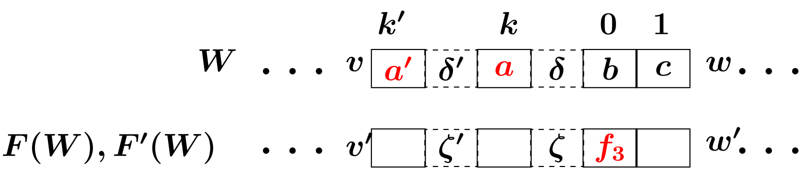
Case 2)

Consider CA and CA' which have the same local function $f_3(x, y, z)$ and different neighborhoods $(-1, 2+k, 1)$ and $(-1, 2+k', 1)$, where $0 \le k < k'$. Then, for configuration $W = vadc\delta b\delta' b' w$, where $W(0) = d$, δ and δ' are words of lengths $k-1$ and $k'-k-1$ and v, w are semi-infinite words over Q, we have $F(W)(0) = f_3(a, b, c) \neq f_3(a, b', c) = F'(W)(0)$. That is $F(W) \neq F'(W)$. In this way, countably many CA $\{\{(\mathbb{Z}, Q, f_3, (-1, 2+k, 1)),\ k \ge 1\}$ are induced from a single local function f_3.



Case 3)

Consider CA and CA' which have the same local function $f_3(x, y, z)$ and different neighborhoods $(-k-1, 0, 1)$ and $(-k'-1, 0, 1)$ where $0 \leq k < k'$. Then, for configuration $W = va'\delta' a\delta bcw$, where $W(0) = b$, δ and δ' are words of lengths $k-1$ and $k'-k-1$ and v, w are semi-infinite words over Q, we have $F(W)(0) = f_3(a, b, c) \neq f_3(a', b, c) = F'(W)(0)$. That is $F(W) \neq F'(W)$. In this way, countably many CA $\{(\mathbb{Z}, Q, f_3, (-1-k, 0, 1)),\ k \geq 1)\}$ are induced from a single local function f_3. ■



Elementary Cellular Automaton

- Elementary Local Function (ELF): $f_3(x, y, z)$ over $GF(2)$. There are 256 ELF by Equation(3).
- Elementary Neighborhood ν_E: $\mathrm{range}(\nu_E) = (-1, 0, 1)$ or $ENB = (-1, 0, 1)$.
- Elementary Cellular Automaton (ECA): $(\mathbb{Z}, GF(2), f_3, \nu_E)$. There are 256 ECA.
- Wolfram number vs. polynomial: Rule 90 = $x + z$ over $GF(2)$.

Corollary 1 *There are countably many 2 states 3 neighbors CA different from ECA.*

Problems arising from this result

What kind of CA is induced from ELF by changing the neighborhood?

For example,

Does an irreducible ECA become reducible by changing the neighborhood?

Does a nonuniversal ECA become universal by changing the neighborhood?

...etc.

Then, finally, what is the neighborhood?

Equivalence of CA

When $\mathbb{Z}$ and $\boldsymbol{Q}$ are understood, we denote $(\mathbb{Z}, \boldsymbol{Q}, \boldsymbol{f_n}, \boldsymbol{\nu})$ simply by $(\boldsymbol{f_n}, \boldsymbol{\nu})$.

Definition 1 *Two CA* $(\boldsymbol{f_n}, \boldsymbol{\nu})$ *and* $(\boldsymbol{f'_{n'}}, \boldsymbol{\nu'})$ *are called equivalent, denoted by* $(\boldsymbol{f_n}, \boldsymbol{\nu}) \cong (\boldsymbol{f'_{n'}}, \boldsymbol{\nu'})$*, if and only if their global maps are equal.*

Note that there is a local function which induces the same CA for different neighborhood functions, while different local functions may induce the same CA by changing the neighborhood function.

For example, $(\boldsymbol{R}85, (-1, 0, 1)) \cong (\boldsymbol{R}51, (-1, 1, 0))$, where R85 and R51 are ELF in Wolfram number which give reversible ECA on ENB, see proof of Theorem 7.

Decidability of equivalence $\cong$

Theorem 2 *The equivalence $\cong$ of CA is decidable.*

Proof: Consider two CA (f_n, ν) and $(f'_{n'}, \nu')$ for the same set Q of states. Let $N = \mathrm{range}(\nu) \cup \mathrm{range}(\nu')$. We will consider finite "subconfigurations" $\ell : N \longrightarrow Q$.

Changing in c the states of cells outside the finite part N has no influence in the computation of $F(c)(0)$ or $F'(c)(0)$. Thus any subconfiguration ℓ determines states $F(c)(0)$ or $F'(c)(0)$ which we denote $G(\ell)$ and $G'(\ell)$.

- Now assume, that the two CA are not equivalent: $(f_n, \nu) \ncong (f'_{n'}, \nu')$, i.e. the corresponding global maps F and F' are not the same. Then there is a configuration c such that $F(c) \neq F'(c)$. Since global maps commute with the shift, it is without loss of generality to assume that $F(c)(0) \neq F'(c)(0)$. Hence in this case there is an $\ell = c|_N$ such that $G(\ell) \neq G'(\ell)$.

- On the other hand, when there exists an ℓ such that $G(\ell) \neq G'(\ell)$, then obviously F and F' will be different for any configuration c satisfying $c|_N = \ell$ and hence the CA are not equivalent.

For deciding the equivalence it is therefore sufficient to check whether for all $\ell : N \to Q$ holds: $G(\ell) = G'(\ell)$. If this is the case, the two CA are equivalent, if not they are not. ■

The following easily proved proposition shows that for CA defined by the neighborhood function ν, there is an equivalent CA' having the ordinary neighborhood of scope $2r+1$.

Proposition 1 *For* (f_n, ν)*, let* $r = \max\{|\nu(i)| \mid 0 \leq i \leq n-1\}$*. Then there is an equivalent* (f'_{2r+1}, ν') *such that* $\mathrm{range}(\nu') = (-r, -r+1, ..., 0, ..., r-1, r)$ *and* f'_{2r+1} *takes the same value as* f_n *on* $\mathrm{range}(\nu)$*, while variables* x_i *are don't care for* i *such that* $\nu'(i) \notin \mathrm{range}(\nu)$*.*

Neighborhood family

Definition 2 *The neighborhood family* $\mathfrak{F}(f_n)$ *of* f_n *is an infinite set of global functions defined by*

$$\mathfrak{F}(f_n) = \bigcup_{\nu \in N_n} \{(f_n, \nu)\}, \tag{4}$$

where N_n *is the set of all injections* $\nu : \{0, \ldots, n-1\} \to \mathbb{Z}$.

Definition 3 *A permutation* π *of* $\mathrm{range}(\nu)$ *is denoted by* $\pi(\nu)$ *or simply* π *when* ν *is known. The permutation family* $\mathcal{P}(f_n, \nu)$ *of* (f_n, ν) *is a finite set of global functions defined by*

$$\mathcal{P}(f_n, \nu) = \bigcup_{i=0}^{n!-1} \{(f_n, \pi_i(\nu))\}. \tag{5}$$

Example: In case of n=3 there are 6 permutations of ENB.

$$\pi_0 = (-1,0,1), \pi_1 = (-1,1,0), \pi_2 = (0,-1,1),$$
$$\pi_3 = (0,1,-1), \pi_4 = (1,-1,0), \pi_5 = (1,0,-1).$$

Proposition 2 *The set of CA* $\{(f_n, \nu) \mid f_n : \text{n-ary function}\}$ *is closed under permutation of the neighborhood. That is*

$$\bigcup_{f_n} \mathcal{P}(f_n, \nu) = \bigcup_{i=0}^{n!-1} \{(f_n, \pi_i(\nu))\} = \bigcup_{f_n} \{(f_n, \nu)\}. \qquad (6)$$

Proof: Since a permutation of the neighborhood amounts to a permutation of the variables of the local function with the neighborhood being fixed to ν, for any f_n there is a function g_n and permutation π_i such that $(f_n, \nu) \cong (g_n, \pi_i(\nu))$ for some $1 \le i \le n! - 1$. ■

Three properties of CA preserved from changing the neighborhood.

Proposition 3 *$f_n(x_1, ..., x_n)$ is called totalistic if it is a function of $\sum_{i=1}^{n} x_i$. If f_n is totalistic, then any $(f_n, \nu) \in \mathfrak{F}(f_n)$ is totalistic.*

Proposition 4 *An affine CA is defined by a local function*

$f_n(x_1, x_2, ..., x_n) = u_0 + u_1 x_1 + \cdots + u_n x_n$, where $u_i \in Q$, $0 \leq i \leq$

If f_n is affine, then any $(f_n, \nu) \in \mathfrak{F}(f_n)$ is affine.

Proposition 5 *A local function $f : Q^n \to Q$ is called balanced if $|f^{-1}(a)| = |Q|^{n-1}$, $\forall a \in Q$. A finite CA is called balanced if any global configuration has the same number of preimages. In case of finite CA, if (f_n, ν) is balanced then $(f_n, \pi(\nu))$ is balanced for any π.*

A property sensitive to permutation of the neighborhood.

Proposition 6 *The number-conserving ECA is sensitive to permutation.*

Proof: The only number-conserving ECA are $(\boldsymbol{R184}, \boldsymbol{\pi_0})$ and its conjugate $(\boldsymbol{R226}, \boldsymbol{\pi_0})$ [1]. It is seen that $(\boldsymbol{R184}, \boldsymbol{\pi_2}) \cong (\boldsymbol{R172}, \boldsymbol{\pi_0})$ which is not number-conserving. A similar relation holds for $\boldsymbol{R226}$. ■

Reversibility of CA

There are 6 reversible ECA and 1800 reversible 3 states CA on ENB, see page 436 of [5].

For 2 states CA, we have

Proposition 7 *The set of (6) reversible ECA is closed under permutation of neighborhoods.*

Proof: There are 6 reversible ECA; R15, R51, R85, R170, R204, R240 expressed by Wolfram numbers, see page 436 of [5]. Their local functions are listed in Table 1. In the sequel such 6 functions are called elementary reversible functions(ERF for short). Note that R204 is the conjugate of R51, R240 is the conjugate of R15 and R170 is the conjugate of R85.

Table 1. Reversible CA with 2 states 3 neighbors

local configuration	000	001	010	011	100	101	110	111
R15	1	1	1	1	0	0	0	0
R51	1	1	0	0	1	1	0	0
R85	1	0	1	0	1	0	1	0
R170	0	1	0	1	0	1	0	1
R204	0	0	1	1	0	0	1	1
R240	0	0	0	0	1	1	1	1

For instance, from R51, by permuting ENB, we obtain R15 and R85.
Summing up, we see that

$$(\boldsymbol{R51}, \boldsymbol{\pi_1}) \cong (\boldsymbol{R85}, \boldsymbol{\pi_0}), \quad (\boldsymbol{R51}, \boldsymbol{\pi_2}) \cong (\boldsymbol{R15}, \boldsymbol{\pi_0})$$
$$(\boldsymbol{R51}, \boldsymbol{\pi_3}) \cong (\boldsymbol{R15}, \boldsymbol{\pi_0}), \quad (\boldsymbol{R51}, \boldsymbol{\pi_4}) \cong (\boldsymbol{R15}, \boldsymbol{\pi_0})$$
$$(\boldsymbol{R51}, \boldsymbol{\pi_5}) \cong (\boldsymbol{R51}, \boldsymbol{\pi_0}).$$

Similarly from R204 we obtain R170 and R240 by permutation. Note, however, that R170 can not be obtained by permutation of R51, but by complementation. In other word, $\mathcal{P}(\boldsymbol{R51}, \boldsymbol{\nu_E}) \cap \mathcal{P}(\boldsymbol{R170}, \boldsymbol{\nu_E}) = \emptyset$. ■

In case of binary CA, reversibility is independent of the neighborhood.

Proposition 8 *Let* f_{RELF} *be an ELF contained by Table 1. Then* (f_{RELF}, ν) *is reversible for any* ν*, in particular for* $\nu \neq ENB$*.*

Proof: $R15 = x+1$, where variables y and z are don't care, and CA $(R15, ENB)$ is essentially a right shift by 1 cell. Now, it is seen that $(R15, (-k, l, m))$ is a right shift by k cells for any integers k, l, m, which is a reversible CA. Since $R51 = y + 1$ and $R85 = z + 1$, we have the same conclusion that they define reversible CAs for any neighborhood functions. As for $R170 = z, R204 = y$ and $R240 = x$, we have the same conclusion. ■

Problem 1 *Is there an irreversible ELF (a function not contained by Table 1) such that* $(\boldsymbol{f_{ELF}}, \boldsymbol{\nu})$ *becomes reversible, when* $\boldsymbol{\nu} \neq \boldsymbol{ENB}$.

In case of 3 states CA, a proposition like Proposition 8 does not hold.

Proposition 9 *There is a 3 states local function* f_{R3} *such that* (f_{R3}, ν) *is reversible if* $\nu = ENB$, *but not if* $\nu \neq ENB$.

Proof: Among $\mathbf{3^{3^3}}$ 3 states 3-ary local functions, 1800 give rise to reversible CA on ENB [5]. However, for example, R[270361043509] is proved not injective nor surjective on $(\mathbf{-1, 0, 2})$.

Injectivity: R[270361043509] on neighborhood $(\mathbf{-1, 0, 2})$ maps both global configurations $\overline{\mathbf{0}}\mathbf{1}\overline{\mathbf{0}}$ and $\overline{\mathbf{0}}\mathbf{11}\overline{\mathbf{0}}$ to $\overline{\mathbf{1}}\mathbf{0}\overline{\mathbf{1}}$. So, it is not injective.

Surjectivity: Clemens Lode [2], student of the University of Karlsruhe, wrote a Java program called catest105, based on the Sutner-Tarjan algorithm, which tests injectivity and surjectivity of CA for arbitrary neighborhoods. The program classifies R270361043509 as not to be injective nor surjective on $(\mathbf{-1, 1, 0})$, $(\mathbf{-1, 0, 2})$ and on other several neighborhoods. ■

Java Applet Program catest105 which tests injectivity and surjectivity of 1-dimensional CA on different neighborhoods

The following slides show the front page of catest105 with example parameters and testing results for 3-ary CA Rule 270361043509 and 277206003607 (3 states) as well as Rule 90 (2 states) respectively.

- The neighborhood size is given by a positive integer k and the significant neighborhood size is a positive integer $1 \leq h \leq k$. Then we have a significant neighborhood $N \subseteq \{0, 1, ..., k-1\}$ of size h.

- By selecting a parameter all neiborhood permutations the program tests CA on every significant neighborhood N of size h contained by the scope k neighborhood $\{0, 1, .., k-1\}$. For instance, there are 12 significant neighborhoods of size 3 in the scope 4 neighborhood.

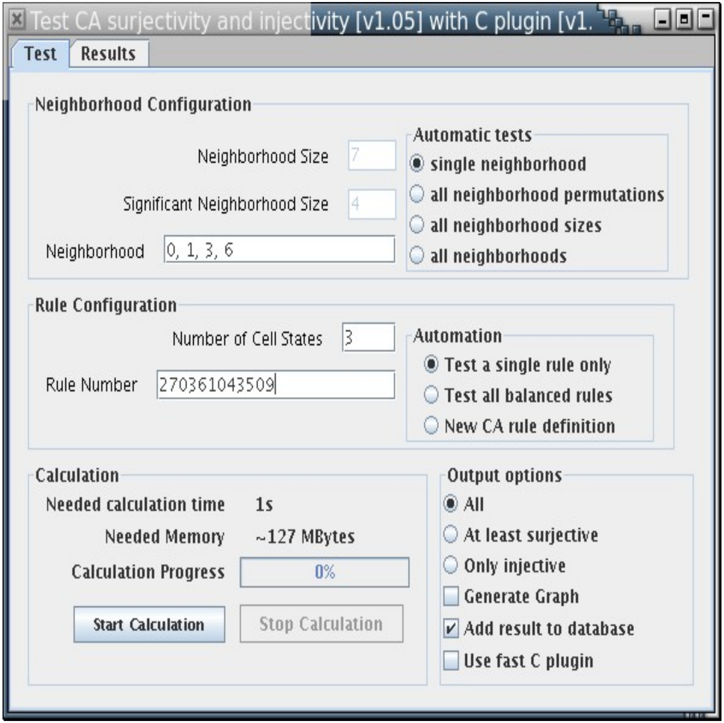
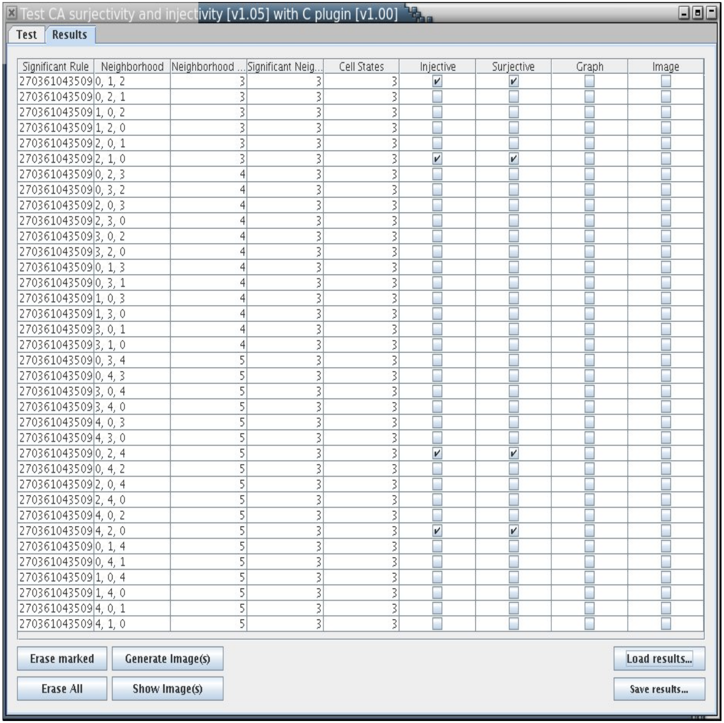


Figure 3: Front page of catest105



Significant Rule	Neighborhood	Neighborhood ...	Significant Neig...	Cell States	Injective	Surjective	Graph	Image
270361043509	0, 1, 2	3	3	3	✓	✓	☐	☐
270361043509	0, 2, 1	3	3	3	☐	☐	☐	☐
270361043509	1, 0, 2	3	3	3	☐	☐	☐	☐
270361043509	1, 2, 0	3	3	3	☐	☐	☐	☐
270361043509	2, 0, 1	3	3	3	☐	☐	☐	☐
270361043509	2, 1, 0	3	3	3	✓	✓	☐	☐
270361043509	0, 2, 3	4	3	3	☐	☐	☐	☐
270361043509	0, 3, 2	4	3	3	☐	☐	☐	☐
270361043509	2, 0, 3	4	3	3	☐	☐	☐	☐
270361043509	2, 3, 0	4	3	3	☐	☐	☐	☐
270361043509	3, 0, 2	4	3	3	☐	☐	☐	☐
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270361043509	0, 1, 4	5	3	3	☐	☐	☐	☐
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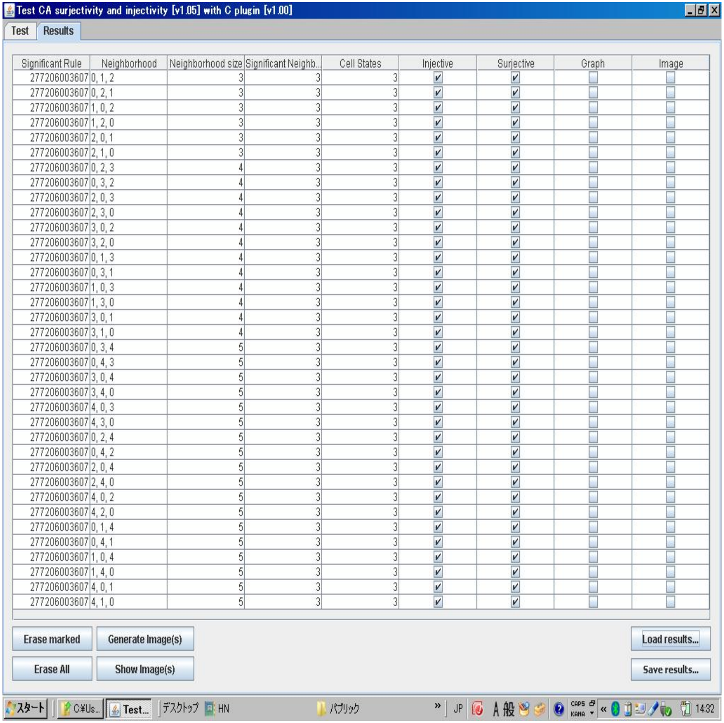
270361043509	1, 0, 4	5	3	3	☐	☐	☐	☐
270361043509	1, 4, 0	5	3	3	☐	☐	☐	☐
270361043509	4, 0, 1	5	3	3	☐	☐	☐	☐
270361043509	4, 1, 0	5	3	3	☐	☐	☐	☐

Figure 4: Test of 270361043509



Significant Rule	Neighborhood	Neighborhood size	Significant Neighb...	Cell States	Injective	Surjective	Graph	Image
277206003607	0, 1, 2	3	3	3	✓	✓		
277206003607	0, 2, 1	3	3	3	✓	✓		
277206003607	1, 0, 2	3	3	3	✓	✓		
277206003607	1, 2, 0	3	3	3	✓	✓		
277206003607	2, 0, 1	3	3	3	✓	✓		
277206003607	2, 1, 0	3	3	3	✓	✓		
277206003607	0, 2, 3	4	3	3	✓	✓		
277206003607	0, 3, 2	4	3	3	✓	✓		
277206003607	2, 0, 3	4	3	3	✓	✓		
277206003607	2, 3, 0	4	3	3	✓	✓		
277206003607	3, 0, 2	4	3	3	✓	✓		
277206003607	3, 2, 0	4	3	3	✓	✓		
277206003607	0, 1, 3	4	3	3	✓	✓		
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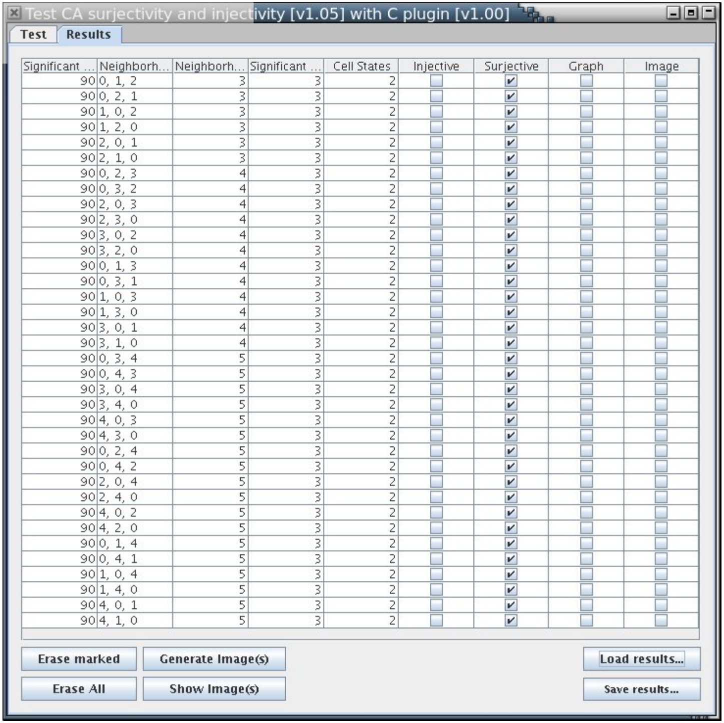
277206003607	1, 4, 0	5	3	3	✓	✓		
277206003607	4, 0, 1	5	3	3	✓	✓		
277206003607	4, 1, 0	5	3	3	✓	✓		

Figure 5: Test of 277206003607



Significant ...	Neighborh...	Neighborh...	Significant ...	Cell States	Injective	Surjective	Graph	Image
90	0, 1, 2	3	3	2	☐	☑	☐	☐
90	0, 2, 1	3	3	2	☐	☑	☐	☐
90	1, 0, 2	3	3	2	☐	☑	☐	☐
90	1, 2, 0	3	3	2	☐	☑	☐	☐
90	2, 0, 1	3	3	2	☐	☑	☐	☐
90	2, 1, 0	3	3	2	☐	☑	☐	☐
90	0, 2, 3	4	3	2	☐	☑	☐	☐
90	0, 3, 2	4	3	2	☐	☑	☐	☐
90	2, 0, 3	4	3	2	☐	☑	☐	☐
90	2, 3, 0	4	3	2	☐	☑	☐	☐
90	3, 0, 2	4	3	2	☐	☑	☐	☐
90	3, 2, 0	4	3	2	☐	☑	☐	☐
90	0, 1, 3	4	3	2	☐	☑	☐	☐
90	0, 3, 1	4	3	2	☐	☑	☐	☐
90	1, 0, 3	4	3	2	☐	☑	☐	☐
90	1, 3, 0	4	3	2	☐	☑	☐	☐
90	3, 0, 1	4	3	2	☐	☑	☐	☐
90	3, 1, 0	4	3	2	☐	☑	☐	☐
90	0, 3, 4	5	3	2	☐	☑	☐	☐
90	0, 4, 3	5	3	2	☐	☑	☐	☐
90	3, 0, 4	5	3	2	☐	☑	☐	☐
90	3, 4, 0	5	3	2	☐	☑	☐	☐
90	4, 0, 3	5	3	2	☐	☑	☐	☐
90	4, 3, 0	5	3	2	☐	☑	☐	☐
90	0, 2, 4	5	3	2	☐	☑	☐	☐
90	0, 4, 2	5	3	2	☐	☑	☐	☐
90	2, 0, 4	5	3	2	☐	☑	☐	☐
90	2, 4, 0	5	3	2	☐	☑	☐	☐
90	4, 0, 2	5	3	2	☐	☑	☐	☐
90	4, 2, 0	5	3	2	☐	☑	☐	☐
90	0, 1, 4	5	3	2	☐	☑	☐	☐
90	0, 4, 1	5	3	2	☐	☑	☐	☐
90	1, 0, 4	5	3	2	☐	☑	☐	☐
90	1, 4, 0	5	3	2	☐	☑	☐	☐
90	4, 0, 1	5	3	2	☐	☑	☐	☐
90	4, 1, 0	5	3	2	☐	☑	☐	☐

Figure 6: Test of 90

Conjecture 1 *By use of catest105, we see that another 3 states reversible CA R277206003607 in [5] is reversible on all permutations of ENB and on permutations of many other neighborhoods such as* $(-1, 0, 2), (-1, 0, 3)$ *and* $(-2, 0, 1)$*. See the previous slide.*

From this, we conjecture that R277206003607 is reversible for arbitrary neighborhoods of size 3 in $\mathbb{Z}$*.*

Java Applet Simulator for 1-dimensional CA on different neighborhoods

We are using a Java Applet simulator of 1-dimensional CA coded by Christoph Scheben for the Institute of Informatics, University of Karlsruhe [4].

It works for arbitrary local function, number of states, neighborhood and initial configuration (including random configurations) up to 1,000 cells with cyclic boundary and 1,000 time steps. The simulator is the first of this kind —arbitrary neighborhoods.

The following figures are outputs of the simulator, where the local function Rule 110 is fixed while the neighborhood is changed. Number of cells $\times$ time is 1000×1000 with cyclic boundary. The initial configuration is random ($p(0) = p(1) = 0.5$) and the same for all cases.

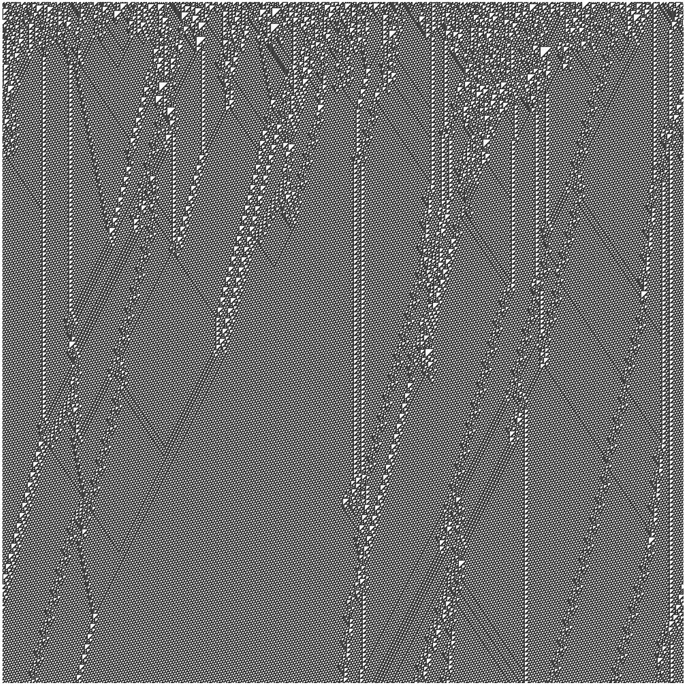


Figure 7: Rule 110 with neighborhood $(-1, 0, 1)$=ENB

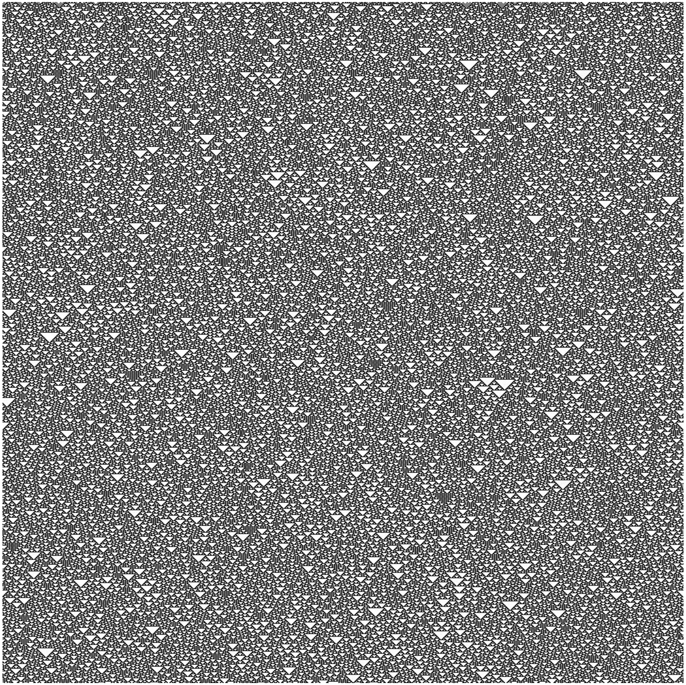


Figure 8: Rule 110 with neighborhood $(0, -1, 1)$

Thank you for your attention!

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as well as to his students for coding Java Applet programs.

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