Hierarchical Relaxations of the Correctness Preserving Property for Restarting Automata[1]

František Mráz[1] Friedrich Otto[2] Martin Plátek[1]

[1] Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

[2] Fachbereich Elektrotechnik/Informatik, Universität Kassel, Kassel, Germany

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A linguistic motivation

(Czech, Russian, German) sentence analysis - - Prague linguistic group (Sgall, Hajičová, Panevová) , Melčuk, Kunze.

This method is different from the Chomskian type of sentence analysis in an essential way.
It is complex and has two basic phases.

- morphological disambiguation, lexico-semantic disambiguation, ... } additional information is inserted into the input sentence – auxiliary symbols
- analysis by reduction - correctness preserving simplifications of a fully disambigued sentence

Formal models

- model for the sentence analysis – restarting automaton
- model for the analysis by reduction – correctness preserving restarting automaton

Outline

1. Restarting automaton
   - Definition
   - Meta-instructions
   - Languages defined by restarting automata
   - Basic properties of restarting automata
2. Relaxations of the Correctness Preserving Property
   - Cyclic relaxation and error relaxation of the Correctness Preserving Property
3. Results
   - The Hierarchy
   - Time-complexity results
   - Technicalities
4. Conclusions

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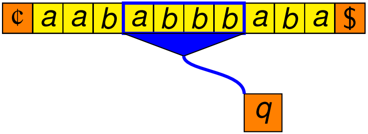
Restarting automaton (RLWW - automaton)

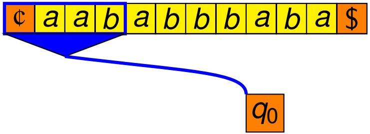
basic model



- restarting automaton consists of:
  - finite control
  - elastic working tape with sentinels
  - read/write window of fixed size
  - operations: move right, move left, rewrite, accept, restart

RLWW - automaton

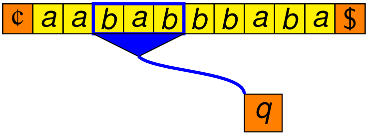
computation, start, restart, rewrite



- computation
  - starts on the left end in the starting state, the same situation after a restart
  - between two (re)starts exactly one rewriting of the content of its window must occur (local change), it must shorten the tape

RLWW-automaton

denotations



$M = (Q, \Sigma, \Gamma, ¢, \$, q_0, k, \delta)$:

- Q is a finite set of states,
- Σ is a finite input alphabet,
- Γ is a finite tape alphabet, $\Sigma \subseteq \Gamma$,
- $¢, \$$ are sentinels, $\{¢, \$\} \cap \Gamma = \emptyset$
- $q_0 \in Q$ is the initial state
- δ is the transition relation = a finite set of instructions.

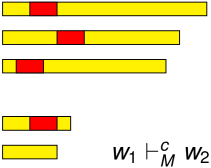
RLWW-automaton as a reduction system

Cycles, tails, reductions

a cycle: any part of a computation starting from a (re)starting configuration and ending by the next restart

a tail: the part of a computation after the last restart

$w_1 =$
$w_2 =$
$w_3 =$
...
$w_{n-1} =$
$w_n =$



$w_1 \vdash_M^c w_2 \vdash_M^c \cdots \vdash_M^c w_{n-1} \vdash_M^c w_n$

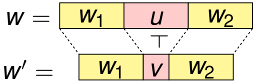
Note: if w_n is accepted by M then all $w_1, \ldots, w_n$ are accepted by M.

Meta-instructions

A more convenient representation

$(E_1, u \to v, E_2)$ a rewriting meta-instruction,

- $E_1, E_2 \subseteq \Gamma^*$ are regular languages called constraints
- $u, v \in \Gamma^*$ such that $|u| > |v|$,
- if $w = w_1 u w_2$, where $¢ \cdot w_1 \in E_1$, $w_2 \cdot \$ \in E_2$ then



(E, Accept) an accepting meta-instruction

- $E \subseteq \Gamma^*$ is a regular language

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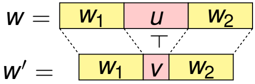
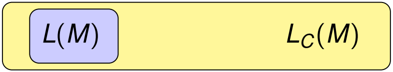


(E, Accept) an accepting meta-instruction

- $E \subseteq \Gamma^*$ is a regular language

Languages defined by a RLWW-automaton

- A word w is accepted by M if there exists a computation which starts by the (re)starting configuration $q_0¢w\$$ and ends by an accepting configuration.
- The set of *all* words accepted by M is denoted as $L_C(M)$ and it is called the complete (characteristic) language accepted by the RLWW-automaton M.
- $L(M) = L_C(M) \cap \Sigma^*$ denotes the *input language* accepted by M



Sample restarting automaton M_s

M_s, with input alphabet $\Sigma = \{a, b\}$, one auxiliary symbol C ($\Gamma = \Sigma \cup \{C\}$) and the following meta-instructions:

- accepts the input language $L(M_s) = \{ww^R \mid w \in \{a,b\}^*\}$,
- the complete language of M_s is $L_C(M_s) = \{ww^R, wCw^R \mid w \in \{a,b\}^*\}$.

1. $(¢ \cdot C \cdot \$, \text{Accept})$
2. $(¢ \cdot (a+b)^*, aCa \to C, (a+b)^* \cdot \$)$
3. $(¢ \cdot (a+b)^*, bCb \to C, (a+b)^* \cdot \$)$
4. $(¢ \cdot (a+b)^*, aa \to C, (a+b)^* \cdot \$)$
5. $(¢ \cdot (a+b)^*, bb \to C, (a+b)^* \cdot \$)$

abbbbbba
abbCbba
abCba
aCa
C
Accept

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$abbbbbba$
$abbCbba$
$abCba$
aCa
C
Accept

A mistake by M_s

- the complete language of M_s is $L_C(M_s) = \{ww^R, wCw^R \mid w \in \{a,b\}^*\}$.

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abbbbbba
abbbbCa

Important notion. A mistake by M - M reduces in a cycle a word from $L_C(M)$ to a word not belonging to $L_C(M)$.

M_s can make in the first cycle many different mistakes

Basic properties

Definition

(Correctness Preserving Property (no mistakes))
An RLWW-automaton M is *correctness preserving* if $u \in L_C(M)$ and $u \vdash_M^{c^*} v$ imply that $v \in L_C(M)$.

Definition

(Error Preserving Property)
An RLWW-automaton M is *error preserving* if $u \notin L_C(M)$ and $u \vdash_M^{c^*} v$ imply that $v \notin L_C(M)$.

Basic facts

- Each RLWW-automaton is error preserving.
- All deterministic RLWW-automata are correctness preserving.
- There are nondeterministic RLWW-automata that are not correctness preserving.

Cyclic relaxation and error relaxation

(Informal) definition. Let i be a non-negative integer. A RLWW-automaton M has *cyclic relaxation* of degree i, if M cannot make a mistake after the first i cycles (on a word w from $L_C(M)$).

(Informal) definition. Let M be an RLWW-automaton, and let j be a non-negative integer. M has *error relaxation* of degree j, if for the first cycle on a word from $L_C(M)$, there are at most j different mistakes that M can possibly make.

Notation

c(i)-RLWW – the class of RLWW-automata with cyclic relaxation of degree i,
e(j)-RLWW – the class of RLWW-automata with error relaxation of degree j,
ce(i, j)-RLWW – the class of RLWW-automata that simultaneously have cyclic relaxation of degree i and error relaxation of degree j.

Here we denote the corresponding classes of complete languages simply by –

c(i), e(j), ce(i, j).

Relaxations

Complete correctness preserving RLWW-languages

= c(0) = e(0) = ce(0, 0)

= complete deterministic RLWW-languages

[Messerchmidt,Otto 2007]

The Complete Hierarchy

$$
\begin{array}{ccccccccccc}
\mathrm{ce}(1,1) & \longrightarrow & \mathrm{ce}(2,1) & \longrightarrow & \mathrm{ce}(3,1) & \longrightarrow & \cdots & \longrightarrow & \bigcup_{i\geq 0}\mathrm{ce}(i,1) & \longrightarrow & \mathrm{e}(1) \\
\downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
\mathrm{ce}(1,2) & \longrightarrow & \mathrm{ce}(2,2) & \longrightarrow & \mathrm{ce}(3,2) & \longrightarrow & \cdots & \longrightarrow & \bigcup_{i\geq 0}\mathrm{ce}(i,2) & \longrightarrow & \mathrm{e}(2) \\
\downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
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\downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
\cdots & & \cdots & & \cdots & & \cdots & & \cdots & & \cdots \\
\downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
\bigcup_{j\geq 0}\mathrm{ce}(1,j) & \longrightarrow & \bigcup_{j\geq 0}\mathrm{ce}(2,j) & \longrightarrow & \bigcup_{j\geq 0}\mathrm{ce}(3,j) & \longrightarrow & \cdots & \longrightarrow & \bigcup_{i,j\geq 0}\mathrm{ce}(i,j) & \longrightarrow & \bigcup_{j\geq 0}\mathrm{e}(j) \\
\downarrow & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
\mathrm{c}(1) & \longrightarrow & \mathrm{c}(2) & \longrightarrow & \mathrm{c}(3) & \longrightarrow & \cdots & \longrightarrow & \bigcup_{i\geq 0}\mathrm{c}(i) & \longrightarrow & \mathcal{L}_C(\mathrm{RLWW})
\end{array}
$$

Time-complexity results

Theorem

If M is a ce(i, j)-RLWW*-automaton, then the membership problems for the languages $L_C(M)$ and $L(M)$ are solvable in time $O((j+1)^i \cdot n^2)$.*

Theorem

If M is a c(i)-RLWW*-automaton for some $i \geq 0$, then the membership problems for the languages $L_C(M)$ and $L(M)$ are solvable in time $O(n^{i+2})$.*

Technicalities

- $L_{1,1} := \{ a^n b^n, a^n c b^n \mid n \geq 1 \} \cup \{ a^n b^{2n}, a^n d b^{2n} \mid n \geq 1 \}$.

Theorem

$L_{1,1} \notin ce(0,0)$ $\quad$ $L_{1,1} \in ce(1,1)$

- $L_{2,1} := L_{1,1} \cdot L_{1,1}$.

Theorem

$L_{2,1} \in ce(2,1)$ $\quad$ $L_{2,1} \notin ce(1,1)$.

- $L_{+,1} := L_{1,1}^{+}$.

Theorem

For all $j \geq 1$, $\bigcup_{i \geq 0} \mathrm{ce}(i,j) \subset \mathrm{e}(j)$, $\quad$ $\bigcup_{i \geq 0} \mathrm{c}(i) \subset \mathcal{L}_C(\mathrm{RLWW})$.

Important separations

Theorem

$L_{s1} \in \mathrm{c}(1) \smallsetminus \bigcup_{j\geq 0} \mathrm{e}(j)$.

- $L_{s2} := \{ ww^R dw_1 w_1^R, wcw^R dw_1 w_1^R, wcw^R dw_1 cw_1^R \mid w, w_1 \in \{a, b\}^* \}$ is accepted by an RLWW-automaton with cyclic relaxation of degree 2

Theorem

$L_{s2} \in \mathrm{c}(2) \smallsetminus (\mathrm{c}(1) \cup \bigcup_{j\geq 0} \mathrm{e}(j))$.

Conclusions

- I believe that we are able to show a similar hierarchy for the input languages.
- I believe that in the close future we will use the cyclic and error relaxations in order to characterize the degree of non-determinism of CFL. We will also consider the non-constant boundaries for the relaxations above.
- The real model for the sentence analysis is a bit more complex. It allows more than one rewriting in a cycle and it uses a more complex partition of the working alphabet (vocabulary) into so called 'levels'.