Simple New Algorithms to Solve the FSSP

Jean-Baptiste.Yunes@liafa.jussieu.fr
http://www.liafa.jussieu.fr/~yunes/

LIAFA
CNRS - Université Paris 7

MCU'07, September 2007, Orléans

The Problem

- An old ``inverse'' problem
- Design a FLCA (Q,δ) such that from **any** initial configuration **Xq***

Xqqqqqqqqqq
initial configuration (time 0)

the computation leads to the synchronized corresponding configuration **F***

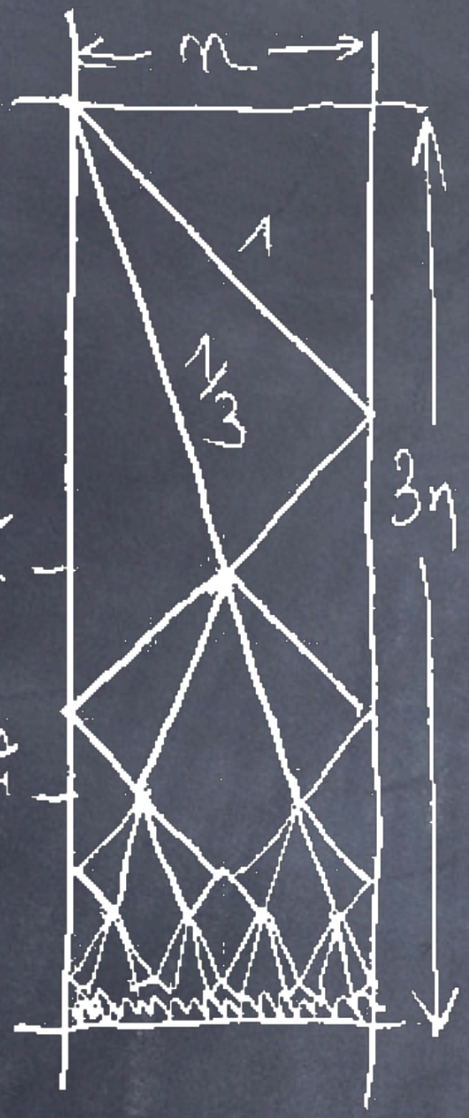
FFFFFFFFFF
synchronized configuration (time T(n))

avoiding forbidden configurations as

.....F.....

The problem

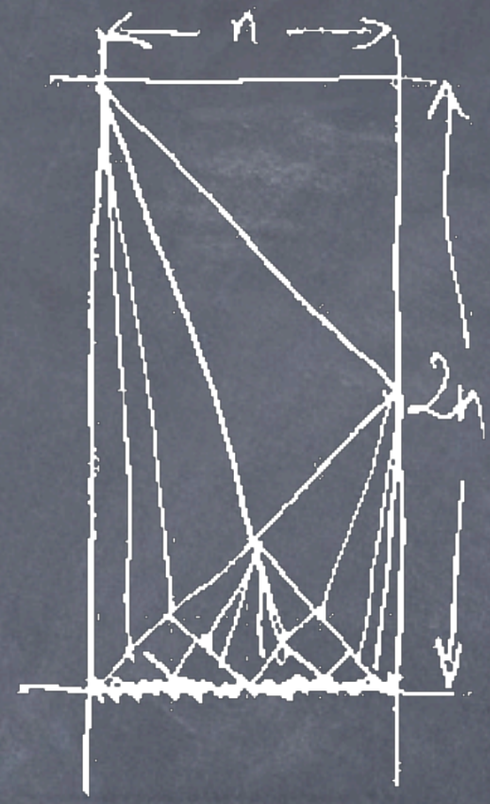
- Related to many others:
  - auto-reproductive machines
  - morphogenesis
  - distributed systems (dolev...)
  - graph traversal algorithms (kobayashi...)
- Recent studies suggest to think about a restatement of the problem:
  - reversible FS (morita...)
  - general insensitive FS (yunes...)
  - position insensitive FS (umeo, yunes...)

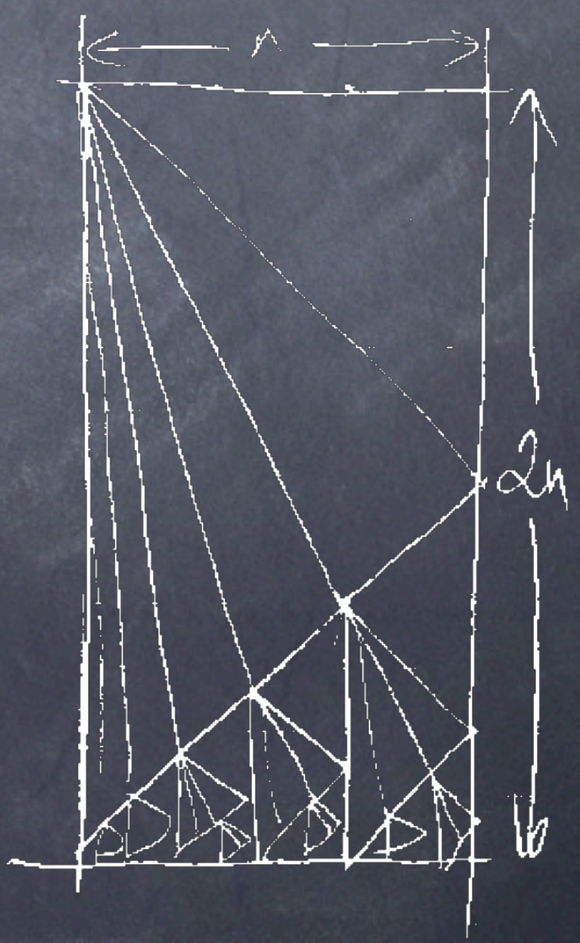


History

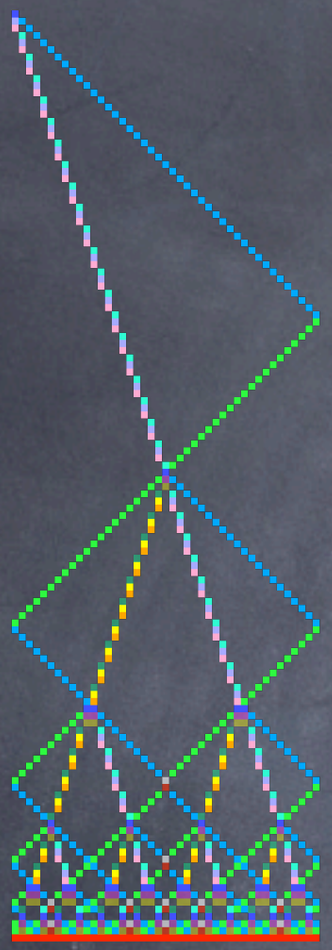
(quest for minimality)

- Stated by Myhill (1957)
  - 1st solution (unpublished) Goto (1962)
- McCarthy-Minsky
  T(n)=3n
- Waksman (1966)
  T(n)=2n
- Balzer (1967)
  T(n)=2n
- Mazoyer (1986)
  T(n)=2n



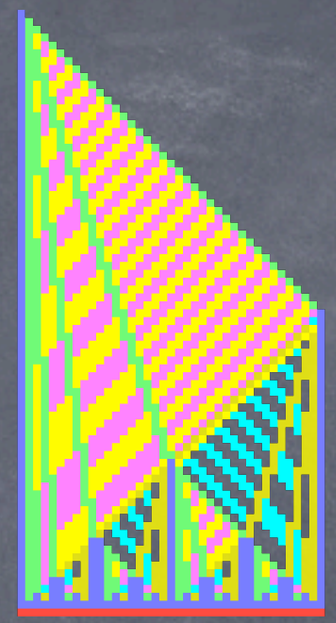


History
(quest for minimality)

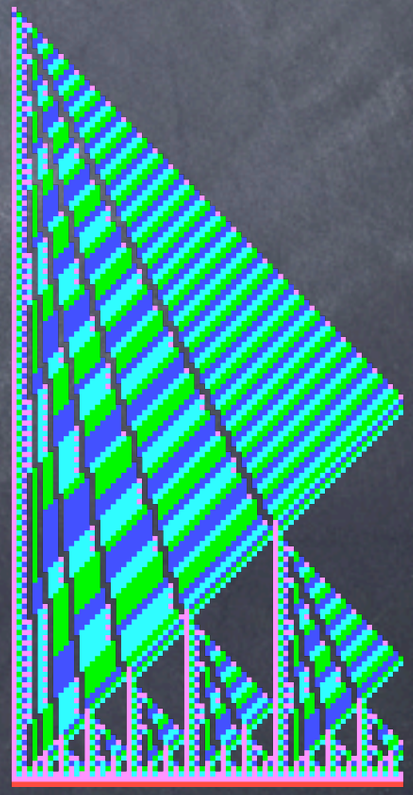


|Q|=15

- Stated by Myhill (1957)
- 1st solution (unpublished) Goto (1962)
- McCarthy-Minsky
  T(n)=3n W(n)=n.log(n)
- Waksman (1966)
  T(n)=2n |Q|=16 W(n)=n^2
- Balzer (1967)
  T(n)=2n |Q|=8 W(n)=n^2
- Mazoyer (1986)
  T(n)=2n |Q|=6 W(n)=n^2



|Q|=8

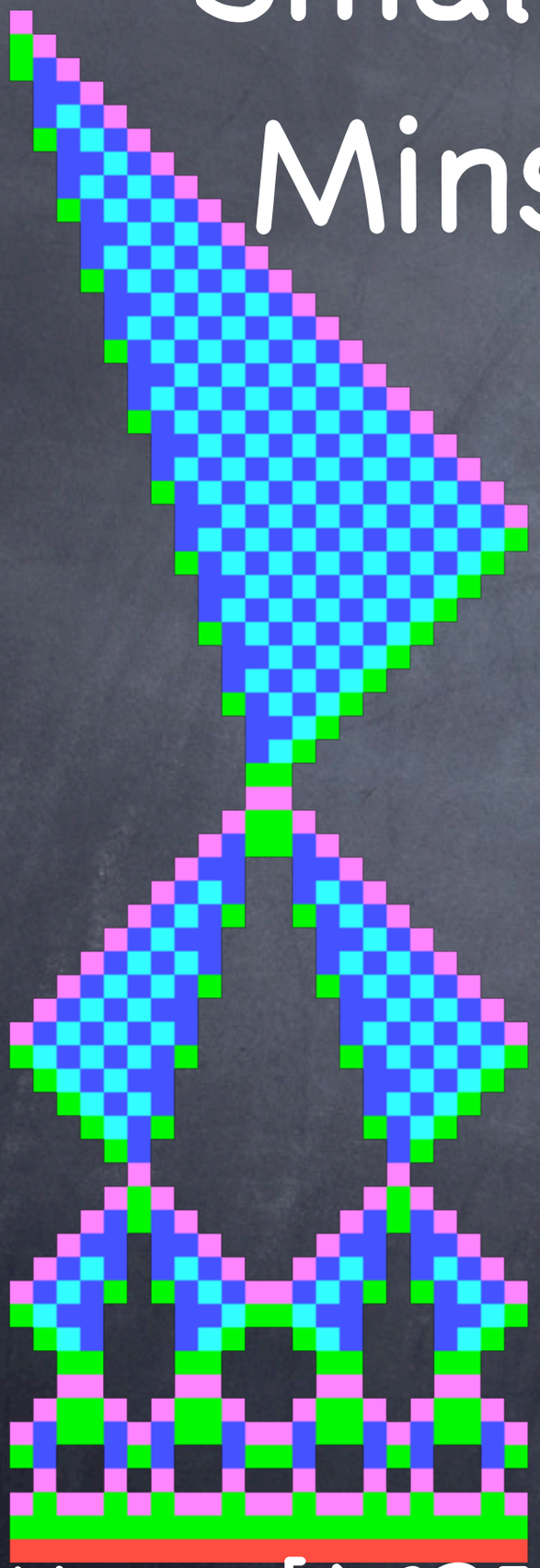


|Q|=6

History (cnt'd)

Author	Year	|Q|	T(n)	W(n)
Goto	1962	>2000	2n	n.log(n)
Minsky	1967	~20	3n	n.log(n)
Waksman	1966	16	2n	n^2
Balzer	1967	8	2n	n^2
Gerken	1986	7	2n	n^2
Mazoyer	1986	6	2n	n^2
Yunès	1993	7	3n	n.log(n)
Settle	2002	6	3n	n^2
Nogushi	2004	8	2n	n^2
Umeo	2006	6	3n	n^2
Yunès	2007	6	3n	n.log(n)

Small solutions using Minsky-McCarthy's scheme



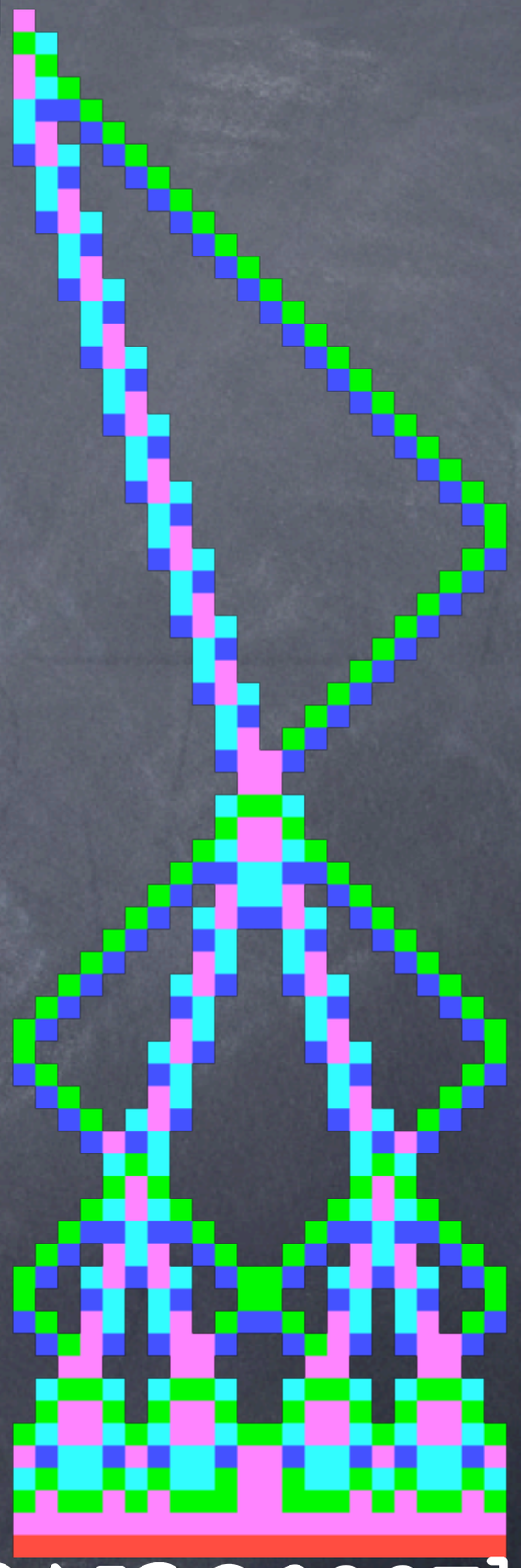
|Q|=6

T(n)=3n

Symmetric

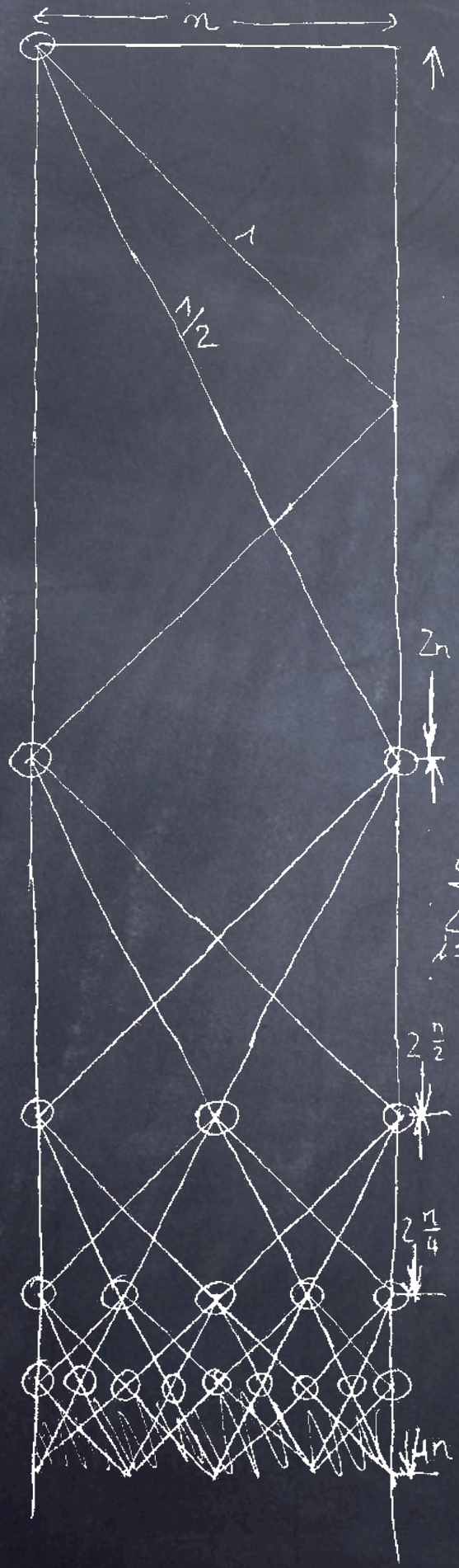
$W(n)=n^2$

Umeo [ACRI2006]



W(n)=n.log(n)

Yunès [RAIRO2007]



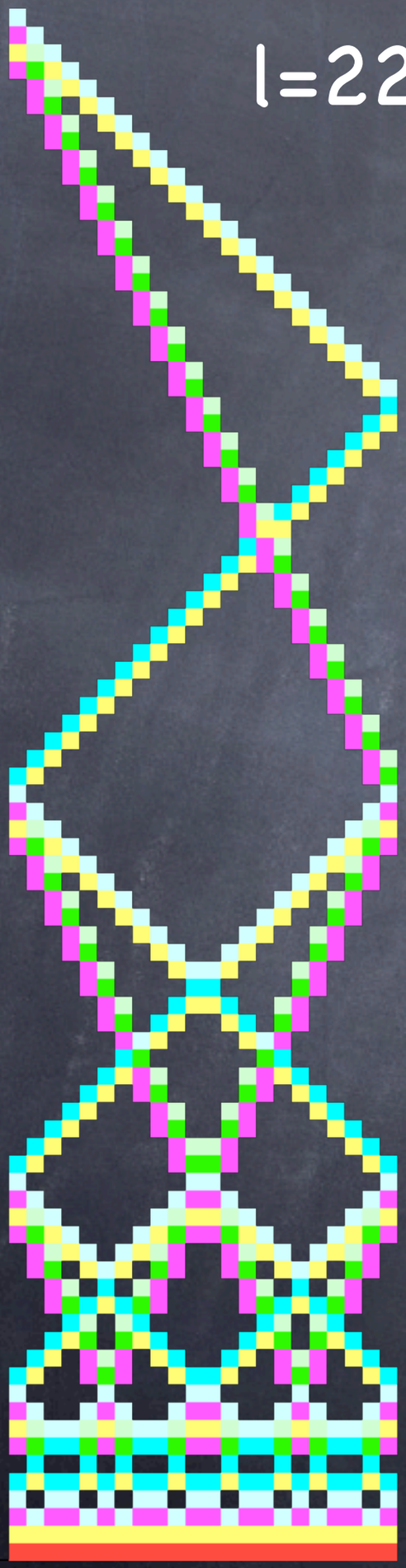
A New Scheme

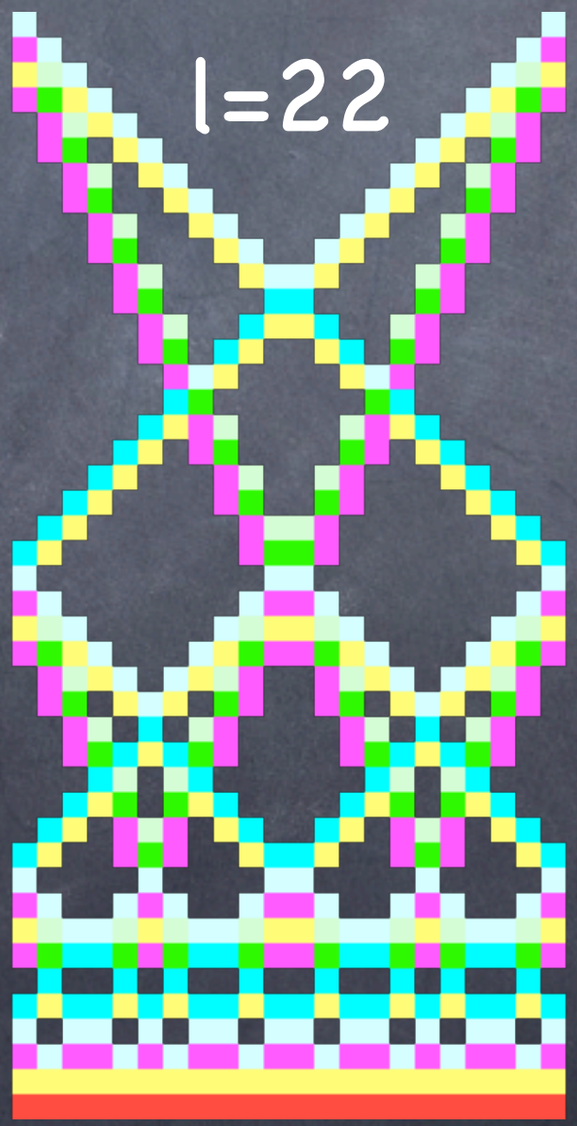
with only 2 signals:
slopes 1 & 2 ⇒ T(n)=4n

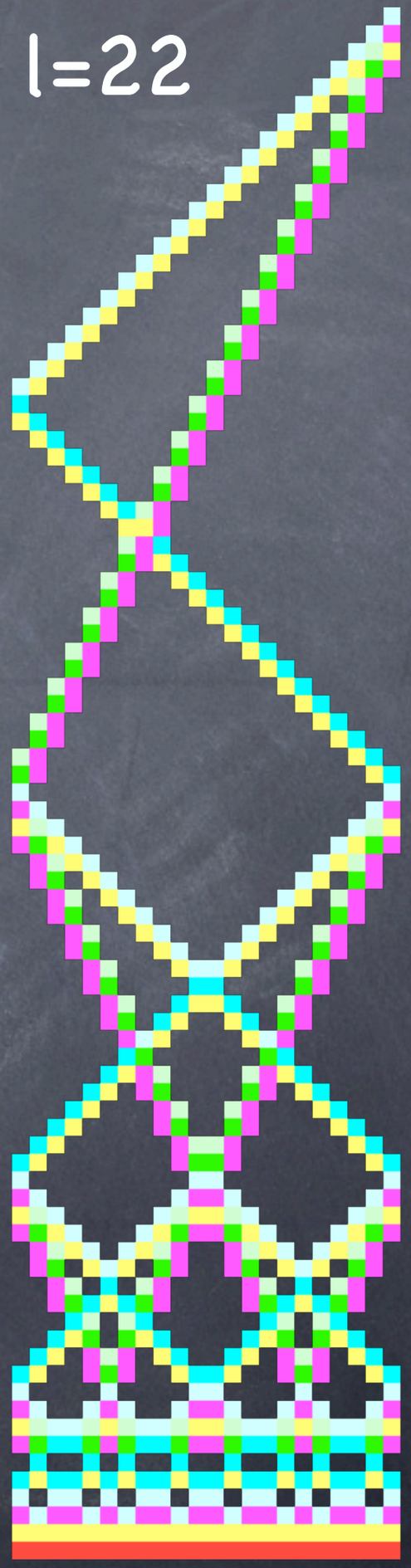
with the same scheme but
different slopes we have
slopes s & 2s ⇒ T(n)=4sn

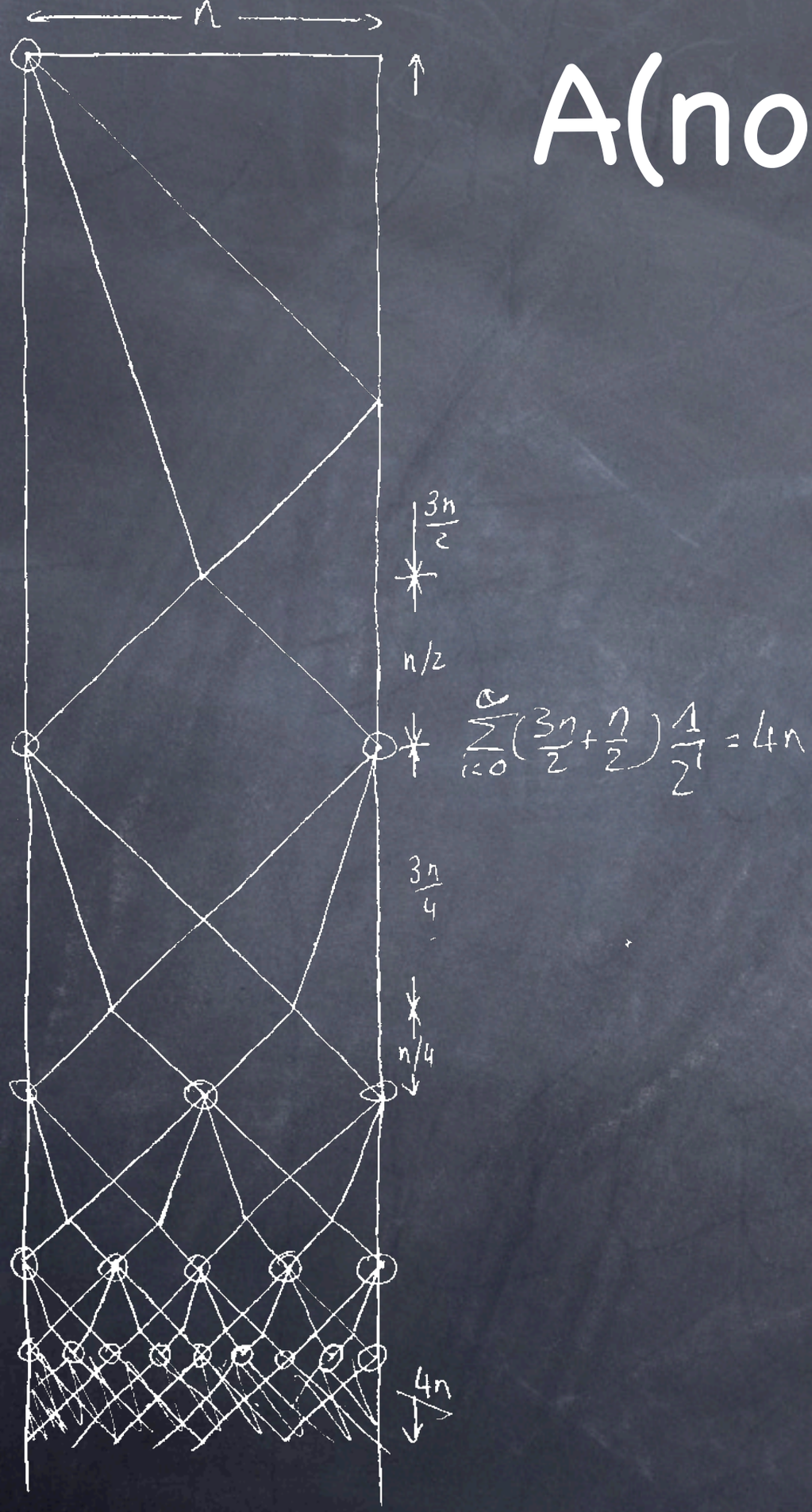
An implementation T(n)=4n |Q|=8

Symmetric, 195 rules









A(nother) new scheme

with 2 signals:
slopes 1 & 3 ⇒ T(n)=4n

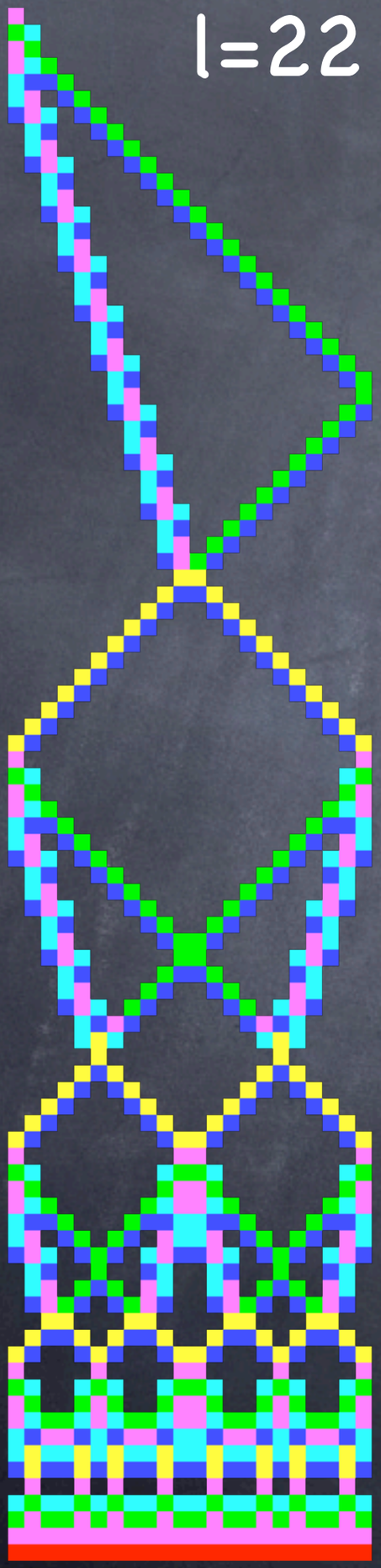
generalizing with three signals of slopes 1, 3 & s we have T(n)=(3+s)n

another generalization could use two signals of slopes s & 3s so that T(n)=4sn

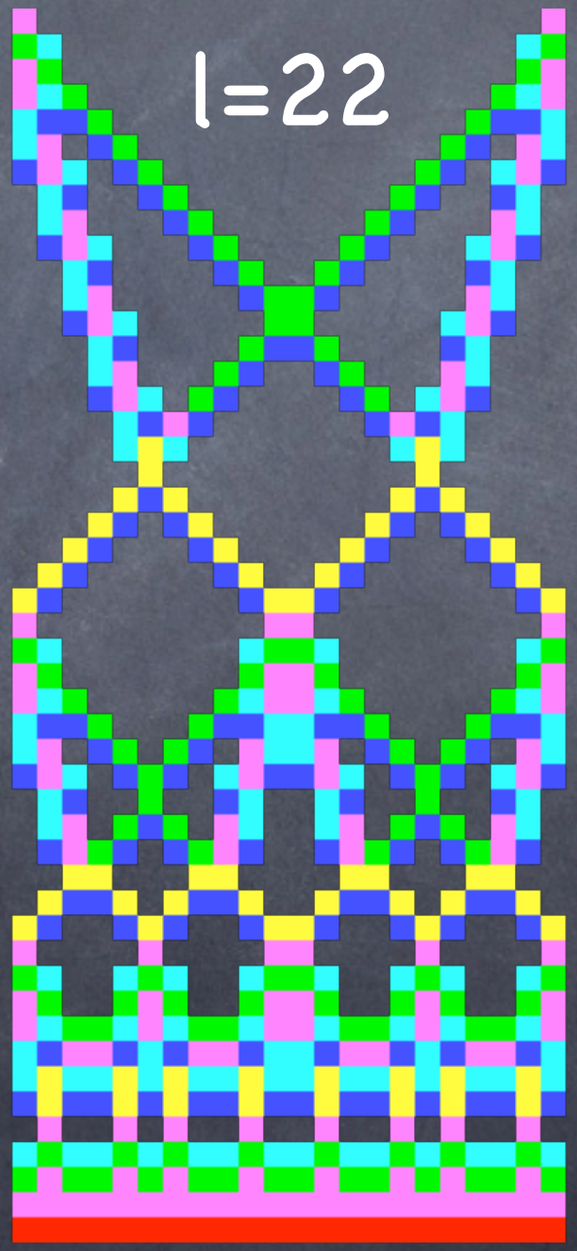
l=22

T(n)=4n |Q|=7

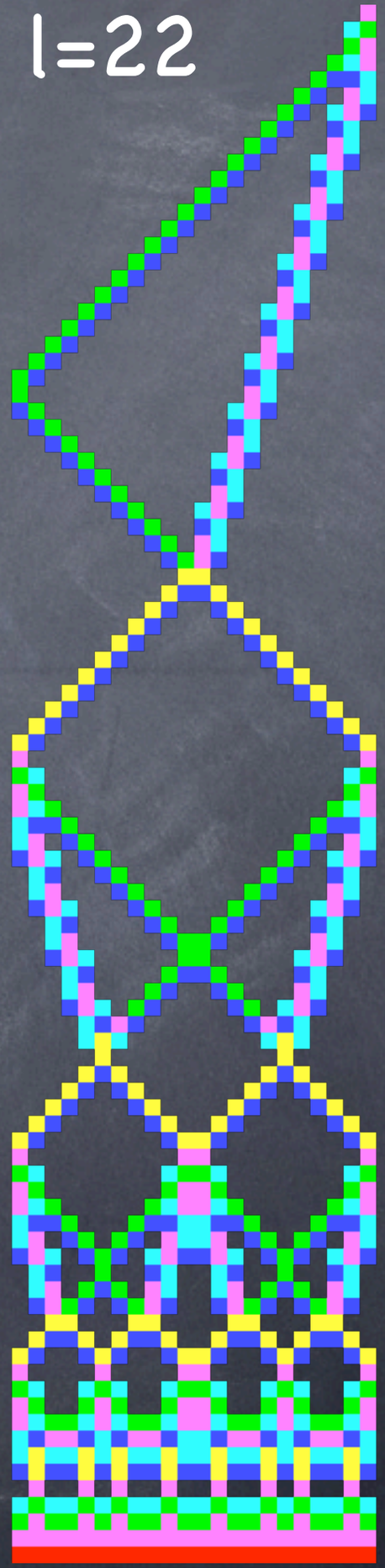
Symmetric, 162 rules



l=22



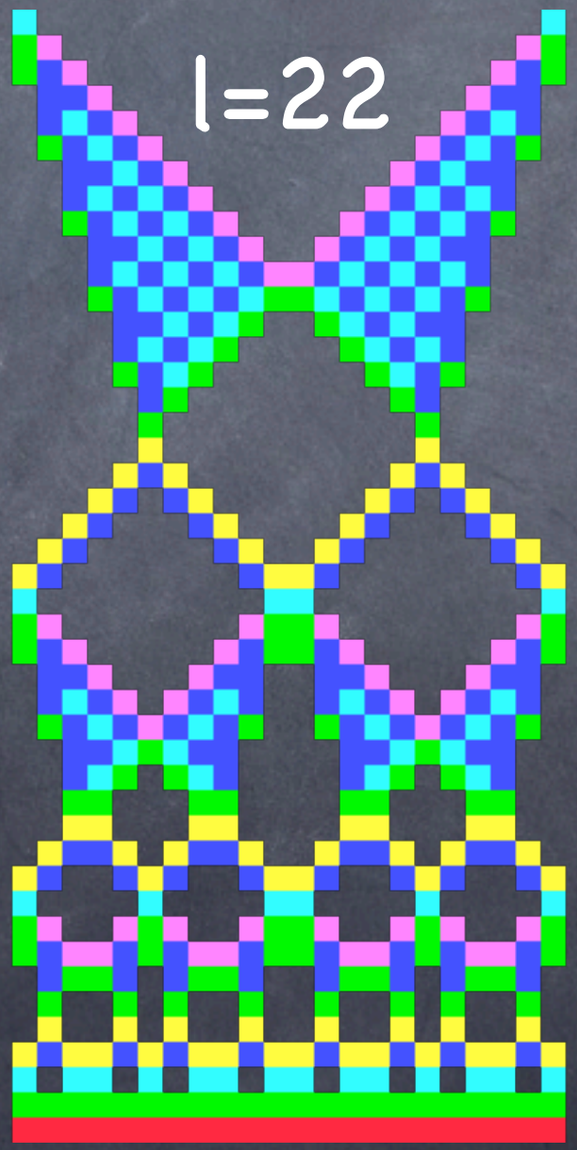
l=22



T(n)=4n |Q|=7

based on Umeo's [ACRI 2006]

Symmetric, 108 rules



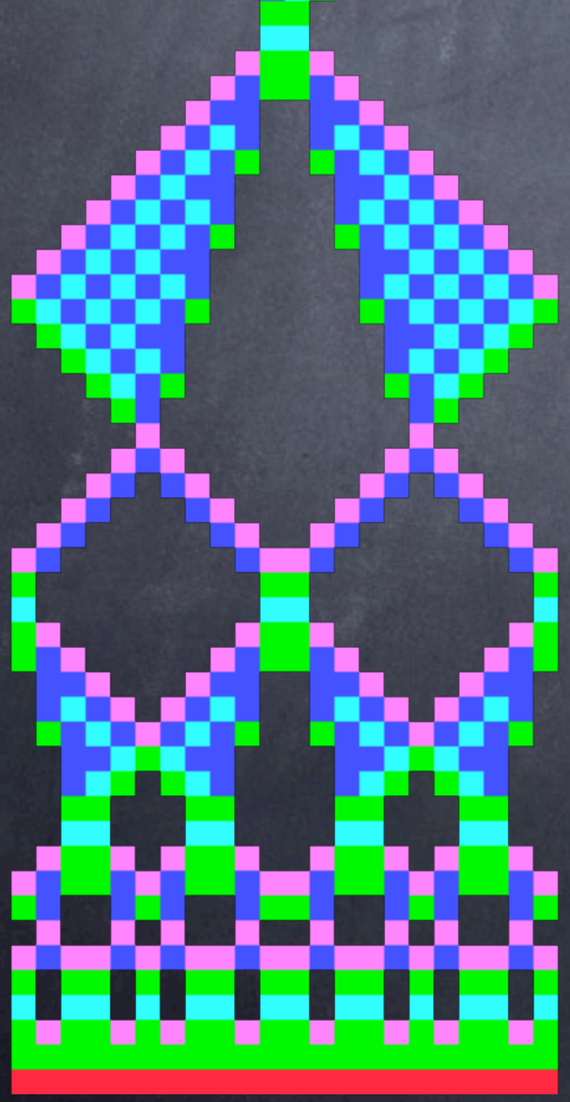
l=22

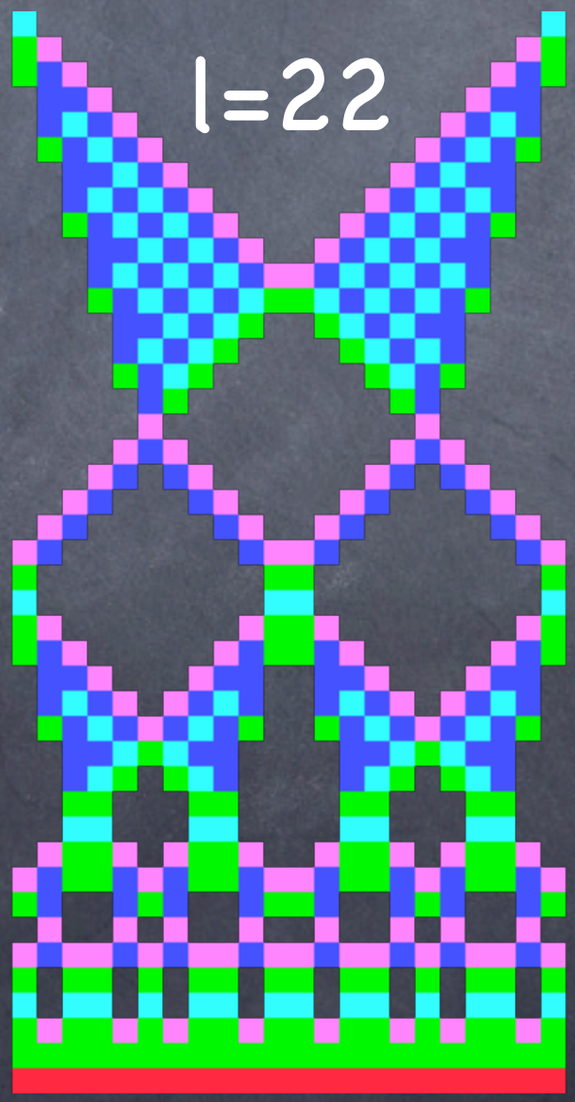
l=22

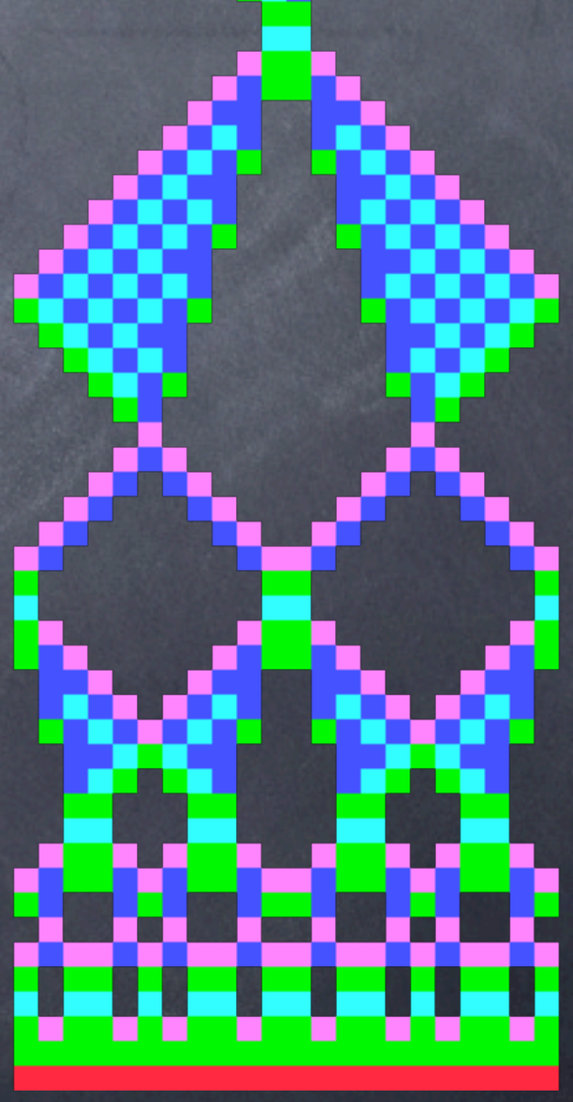
$3n \leq T(n) \leq 4n \quad |Q|=6$

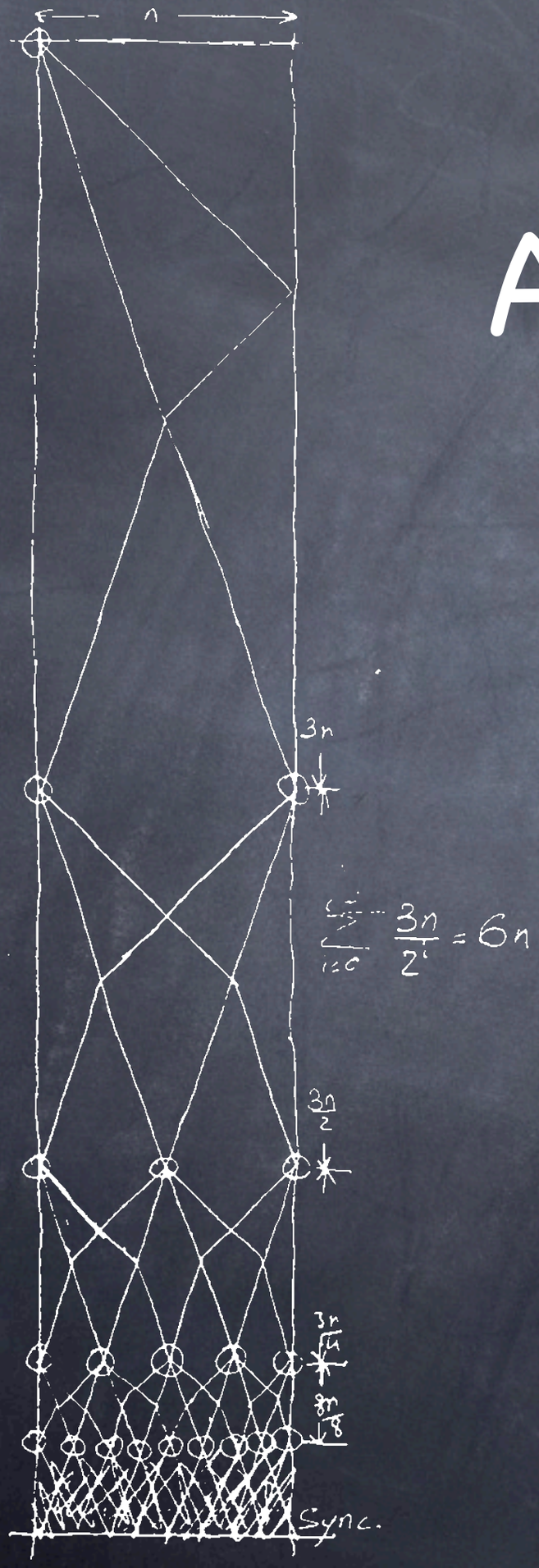
Symmetric, 108 rules

l=22







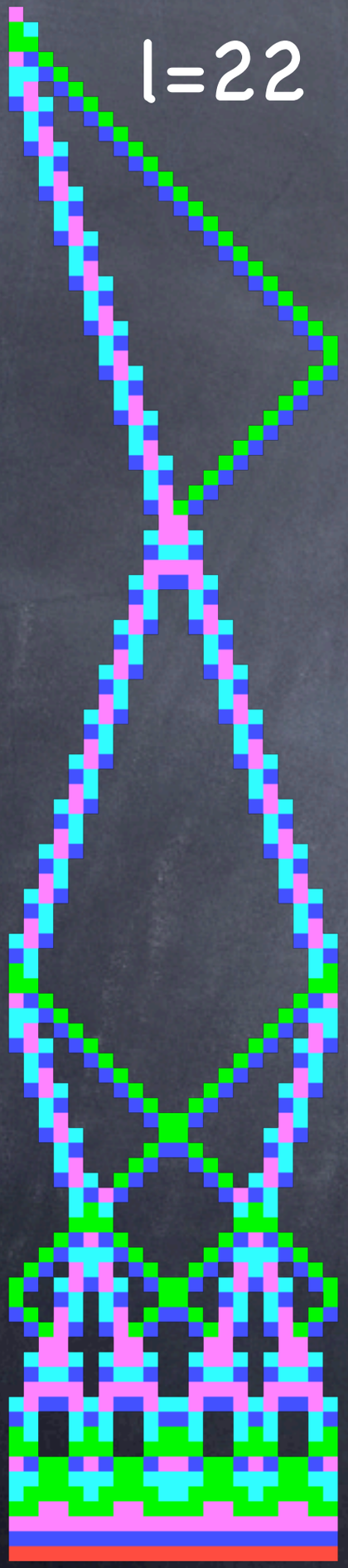


A(nother) new scheme

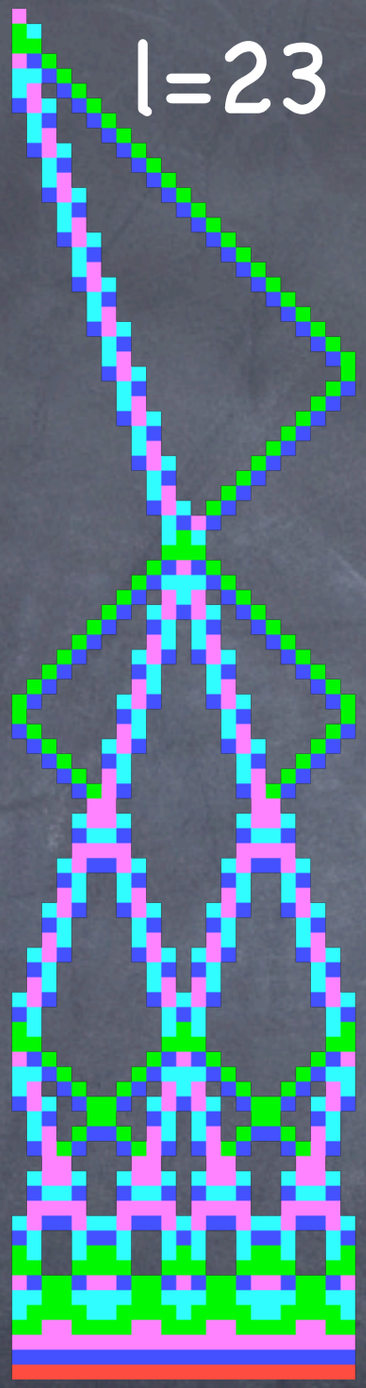
2 signals with slopes 1 & 3

T(n)=6n

l=22



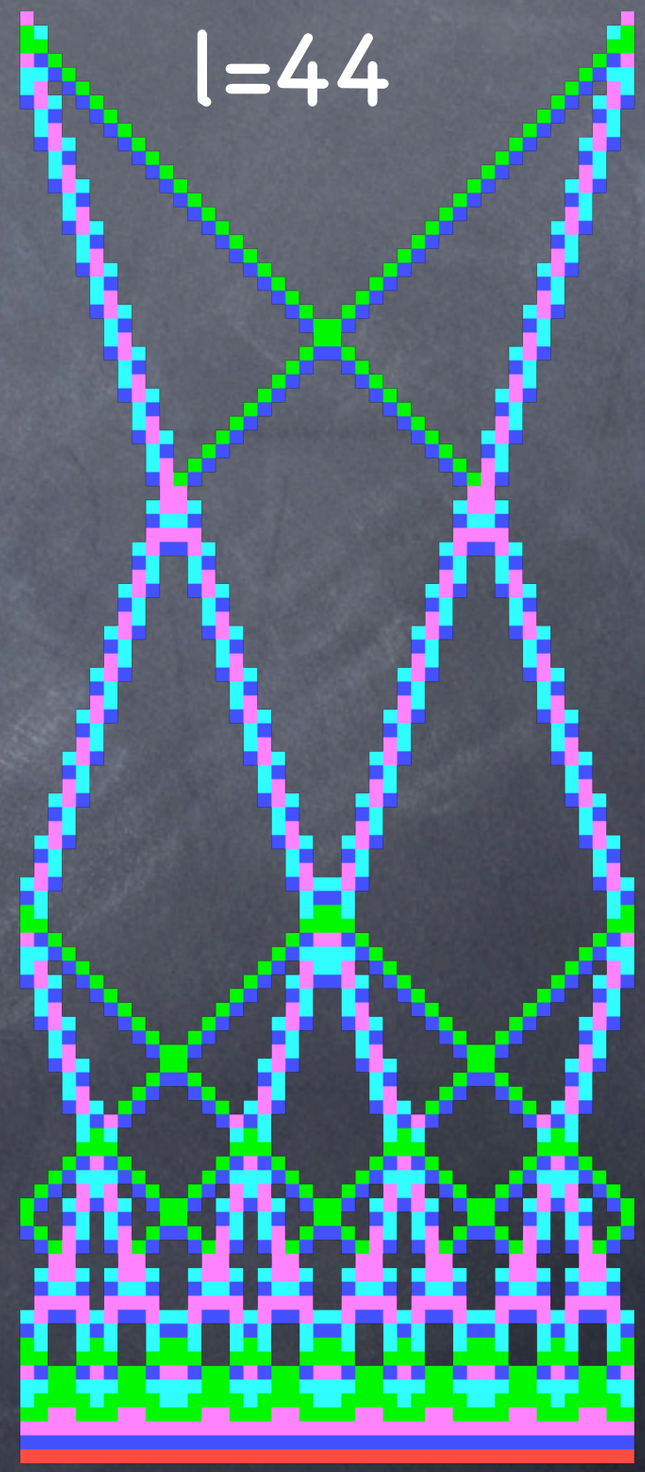
l=23



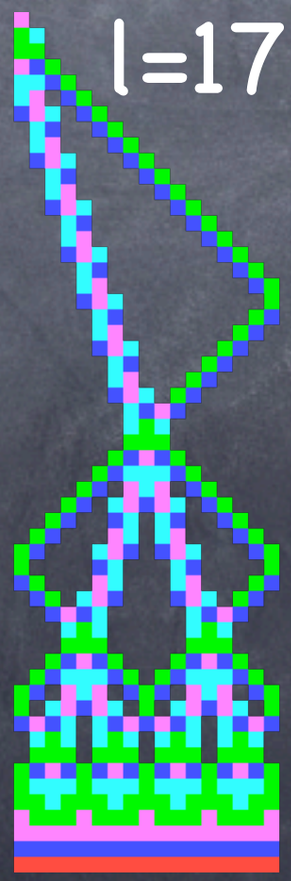
$$3n \leq T(n) \leq 6n$$

$$|Q| = 6$$

l=44



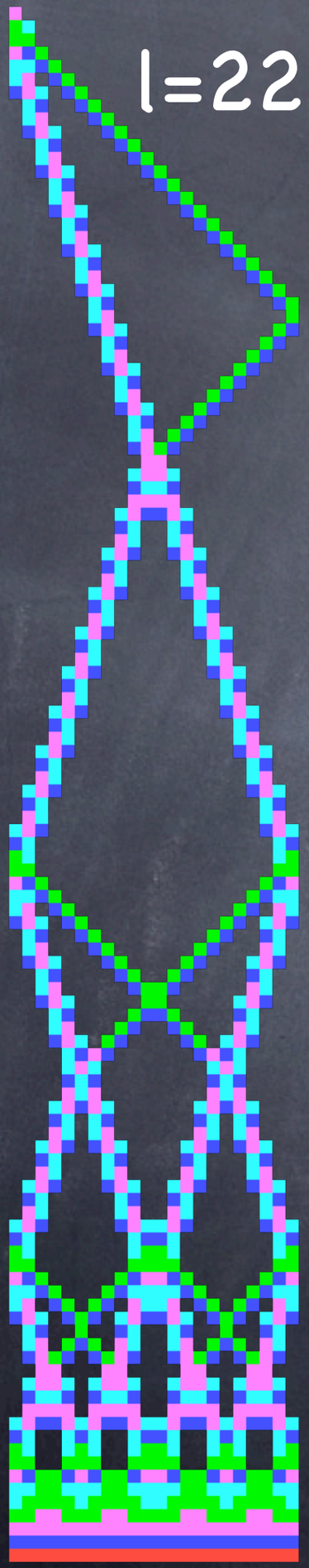
l=17



Symmetric, 134 rules

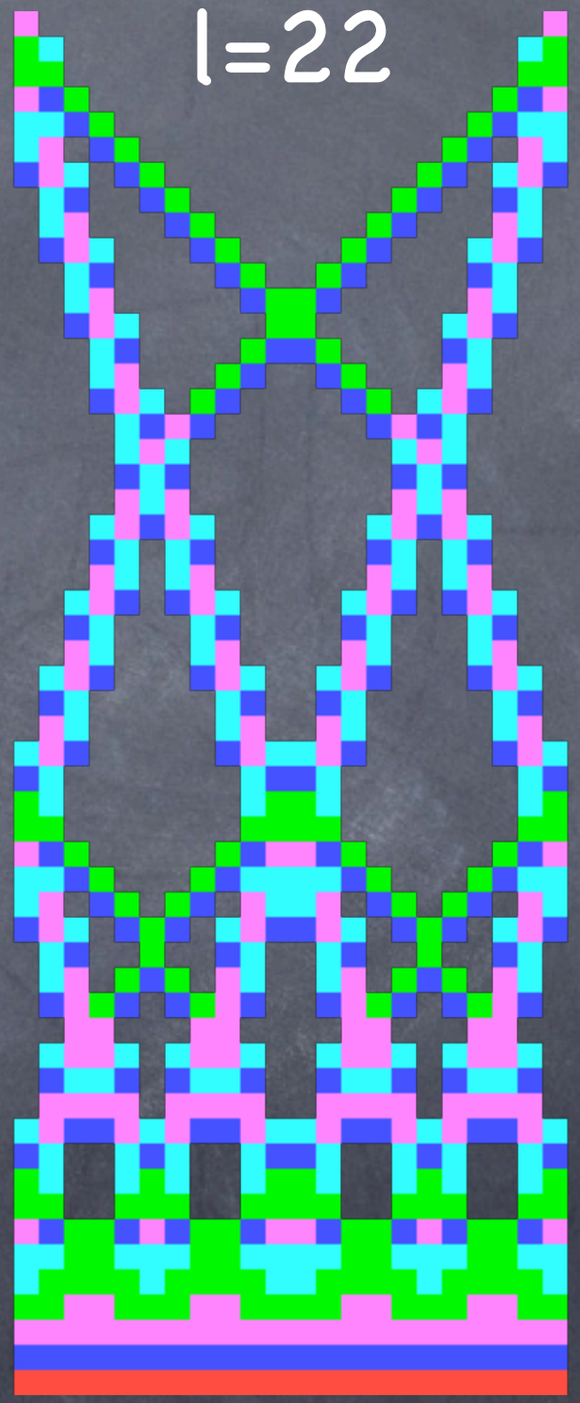
T(n)=6n |Q|=6

l=22



l=23

l=22



l=22

Symmetric, 136 rules

History (updt'd)

Author	Year	|Q|	T(n)	W(n)
Goto	1962	>2000	2n	n.log(n)
Minsky	1967	~20	3n	n.log(n)
Waksman	1966	16	2n	n^2
Balzer	1967	8	2n	n^2
Gerken	1986	7	2n	n^2
Mazoyer	1986	6	2n	n^2
Yunès	1993	7	3n	n.log(n)
Settle	2002	6	3n	n^2
Nogushi	2004	8	2n	n^2
Umeo	2006	6	3n	n^2
Yunès	2007	6	3n	n.log(n)
		8	4n	n.log(n)
		7	4n	n.log(n)
		6	3n/4n	n^2
		6	6n	n.log(n)

Some questions...

- Other simple linear-time scheme?
- Other 6 states (minimal-time) solutions?
- Zoo of few states linear-time solutions, what about polynomial-time, exponential-time?
- What kind of relation is there in between time, #states and #signals?