Complexity of Self-Assembling Tilings

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Your next twenty minutes

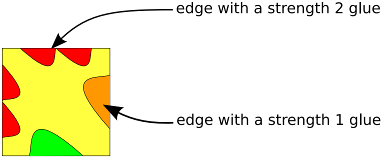
1. Self-assembly
2. Temperature 1
3. Temperature 2 and higher

Introduction and Context

- DNA computing
- Growth of a complex pattern from simple parts
- expressing algorithms in a new formalism
- Finite shapes: Turing
- Tilings of the whole plane: ?

Definition

- A set of colored squares (Wang tiles).



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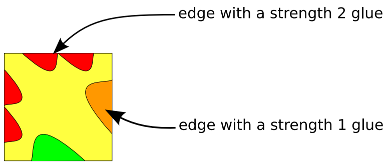

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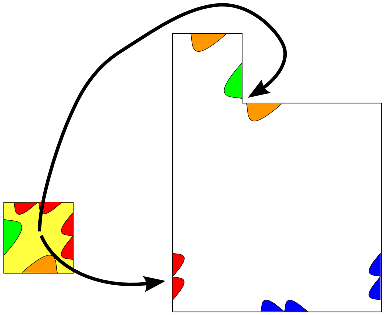

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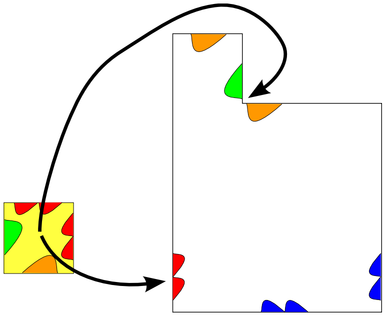

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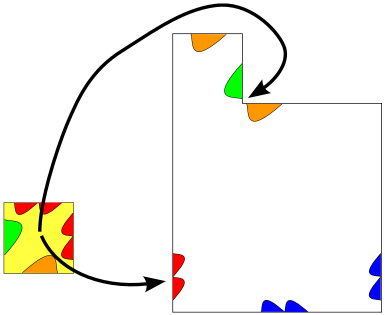
- Starting with a single tile, the seed.

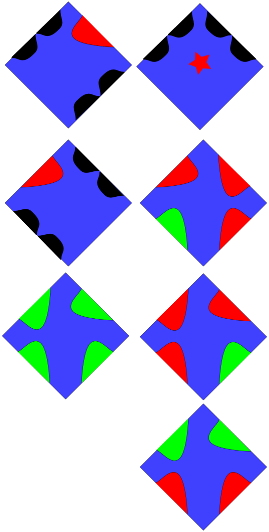


Example

Productions and Dynamic

- Adding a tile = transition
- Pattern we get by transitions = production

An example: a Sierpinsky Triangle (Temperature 2)



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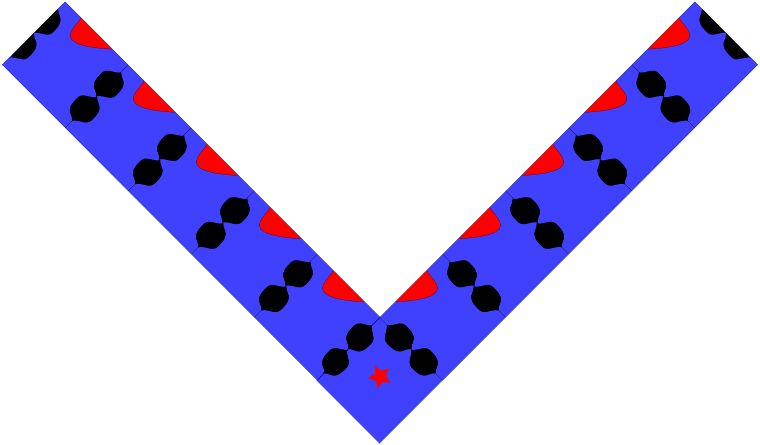

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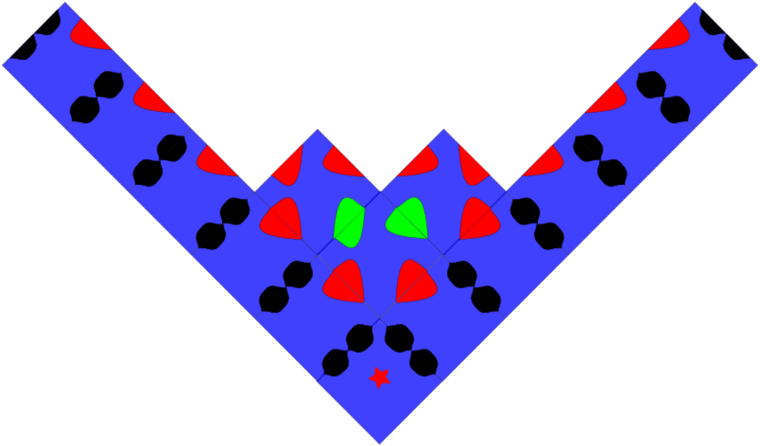

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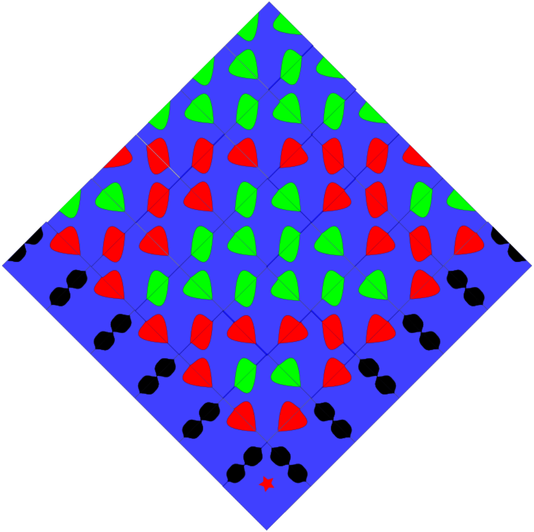

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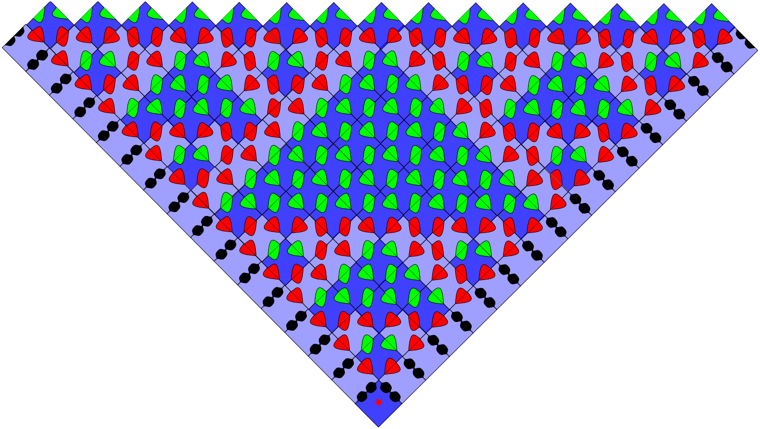

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- If convergence time = 0, no intermediary states, CA = self-assembly.

Infallible assembly and limits

Definition (Infallible assembly)

A self-assembling system is infallible if for any $z \in \mathbb{Z}^2$, from any production p, there is a sequence of productions which covers z.

Definition

We say that an infallible system assembles a set of tilings T if there is a tilewise projection from the sets of all limits of sequences which covers $\mathbb{Z}^2$ to T.

- We want to (eventually) cover the whole plane
- We might need several shades of each color

What tilings are self-assemblable?

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- Really different from a Kolmogorov angle.
- Common point: no information transmission.
- Two tiles are equivalent if substituting one for the other only changes the immediate neighbors.
- Random tilings are uniform under equivalence.

Only Trivial Tilings can be Assembled

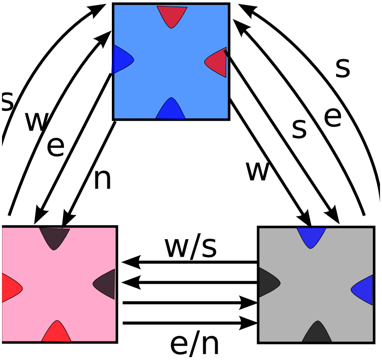
Theorem

Let T be a set of tilings assembled by an infallible temperature 1 system. Then T only has one (periodic) pattern under equivalence.

Idea: temperature 1 self-assembly works like a finite automaton.

Proof Sketch

- In the determinist case, every tile only depends on one neighbor.
- Finite automaton compatible with $\mathbb{Z}^2$ (states = tiles, transition = adjacencies).
- One periodic pattern.



Proof Sketch

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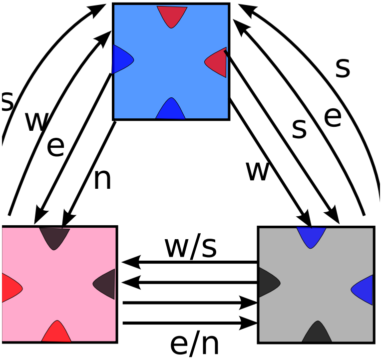
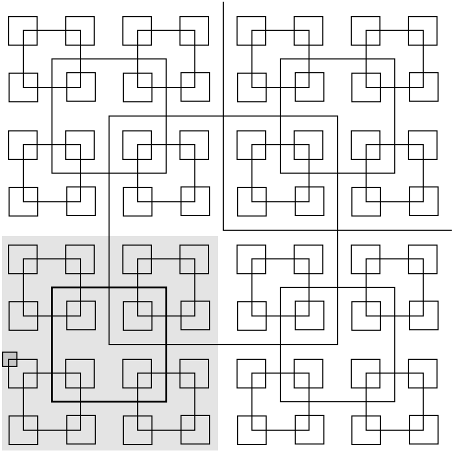
- Finite automaton compatible with $\mathbb{Z}^2$ (states = tiles, transition = adjacencies).
- One periodic pattern.
- General case: non-deterministic finite automaton.
- Determinisation/minimisation = take equivalence on tiles.



Robinson's Tiling is Assemblable

- To show the undecidability of tiling problems, one uses Robinson's patterns
- Quasi-periodic $\Rightarrow$ computation everywhere, uniformly
- The base pattern can be self-assembled
- We can use quasi-periodic patterns to get complexity bounds

More details on that



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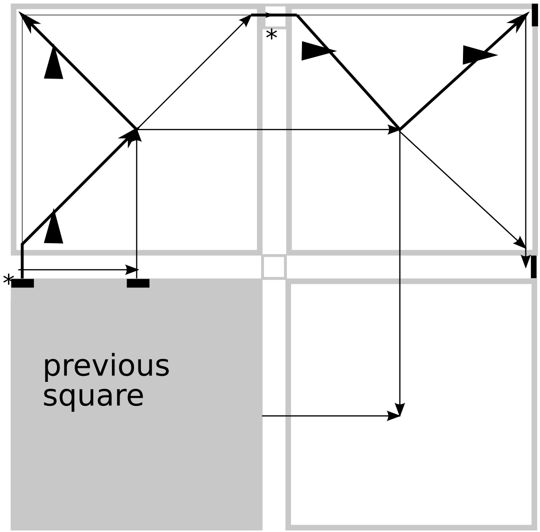

Tiling Complexity

- Complexity of a set of patterns = complexity of the language of extracted squares
- What if we start assembling in a region with “easy” squares?
- Unavoidable complexity = complexity $\circ$ quasi-periodicity
- Note: quasi-periodic set of tilings: a pattern that appears in one of the tilings appears in every big enough square of each of the tilings.

Temperature 2 complexity

Theorem

Let T be a quasi-periodic set of tilings that is assembled by a temperature 2. Then the complexity of deciding if a given $n \times n$ square appears in the tilings is $NEXPTIME(q(n))$.

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- Just before you assemble a $n \times n$ square, you have a band of width n.
- This band has length at most $2^{O(n)}$, otherwise it would be periodic.
- In order to decide if a $n \times n$ square appears in a limit of the system, try all ways to tile a $2^{O(q(n))} \times q(n)$ band, and see if that square appears.

Higher temperature (general case)

- Largest shape before all information on a $n \times n$ square is available: any graph of treewidth n.
- What does this mean in terms of complexity?

Conclusion

- Self-assembly = less powerful model than Turing: a model of complexity?
- At temperature 2 (and more?), complexity bounds
- Are they tight?
- Are there other restrictions (communication schemes?)