About a message system for the tiles of the heptagrid of the hyperbolic plane

Maurice Margenstern

professor emeritus of Université Paul Verlaine − Metz
LITA, EA 3097, UFR-MIM, and CNRS, LORIA
Campus du Saulcy, 57045 Metz Cedex, France
margens@univ-metz.fr

This paper is a continuation of a topic considered in [1, 3] by the author.

In those papers, the author considered the possiblity for cells of a cellular automaton in the pentagrid or in the heptagrid of the hyperbolic plane to communicate. In this talk, we shall present a refinment of the protocol described in these papers and we shall present a small experiment performed in order to test several assumptions about a possible 'realization' of this message system.

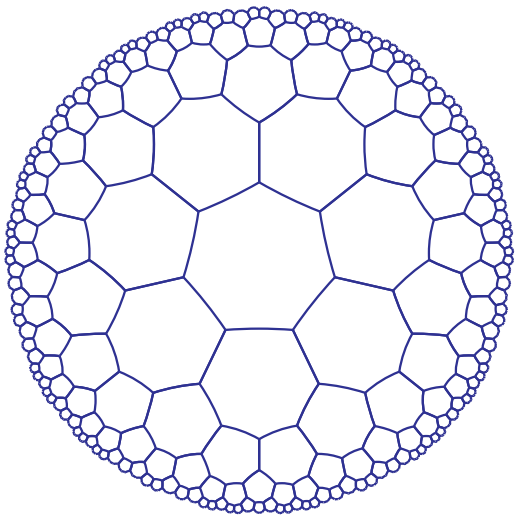

Figure 1. *The heptagrid, an illustration.*

We remind the reader that the heptagrid, see Figure 1, is one of the infinite family of tilings of the hyperbolic plane whose existence was established by Poincaré, see [2, 4]. We chose this tiling as it looks very close to the classical hexagonal tiling and as among the above mentioned tilings of the hyperbolic plane, it is the tiling with regular tiles whose interior angle is $\frac{2\pi}{3}$ for which the number of sides is the smallest.

The new protocol takes advantage of the implementation of an algorithm described in [3] in order to compute a shortest path between two tiles of the heptagrid. This implementation was performed by a computer program written in *ADA*95. This was one of the key points to implement the protocol and to proceed to the experiment. It is important to note that the experiment is necessarily very limited as the growth of the area of observation is exponential in the diameter of this area.

References

[1] M. Margenstern, On the communication between cells of a cellular automaton on the penta- and heptagrids of the hyperbolic plane, *Journal of Cellular Automata* **1**(3), (2006), 213-232.
[2] M. Margenstern, *Cellular Automata in Hyperbolic Spaces, Volume* 1*, Theory*, *OCP*, Philadelphia, (2007), 422p.
[3] M. Margenstern, *Cellular Automata in Hyperbolic Spaces, Volume* 2*, Implementation and computations*, *OCP*, Philadelphia, (2008), 360p.
[4] H. Poincaré, Théorie des groupes fuchsiens, Acta Math. **1** (1882), 1-62.