Complexity and Expressive Power of Ontology-Mediated Queries

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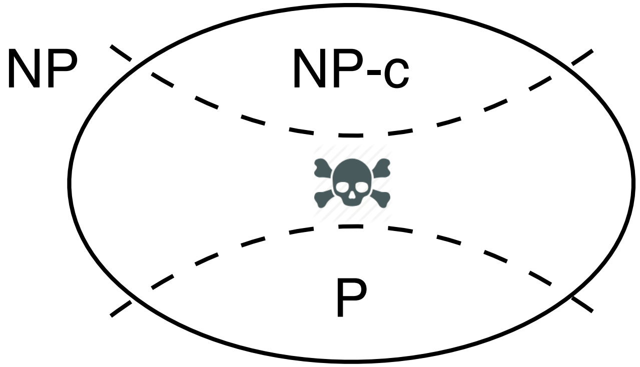
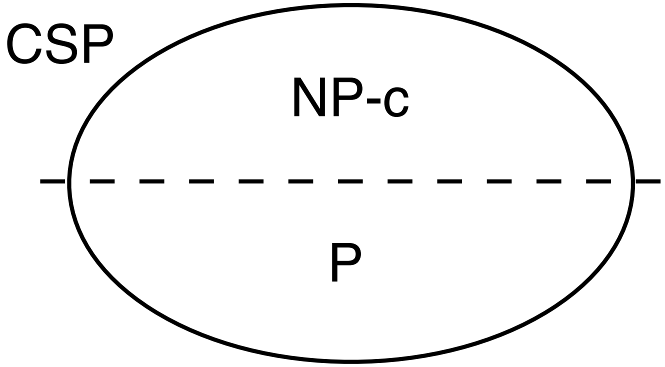
A Teaser



NP is an interesting class of problems

…and a difficult one! [Ladner73]

Subclasses might be more approachable:



FederVardi [93] conjecture: no

Concrete conjecture on frontier from univ. algebra [BulatovJeavonsKrokhin05]

Many subclasses arise in database theory: consistent query answering, ontology-mediated queries, view-based deletion propagation, etc.

Where can we classify complexities, where decide associated meta-problems?

Ontology-Mediated Queries

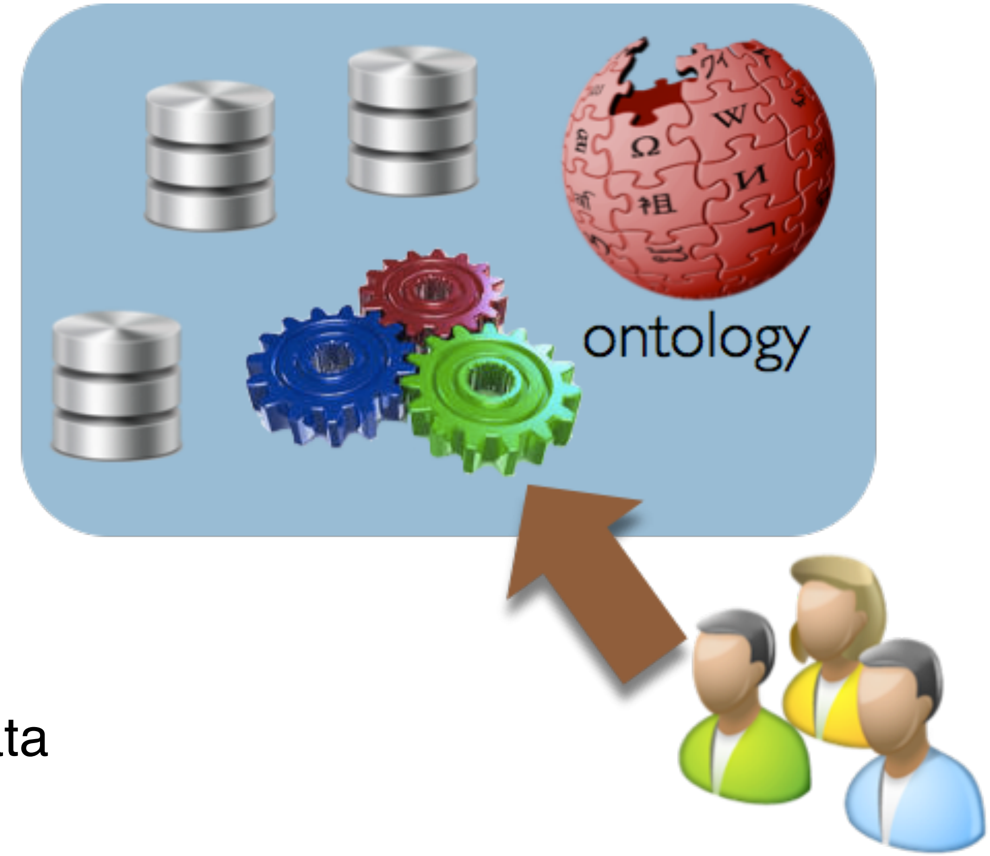
Today data is often highly incomplete and very heterogeneous

Examples include web data and large-scale data integration

Querying such data can be a challenging problem

Ontology is logical theory that

- adds domain knowledge
  (@ incompleteness)
- interrelates diverging vocabularies
  (@ heterogeneity)
- provides unified view of the data



Ontology-Mediated Query: Example

Ontology $\mathcal{O}$:

$$\forall x(\ \mathsf{Director}(x) \to (\mathsf{Person}(x) \wedge \exists y(\mathsf{directed}(x,y) \wedge \mathsf{Movie}(y)))\ )$$

Data D:

Person: jj, Movie: dbl

$\text{jj} \xrightarrow{\text{directed}} \text{dbl}$

Director, Person: ww, Movie: $\bullet$

$\text{ww} \xrightarrow{\text{directed}} \bullet$

Query $q(x)$: $\exists y(\ \mathsf{Person}(x) \wedge \mathsf{directed}(x,y) \wedge \mathsf{Movie}(y)\ )$

Answers: jj ww

Ontology-Mediated Query: Example

Ontology $\mathcal{O}$:

$$\forall x(\ \mathsf{Director}(x) \rightarrow (\mathsf{Person}(x) \wedge \exists y(\mathsf{directed}(x,y) \wedge \cancel{\mathsf{Movie}(y)))}\ )$$

$$\mathsf{Movie}(y) \vee \mathsf{TVseries}(y)))\ )$$

Data D:

$$\overset{\text{Person}}{\text{jj}} \xrightarrow[\text{directed}]{} \overset{\text{Movie}}{\text{dbl}} \qquad \overset{\text{Director}}{\underset{\text{Person}}{\text{ww}}} \xrightarrow[\text{directed}]{} \overset{\cancel{\text{Movie}}}{\bullet}\ \text{TVseries}$$

Query $q(x)$: $\exists y(\ \mathsf{Person}(x) \wedge \mathsf{directed}(x,y) \wedge \mathsf{Movie}(y)\ )$

Answers: jj ~~ww~~

Semantics: consider all extensions of the data that satisfy ontology

return answers valid in all of them (certain answers)

Description Logic

The World Wide Web Committee (W3C) has standardized a family of ontology languages for the web called OWL2

OWL2 is based on a family of logics from KR/AI: description logics

I will consider

- ontologies formulated in description logics
- queries that are unions of conjunctive queries (UCQs)

  in other words: existential positive FO sentences

Description logics are…

…decidable fragments of FO

…related to modal logics

…often contained in the guarded fragment and in FO2

A Basic Description Logic: $\mathcal{ALC}$

Only unary and binary predicates (concept names and role names)

Operators available in $\mathcal{ALC}$: (attribute concept language with complement [Schmidt-SchaußSmolka91])

A	$A(x)$
$\neg C,\ C \sqcap D,\ C \sqcup D$	$\neg C(x),\ C(x) \land D(x),\ C(x) \lor D(x)$
$\exists r.C$	$\exists y\, r(x,y) \land C(y)$
$\forall r.C$	$\forall y\, r(x,y) \rightarrow C(y)$

Ontology: finite set of $C \sqsubseteq D$ $\quad \forall x\, C(x) \rightarrow D(x)$

For example: $\mathsf{Director} \sqsubseteq \mathsf{Person} \sqcap \exists \mathsf{directed}.(\mathsf{Movie} \sqcup \mathsf{TVseries})$

Theorem. An FO-sentence is equivalent to an $\mathcal{ALC}$-ontology iff it is preserved under global bisimulation and disjoint union.

Ontology-Mediated Queries

Ontology-mediated query (OMQ): triple $(\mathcal{O}, \Sigma, q)$ where

- $\mathcal{O}$ is ontology
- q is query
- Σ is signature of the data (set of allowed symbols)

Example: $\Sigma = \{e\}$, e binary (we speak about digraphs)

Ontology $\mathcal{O}$:

$\top \sqsubseteq R \sqcup G \sqcup B$	$\forall x\,(R(x) \vee G(x) \vee B(x))$
$C \sqcap \exists e.C \sqsubseteq D$ for $C \in \{R, G, B\}$	$\forall x \forall y\,(C(x) \wedge e(x,y) \wedge C(y) \rightarrow D(x))$

Query: $q() = \exists x\, D(x)$

Expresses non-3-colorability, thus coNP-complete

Ontology-Mediated Queries

Ontology-mediated query (OMQ): triple $(\mathcal{O}, \Sigma, q)$ where

Central questions in the field:

how to distinguish tractable from intractable OMQs?

when is an OMQ rewritable into FO / into Datalog?

=

SQL

how to decide whether OMQ belongs to these classes?

how to compute rewritings when they exist?

Query: $q() \leftarrow \exists x\, D(x)$

Expresses non-3-colorability, thus coNP-complete

Ontology-Mediated Queries

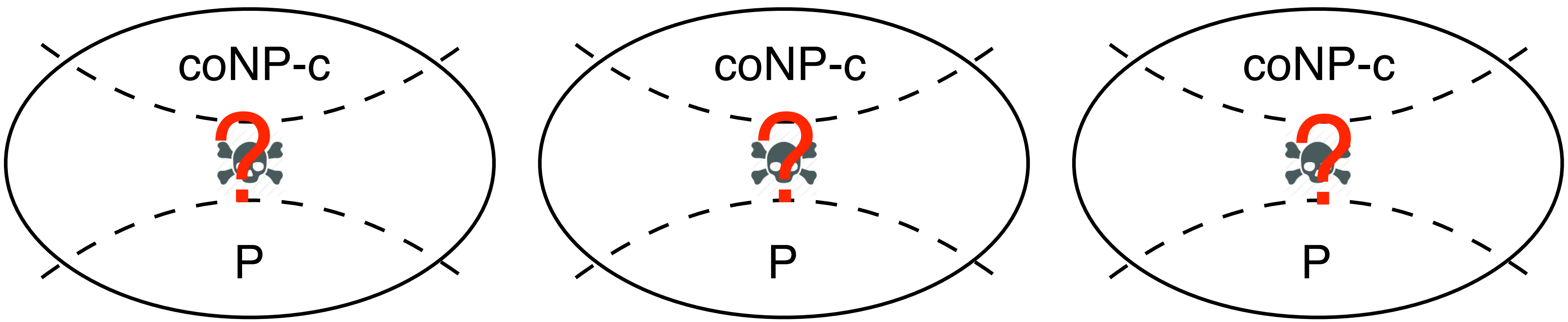
OMQ language:

pair $(\mathcal{L}, \mathcal{Q})$ with $\mathcal{L}$ ontology language and $\mathcal{Q}$ query language

for example $(\mathcal{ALC}, \text{UCQ})$, but many other choices:

- extensions of $\mathcal{ALC}$ such as $\mathcal{ALCI}$, $\mathcal{ALCF}$, the guarded fragment of FO
- other queries such as tree-shaped UCQs (tUCQs)

All of these are subclasses of coNP



CSP Equivalence

We start with tree-shaped queries: $(\mathcal{ALC}, \text{tUCQ})$ = coCSP

CSP is homomorphism problem:

for finite relational structure T (template), CSP(T) = $\{S \mid S \rightarrow T\}$

Theorem [BienvenuTenCateL_Wolter13]

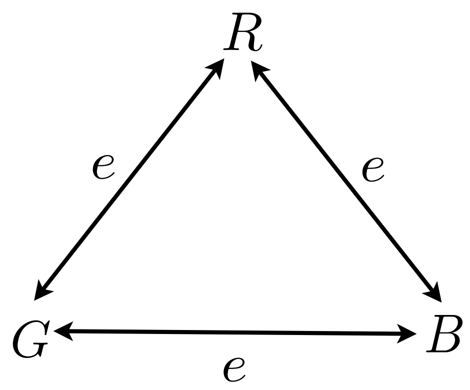
Every OMQ from ($\mathcal{ALC}$,tUCQ) is equivalent to the complement of a CSP and vice versa.

From CSP to OMQ:

$$\top \sqsubseteq R \sqcup G \sqcup B$$

$$C \sqcap \exists e.C \sqsubseteq D \quad \text{for } C \in \{R, G, B\}$$

$$q() = \exists x \, D(x)$$



From OMQ to CSP

Take disjoint union of all finite models of $\mathcal{O}$ that make q false

then make finite by applying filtration:

- 1-type of an element is set of all subformulas of $\mathcal{O}$ and q that are true at that element
- identify all elements with same 1-type

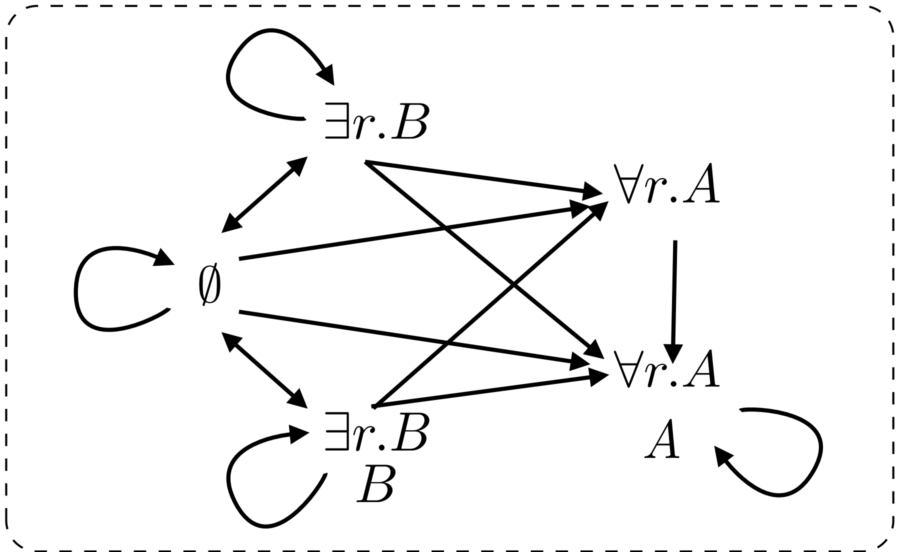
Ontology:

$$A \sqsubseteq \forall r.A$$

$$B \sqsubseteq \exists r.B$$

Query:

$$q(x) = \exists x\, A(x) \wedge B(x)$$



Meta-Problems

Many relevant results transfer from CSP to OMQ, e.g.:

- (co)NP/PTime dichotomy iff the FV conjecture is true
- Tractability via Datalog + group theory / certain polymorphism [FederVardi93,CohenJeavonsGyssens97,many others]
- No complexities between FO=AC_0 and LogSpace [LaroseTesson07]
- FO- and Datalog-rewritability decidable [LaroseLotenTardiff07,BartoKozik09]

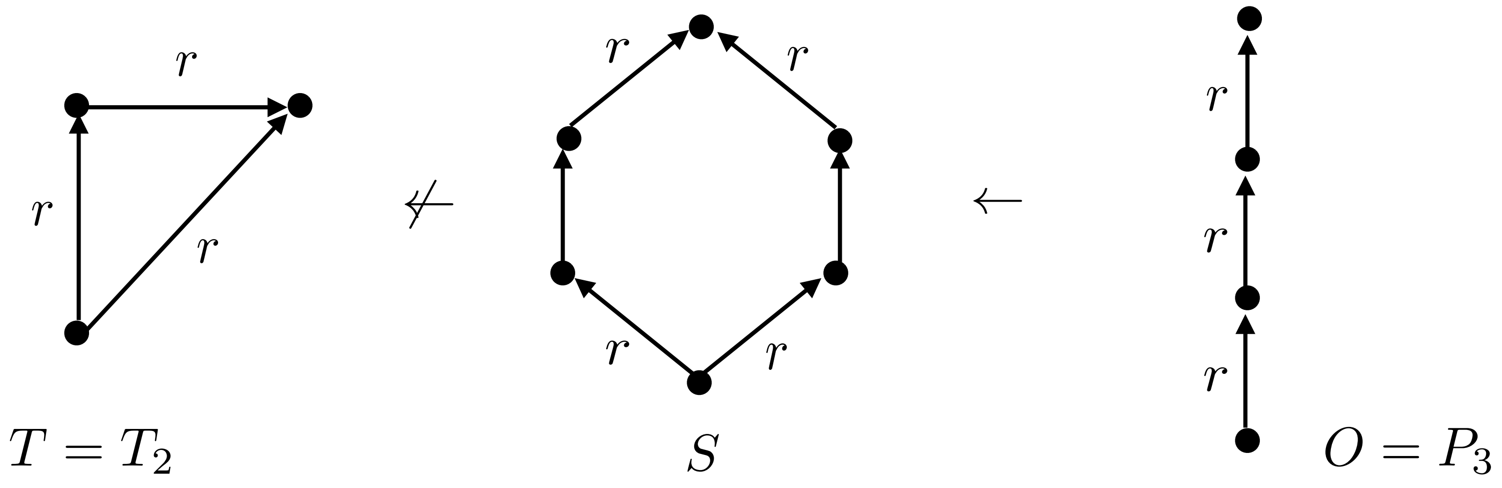
Theorem [BienvenuTenCateL_Wolter13]

FO-rewritability and Datalog-rewritability in $(\mathcal{ALC}, \text{tUCQ})$ is decidable and NExpTime-complete.

Upper bound from CSP translation, lower bound from tiling problem

Obstructions

Structure O is obstruction for T if $O \to S$ implies $S \not\to T$ for all S



Obstruction set $\mathcal{O}$ complete if $S \not\to T$ implies $O \to S$ for some $O \in \mathcal{O}$

For example $T = T_2$ and $\mathcal{O} = \{P_3\}$.

($(\mathcal{O}, T)$ also called homomorphism duality)

$\mathcal{O}$ complete obstruction set for $T_Q \Rightarrow \bigvee \mathcal{O}$ existential positive rewriting of Q

Computing Rewritings

[Atserias05, NesetrilTardiff00]:

FO rewritability = finite obstruction set of finite trees

Can guide the computation of FO-rewritings (tUCQ suffices!)

Datalog-rewritability connected to obstructions of bounded tree width

But there is something better:

(2,3)-canonical Datalog program is most complete Datalog-approximation of coCSP, complete if Datalog-rewritable

[FederVardi93, BartoKozik09]

None of this immediately practical (large blowups involved)

Beyond Trees

Consider $(\mathcal{ALC}, \text{UCQ})$ where queries are no longer trees

For special case $(\emptyset, \Sigma, q)$, we need template T_q such that

$$q \to D \quad \text{iff} \quad D \not\to T_q \quad \text{for all data sets } D$$

Such singleton duality exists iff q is tree-shaped [NesetrilTardiff00]

We thus need generalization of CSP: MMSNP to the rescue!

$$\begin{array}{cccc} \text{NP} & = & \exists\text{SO} & \text{(Fagin74)} \\ \cup\!\!| & & \cup\!\!| & \\ \text{CSP} & \approx_{\text{PTime}} & \text{MMSNP} & \text{(FederVardi93,Kun13)} \end{array}$$

$\exists S_1 \cdots \exists S_n \forall x_1 \cdots \forall x_m \, \varphi$ with φ conjunction of $\bigwedge_i P_i(\overline{x}_i) \to \bigvee_i S_i(x_i)$

Theorem [BienvenuTenCateL_Wolter13]

Every OMQ from ($\mathcal{ALC}$,UCQ) is equivalent to the complement of an MMSNP sentence and vice versa.

Meta-Problems

Thus ($\mathcal{ALC}$,UCQ) has coNP/PTime dichotomy iff FV conjecture holds.

Meta-problems can be attacked via FV-translation of MMSNP to CSP:

- FO-rewriting of CSP yields FO-rewriting of MMSNP sentence
- conversely, we get a rewriting of the CSP that is complete only on inputs whose girth exceeds rule diameter of MMSNP sentence

Fill gap by analysing obstructions for FO-rewritable MMSNP sentences: finite sets of structures of treewidth (1,*k*), *k* diameter of sentence

Theorem [BourhisL_16,unpublished]

FO- and monadic Datalog-rewritability are decidable and 2NExpTime-complete in MMSNP and in $(\mathcal{ALC}, \text{UCQ})$.

Currently working on Datalog-rewritability (2NExpTime-hardness clear)

Guarded Fragment

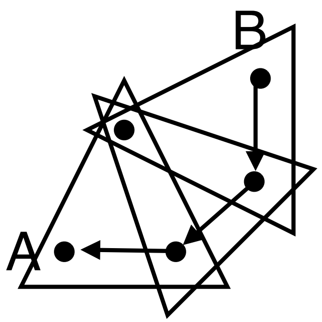
Consider $(\mathrm{GF}, \mathrm{UCQ})$ where GF is guarded fragment of FO

Theorem [BienvenuTenCateL_Wolter13]

(GF,UCQ) is strictly more expressive than the complement of MMSNP

Non-expressible property:

"cartwheel" reachability



(Proof via coloured forbidden patterns)

GMSNP: non-monadic MMSNP, but implications must be guarded

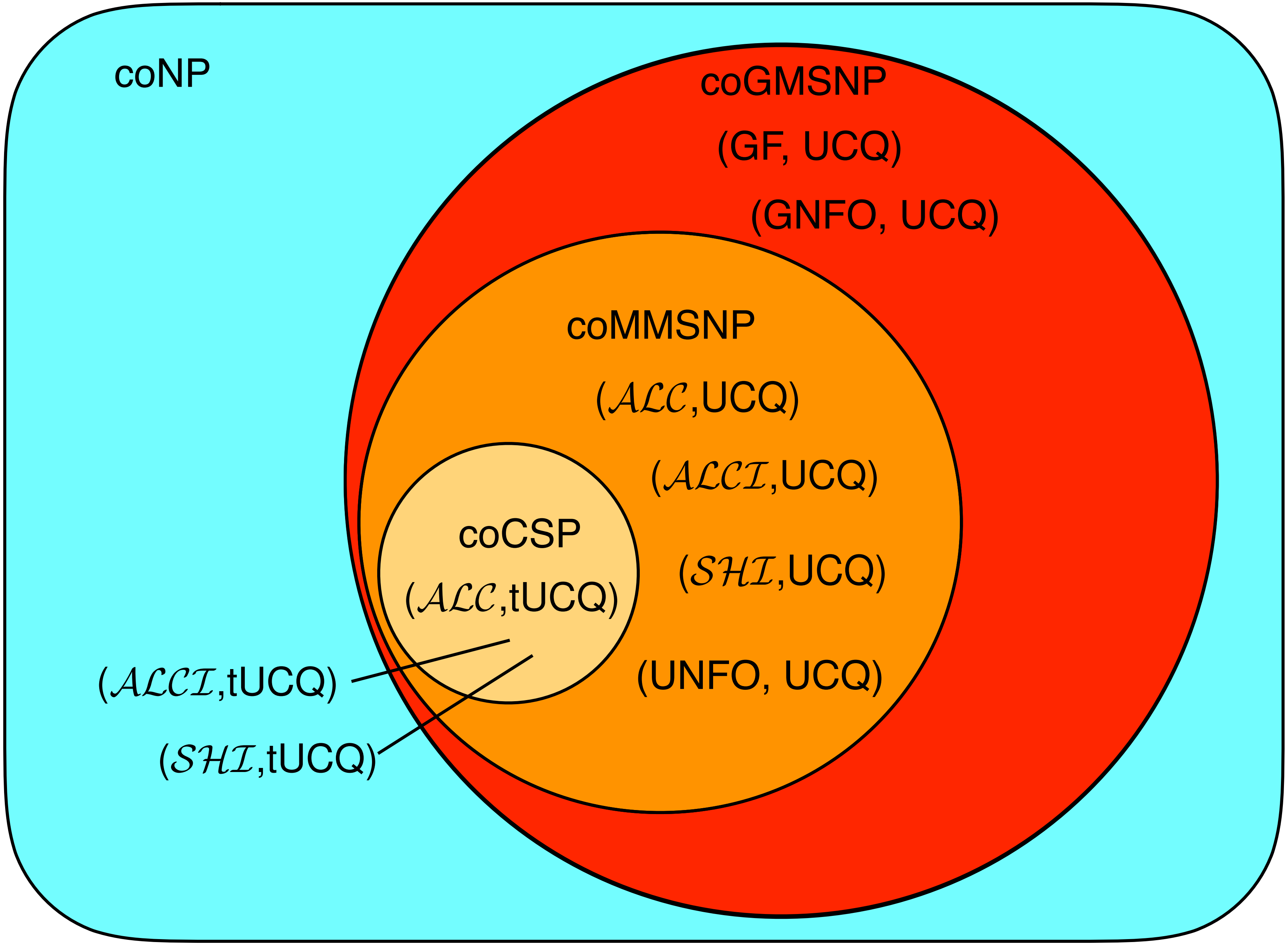
such as $\exists S \forall x \forall y R(x,y) \rightarrow S(x,y) \vee S(x,x)$

Theorem [BienvenuTenCateL_Wolter13]

Every OMQ from (GF,UCQ) is equivalent to the complement of a GMSNP sentence and vice versa.

PTime / NP dichotomy status of GMSNP is interesting open problem

Overview


coNP
coGMSNP
(GF, UCQ)
(GNFO, UCQ)
coMMSNP
($\mathcal{ALC}$,UCQ)
($\mathcal{ALCI}$,UCQ)
($\mathcal{SHI}$,UCQ)
(UNFO, UCQ)
coCSP
($\mathcal{ALC}$,tUCQ)
($\mathcal{ALCI}$,tUCQ)
($\mathcal{SHI}$,tUCQ)

Counting

$\mathcal{ALCF}$ extends $\mathcal{ALC}$ with functional role declarations:

func(hasMother)   func(hasFather)

$\mathcal{ALCF}$ does not admit filtration, so template construction fails!

Theorem [L_Wolter12]

$(\mathcal{ALCF}, \text{tCQ}) \approx_{\text{PTime}} \text{coNP}$

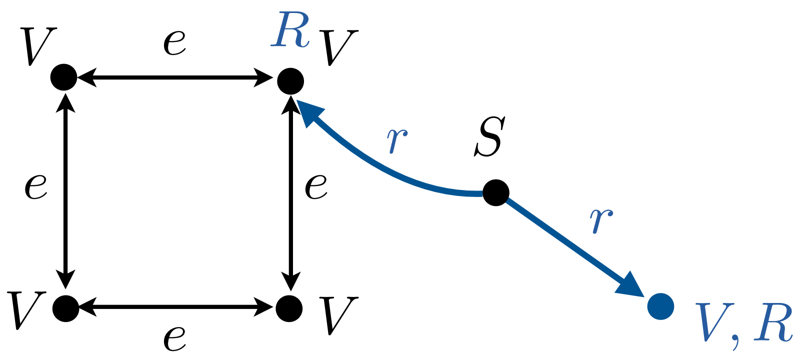
trees, no disjunctions!

Thus $(\mathcal{ALCF}, \text{tCQ})$ has no PTime/coNP dichotomy (unless PTime $=$ NP)

The proof requires the ontology to `verify' a grid-structure in the data which relies on some relations being partial functions

Closed Predicates

Back to $(\mathcal{ALC}, \text{tUCQ})$, but now some predicates can be closed in data



Ontology: $S \sqsubseteq \exists r.(V \sqcap R)$

monadic predicate V closed

Natural setup: corresponds to partially complete data

Theorem [SeylanL_Wolter15]

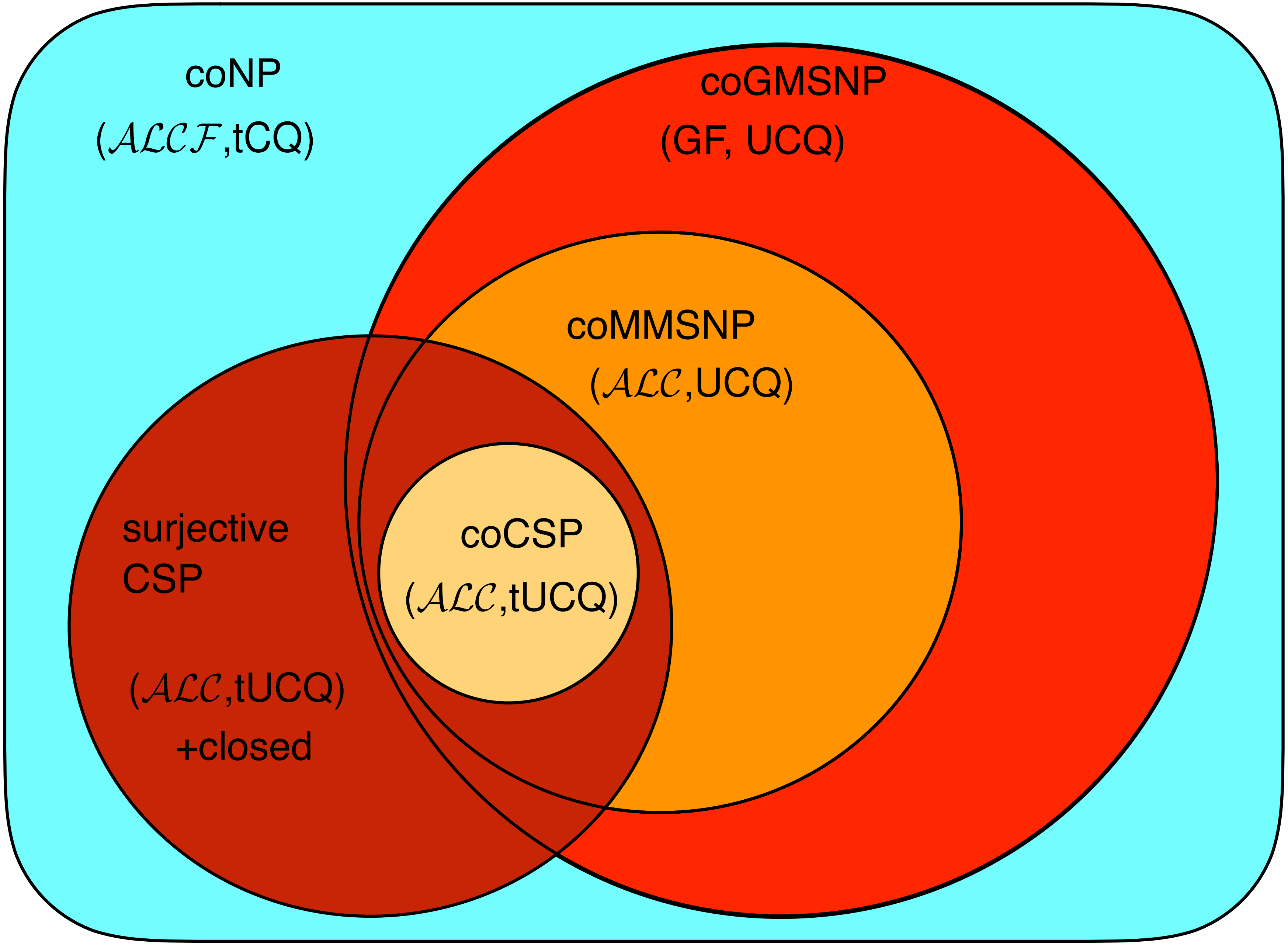
($\mathcal{ALC}$,tUCQ) w. closed preds “equivalent” to surjective CSPs, but

- not same expressivity, only same complexities up to FO-reductions
- small gap: OMQs translate into surjective multi-template CSPs

Surjective CSPs are difficult and little is known about them:

simple templates of unknown complexity (6-cycle), no dichotomy conjecture

Overview



coNP
($\mathcal{ALCF}$,tCQ)
coGMSNP
(GF, UCQ)
coMMSNP
($\mathcal{ALC}$,UCQ)
surjective
CSP
($\mathcal{ALC}$,tUCQ)
+closed
coCSP
($\mathcal{ALC}$,tUCQ)

Beyond Ontology-Mediated Queries

Other areas of database theory provides more subclasses of (co)NP:

> consistent query answering, deletion propagation, peer data exchange, causality, resilience, disjunctive query languages, etc

All based on positive existential queries, thus homomorphisms

There should be some kind of connection to CSP!?

Quick look at Consistent Query Answering (CQA)

Assume a set of constraints C and data D that violates C

- Repair is data D' that satisfies C and with $D' \,\Delta\, D$ minimal
- Given C, D, and query q, we want to compute the answers to q on which all repairs agree

A CQA problem is a pair (C, q).

Consistent Query Anwering (CQA)

First connection to CSP established in [Fontaine13]

Theorem [Fontaine13]

For every CSP, there is a CQA problem that has the same complexity up to PTime reductions.

Constraints take form $\forall \overline{x}\, \neg\varphi(\overline{x})$ with φ a UCQ, queries are UCQs too

How close is the connection between CQA and CSP?

No equivalence-preserving translations can be expected!

Important reason for clean connection between OMQ and CSP:

Relevant ontology languages are essentially unary in the sense that 1-types “largely suffice to describe modes” (recall filtration!)

First Crack at CQA

Theorem [L_Wolter15]

- coCSP $\approx_{\text{FO}}$ CQA: constraints $\forall x\, \neg(A_1(x) \wedge \cdots \wedge A_n(x))$ + tUCQs
- coMMSNP $\approx_{\text{FO}}$ CQA: same constraints + UCQs
- coGMSNP $\approx_{\text{FO}}$ CQA: constraints $\forall x\, \neg(R_1(\overline{x}) \wedge \cdots \wedge R_n(\overline{x}))$ + UCQs

  where all $R_i(\overline{x})$ have same arity and use same vars in same order

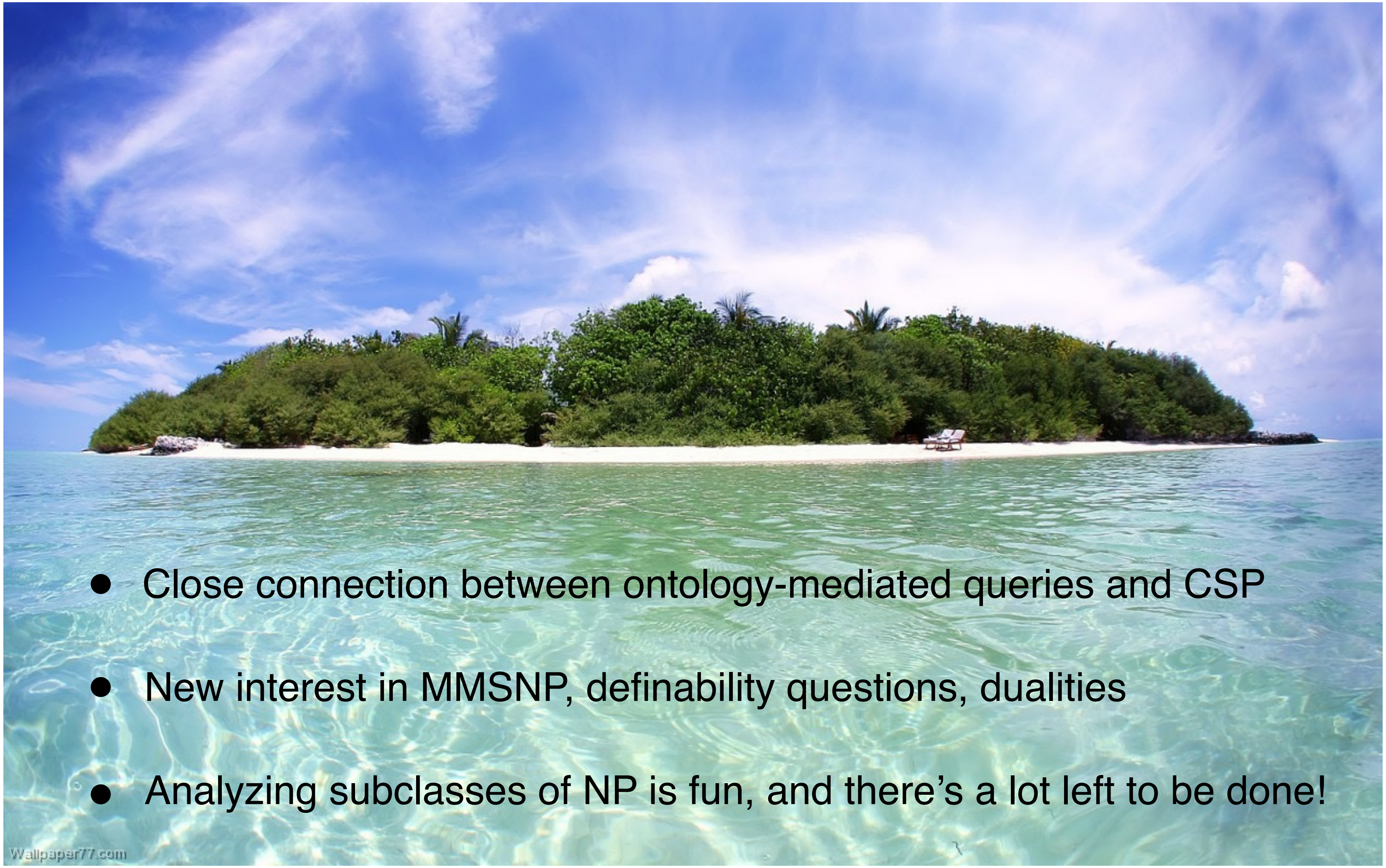
More important are key constraints (functional relations)

Known:

- PTime / coNP dichotomy for "self-join free queries" [KoutrisWijsen15]
- at least as hard to classify as conservative CSPs [Fontaine13]

No full understanding of relation to CSP yet…

Thank you!



- Close connection between ontology-mediated queries and CSP
- New interest in MMSNP, definability questions, dualities
- Analyzing subclasses of NP is fun, and there's a lot left to be done!