The Intrinsic Universality Problem of One-Dimensional Cellular Automata

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Abstract. Undecidability results of cellular automata properties usually concern one time step or long time behavior of cellular automata. Intrinsic universality is a dynamical property of another kind. We prove the undecidability of this property for one-dimensional cellular automata. The construction used in this proof may be extended to other properties.

Cellular automata are simple discrete dynamical systems given by a triple of objects: a *regular lattice of cells*, a *neighborhood vector* on this space, and a finitely described *local transition function* defining how the state of a cell of the lattice evolves according to the states of its neighbors. A *configuration* of a cellular automaton is a mapping from the lattice of cells to the finite set of states which assigns a state to each cell. The *global transition function* of the cellular automaton, which defines its *dynamics*, transforms a configuration into another one by applying the local transition function uniformly, and in parallel, to each cell. A *space-time diagram* is an infinite sequence of configurations obtained by iteration of the global transition function starting from an initial configuration. A main concern of the study of cellular automata is to understand the links between local and global properties of cellular automata: how can very simple local transition functions provide very rich dynamics?

Recently, an algebraic framework was proposed by J. Mazoyer and I. Rapaport [9] to induce an order on cellular automata which is somehow relevant from the point of view of global behavior study. A cellular automaton is said to simulate another one if, up to some rescaling of both cellular automata, the set of space-time diagrams of the former includes the set of space-time diagrams of the later. This relation is a quasi-order on the set of cellular automata and the induced equivalent classes and order on these classes provide a natural way to compare cellular automata. In [9], this order was proven to admit a global minimum and simple known dynamical properties were used to characterize classes at the bottom of the order. In [10], we introduced a generalization of this order by allowing a more general notion of cellular automata rescaling. Thanks to these new geometrical transformations on cellular automata space-time diagrams, we were able to take new phenomena into account. In particular, contrary to the original one, our new order admits a global maximum. This maximum

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corresponds to the set of intrinsically universal cellular automata, that is cellular automaton which can simulate any other cellular automaton step by step, already present in the work of E. R. Banks [3] and formalized for the first time by J. Albert and K. Čulik [1].

In the present article, we prove that the intrinsic universality property is undecidable. Contrary to the computation universality for Turing machines, this result is not a direct consequence of some Rice's theorem because cellular automata behavior lacks such tool. This result explains the difficulty to exhibit intrinsically universal cellular automata with a few states as discussed in [11]. Moreover, when interpreted in the framework of cellular automata comparison, this implies that there exist no class of cellular automata that lie at the limit downside intrinsically universal ones.

1 Definitions

A *cellular automaton* $\mathcal{A}$ is a quadruple $\left(\mathbb{Z}^d, S, N, \delta\right)$ such that $\mathbb{Z}^d$ is the d-dimensional regular grid, S is a finite set of states, N is a finite set of ν vectors of $\mathbb{Z}^d$ called the neighborhood of $\mathcal{A}$ and δ is the local transition function of $\mathcal{A}$ which maps S^ν to S. Two classical neighborhoods for one-dimensional cellular automata are the one-way neighborhood $N_{\mathsf{OCA}} = \{-1, 0\}$ and the first-neighbors neighborhood $N_{\mathsf{vN}} = \{-1, 0, 1\}$.

A *configuration* $\mathcal{C}$ of a cellular automaton $\mathcal{A}$ maps $\mathbb{Z}^d$ to the set of states of $\mathcal{A}$. The state of the i-th cell of $\mathcal{C}$ is denoted as $\mathcal{C}_i$. A configuration $\mathcal{C}$ is *periodic* if there exists a basis $(v_1, \ldots, v_d)$ of $\mathbb{Z}^d$ such that for any index k and any cell i of $\mathbb{Z}^d$, $\mathcal{C}_{i+v_k} = \mathcal{C}_i$. In the case of one-dimensional configurations, the smallest strictly positive value for v_1 is the *period* of the configuration.

The local transition function δ of $\mathcal{A}$ is naturally extended to a *global transition function* $G_\mathcal{A}$ which maps a configuration $\mathcal{C}$ of $\mathcal{A}$ to a configuration $\mathcal{C}'$ of $\mathcal{A}$ satisfying, for each cell i, the equation $\mathcal{C}'_i = \delta\left(\mathcal{C}_{i+v_1}, \ldots, \mathcal{C}_{i+v_\nu}\right)$, where $\{v_1, \ldots, v_\nu\}$ is the neighborhood of $\mathcal{A}$. A *space-time diagram* of a cellular automaton $\mathcal{A}$ is an infinite sequence of configurations $\left(\mathcal{C}_t\right)_{t\in\mathbb{N}}$ such that, for every time t, $\mathcal{C}_{t+1} = G_\mathcal{A}\left(\mathcal{C}_t\right)$. The usual way to represent space-time diagrams is to draw the sequence of configurations successively, from bottom to top.

The *limit set* $\Omega(\mathcal{A})$ of a d-dimensional cellular automaton $\mathcal{A}$ with set of states S is the non-empty set of configurations of $\mathcal{A}$ that can appear at any time step in a space-time diagram. Formally,

$$\Omega(\mathcal{A}) = \bigcap_{t\in\mathbb{N}} G^t_\mathcal{A}\left(S^{\mathbb{Z}^d}\right).$$

A *fixed point* $\mathcal{C}$ of a cellular automaton $\mathcal{A}$ is a configuration of $\mathcal{A}$ such that $G_\mathcal{A}(\mathcal{C}) = \mathcal{C}$. Thus, we say that a configuration $\mathcal{C}$ evolves to a fixed point if there exists a time t such that $G^t_\mathcal{A}(\mathcal{C})$ is a fixed point.

A cellular automaton $\mathcal{A}$ is *nilpotent* if any configuration of $\mathcal{A}$ evolves to a same fixed point, *i.e.* $\Omega(\mathcal{A})$ is a singleton. Symmetrically, a cellular automaton $\mathcal{A}$ is *nilpotent for periodic configurations* if any periodic configuration of $\mathcal{A}$ evolves to a same fixed point. Notice that in both cases, the fixed point configuration has to be monochromatic, *i.e.* consisting of only one state s. To emphasize the choice of s, we will speak of s-nilpotency.

A *sub-automaton*[1] of a cellular automaton corresponds to a stable restriction on the set of states. A cellular automaton is a sub-automaton of another cellular automaton if (up to a renaming of states) the space-time diagrams of the first one are space-time diagrams of the second one. To compare cellular automata, we introduce a notion of rescaling space-time diagrams. To formalize this idea, we introduce the following notations:

σ^k. Let S be a finite set of states and k be a vector of $\mathbb{Z}^d$. The shift σ^k is the bijective map from $S^{\mathbb{Z}^d}$ onto $S^{\mathbb{Z}^d}$ which maps a configuration $\mathcal{C}$ to the configuration $\mathcal{C}'$ such that, for each cell i, the equation $\mathcal{C}'_{i+k} = \mathcal{C}_i$ is satisfied.

o^m. Let S be a finite set of states and $m = (m_1, \ldots, m_d)$ be a finite sequence of strictly positive integers. The *packing map* o^m is the bijective map from $S^{\mathbb{Z}^d}$ onto $(S^{m_1 \cdots m_d})^{\mathbb{Z}^d}$ which maps a configuration $\mathcal{C}$ to the configuration $\mathcal{C}'$ such that, for each cell i, the equation $\mathcal{C}'_i = \big(\mathcal{C}_{mi}, \ldots, \mathcal{C}_{m(i+1)-1}\big)$ is satisfied. The principle of $o^{(3,2)}$ is depicted on Fig. 1.

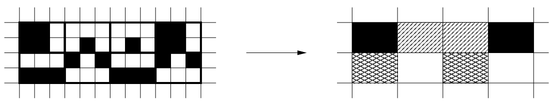

Fig. 1. The way $o^{(3,2)}$ cuts $\mathbb{Z}^2$ space

Definition 1. *Let $\mathcal{A}$ be a d-dimensional cellular automaton with set of states S. A* $\langle m, n, k \rangle$-rescaling *of $\mathcal{A}$ is a cellular automaton $\mathcal{A}^{\langle m,n,k \rangle}$ with set of states $S^{m_1 \cdots m_d}$ and global transition function $G_{\mathcal{A}}^{\langle m,n,k \rangle} = \sigma^k \circ o^m \circ G_{\mathcal{A}}^n \circ o^{-m}$.*

Definition 2. *Let $\mathcal{A}$ and $\mathcal{B}$ be two cellular automata. Then $\mathcal{B}$* simulates *$\mathcal{A}$ if there exists a rescaling of $\mathcal{A}$ which is a sub-automaton of a rescaling of $\mathcal{B}$.*

The relation of simulation is a quasi-order on cellular automata. It is a generalization of the order introduced by Mazoyer and Rapaport [9]. In [10], we motivate the introduction of this relation and discuss its main properties. In particular, it induces a maximal equivalence class which exactly corresponds to the set of intrinsically universal cellular automata as described by Banks [3] and Albert and Čulik II [1].

[1] The prefix *sub* emphasizes the fact that $(S'^{\mathbb{Z}}, G)$ is an (algebraic) sub-structure of $(S^{\mathbb{Z}}, G)$. One could have also used the terminology *divisor* as the set of space-time diagrams of one automaton is included into the one of the other.

Definition 3. *A cellular automaton $\mathcal{A}$ is* intrinsically universal *if, for each cellular automaton $\mathcal{B}$, there exists a rescaling of $\mathcal{A}$ of which $\mathcal{B}$ is a sub-automaton.*

As any one-dimensional cellular automaton can be simulated by a one-way cellular automaton, that is a cellular automaton with neighborhood N_{OCA}, there exist intrinsically universal one-way cellular automata. Therefore, to prove that a particular one-dimensional cellular automaton is intrinsically universal, it is sufficient to prove that it can simulate any one-way cellular automaton. In particular, the intrinsically universal cellular automaton of section 3 is constructed by mean of one-way cellular automata simulation.

2 Deciding Properties of Cellular Automata

One time step behavior of cellular automata, like the injectivity or the surjectivity of the global transition function (for cellular automata, injectivity implies bijectivity) have been well studied. Both properties have been proved decidable for one-dimensional cellular automata by Amoroso and Patt [2]. However, in the case of higher dimensions, both properties have been proved undecidable by Kari [6]. There is a gap of complexity between dimensions one and two. The proofs of Kari rely on reductions to tiling problems of the plane and aperiodic tilings studied by Robinson [12].

Long time behavior of cellular automata deals with the limit set of cellular automata: the set of configurations that can appear after any possible number of time steps. This set is known to be either a singleton either an infinite set. In the first case, the unique configuration of the limit set is monochromatic and the cellular automaton is said nilpotent. Nilpotency has been proved undecidable for two-dimensional cellular automata by Čulik II, Pachl, and Yu [4]. This result has been extended to one-dimensional cellular automata by Kari [5] using a reduction to the tiling problem of the plane for NW-deterministic tile sets. Eventually, Kari [7] has generalized his result to an analog of Rice's theorem for limit set properties of cellular automata: every non-trivial long time behavior properties of cellular automata are undecidable.

Long time behavior of cellular automata on periodic configurations has also its literature because previous results do not automatically extend to periodic configurations. In the case of one-dimensional cellular automata, Sutner [13] proved that it is undecidable to know whether each periodic configuration of a one-dimensional cellular automaton evolves to a fixed point. This result was recently extended by Mazoyer and Rapaport [8] as follows: it is undecidable to know whether each periodic configuration of a one-dimensional cellular automaton evolves to a same fixed point. The proof still uses a reduction to a particular tiling problem. In the present article, we prove an undecidability result by reduction to a simple variant of Mazoyer and Rapaport's result, CA-1D-NIL-PER.

CA-1D-NIL-PER	
Input	A one-dimensional cellular automaton $\mathcal{A}$ and a particular state s from $\mathcal{A}$
Question	Is $\mathcal{A}$ s-nilpotent for periodic configurations ?

Some dynamical properties of cellular automata are neither one time step nor long time properties. An example of such properties is the intrinsic universality. Briefly, a cellular automaton is intrinsically universal if it can simulate, in a particular sense, any other cellular automaton step by step. In the classical case of Turing machines, computational universality, even if not formally defined, is undecidable considering Rice's theorem. In the case of cellular automata, there is no such tool. We will now prove that the problem CA-1D-UNIV is undecidable using a new technique which should work for other dynamical properties.

CA-1D-UNIV	
Input	A one-dimensional cellular automaton $\mathcal{A}$
Question	Is $\mathcal{A}$ intrinsically universal ?

3 An Intrinsically Universal Cellular Automaton

Our proof of the undecidability of the intrinsic universality problem of one-dimensional cellular automata proceeds by reduction to the nilpotency problem for periodic configurations and relies on the existence of a particular intrinsically universal cellular automaton $\mathcal{U}$. We briefly describe its structure and properties.

The cellular automaton $\mathcal{U}$ is defined by simulation of a multi-head Turing machine $\mathcal{M} = (Q_{\mathcal{U}}, \Sigma_{\mathcal{U}}, \pi)$ with set of states $Q_{\mathcal{U}}$, alphabet $\Sigma_{\mathcal{U}}$ and whose transition function π maps $Q_{\mathcal{U}} \times \Sigma_{\mathcal{U}}$ to $Q_{\mathcal{U}} \times \Sigma_{\mathcal{U}} \times \{\leftarrow, \downarrow, \rightarrow\}$. Notice that the behavior of $\mathcal{M}$ on configurations where several heads share a same position is undefined. By simulation, we mean here that $\mathcal{U}$ is defined as

$$\mathcal{U} = (\mathbb{Z}, (\{\cdot\} \cup Q_{\mathcal{U}}) \times \Sigma_{\mathcal{U}}, N_{\mathsf{vN}}, \delta_{\mathcal{U}})\,.$$

A state of $\mathcal{U}$ is a pair constituted of a head or a blank and a letter. A configuration of $\mathcal{U}$ looks like a configuration of $\mathcal{M}$:

$$\begin{array}{cccccccccccccccccccccc} a & a & a & a & b & b & a & b & a & b & a & a & a & b & b & a & a & a & b & a & b & b \\ \cdot & \cdot & \cdot & \cdot & \cdot & s & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s' & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

The local transition function $\delta_{\mathcal{U}}$ is defined in order to emulate $\mathcal{M}$ according to π on locally valid configurations of $\mathcal{M}$. By locally valid configurations of $\mathcal{M}$ we mean configurations where no two heads are in the von Neumann neighborhood of each other. To fully define $\delta_{\mathcal{U}}$, we simply ask that no new head is created, for example by destroying heads that locally invalidate a configuration.

We also give some constraints on intrinsic simulation. For each one-way cellular automaton $\mathcal{A}$, there must exist a positive integer m and an injective map

φ from $S_{\mathcal{A}}$ into $S_{\mathcal{U}}^m$ such that $\mathcal{U}$ simulates $\mathcal{A}$ according to φ without shift: there exists some n such that $\overline{\varphi} \circ G_{\mathcal{A}} = G_{\mathcal{U}}^{\langle m,n,0\rangle} \circ \overline{\varphi}$ where $\overline{\varphi}$ is the extension of φ to $S_{\mathcal{A}}^{\mathbb{Z}}$, *i.e.* the following diagram commutes:

$$\begin{array}{ccc} C & \xrightarrow{o^{-m}\circ\overline{\varphi}} & (o^{-m}\circ\overline{\varphi})(C) \\ {\scriptstyle G_{\mathcal{A}}}\big\downarrow & & \big\downarrow{\scriptstyle G_{\mathcal{U}}^n} \\ G_{\mathcal{A}}(C) & \xrightarrow{o^{-m}\circ\overline{\varphi}} & (o^{-m}\circ\overline{\varphi})(G_{\mathcal{A}}(C)) \end{array} .$$

Moreover φ has the following three properties:

First, each macro-cell (a block of cells encoding one cell of the simulated cellular automaton) is driven by a Turing head. Formally, there exists a state s_0 of $\mathcal{M}$ such that, for each state s of $\mathcal{A}$, its image $\varphi(s)$ contains s_0 as the head component of its first cell and no head elsewhere.

Second, during the simulation, the Turing heads move like a comb. Formally, for each configuration $\mathcal{C}$ of $\mathcal{A}$ and for each time t, the head components of the configuration $\mathcal{C}^{(t)} = G_{\mathcal{U}}^t\left(\overline{\varphi}(\mathcal{C})\right)$ of $\mathcal{U}$ contains heads exactly at positions $(mi + l_t)_{i\in\mathbb{Z}}$ for some l_t.

Third, we can extend the simulation macro-cells to bigger m by padding them and still have the same properties. Formally, there exists a letter a of $\mathcal{M}$ such that, for each positive integer l, the sum $m + l$ and the injective map

$$\begin{aligned} \varphi_l : S_{\mathcal{A}} &\longrightarrow S_{\mathcal{U}}^{m+l} \\ s &\longmapsto (\varphi(s))\,(\cdot, a)^l \end{aligned}$$

are valid choices to replace m and φ and keep the same simulation properties.

Let us now sketch what the behavior of such a cellular automaton $\mathcal{U}$ could be. The following ideas are depicted on Fig. 2, where the head movements during the times that are not depicted are represented by straight segments.

The first idea of the construction is to cut the line of cells regularly into blocks of cells. Each block corresponds to a macro-cell which encodes one cell of the simulated one-way cellular automaton. The border line between two such blocks is materialized by two border letters `#` separated by a void of letters `.`.

Inside the block, several regions are distinguished and separated by a letter `@`. In a first region is stored the state of the macro-cell. A second region is used during the transition to store the state of the neighbor macro-cell state. A third region is used to temporarily store the next state of the macro-cell. Finally, a last region is required to store the transition table of the simulated cellular automaton.

One transition of the simulated macro-cell is operated thanks to the following steps. First, the head copies the state of the neighbor macro-cell in the appropriate storage region. Next, the transition table is read entirely. At each step,

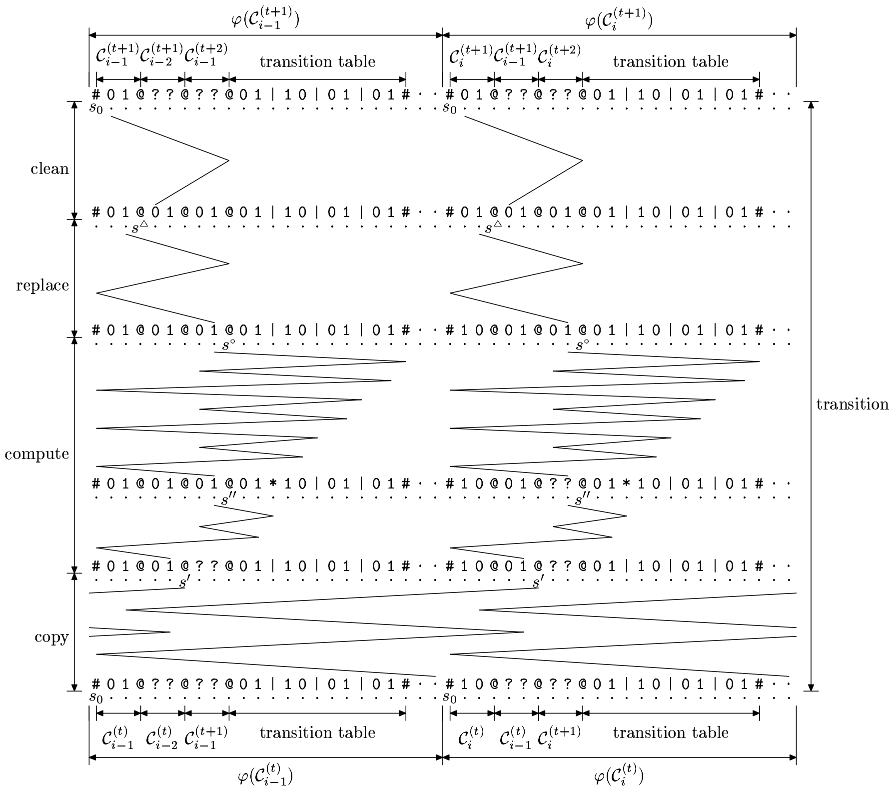

Fig. 2. Typical behavior of the intrinsically universal cellular automaton $\mathcal{U}$

the values of the stored states are compared to the current position inside the transition table and the result of the transition is copied into the next state storage region, or this copy is emulated if the values do not match. Once the reading of the transition table is achieved, the head replaces the content of the state storage region by the content of the next state storage region and cleans up the storage areas. Afterwards, the head goes back to its initial position and enters state s_0.

4 Undecidability of the Intrinsic Universality Problem

Once convinced that such a $\mathcal{U}$ exists, consider the following transformation which is the key idea of our reduction. For each cellular automaton $\mathcal{A}$ and each state s of $\mathcal{A}$, we define a product cellular automaton $\mathcal{A} \circledast_s \mathcal{U}$ by

$$\mathcal{A} \circledast_s \mathcal{U} = (\mathbb{Z}, S_{\mathcal{A}} \times \{\cdot, *\} \times S_{\mathcal{U}}, N_{\mathcal{A}} \cup N_{\mathsf{OCA}} \cup N_{\mathcal{U}}, \delta)\,,$$

a three layers automaton whose local transition rule δ is described layer by layer.

The bottom layer is the energy production layer. It consists of a configuration of $\mathcal{A}$ which evolves according to the local transition function $\delta_{\mathcal{A}}$ of $\mathcal{A}$.

The middle layer is the energy diffusion layer. It consists of a configuration on states $\cdot$ (no energy) and $*$ (an energy dot). The evolution of this layer is the one of a shift but: if the bottom layer of a cell does not contain state s the middle layer produces an energy dot $*$; if the upper layer of a cell contains a head state and its middle layer receives an energy dot $*$ from its neighbor, the middle layer dissipates the incoming energy dot and receives a no energy state.

The upper layer is the energy consumption layer. It consists of a configuration of $\mathcal{U}$ which evolves according to the local transition function of $\mathcal{U}$ under the control of the middle layer. If the upper layer of a cell contains a head state in a cell of its neighborhood then, if the middle layer of the cell containing the head receives an energy dot $*$, the upper layer evolves according to $\mathcal{U}$.

Lemma 1. *For any cellular automaton $\mathcal{A}$, the automaton $\mathcal{A} \circledast_s \mathcal{U}$ is intrinsically universal if and only if $\mathcal{A}$ is not s-nilpotent on periodic configurations.*

Proof. Let $\mathcal{A}$ be a cellular automaton and s a state of $\mathcal{A}$. The proof is by discrimination on the s-nilpotency on periodic configurations of $\mathcal{A}$.

If $\mathcal{A}$ is not s-nilpotent on periodic configurations, there exists a configuration $\mathcal{C}$ of $\mathcal{A}$ which is periodic both in space and time and is not s-monochromatic. Let p be a spatial period of $\mathcal{C}$. To prove that $\mathcal{A} \circledast_s \mathcal{U}$ is intrinsically universal, we prove that it in the maximal equivalence class for the simulation relation by proving that it can simulate any one-way cellular automaton. Consider any one-way cellular automaton $\mathcal{B}$. Let m, n and φ be the parameters of one of our particular intrinsic simulations of $\mathcal{B}$ by $\mathcal{U}$ such that p divides m (use padding property if necessary). Then, $\mathcal{A} \circledast_s \mathcal{U}$ simulates $\mathcal{B}^T$ for some strictly positive T with parameters m, Tn and ψ where ψ is obtained from φ using the following ideas. The bottom layer of ψ consists of periods of $\mathcal{C}$. The upper layer of ψ is directly given by φ. As $\mathcal{C}$ is periodic and its period divide m, it periodically produces energy for the universal simulation, preserving the comb structure, thus allowing the simulation to take place with a given slowdown. By a simple pigeonhole reasoning, it is possible to choose a middle layer for ψ such that during any simulation this layer appears on a periodic duration basis t (remember that each macro-cell produces and consums energy the same way whatever state it encodes). As it is straightforward to see, by choosing for T a value such that t divides Tn, the following two properties hold: ψ is injective and $\mathcal{A} \circledast_s \mathcal{U}$ simulates $\mathcal{B}^T$ with parameters m, Tn and ψ.

If $\mathcal{A}$ is s-nilpotent on periodic configurations, we prove that $\mathcal{A} \circledast_s \mathcal{U}$ cannot simulate the cellular automaton $\mathcal{B} = (\mathbb{Z}, \{\circ, \bullet\}, N_{\mathsf{OCA}}, \oplus)$ where $(\{\circ, \bullet\}, \oplus)$ is the cyclic group $(\mathbb{Z}_2, +)$ where $\circ$ corresponds to 0 and $\bullet$ to 1. Assume that $\mathcal{A} \circledast_s \mathcal{U}$ is intrinsically universal. In particular, it simulates $\mathcal{B}$: let $w_\circ$ and $w_\bullet$ be the respective encoding of its states. The automaton $\mathcal{B}$ admits a space-time diagram Δ periodic in both space and time with the following filling pattern:

As $\mathcal{A}$ is s-nilpotent on periodic configurations, the bottom layer of the space-time diagram of the simulation of $\mathcal{B}$ on Δ stops creating energy on the middle layer after a finite time. Thus, either $w_\circ$ and $w_\bullet$ cannot contain both energy cells on their middle layer and heads on their upper layer (because the heads would consume the whole energy in finite time which would be a problem to preserve the injectivity of the encoding).

If there is a head in the upper layer of $w_\circ$ or $w_\bullet$ then the middle layer is empty of energy cells: the bottom layer behaves periodically and the other layers are constant, thus $\mathcal{A} \circledast_s \mathcal{U}$ behaves like a periodic automaton on the simulation configurations.

If there is no head in the upper layer of $w_\circ$ and $w_\bullet$ then the first layer is periodic, the second layer is a shift and the third layer is constant, thus $\mathcal{A} \circledast_s \mathcal{U}$ behaves like a cartesian product of a shift and a periodic automaton on the simulation configurations.

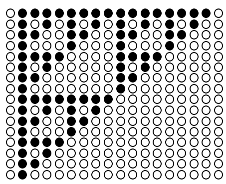

Fig. 3. Pascal triangle modulo two produced by $\mathcal{B}$

In both cases, these behaviors fail to capture the behavior of $\mathcal{B}$ as the behavior of $\mathcal{B}$ on the configuration $\circ$-monochromatic but on cell 0 which has state $\bullet$ cannot be simulated by such cellular automaton as it produces a Pascal triangle modulo two (as depicted on Fig. 3) which cannot be obtained by the product of a shift and a periodic cellular automaton. ∎

Theorem 1. *The intrinsic universality problem of one-dimensional cellular automata is undecidable.*

Proof. This proposition is a corollary of the previous lemma. As the computation of the cellular automaton $\mathcal{A} \circledast_s \mathcal{U}$ from the cellular automaton $\mathcal{A}$ is recursive, the decidability of the intrinsic universality problem of one-dimensional cellular automata would imply the decidability of s-nilpotency problem on periodic configurations. ∎

5 Cellular Automata Dynamics and Computation

The dynamical properties of cellular automata known to be undecidable are one time step or long time behavior properties. We were able to prove the undecidability of the intrinsic universality of one-dimensional cellular automata, which is a dynamical property of another kind, using a particular reduction to a long time behavior property.

The key idea of our proof is to combine an intrinsically universal cellular automaton with a second cellular automaton which acts as an energy provider. The first automaton consumes energy provided by the second one. The product cellular automaton is intrinsically universal if and only if the energy provider is not nilpotent for periodic configurations. It should be possible to replace intrinsic universality by other dynamical properties to prove other undecidability results.

Intrinsic universality plays the same role for cellular automata dynamics as computation universality for Turing machines. A formalization of this idea seems worth studying as it would certainly lead to a better understanding of the way computation occurs inside space-time diagrams and provide undecidability results of the same kind as Rice's theorem.

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